Technische Universität Berlin
Institut für Mathematik

Summer Semester 2012

Mathematics of Infrastructure Planning

PD Dr. Ralf Borndörfer
Prof. Dr. Martin Grötschel

Exercise sheet 8

Deadline: Thursday, June 7, 2012, 23:59 (please submit your solution by email to: `borndoerfer@zib.de`)

Exercise 26. **10 points**

Prove the following:
Theorem. For a graph $G = (V, E)$, the following are equivalent:

(A) G is 2-edge-connected.

(B) For any pair of nodes, there are two edge-disjoint paths connecting them.

(C) G is connected and cyclic (where *cyclic* means that every edge is contained in a circuit).

(D) There is a sequence $G_0, G_1, ..., G_q = G$ of subgraphs of G, to be called an **ear decomposition** of G, where G_0 consists of one node and no edge, and each G_i arises from G_{i-1} by adding a path P_i (called an **ear**) for which the (not-necessarily-distinct) endnodes belong to G_{i-1} while the (possibly empty) set of inner nodes of P_i do not.

(E) G can be built up from a node by sequentially adding edges (allowing loops) connecting two existing nodes and subdividing edges.

Exercise 27. **10 points**

For a given graph $G = (V, E)$ with weights $c(e)$, $e \in E$, the problem of finding a smallest spanning 2-edge-connected subgraph means that one has to find a subset $F \subseteq E$ of smallest weight $c(F)$ such that (V, F) is 2-edge connected.
Prove the following:
Theorem. The problem of finding a smallest spanning 2-edge-connected subgraph of a graph G is NP-hard.

Exercise 28. **10 points**

Let $G = (V, E)$ be a graph with nodes s and t, $s \neq t$. Let $\mathcal{P}$ be the collection of all subsets of E that contain an $s-t$-path and let $P_{s-t}(G)$ be the convex hull of the incidence vectors of the elements of $\mathcal{P}$. Find a complete description of $P_{s-t}(G)$ by means of linear inequalities and equations.

Exercise 29. **10 points**

Compute (numerically) a complete linear description of the convex hull of the incidence vectors of the spanning 2-edge-connected subsets of the complete graph K_5.