Network Design and Operation (WS 2015)

Excercise Sheet 4
Submission: Mo, 16. November 2015, tutorial session

Exercise 1. 10 Points

Prove that the smallest enclosing circle of \( n \) points in the plane is uniquely determined by 3 points.

Exercise 2. 5+5 Points

Solving the 1-center network location problem \( 1/N/\cdot/sp/\max \) requires the solution of minimization problems

\[
(P_{uv}) = \min_{w \in V} \max_{\lambda \in [0,1]} \{ \text{sp}(u, w) + \lambda c_{uv}, \text{sp}(v, w) + (1 - \lambda)c_{uv} \}
\]

for all undirected edges \( uv \in E \).

a) The functions

\[
\lambda \mapsto \max_{w \in V} \min_{\lambda \in [0,1]} \{ \text{sp}(u, w) + \lambda c_{uv}, \text{sp}(v, w) + (1 - \lambda)c_{uv} \}
\]

are continuous and piecewise affine with at most \( 2|V| \) pieces.

b) \( (P)_{uv} \) can be solved in linear time.

Exercise 3. 5+3+2 Points

Consider a modification of the select median finding algorithm that subdivides \( m \) given numbers into groups of \( k \) elements, where \( k \) can be different from 5.

a) Ignoring integrality issues, derive a recursion for the run time.

b) What is the ratio in the resulting geometric series?

c) What is special about \( k = 5 \)?

Exercise 4. 10 Points

Consider real number \( a_1, \ldots, a_m \) and positive weights \( w_1, \ldots, s_m \); let \( W := \sum_{i=1}^m w_i \).

The weighted median of \( \{a_i\} \) w.r.t. \( w_i \) is

\[
w\text{-med} \{a_i\} := \left[ a_k : \sum_{a_i < a_k} w_i < W/2, \sum_{a_i \geq a_k} w_i \leq W/2 ; a_{\ell} : \sum_{a_i < a_{\ell}} w_i \leq W/2, \sum_{a_i \geq a_{\ell}} w_i < W/2 \right].
\]

Prove that the median of \( \{a_i\} \) is the weighted median of \( \{a_i\} \) w.r.t. weights \( w \equiv 1/m \).
Exercise 5.

Consider the 6-node graph $N = (V, E)$ in Fig. 1 with distances $d_{ij}$ and demands $w_i$ as drawn next to the edges and nodes.

a) Solve the warehouse location problem $1/V/\cdot/sp/\sum w_i$.

b) Solve the warehouse location problem $2/V/\cdot/sp/\sum w_i$ by fixing the solution of a) and adding a second warehouse in a best possible way.

c) Develop an IP formulation for $2/V/\cdot/sp/\sum w_i$.

d) Solve your formulation from c).

e) Did b) produce the optimum?

f) Solve the network center problem $1/V/\cdot/sp/\max$.

g) Solve the network center problem $2/V/\cdot/sp/\max$ by fixing the solution of f) and adding a second center in a best possible way.

h) Develop an IP formulation for $2/V/\cdot/sp/\max$.

i) Solve your formulation from h).

j) Did g) produce the optimum?

Figure 1: Warehouse location/network center problem.