

# Computational Integer Programming

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## Exercise sheet 5

Deadline: Thu, 24 Nov. 2011, by email to [borndorfer@zib.de](mailto:borndorfer@zib.de)

### Exercise 1.

**10 points**

Prove the following

**Theorem:** Let  $D = (V, A)$  be a digraph and  $M$  its node-arc incidence matrix, i.e.,  $m_{i,jk} = +1$  for  $j = i$ ,  $m_{i,jk} = -1$  for  $k = i$ , and  $m_{i,jk} = 0$  otherwise. Then:

$$\{\lambda \in \mathbb{R}^A : M\lambda \geq 0, \lambda \geq 0\} = \text{cone}\{\chi^C : C \text{ is a directed cycle in } D\}.$$

Hint: Start proving  $\{\lambda \in \mathbb{R}^A : M\lambda \geq 0, \lambda \geq 0\} = \{\lambda \in \mathbb{R}^A : M\lambda = 0, \lambda \geq 0\}$ .

### Exercise 2.

**10 points**

Use ZIMPL to model the capacitated vehicle routing problem described in the file `WS11-CIP-UE-05.VRP.pdf`. Sample trip data for an instance can be found in file `trips.dat`. The model file `vrp-skeleton.zpl`. All files are posted on the webpage.

### Exercise 3.

**10 points**

Try to solve the following variants of the VRP problem of exercise 2 for instance `trips.dat`:

- The maximum distance is 30 (the default).
- The maximum distance is increased to 40.
- The maximum distance is increased to 50.
- The maximum distance is increased to  $\infty$ .
- The fixed cost is reduced to 0.

**Note:** Each group should email their ZIMPL models and their computational results for exercise 3 to [borndorfer@zib.de](mailto:borndorfer@zib.de) by Thu, 24 Nov 2011.