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Project B10

Describing polyhedra by polynomial inequalities

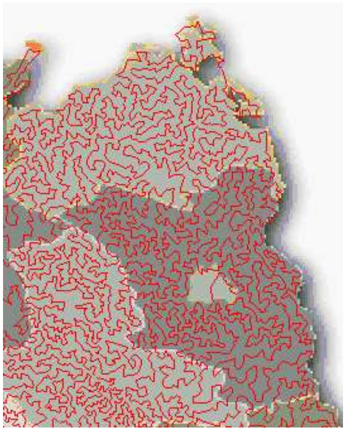
Hartwig Bosse   Martin Grötschel

Konrad-Zuse-Zentrum
für Informationstechnik
Berlin
www.zib.de

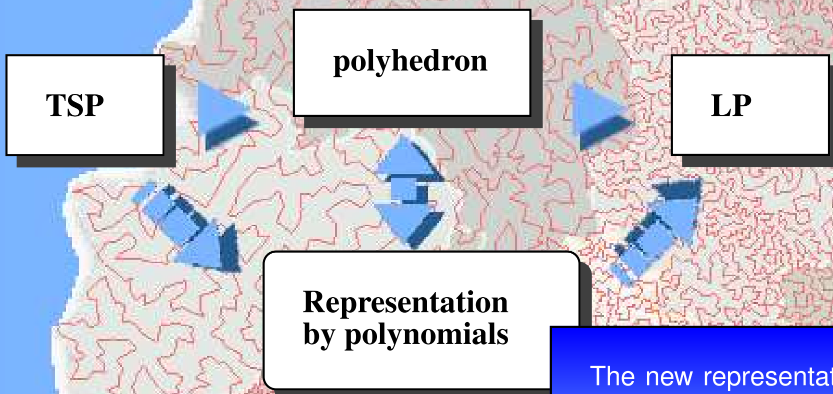
Technische Universität
Berlin
www.tu-berlin.de

Combinatorial optimization

Hard combinatorial problems – such as the “Traveling Salesman Problem” (TSP) – are often attacked using the so-called polyhedral approach: all feasible solutions are represented as vertices of a polyhedron, linear programming (LP) is used to find an optimal solution. Efficient algorithms solving linear programs arising this way exploit geometric properties and special analytic or algebraic representations of such polyhedra. This complex interplay between geometry, algebra, and numerics has helped solve many instances of notoriously hard combinatorial optimization problems in practice.



Optimal solution of a TSP.



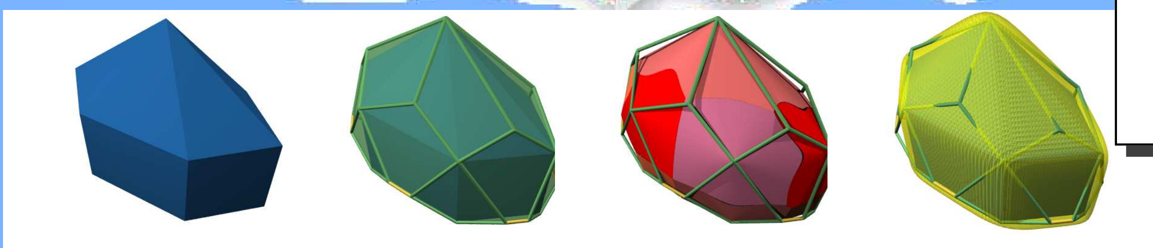
The new representation for polyhedra as solutions of **few** polynomial inequalities, possibly offers new roads of attack for combinatorial optimization problems, involving techniques from nonlinear optimization.

Representing polyhedra by polynomial inequalities

Inspired by a deep result in semi-algebraic geometry due to Bröcker and Scheiderer, this project has developed a new representation for polyhedra:

Theorem: Every polyhedron in $\mathbb{R}^n$ can be represented by at most $2n$ polynomial inequalities.

For every polyhedron, the polynomials needed can be constructed *explicitly*.



Specific polynomial inequalities form these approximative sets to a given polyhedron.