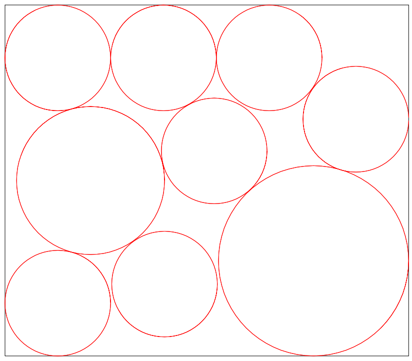
5th Porto Meeting on Mathematics for Industry, April 10–11, 2014
Ambros M. Gleixner, Zuse Institute Berlin

Exercise: Cutting circles from area-minimizing rectangles

In this exercise, we will formulate and solve MINLP models for the problem of cutting circles from rectangles with minimal area. This may occur in the manufacturing industry when trying to minimize trimloss, see, e.g., [1].



1. Suppose we are given n circles of radii $r_1, \dots, r_n > 0$. The task is to find a rectangle of width w and height h such that all circles fit into the rectangle without overlap and the area hw is minimized.

(a) Model this problem as a (quadratic) NLP. Is it convex?

(b) Implement this model in Zimpl. Assume that the input data are given in an external text file listing each radius on a separate line, the first line giving the number of circles. As an example, consider a file `rad.dat`

```
4
1        3.5
2        13
3        7
4        13
```

for an instance with four circles of radii 3.5, 7, and twice 13. You can read this file with the Zimpl code

```
param n := read "rad.dat" as "1n" use 1;
set N := { 1 to n };
param r[N] := read "rad.dat" as "<1n> 2n" skip 1 use n;
```

See the Zimpl manual at http://zimpl.zib.de/download/zimpl.pdf for how to declare the variables and constraints of your model.

2. Hands on: Download a SCIP binary for your platform from http://scip.zib.de, unzip it, and start the interactive shell. You can find a tutorial on how to use the interactive shell under http://scip.zib.de/doc/html/SHELL.php.

(a) Now try to solve your Zimpl model `nlp.zpl`:

```
SCIP> read nlp.zpl
...
SCIP> optimize
...
```

(b) Check the correctness of your model in SCIP with the following input:

- 16, 9, 13 (solution: 2323.069)
- 1, 2, 3, 4 (solution: 132.3641)
- 1, 1, 2, 3, 5, 8, 13 (solution: 1076.298)

Which is the largest instance of type $1, 2, 3, \ldots, n$ you can solve to global optimality (within which time)? You can stop earlier by changing the limit on the relative optimality gap:

```
SCIP> set limits gap
```

(c) Add constraints that break symmetry from

- switching height and width,
- rotating by 180°,
- flipping.

Are these constraints valid together?

How do they affect solution times?

2. Suppose that the centers of the circles are required to lie on a grid, i.e., to take integral coordinates. As before, we wish to minimize hw.

(a) Model this extension as an MINLP.

(b) Are the symmetry breaking constraints still valid?

(c) Do the integrality restriction make the problem harder or easier for SCIP?

Good luck!

References

[1] Josef Kallrath, *Cutting Circles and Polygons from Area-Minimizing Rectangles.* Journal of Global Optimization, Volume 43, Issue 2-3, pp. 299–328, 2009. doi:10.1007/s10898-007-9274-6.