Technische Universität Berlin
Institut für Mathematik

Wintersemester 11/12

Computational Integer Programming

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Exercise sheet 4

Deadline: Thu, 17 Nov. 2011, by email to `borndoerfer@zib.de`

Exercise 1. **10 points**

Use ZIMPL to model the TSP using the Van Vyve Wolsey (VVW) formulation. The VVW formulation includes the entire Miller Tucker Zemlin formulation, and adds additional variables and constraints in order to strengthen the LP relaxation of the model. The VVW formulation involves a parameter $k \in \mathbb{N}$, $2 \leq k < n$, the size of a "local neighborhood" for each node. Denote by N_v the set of the k nearest neighbors of node v, including v itself. The formulation adds for each such neighborhood a set of additional flow variables w_{ij}^v, $i \in N_v$ or $j \in N_v$, that are used to route a flow of value 1 from outside of N_v to v. Flow conservation constraints are added for all nodes in N_v except v, which acts as a sink with deficit 1. Additional coupling constraints are added that only allow flow to travel on arcs that are selected by the TSP tour, ensuring that there can be no subtours in each local neighborhood. Create a ZIMPL model of this formulation. The model file `tspvvw-tpl.zpl`, posted on the webpage, gives you a start (you can also try all by yourself!). Hint: Use ZIMPL's `argmin` operator to construct the local neighborhoods.

Exercise 2. **16 points**

We investigate in this exercise the effects of a number of seemingly minor variations of the VVW formulation on IP solver performance. Here are the variations:

- (V1) In the original version of the VVW formulation, the case $k = n$ was included. To make the model feasible in this case, VVW excluded node 1 from all neighborhoods, i.e., the neighborhood N_1 was not considered, and the local neighborhoods of the other nodes were computed as if node 1 was non-existent.
- (V) Set $1 < k < n$ and include node 1 in all neighborhoods.
- (CE) The flow conservation constraints are formulated as equations.
- (CG) The flow conservation constraints can be relaxed to $\geq$ to stipulate an inflow of at least 1 into each node v.
- (F) The local flow is defined on all arcs that touch a local neighborhood.
- (FI) The arcs going out of the local neighborhood can be fixed to zero.

- (UI) The MTZ position variables can be declared as non-negative integers.
- (UII) The MTZ position variables can be declared as implicitly integer.
- (UR) The MTZ position variables can be declared as non-negative reals.
- (WR) The local flow variables can be declared as reals in $[0, \infty]$.
- (WR1) The local flow variables can be declared as reals in $[0, 1]$.
- (WI) The local flow variables can be declared as non-negative integers.
- (WII) The local flow variables can be declared as non-negative implicit integers.
- (WB) The local flow variables can be declared as binaries.
- (WBI) The local flow variables can be declared as implicit binaries.
- (K) The size of the local neighborhood can be varied.

Denoting the variations by (V*,C*,F*,U*,W*,K), the "standard version" would be (V,CE,UI,WR,F,13). Try the listed variations on instance `berlin52.dat`. Hint: There are some choices that are clear winners or loosers, such that you don't have to try everything. Make a table, listing the impact on the root LP bound, the number of nodes in the branch-and-bound tree, and the overall solution time. What can you observe? Try to find an explanation (short). Choose a best possible model version and justify your choice.

Exercise 3. **4 points**

Use your best TSP model to solve (possibly with a gap) the following instances from TSPLIB:

- `germany18.tsp`
- `att48.tsp`
- `berlin52.tsp`
- `pr76.tsp`

TSPLIB data can be downloaded from `http://elib.zib.de/pub/mp-testdata/tsp/tsplib/tsplib.html`. Note that you have to manually comment out everything except the data in the `NODE_COORD_SECTION`.

Note: Each group should email their ZIMPL models for exercises 1 and 3, and their computational results for exercises 2 and 3 to `borndoerfer@zib.de` by Thu, 17 Nov 2011.