are very important and maybe it would be good if the European Mathematical Society organised some committee devoted to mathematical education, a committee which selects some opinions before the next mathematical congress.

Participant D

I will be very brief. I only want to draw your attention to the following question. You can open almost every textbook in mathematics and, if you look at the problems and the exercises, you will see that 95% of them begin with the words "prove that", "show that". I think this point is one of the biggest problems. In my own experience as a mathematician in Mexico and then in Russia, in the hands of Arnold and Ilyashenko, in this change of schools, I learnt that it is very important to change the words "prove that" or "show that" to "is it true that?". I think that this is the very big difference for making the student a thinking person.

Participant E

The Education Committee of the European Mathematical Society is working on a project that is called Reference Levels for Age 16. So we do not tackle the problem for university students, but in principle the questions are the same. We hope within one year to come out with a longer report, not only about the mathematical content, but about the quality of mathematical thinking and doing mathematics. It is a project that this enquiry can be followed by another one for later ages, and so in the passage from school to university or even in the first years of university. This has not yet started, but we plan to do it if we have the possibility to do so, and therefore there will then be specific conferences on this subject.

Oleg Viro

I just want to suggest one thing. Instead of doing this, which is probably very good, why does the EMS not make a discussion page about this situation?

Moderator

You are right. Thank you very much to all for participating.

The Impact of Mathematical Research on Industry and Vice Versa

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Introduction

by Irene Fonseca

Mathematics and industry have everything to gain from working together. Exciting technological developments are motivating remarkable advances in mathematics which, in turn, is contributing significantly to industrial competitiveness. The panel will focus on how this two-way process works and how it could work better, by addressing the following questions:

Q1. Which industrial areas provide scope for interaction? The panelists themselves have personal experience in telecommunications, production planning, manufacturing, logistics and transportation, finance and investment banking. We will identify 'mainstream' areas of application and new areas that are appearing on the horizon.

Q2. What areas of mathematics are, or are likely to be, involved? Bearing in mind the role of number theory in cryptography, it is hard to say that *any* part of mathematics is immune from the threat of being useful! But do we see here, in addition to traditional disciplines, the emergence of a brand of mathematics and fundamentally new science —the science of complex phenomena?

Q3. How can collaboration be secured most effectively? It is common experience that mathematics is a key ingredient in the solution of some hard real problems, but also that close cooperation with many other disciplines is necessary in order to understand and model the phenomena correctly. In fact, such models very often challenge traditional mathematics. The introduction of radical ideas, new ways of doing mathematics with computers, and close collaboration with other scientists and the industry frequently appear necessary in order to match mathematical theories with demanding questions and available experimental data. How can individual mathematicians best equip themselves for this challenging task?

Q4. What is the role of computing and software engineering? Many industrial projects involve highly complex calculations or huge amounts of data, or both. Modern database design, object-oriented programming and languages for scientific computation and data analysis are a partial answer, but will the whole style of mathematics change in some more drastic way?

Q5. What are the implications for education and training? Vital applied problems need to be addressed in a rapid, cost-effective and interdisciplinary manner. This will require new curricula in education and effort on the part of mathematicians to bring fundamental ideas and techniques to a wider audience. Do we need a new breed of mathematicians able to mediate between pure and applied science and the industry?

Q6. Do mathematicians need to be more entrepreneurial? The old model in which scientists from established institutions form collaborations with development teams from large companies is giving way to a new paradigm in which individual scientists seek to exploit their own ideas. What new educational, scientific and business infrastructure is required to support this trend?

The Lisbon summit of the EU in March 2000 set forward an agenda for developing the EU as a 'knowledge-based' economy. The above questions all relate to the effective participation of the mathematical community in this process.

In recent years exciting technological and industrial developments have motivated remarkable advances in applied mathematics. In turn, the profusion of advanced industries and technologies yielded by research endeavors have led to improved standards of living, and underpinning much of this progress has been the emergence of scientific computing. Today's challenges faced by science and engineering are so complex that they can only be solved through the help and participation of mathematicians and engineers allied on three fronts: observation and experiment, modeling, and theory.

Remarks on Q1

We expect mathematics to play a fundamental role in major unpredictable scientific breakthroughs that will radically transform our lives in communication, environment, finance, manufacturing and business, the recurring themes being modeling, complexity and size, uncertainty, multiple scales, computation and large data sets. These include the following:

- Computing capabilities grow exponentially. Scientists at Georgia Tech are building one chip with 200 billion transistors for the current year (as opposed to the 7.5 million transistors of a current Pentium II computer), and we are approaching the era of quantum computing and its fascinating implications!
- Nanotechnology, the science of developing tools and machines as small as one molecule, will have an enormous impact on our lives, just as the transistor did 40 years ago. There will be systems for security smaller than a piece of dust, intelligent household appliances and cars, and thin film devices will allow for non-invasive surgery. The latter are amongst some of the most contemporary issues in materials science: the production of new materials and underlying multiscale phenomena (thin films, composites, microscale and nanoscale devices, and electronic, magnetic, and dielectric materials). Condensed matter physics is also being used in materials design, nanotechnology and high temperature superconductivity, each containing seeds of new industries and new scientific understanding. The implications for public safety and health are astounding.
- The human genome revolution and DNA-based molecular design will have overwhelming implications in human health, food production, and the environment.
- The study of the brain, neuroscience (neural networks), magnetic resonance imaging (nuclear magnetic resonance) techniques (modeling quantum computers), are also leading to the development of new, non-invasive instrumentation.
- Combustion is the principal source of energy for transportation, electric power production, and several industrial processes. Some of the main issues have to do with the prediction of operating characteristics such as safety, efficiency, and emissions. Mathematicians allied with engineers have created the analytical and computational tools to model combustion systems (e.g., in the modeling of chemistry of combustion and engineering-scale simulation).
- Cosmology, a discipline which was previously based mostly on speculation and scarce observations, has become a science rich in both data and theory (e.g., the relativistic 'hot big bang' model is widely accepted nowadays). Massive data sets are a major challenge, and as cosmology moves towards becoming an exact science, major mathematical barriers arise in coping with displaying, understanding, and explaining the overwhelmingly large set of high-quality data expected during the next few years and collected by NASA and by the European Space Agency. Mastering modeling and simulation will become imperative.
- In the finance world, connections with mathematics are tight in the areas of derivative securities and risk management (the classical Black-Scholes model).
- Air traffic management is an area in which operations research will play a crucial role. Air traffic in the US is expected to grow by 5% annually for the next 15 years

and rates of growth across the Pacific Rim are expected to be more than 15%. Recent studies indicate that if there is no change in the actual structure of air traffic control then by year 2015 there could be a major accident every 7 to 10 days...

— Internet analysis, reliability and security, and the involvement of mathematics in voice communication networks.
— Climate, mixing in the oceans and in the atmosphere.
— Diagnosis using variational probabilistic inference.

Remarks on Q2

The impact of science, technology and industry on society has been accompanied by greater expectations of benefits to society. The opening of the global world economy and the manufacturing of new (smart) materials have forced the shortening of the time from concept to product. These expectations can only be met through strong alliances between science, technology and industry. In scientifically advanced countries, mathematical research projects with industry range from control and scheduling of machines, telecommunications, logistics, and health sciences, to public transportation and energy optimization. Mathematics research has had a great impact on materials science (multiscale problems, materials instabilities), waves and scattering, high performance computing environmental science, molecular biology and mathematical biology (ordinary differential equations, discrete mathematics, combinatorics), adaptive computational methods, signal processing (fast Fourier transforms), computer vision (partial differential equations, calculus of variations), and economics (stochastic partial differential equations, optimization, mathematical finance). Also, integral geometry is at the base of x-ray tomography (CAT scan), arithmetic over prime numbers leads to the generation of perfect codes, and infinite-dimensional representations of groups suggests a design of large, economically efficient networks of high connectivity.

Mathematical models needed in the semiconductor industry involve scales ranging from the centimeter to the sub-micron level. The solutions of these models are computationally unfeasible, and the goal is to search for short cuts without sacrificing the physics. Also, in a wide class of problems typically coming from experimental science (biology, chemistry, geophysics, medical sciences, etc.), one has to deal with huge amounts of loosely structured data. These challenge traditional mathematics (probability theory and mathematical statistics, numerical analysis) which work well when the structure in question is essentially absent. Theoretical developments in these areas will have a substantial impact on industry and technology. For example, an efficient inverse scattering algorithm would revolutionize medical diagnostics, making ultrasonic devices at least as efficient as current x-ray analysis.

The need to model or compute on multiple scales arises when occurrences on vastly disparate scales (space, time, or both) contribute simultaneously to an observable outcome. For example, in turbulent combustion the shape of the vessel is important and so are

the very small fluctuations in temperature which control the various chemical reactions. Multiscale problems arise in the study of advanced materials, climate prediction, stochastic decision-making in finance, statistical modeling and development of algorithms, complex phenomena described by partial differential equations of classical physics (e.g., flow in porous media, a problem which arises in studies of the transport of groundwater pollutants and of oil in petroleum reservoirs), the need for less costly and more rapid means of designing and testing therapeutic drugs, and issues of learning and adaptation motivated in part by progress in neuroscience and by bringing the visions of artificial intelligence closer to reality.

These are all pieces of the great puzzle of a science for predicting complex phenomena that must be met with large data bases, large-scale computing... and large ideas!

Remarks on Q3

With new problems and challenges come new opportunities, and these generate excitement and attract the interest of young people, calling for a carefully thought-out strategy for investment of this scientific potential. Strong departments in the natural sciences — physics, chemistry, mathematics and biology— are a sine qua non for any fine university to stand out in the areas of engineering and science; and mathematics plays a privileged role as a common denominator among the so-called basic disciplines. It is here that decision making at the highest level of scientific administration and government policy calls for scientific vision: it is of critical importance to succeed in providing substantial resources to programs while reaching a compromise between immediate goals and broader objectives. Progress of science depends on the depth of understanding in individual disciplines, on the support for work with long-term horizons, and on considerable breadth across many areas.

The accelerated development of science in this century has widened the gap between theory and applications. Nowadays there are fewer people in industry with the time and skills needed to recognize and apply cutting-edge science to business problems. In the US there are virtually no resources to carry out long-term research in the industrial setting, which once led all the way from basic research to new industries and new markets. A key requirement for success in this arena is downstream planning of research, so that work is organized with a focus on solving problems that need to be solved. This approach will work best if scientists are allowed and agree to play a major role in the planning process. It should be stressed, however, that introducing downstream planning, or costumer focus, does not subtract from science, it adds to it.

Remarks on Q4

The role of mathematics in industry would be significantly reduced without computers. To a large extent, computers are the enabling technology for mathematics. They transform mathematical theories and algorithms into useful tools. In addition, the increasing computing power brings many complex models, considered hitherto unsolvable, into the realm of solvability, fertilizing at the same time mathematical research on problems that would not have been addressed without the availability of computers. Computational experiments and simulation support both theory building and the development of good practice.

Modern applications often need huge amounts of data and are frequently based on the interaction of complex software systems. The time of standalone software programs for special applications is over. Software engineering, database technologies and other concepts and tools play a very important role in industry, requiring mathematicians in industry or interacting with industry to acquire excellent knowledge in these areas. Although important, mathematical algorithms are just one of many components of a successful application system. Software engineering, systems design, and modern programming languages are among the tools that enable the interaction of the various components and pave the road for mathematics to enter the industrial world.

The style of doing mathematics will change, though not fundamentally. Mathematicians will take up problems from the real world, formulate them in mathematical terms, study them from a theoretical point of view and make the theory work via computer programs. The latter steps of this process will be supported by computers and software —in the future to a much larger extent than at present.

Remarks on Q5

Do we need a new breed of mathematicians, the industrial mathematicians, able to mediate between pure and applied science and the industry? The concept of industrial mathematics has existed in the US for over 10 years, and the success of the Minnesota Center for Industrial Mathematics, for instance, depends greatly on rendering faculty interested in investing the time necessary to start these programs and to ensure that their institutions are interested in investing in their faculty by providing incentives. Start-up processes are lengthy, and results will only show after a couple of years. In the US there are programs sponsored by federal agencies such as the NSF and ARPA which promote and enhance university-industry collaborations. As examples, we single out Mathematical Sciences University-Industry Postdoctoral Research Fellowships, Mathematical Sciences University-Industry Senior Research Fellowships, Industry-Based Graduate Research Assistantships and Cooperative Fellowships in the Mathematical Sciences, and NSF Research Experience for Undergraduates (REU).

We must, therefore, rethink how we train future scientists, engineers and mathematicians, and the key words here are multidisciplinarity and interdisciplinarity. Interdisciplinary and multidisciplinary research and teaching are critically important in the training of students. Such programs require immense investments of time, and explicit rewards for successful entrepreneurial behavior. Incentives to lead people to break out of their comfortable disciplinary molds are imperative, since the natural tendency is to build within the discipline. These incentives might include funding for laboratories and positions, research and time, and must revamp assessment criteria for raises and promotions. Postdoctoral and graduate fellowships in partnership with industry are an effective approach, and opportunities for undergraduate research (vertical integration) should be created. Projects that are beneficial to industry, and dissertations based on realistic industrial questions and co-advised between academia and industry, should be strongly encouraged.

In what sense do industrial mathematicians differ from engineers? Are we producing here a second rate class of mathematicians, rather than providing incentives to those fully involved in their research/academic careers to diversify and enlarge their agendas so as to include industrial consulting and some curiosity towards industrial applications of their scientific expertise? Mathematicians are useful in industry because they can move quickly from problem to problem and because they can penetrate deeply and rationally. Industrial mathematicians get their research problems from industry, and often their results are not published in scientific journals of the discipline due to the fact that their degree of formalism may be unacceptable for an applied mathematics journal. These are problems where mathematics are usually not very hard but the underlying motivations are significant. Industrial mathematicians also differ from applied mathematicians in that their scope is usually broader, and they are tied up to a tight frame of milestones imposed by their company/consumer/costumer. The immediate goal of an industrial mathematician is to increase resources, while applied mathematicians seek to improve theoretical results in the absence of time constraints.

What is needed to succeed in industry? Computational skills are essential and mathematical modeling, numerical methods, statistics, probability, differential equations, discrete mathematics, optimization, and operations research enter theseprojects to varying degrees. It requires communicating effectively, being a problem solver, and knowing how to combine depth and breadth while being flexible.

University departments that want to build bridges to industry should form networks built around their own graduates and exploit other contacts within their institutions, building the relationship 'from the bottom-up'.

Remarks on Q6

The chain usually followed in the dissemination of scientific knowledge and skills is 'basic science to applied science to engineering to product'. This is the trickle-down approach to the application of science and technology to industry. It is too slow and costly, too subdivided, and it is sometimes defended on the grounds of a need for specialization and division of labor. This approach is no longer adapted to today's world in view of the need for specialized skills to be applied to challenging problems in a rapid, flexible, cost-effective, and interdisciplinary manner. The volume, depth, and structural complexity of the present body of mathematics make it imperative to find new approaches to communicating mathematical discoveries from one domain to another, and to drastically improve the accessibility of mathematical ideas to non-mathematicians. Mathematicians often have little grasp of what is going on in science and engineering, while experimental scientists and engineers are, in many cases, unaware of opportunities offered by progresses in pure and applied mathematics. We must attempt to introduce faster and cheaper ways to bring science and technology to bear on industrial and business problems, while preserving the capacity of researchers for deep, academic innovation.

Governments and major sponsors should consider funding scientific research as a high investment priority: no other government expenditure will create more jobs and wealth, promote better health and education for all, and be as successful in the protection of the environment.

Problem solving in industry: the mathematical approach

by Martin Grötschel

I have been involved in projects with industry for more than 15 years and I am currently leading a research team at Konrad-Zuse-Zentrum of about 20 members, most of whom are participating in one or more practical projects in optimization. More than half of the team members are financed by industry directly.

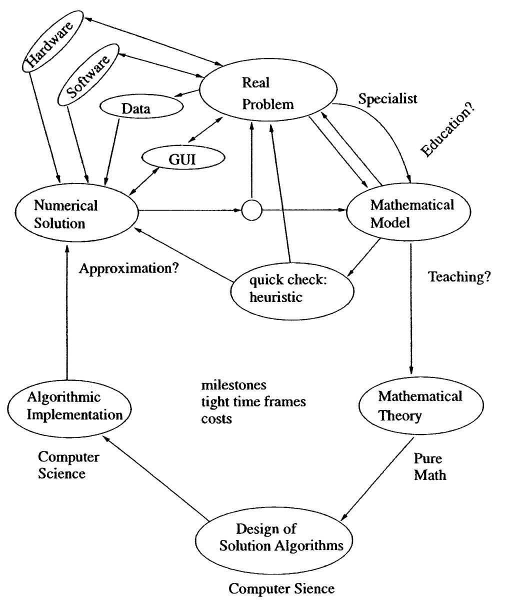
The application areas covered include production and manufacturing, telecommunications, logistics and transportation, VLSI-design, and the energy sector. The projects range from "small scale", such as the control of CNC-machines, or the job scheduling of printing machines, through "medium size" optimization of logistic systems within factories or the production scheduling of whole factory floors, to "truly large and difficult" problems such as the design of the telecommunications system of a whole country or the scheduling of buses in big cities such as Berlin.

Partners of our projects have been industrial giants such as Siemens, IBM, BASF, Deutsche Telekom, Bosch, BMW; medium to large companies like e-Plus, Telekom Austria, Norwegian Telecom, Ruhrgas, VEAG, Berliner Verkehrsbetriebe, Hamburger Hochbahn, ADAC, and Herlitz; and small to medium firms such as IVU, HanseCom, CARMEN, BZA, Intranetz, Grob.

This contribution to the topic of the Round Table is based on some of the experiences gained in the projects mentioned above. It will not be a detailed account. I will describe aspects that I consider important for research and education in mathematics and for the competitiveness of the industries involved.

The following diagram, called "The Problem Solving Cycle in Modern Applied Mathematics", is a pictorial summary of my presentation:

The Problem Solving Cycle



in Modern Applied Mathematics

I will describe its contents, i.e., its "boxes", arrows and the relations between the boxes.

From the real problem to the mathematical model

The focus of industrial mathematics is the "real problem", the aim is its solution. Therefore, the "real problem" is located at the top position of the diagram. The "user" does not care what mathematics is employed. The result is important for him (or her). But what is the real problem?

My institution (the Konrad-Zuse-Zentrum) has a broad range of industry customers who know that our specialty is optimization. Some even know that we are particularly good at integer programming. It, thus, happened several times that customers wanted to gain our interest by starting a request with: "I have a Traveling Salesman Problem for you", knowing that I have worked on this topic. Of course, in almost all cases the problem was not a TSP. Sometimes it was even a nonlinear or stochastic optimization problem.

Finding this out is quite a difficult process. I call this the "modeling phase". We start with a somewhat vague description provided by the customer. By asking detailed questions we try to arrive at a more precise understanding of the question until we feel able to state the problem in mathematical terms. We then go back to the customer, ask more questions, look for further side constraints and iterate this process (possibly several times) until both parties have the impression that a solution of the current "mathematical model" will yield a solution of the real question.

This modeling phase needs a close interaction between the specialists (there may be several with different backgrounds) and the mathematicians. It is sometimes psychologically difficult. In fact, in some cases we have been unsuccessful due to fears of employees on the user's side, who were afraid that the new approach may reveal former mistakes, or some amount of incompetence, or may even make their job insecure. Both sides have to go through a learning process in order to arrive at a fertile cooperation.

Our university curricula rarely teach the aspects indicated above. We usually consider a problem as "given". Real problems are not "given". Their mathematical version is the end of a long process of interaction. What is not taught at all is the psychology involved. Even if a company has decided to attack a certain problem by mathematical means, some of the persons participating may have different views or second thoughts. Mathematicians have to learn to cope with this, to compromise on certain issues, and to share success. Otherwise application projects are likely to fail.

Rapid prototyping: simple heuristics

Once a mathematical model is found, there are two possibilities. The cheap alternative: The model is standard and (commercial) software exists to solve it. In this case (e.g., the problem is a linear program) one simply employs (buys) available software and the project is over. If the problem is difficult, i.e., mathematically new or much larger than commercial software can handle, the customer would like to know whether it is worth investigating the problem in more detail (and, thus, spending time and money).

Mathematicians usually hesitate to try simple heuristics (or the like) to see whether and what gains can be expected. Rapid prototyping is, however, indispensable for a customer to estimate whether it makes sense for him to enter a real project. It may, in fact, turn out that the expected gains or cost reductions are marginal and that extensive mathematical modeling and the employment of sophisticated solution technologies is not relevant from an economical point of view. The project may, thus, be more expensive than the expected cost reductions. In this case "new technology" is not necessary. Once the heuristics indicate substantial improvements over the current state, a serious discussion should start to specify the goals of a joint project to solve the questions at hand. If both parties agree on the goals, the time frames and milestones, and on the financial contributions, an application-oriented research project begins.

From the mathematical model to new theory

This phase is typical research in mathematics (usually called pure mathematics). A clean mathematical question has been specified and mathematical research is performed in order to understand the problem from a mathematical point view, to analyse the structure, to prove theorems, etc. There is one difference to (general) research. However tempting it is to follow exciting sidelines, the goal is to address the model specified and not some variation of it. I think that the determination to address a particular issue without escaping to some other problem of interest distinguishes application-oriented research from the more traditional way of doing pure mathematics.

Design of solution algorithms

Part of the theoretical investigation of models from practice is always of an algorithmical nature. While proving theorems one has to keep in mind how these theorems can be made to work algorithmically. Every area in applied mathematics has its own solution paradigms and algorithmic concepts. One usually tries to fit it into a framework such as the design of a branch-and-bound algorithm, a cutting-plane algorithm, a Lagrangian relaxation technique, etc. However, it has to be kept in mind that a theorem alone is rarely of value for practice unless it can be turned into some useful algorithmic tool that helps solve the practical problem. Thus, algorithm design is an important integral part of the whole solution process, and interactions with computer science are indispensable.

Algorithmic implementation

Once the basic algorithms have been designed, the algorithmic implementation has to be started. Of course, this often goes in parallel with the two previous phases, and lots of experiments are conducted in order to see which of the possible approaches appears to work best in the current situation. Here one often relies on software libraries of colleagues or of commercial vendors that help in the process. A modern solution algorithm for a complex application does not just consist of one algorithm. Typically, the overall procedure is a combination of many algorithms designed for the particular situation at hand. Here one often needs the help of experts in numerical analysis or computer science, who do know which of the available algorithmic techniques works best for certain data or structures.

Numerical solution

I have distinguished here the algorithmic implementation phase from the numerical solution phase, although they often go hand in hand. It may happen that the implementation is done on powerful computers first, but has to be scaled down afterwards to run on machines that are used in practice. Sometimes it is the other way around. Implementations are performed on small machines and the final applications are run on supercomputers.

In any case, it is necessary to address the characteristics of the machines, the sizes of the real-world problems and the algorithms available. One may have to settle for more realistic goals than optimum solutions or exact solutions due to limited computing power, limited time, etc. One has to discuss whether approximate solutions are acceptable for the customers and to make the consequences clear.

From the numerical solution to the real problem

Typically, a customer provides a handful of data sets of real applications and wants to see how the new algorithms work on these data sets and compare them with the solutions used in practice. Once these practical instances have been solved with the newly designed algorithms, a critical stage is reached. One usually makes a presentation to the customer, outlining the results that have been achieved with the help of mathematics.

Now there are various possibilities. It can happen that the customers have "better solutions" than the mathematicians were able to obtain. This has happened to me several times. In each case, a detailed analysis of such "solutions" revealed that the company violated many of its own rules and side constraints. The reason for this is typically that the persons in charge of the problem were unable to find feasible solutions and implemented unfeasible solutions without telling others (openly). It then has to be discussed whether the side constraints have to be relaxed or enforced and what to do.

More often, however, the "mathematical solution" (as it is usually called) is unfeasible in the real world. The reason is usually that some side constraints have been "forgotten". Nobody can be blamed for this, in general. The practitioners assume that the mathematicians know many of these implicit side constraints and did not bother to tell them. Many side constraints are so "natural" that the practitioners assume that the mathematicians would automatically take care of them. In this case, the whole circle may have to begin again, hopefully arriving at a feasible solution in the second round. Another iteration is usually unfeasible since all milestones and time constraints have been violated, and the whole project has to be considered a failure. This shows that adequate modeling is extremely important.

Transfer and implementation

Fortunately, in the application projects we carried out at ZIB, in most cases we could arrive at reasonable solutions usable in practice. The project is not over then. The customer now has to evaluate whether it is useful to integrate the mathematical software into his current software system. If this is positively decided, a new phase has to be started to turn the current implementation into a useful software tool with a nice user interface, etc. Existing software and hardware has to be taken into account, and data handling to be considered. This is often costly and time-consuming, and I will not go into the details of it here. If a project is really successful, a spin-off company may form that takes over the software, develops it on a professional basis, and makes it a commercial tool. For a

research institute, it is very difficult to maintain a software system in the long run, give customer support and guarantees, etc.

I do think that the whole problem-solving cycle that I described above is a sound picture of a typical application of mathematics in industry. I think we should much more strongly focus on this type of problem solving in our university education. Involvement in such projects gives students perspectives for their future professional career. The problems coming up in such applications often give rise to many interesting and new mathematical questions and help in this way to shape the mathematics of the future.

I believe that it is quite healthy for mathematics to make a strong effort to support such a development. Mathematics is part of our society, and it should be an important part of the global development. Global competition, new markets, high production speed, fast technological development —all these trends cry for making optimal use of scarce resources. Mathematics has the tools to solve some of these issues. In most cases it is an auxiliary technique that helps progress within a broader frame. However, mathematics has a very wide range of applications, and I see a bright future for students educated in this type of mathematical approach to solving of real-world problems.

Success profiles of mathematicians in industry

by Mark Davis

I intend to approach this more from a personal prospective than from the point of view of institutions. I will start by saying something about my own experiences as a mathematician in industry, then go on to say something about qualities that I think are required for success. After that I will say something about educational programs and co-operative research, before moving on to cover specifically the questions we listed for this discussion.

From 1995 to 1999 I was head of research and product development at Tokyo-Mitsubishi International, the London-based investment banking subsidiary of the Bank of Tokyo-Mitsubishi. I ran a front office research team of six or seven people, all of them PhD scientists —mostly mathematicians, but some physicists and engineers. Our job was to provide pricing models, risk analysis and general statistical and quantitative assistance to the various front office trading groups. The time scales for these projects varied from "short" —anything from an hour to few days— to "long term", meaning anything up to three months. The output of our endeavours was always a piece of code, either compiled as a spreadsheet function or written as a module to be incorporated in the company's main trading system. The pricing models are generally based on the Black-Scholes methodology, implemented by analytic expressions, finite difference algorithms or Monte Carlo, as appropriate.

To succeed in this area, an individual

- Must be a good mathematician. The theory is often quite subtle and it is easy to make mistakes if one fails to think things through from first principles.
- Must have a personal "mathematical tool-kit". Fast answers are often required and it is very inefficient to start from scratch every time.
- Must be a good problem solver. It is important to be able to recognise which factors are the important ones and to come up with a rough answer against which more precise calculations can be checked. In general, the job is to get the best answer possible given the time and information available —information that may be far from complete.
- Must listen. It is important to understand exactly what traders need, and in many cases their intuition is excellent. If one is going to override it, one had better be able to explain why!
- Must be at home with the technology. Not everybody will be a specialist in software engineering but nowadays there is no such thing as a pen-and-paper industrial mathematician, if indeed there ever was.

To get people with these qualities, I always hired PhDs. Why? Because they have that extra maturity, they are used to independent thinking, and they have some idea what to do when they hit a brick wall.

Undoubtedly the best thing university programmes can do is to turn out good mathematicians. Everything else is secondary. As regards cooperative research, there is —at least in investment banking— a time scale problem. For the bankers, six months is already a long time horizon so it is hard to organise sponsorship for one-year Masters degree courses or three-year PhD programmes. On the other hand, three months internship are more easily organised and clearly benefit both sides.

Now let me turn to the six questions with which we started this discussion. As regards the first two, I will only say that there will always be heavy emphasis on computation so, whatever the area of mathematics, the "deliverable" is sure to be a piece of software.

Question three asks how collaboration can be secured effectively. Several different forms of university/industry interaction work well. For example:

- Individual consulting.
- Supervision of PhD students working in industry.
- Industrially supported academic research centres.
- Joint funding agency/industry supported projects.

Examples that I know to be working well are RiskLab at ETH Zürich, a centre jointly sponsored by the Swiss banks to study questions of risk management, and the Centre for Process Systems Engineering at Imperial College London, which has negotiated long-term collaborative research agreements with a small number of large companies in the chemical processing sector.

Turning to question four, the role of computing and software engineering is pervasive, as mentioned above. This also affects the style of the mathematics: less emphasis on closed form solutions, more emphasis on efficient algorithms.

Questions of education and training I dealt with to some extent earlier. An additional point concerns the transition from industry to academia and back in the course of one's career. Such transitions are still not as easy as they should be, often for absurd bureaucratic reasons. More flexibility is needed here. Do we need special purpose "industrial PhDs"? No thanks!

Finally, do mathematicians need to be more entrepreneurial? Here I entirely agree with Martin Grötschel's remarks during the discussion. Certainly they do, but this is not something we greybeards need to teach the students: much more likely, they will teach us.

Contributions from the audience

Abul Hasan Siddiqi (King Fahd University, Saudi Arabia)

For the last ten years I have been engaged in promoting the teaching and research of industrial mathematics in India, and presently I have been assigned to draft a degree on industrial mathematics at my institution in Saudi Arabia. Although this proposal encountered some resistance among mathematicians, several parties, including the government, want to pursue this initiative.

The question I ask is: should we start offering degrees in industrial mathematics, or is it better to keep industrial mathematics at the level of specialisation built around a solid mathematics training?

Martin Grötschel

I could comment on this based on the experience at my university, the Technische Universität Berlin. We offer three degrees in mathematics: a traditional mathematics degree, one in techno-mathematics, and the third in business mathematics. I want to stress that all of them are mathematics degrees. The basic principle is that those who go for techno-mathematics are required to take courses in certain areas of engineering, and those in business mathematics should acquire some expertise in certain areas of business and economics. But we keep a close eye on their mathematical education.

Our experience is that we have a variety of students. Typically those who are more inclined to go to industry, who are not aiming for academic achievement, opt for mathematics degrees with a specialisation in technology or business. These students tend to be more goal oriented; they want to finish their studies more quickly in order to start a career in industry. It is important to keep these targeted mathematics degrees "under mathematical control" so as to avoid ending up offering soft mathematics within business or something of the sort.

Moderator

Let me just add that in the United States, and for over a decade now, the IMA (Institute for Mathematics and its Applications) at the University of Minnesota, Minneapolis, has been very successful in bridging industry and academic mathematics. The Minnesota Center for Industrial Mathematics is playing an important role in the training of masters and PhDs in Industrial Mathematics.

Claudio Pedrini (Università di Genova, Italy)

I have very little to do with applied mathematics because I consider myself to be a pure mathematician. I am the president of a public agency which channels European Community funds to help building technology in smaller industries.

I think that it is important to address the competition between Europe and the United States, and their differences in funding systems. The experience I have is the following: we have been spending more than one million dollars in financing projects to help expanding technology in small industries inside the Fourth and Fifth Frameworks of the European project. The Fifth Framework is operating, and a proposal for the Sixth Framework has already been submitted to Brussels. The numbers are not very encouraging, not only with regard to the total amount of money spent in Europe in research and technology, but also all the parameters which indicate a widening of the gap between Europe and the United States.

If you look at the database of the European Community concerning funded projects in areas of applied mathematics, you will realise that the number of applied mathematicians who have been inside this project is rather poor. There are more engineers than mathematicians. Essentially these projects are made up by partnership between universities, small industries, research centers or individuals, with little, if any, involvement of mathematicians. In the few instances were they participated, their projects where poor in comparison with those of the engineers.

Are there any suggestions to the European parliament in order to better develop the Sixth Framework?

What are your suggestions in order to make mathematicians more aware that this is one of the best financial resources? The rigidity of our academic system, the lack of private investment for research, and the role of the small industries in Europe render the funding and research landscapes completely different from those of the United States.

Martin Grötschel

I could say a few words on that. I think that mathematicians are, in general, still working in some sort of isolation, in small groups of people, e.g. one chair with one or two assistants —a typical situation in Germany. They have not learned to collaborate with partners from other disciplines and to work in bigger groups. Engineers, on the other hand, have huge chairs in Germany, and they often have a couple of people who do nothing but write applications.

I will give you a personal perspective. I myself went out of all European projects because of the high degree of bureaucracy and, compared to the refereeing process by the Deutsche Forschungsgemeinschaft, its rather poor refereeing system. It is a waste of my personal time to apply for European money. I rather get money from industry directly. I am not saying that this is a good strategy; it is simply my personal approach. I have been involved in three or four of these projects. In some cases we were not successful because we did not include any institution from less-favoured countries. The whole handling was poor and disappointing. The funding process is very political and not as research-oriented as I would like it to be.

In Germany, the science ministry (BMBF) is currently funding about sixty projects in which mathematicians are working together with industry. There have been two hundred and fifty applications for these sixty projects. This shows that there is interest. This line of financial support has really changed the way mathematics is being taught at certain institutions. Students like it, and it is a successful way of training the younger generation.

Joaquim Bruna (Universitat Autònoma de Barcelona, Spain)

I just wanted to add that within the European programme there is the Leonardo programme, which addresses this area: the promotion of industrial mathematics. It has been noticed that there have been very few applications to that programme, and that some of the funds have not been used at all.

Abul Hasan Siddiqi

Are there any guidelines for the minimum content of mathematics that should be taught to industrial mathematicians?

Moderator

In the United States the branching out to industrial mathematics only takes place at the level of graduate schools, not at the level of undergraduate education. So the basic training includes a general education just like any other mathematics student has.

Martin Grötschel

My experience at the Technische Universität Berlin —I think there are now ten to twenty universities in Germany doing techno-mathematics— is that the course design depends on the human resources available, how many faculty members an institution has to teach such courses and the related mathematics. In Germany it is very hard to renew staff. Once a professor has his position, he will stay there for life. So you have to compromise. There are disciplines that should be offered at any rate, like stochastics and optimisation. Optimisation is not taught at German universities in general, while stochastics is.

Simultaneously we are talking to engineers, how to change their courses of study. Engineers can usually solve differential equations, and they are probably better at numerical analysis than many of our colleagues, but they do not know discrete mathematics, for instance. Discrete mathematics comes up more and more often in all the planning processes. I am currently discussing such issues with the engineers at my university, asking them to consider requiring courses of that type, so that they learn about"mathematical modeling".

Another remark. With many computer algebra systems available, we should really change our courses and use these tools in education. Understanding concepts of mathematics and knowing that one can use mathematics may be much more important for an engineer than knowing three different methods to solve a partial differential equation. In the education of engineers we have to teach them that the concepts they are learning in ODEs, PDEs, optimisation, stochastics, etc., do have an impact on the real world. The issue we must address, therefore, is how to teach the non-mathematicians to understand the modeling of their areas in mathematical terms. We also have to teach them to seek for support from mathematics. And this is where interaction starts.

Joaquim Bruna

Concerning the previous question, there are two educational programmes in industrial mathematics run by the European Consortium for Mathematics and Industry (ECMI): one is in techno-mathematics and another in econo-mathematics. You may look at the curricula on their web page.

Participant A

I am from the industry, and from your presentation I had the feeling that you faced some unique problems suitable for one customer. I have a question: have you developed products or solutions suitable for a variety of customers? When you solve problems for many customers, you face another challenge: competition from other inventors of similar solutions or products.

Mark Davis

In the finance industry this is a really hard issue. For example, can you market some generic piece of software, or should you try and make your money by selling yourself as a consultant on an individual basis?

In the finance industry it is difficult to say when a piece of software is going to be used across the industry. It costs a lot of money to develop that; it is a level of expenditure way beyond simple in-house development. And then, what are you going to get for it? What are people going to pay for something that they know you are selling to as many other people as you can? Therefore, typically, the consulting companies in the finance industry do not do that. On the whole, I think the best way of developing anything like a generic product is to couple yourself up with somebody who is selling, say, trading systems — which, you know, every bank must have, if they do not write it for themselves— and this way you can shoehorn your way into different companies as a sort of consultant on the side. So when a company sells a particular trading system to a bank, then they will also bring in consultancy services to configure it in the way that the customer wants it. And this is your chance to have some input in the kind of software and modeling area. However, marketing something like a Matlab toolbox is really difficult to do, if not just from the cost standpoint.

Martin Grötschel

Everything is feasible, possible and doable, depending on what target you set. Typically in consulting, we come up with the problem and solve the problem. If it is general enough, we may have a spin-off company. Our initial business in telecommunication network design, for instance, was very specialised. Now we have done projects with four companies, and the students who have developed the software are going to form their own company. They have enough experience, but they will not design one general tool because all companies have different side constraints. So the spin-off company will offer a mix of consulting and software. This development is a real source for future work, because the underlying mathematical problems are hard. That is why mathematicians have to be around.

Moderator

Any further questions? If not, I would like to thank you all for participating in this debate.