Preface

For quite a number of years the rapid progress in the development of both *computers* and *computing* (algorithms) has stimulated a more and more detailed scientific and engineering modeling of reality. New branches of science and engineering, which had been considered rather closed until recently, have freshly opened up to mathematical modeling and to simulation on the computer. There is clear evidence that our present problem-solving ability does not only depend on the accessibility of the fastest computers (hardware), but even more on the availability of the most efficient algorithms (software).

The construction and the mathematical understanding of numerical algorithms is the topic of the academic discipline *Numerical Analysis*. In this introductory textbook the subject is understood as part of the larger field *Scientific Computing*. This rather new interdisciplinary field influences smart solutions in quite a number of industrial processes, from car production to biotechnology. At the same time it contributes immensely to investigations that are of general importance to our societies—such as the balanced economic and ecological use of primary energy, global climate change, or epidemiology.

The present book is predominantly addressed to *students of mathematics, computer science, science*, and *engineering*. In addition, it intends to reach computational scientists already on the job who wish to get acquainted with established modern concepts of Numerical Analysis and Scientific Computing on an elementary level via personal studies.

The field of Scientific Computing, situated at the confluence of mathematics, computer science, natural science, and engineering, has established itself in most teaching curricula, sometimes still under the traditional name Numerical Analysis. However, basic changes in the contents and the presentation have taken place in recent years, and this already at the introductory level: classical topics, which had been considered important for quite a time, have just dropped out, new ones have entered the stage. The guiding principle of this introductory textbook is to explain and exemplify essential concepts of modern Numerical Analysis for ordinary and partial differential equations using the simplest possible model problems. Nevertheless, readers are only assumed to have basic knowledge about topics typically taught in undergraduate Linear Algebra and Calculus courses. Further knowledge is definitely not required.

The primary aim of the book is to develop *algorithmic feeling and thinking.* After all, the algorithmic approach has historically been one of the roots of today's mathematics. It is no mere coincidence that, besides contemporary names, historical names like Gauss, Newton, and Chebyshev are found in numerous places all through the text. The orientation toward algorithms, however, should by no means be misunderstood. In fact, the most efficient algorithms often require a substantial amount of mathematical theory, which will be developed in the book. As a rule, elementary mathematical arguments are preferred. In topics like interpolation or integration we deliberately restrict ourselves to the one-dimensional case. Wherever meaningful, the reasoning appeals to geometric intuition—which also explains the quite large number of graphical representations. Notions like scalar product and orthogonality are used throughout—in the finite dimensional case as well as in infinite dimensions (functions). Despite the elementary presentation, the book contains a significant number of otherwise unpublished material. Some of our derivations of classical results differ significantly from traditional derivations—in many cases they are simpler and nevertheless more stringent. As an example we refer to our condition and error analysis, which requires only multidimensional differentiation as the main analytical prerequisite.

Compared to the first English edition, a polishing of the book as a whole has been performed. The essential new item is Section 5.5 on stochastic eigenvalue problems—a problem class that has gained increasing importance and appeared to be well-suited for an elementary presentation within our conceptual frame. As a recent follow-up, there exists an advanced textbook on numerical ordinary differential equations [22].

Of course, any selection of material expresses the scientific taste of the authors. The first author founded the Zuse Institute Berlin (ZIB) as a research institute for Scientific Computing in 1986. He has given Numerical Analysis courses at the Technical University of Munich and the University of Heidelberg, and is now teaching at the Free University of Berlin. Needless to say, he has presented his research results in numerous invited talks at international conferences and seminars at renowned universities and industry places all over the world. The second author originally got his mathematical training in pure mathematics and switched over to computational mathematics later. He is presently working in the communication industry. We are confident that the combination of a senior and a junior author, of a pure and an applied mathematician, as well as a member of academia and a representative from industry has had a stimulating effect on our presentation.

At this point it is our pleasure to thank all those who have particularly helped us with the preparation of this book. The first author remembers with gratitude his early time as an assistant of Roland Bulirsch (Technical University of Munich, retired since 2001), in whose tradition his present views on Scientific Computing have been shaped. Of course, our book has significantly profited from intensive discussions with numerous colleagues, some of which we want to mention explicitly here: Ernst Hairer and Gerhard Wanner (University of Geneva) for discussions on the general concept of the book; Folkmar Bornemann (Technical University of Munich) for the formulation of the error analysis, the different condition number concepts, and the definition of the stability indicator in Chapter 2; Wolfgang Dahmen (RWTH Aachen) for Chapter 7; and Dietrich Braess (Ruhr University Bochum) for the recursive derivation of the Fast Fourier Transform in Section 7.2.

The first edition of this textbook, which already contained the bulk of material presented in this text, was translated by Florian Potra and Friedmar Schulz—again many thanks to them. For this, the second edition, we cordially thank Rainer Roitzsch (ZIB), without whose deep knowledge about a rich variety of fiddly TEX questions this book could never have appeared. Our final thanks go to Erlinda Körnig and Sigrid Wacker for all kinds of assistance.

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