Network Design and Operation (WS 2015)

Excercise Sheet 1
Submission: Mo, 26. October 2015, tutorial session

Exercise 1. 8+2 Points
Show that connectedness induces an equivalence relation on the nodes of an undirected graph. What are the equivalence classes?

Exercise 2. 10 Points
Show that an edge is a bridge of an undirected graph if and only if it is not contained in a cycle.

Exercise 3. 10 Points
Show that an undirected graph is bipartite if and only if it doesn’t contain an odd cycle (with an odd number of nodes = number of edges).

Exercise 4. 10 Points
Show that a tree with maximum degree $\Delta(G) := \max_{v \in V} \delta(v)$ has at least $\Delta(G)$ leaves (nodes of degree 1).
**Exercise 5.**

The *assignment problem* involves a complete bipartite graph $G = (U, V, E)$ with the same number of nodes $|U| = |V|$ on both sides and a matrix of edge weights $c \in \mathbb{Q}^E = \mathbb{Q}^{U \times V}$. An *assignment* is a set of edges $M \subseteq E$ such that each node $u$ is contained in exactly one edge $uv$ that matches or assigns it to $v$. The assignment problem is to find an assignment $M$ of minimum cost $c(M)$.

Solve the assignment problem given by the matrix in Table 1.

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Table 1: Assignment problem

**Exercise 6.**

Solve the problem in exercise 5 by integer programming using the programs *zimpl* and *scip*.

a) Download *zimpl* and *scip* from zibopt.zib.de and have a look at the example section in the *zimpl* manual.

b) Formulate the assignment problem as an integer linear program (IP).

c) Translate your IP model into a *zimpl* model.

d) Solve the model using *scip*.

e) Construct the LP relaxation of the model.

f) Solve the LP relaxation using *scip*.

g) Prove that the solution of the LP relaxation is always integer.