Technische Universität Berlin Institut für Mathematik

# **Computational Integer Programming**

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# Exercise sheet 3

Deadline: Thu, 10 Nov. 2011, by email to steffy@zib.de

#### Exercise 1.

# 10 points

Use ZIMPL to model the TSP using the Miller Tucker Zemlin (MTZ) formulation. The MTZ formulation adds one additional variable  $u_i$  for each city i and some additional constraints. These new constraints, along with the degree inequalities (the tour enters and leaves each vertex exactly once) give a correct model for the TSP, without requiring any of the subtour elimination constraints. The MTZ constraints are added to ensure that the u variables define an ordering of cities in the tour. One city is fixed as the starting city (e.g. i = 1) and  $u_1 := 1$  and then constraints are added to ensure that  $u_j \ge u_i + 1$  whenever arc (i, j) is followed by the tour and  $j \ne 1$ . These constraints can be expressed by assigning variable bounds  $u_1 = 1$ ,  $2 \le u_j \le n$  for  $j \ne 1$  and constraints  $u_i - u_j + 1 \le n(1 - x_{ij}) \forall i, \forall j \ne 1$ . The model file tspmtz.zpl, posted on the webpage, will give you a start.

#### Exercise 2.

## 10 points

Another way to model the TSP as an integer program with a polynomial number of constraints is by using network flows. We can embed a single commodity flow problem in the network with a new set of flow variables. An arbitrary vertex is selected as a source with a surplus of (n - 1) units and all additional vertices have a demand of 1 unit. Constraints are added that only allow flow to travel on arcs that are selected by the TSP tour, ensuring that there can be no subtours. Create a ZIMPL model of this formulation. The model file tspflow.zpl gives you a start.

#### Exercise 3.

### 10 points

Create a model that can solve the instance given by berlin52.dat as quickly as you can. For example, try including some subtour elimination constraints to strengthen the formulations you implemented in Exercises 1 and 2.

Note: Each group should email the three ZIMPL models to steffy@zib.de by Thu, 10 Nov 2011. There is no written part to hand in for this assignment.