FREIE UNIVERSITÄT BERLIN
FB MATHEMATIK UND INFORMATIK

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Excercise Sheet 5

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Exercise 1. **8+2 Points**

a) Suppose the smallest circle C enclosing some set V of points in the plane is defined by a two or three point subset $U \subseteq V$ of V and that p is a point outside of C. Show that it is not true that the smallest circle enclosing $V \cup \{p\}$ is defined by a two or three point subset of $U \cup \{p\}$.

b) The algorithm of Elzinga & Hearn constructs a sequence of circles C_i defined by two or three points p_i^j, $j = 1,2$ or $j = 1,2,3$, adding and/or dropping a point in each iteration. Show that it is possible that a point that has been dropped is added again later.

Exercise 2. **10 Points**

From an expected complexity point of view, is it a good idea to initialize the algorithm of Elzinga & Hearn with the two points farthest apart? (No rigorous proof is required.)

Exercise 3. **10 Points**

Consider the following discrete stop location problems in a network $N = (S \cup T, E)$ with demand points V and covering radius r:

(DSL)	$p = \|U\|/S/\operatorname{cov}_r(U) = V/\ell_2/p$	(planning from scratch)
(DSL1)	$p = \|U\|/T/\operatorname{cov}_r(U) = V/\ell_2/p$	(closing stops)
(DSL2)	$p = \|U\|/S/\operatorname{cov}_r(U \cup T) = V/\ell_2/p$	(opening stops)
(DSL3)	$p = \|U\|/S \cup T/\operatorname{cov}_r(U) = V/\ell_2/p$	(closing and opening stops).

Prove that (DSLi) can be reduced to (DSL), $i = 1,2,3$.

Exercise 4. **10 Points**

Consider a set covering problem (SCP) $\min c^T x$, $Ax = \mathbb{1}, x \in \{0,1\}^n$ with constraint matrix $A \in \{0,1\}^{m \times n}$ and objective $c \in \mathbb{R}^n_{>0}$. Prove the validity of the following preprocessing rules:

a) $A_{i\cdot} = e_j \Longrightarrow x_j = 1$ in every solution of (SCP).
b) $A_{\cdot j} \leq A_{\cdot k}$ and $c_j < c_k \Longrightarrow x_j = 0$ in every optimal solution of (SCP).
c) $A_{i\cdot} \leq A_{k\cdot} \Longrightarrow A_{k\cdot} x \geq 1$ is redundant.
d) Find, formulate, and prove another preprocessing rule.

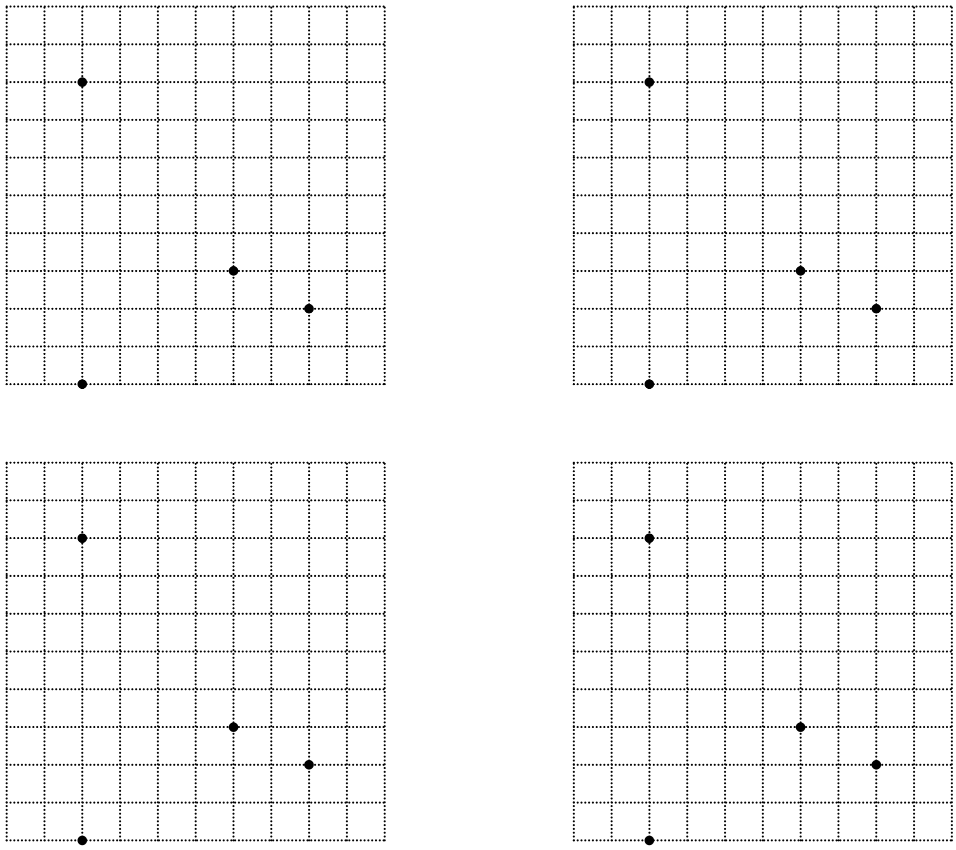

Figure 1: 1-center ℓ_2-problem.

Exercise 5. **Tutorial Session**

Solve the 1-center problem $1/\mathbb{R}^2/\cdot/\ell_2/\max$ w.r.t. Euclidean distances for the set of points $V = \{(2,0),(2,8),(6,3),(8,2)\}$ in the plane graphically by reducing to all 2- and 3-point configurations; use Fig. 1.

Exercise 6. **Tutorial Session**

Solve the 1-center ℓ_2-problem in Fig. 2 using the algorithm of Elzinga & Hearn, starting with the two closest points, always adding the outside point closest to the current circle.

Exercise 7. **Tutorial Session**

Solve the restricted 1-center ℓ_2-problem in Fig. 3. **Hint:** Start with the solution of the unrestricted problem. Consider circles at the three defining points meeting in the center. What happens when you blow up the circles?

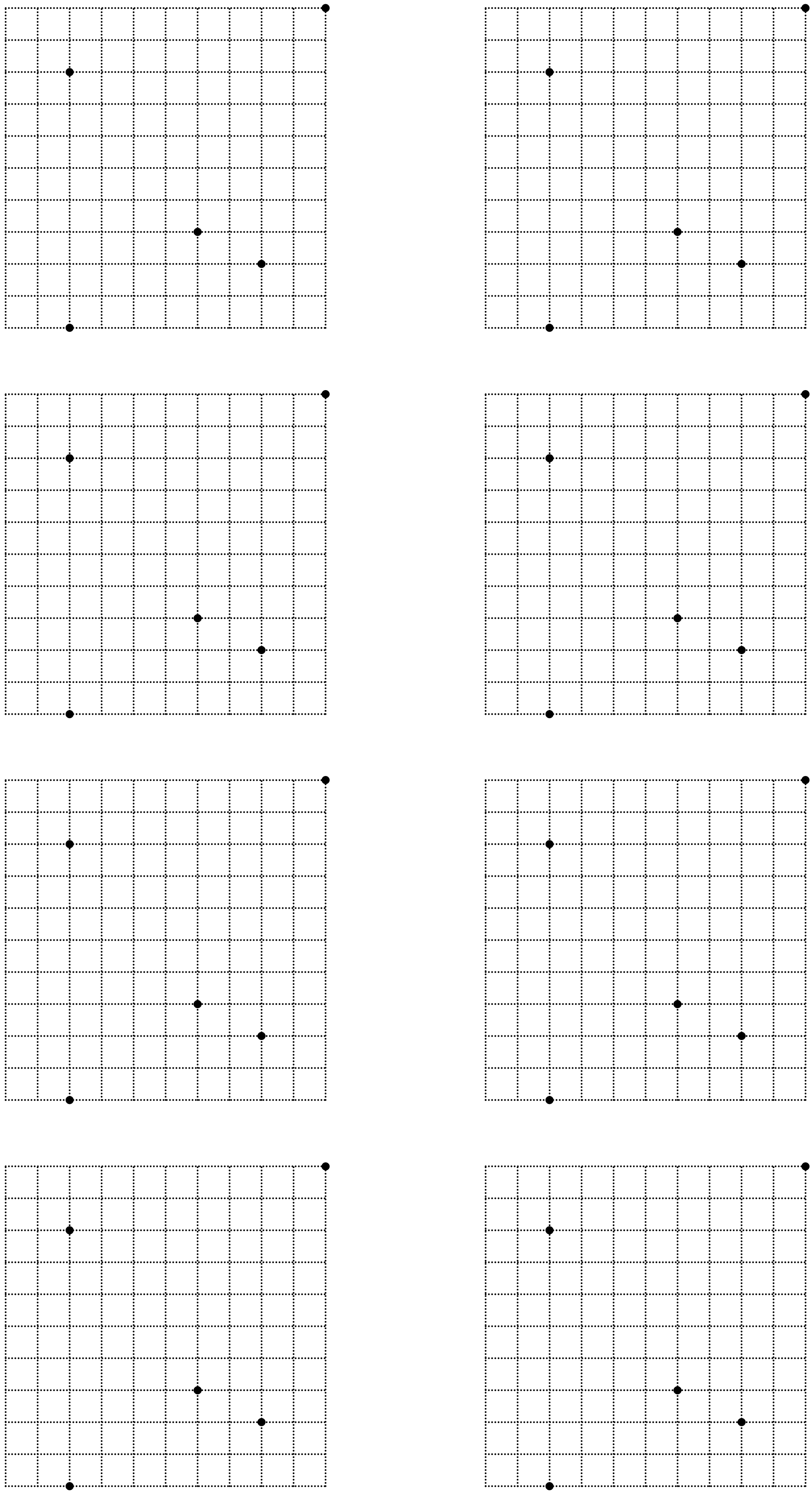

Figure 2: 1-center ℓ_2-problem.

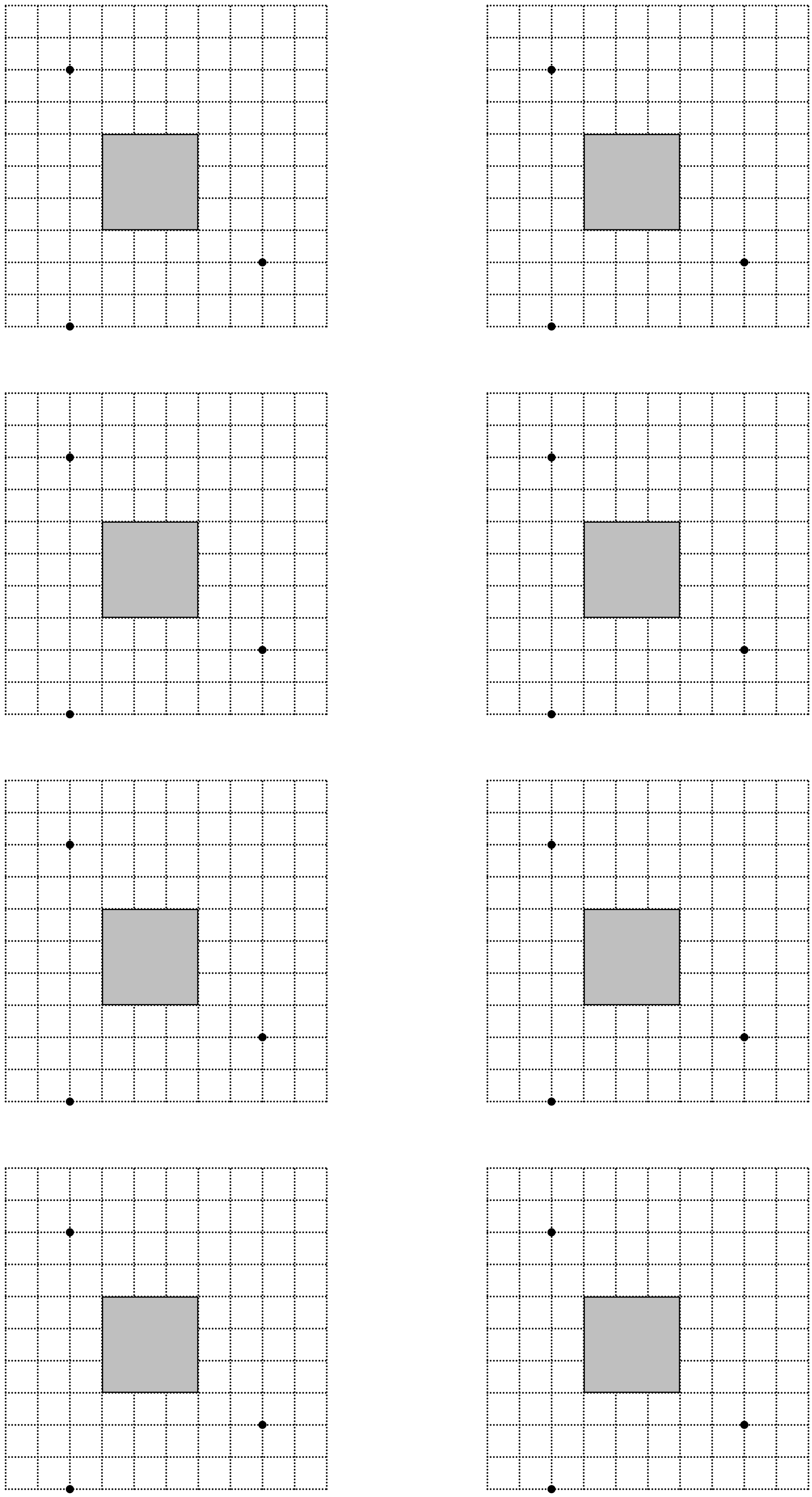

Figure 3: Restricted 1-center ℓ_2-problem.