Technische Universität Berlin
Institut für Mathematik

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Mathematics of Infrastructure Planning

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Exercise sheet 7

Deadline: Thu, May 31, 2012, **23:59**, `mailto:borndoerfer@zib.de`

Exercise 20. **10 points**

Consider the frequency assignment problems of exercises 18 and 19. Suppose that there are at least two frequencies available for every cell. What is the optimal value of the LP relaxation? Formulate a claim and prove it.

Exercise 21. **10 points**

Solving the 1-center network location problem $1/N/\cdot/\operatorname{sp}/\max$ requires the solution of the minimization problems

$$\min \max_{w \in V} \min_{\lambda \in [0,1]} \{\operatorname{sp}(u,w) + \lambda c_{uv}, \operatorname{sp}(v,w) + (1-\lambda) c_{uv}\}$$

for all undirected edges $uv \in E$. Show that this problem can be solved in $O(|V|)$ time for any fixed edge $uv \in E$.

Exercise 22. **10 points**

Consider the following algorithm by Elzinga & Hearn [1971] for the 1-center problem in the Euclidean plane:

Data: set of points $V = \{v1, \ldots, v_n\}$ in the plane
Result: $p^* = \operatorname{argmin} 1/\mathbb{R}^2/\cdot/\ell_2/\max$

1 Choose any two different points $a, b \in V$;
2 Let a, b define the diameter of a circle C with center p; if C covers all points, output p and stop; otherwise choose some point $c \in V$ outside of C;
3 If a, b, c define a right or obtuse triangle (including the collinear case), drop the point at the right or obtuse angle (say c) and go to step 2;
4 Let a, b, c define a circle C with center p; if C covers all points, output p and stop; otherwise choose an outside point d; relabel a, b, c such that a is a point farthest from d; extend the line ap to divide the plane into two halfspaces; relabel b, c such that b is in the same halfspace as d and c the remaining point; replace b by d and go to step 3;

Prove that this algorithm is correct by showing that it produces a sequence of circles with increasing diameters:

R_1 d_1

a

R_2 d_2

b

R_3 d_3

Figure 1: Algorithm of Elzinga & Hearn.

a) If the circle C defined by points a, b in step 2 does not cover all points, there is an outside point d_1, d_2, or d_3 in region R_1, R_2, or R_3, respectively, as depicted on the left of Fig. **??**. The new circle defined by points a, b, d_i, $i = 1, 2, 3$ has a larger diameter than C.
b) If a, b, c form an acute triangle, c lies on the circle between points x and y, where x and y are diametrically opposed to a and b, respectively.
c) If the circle C defined by points a, b, c in step 4 does not cover all points, there is an outside point d_1 or d_2 in region R_1 or R_2, respectively, as depicted on the right of Fig. **??**.
d) If the outside point is d_1 in region R_1, then c is dropped and the new circle defined by a and d_1 is larger than C.
e) If the outside point is d_2 in region R_2, then b is dropped an the new circle defined by a, c, d_2, having a center point p', is larger than C.

Exercise 23. **(Tutorial session)**

Solve the 1-center ℓ_2-problem in Fig. **??** using the algorithm of Elzinga & Hearn described in exercise 22, starting with the two closest points, always adding the outside point closest to the current circle.

Exercise 24. **(Tutorial session)**

Solve the restricted 1-center ℓ_2-problem in Fig. **??**. **Hint:** Start with the solution of the unrestricted problem. Consider circles at the three defining points meeting in the center. What happens when you blow up the circles?

Exercise 25. **(Tutorial session)**

Consider the 6-node graph $N = (V, E)$ in Fig. **??** with distances d_{ij} as drawn next to the edges and nodes. Solve the 1-center location problem $1/N/\cdot/\text{sp}/\max$.

Figure 2: 1-center ℓ_2-problem.

Figure 3: Restricted 1-center ℓ_2-problem.

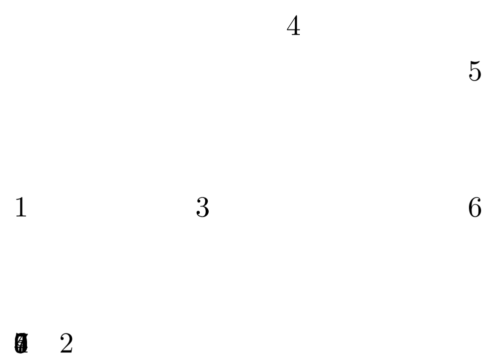

Figure 4: Network center problem.