## FTTx network planning

# Mathematics of Infrastructure Planning (ADM III) 

14 May 2012
$\triangleright \quad$ Fiber To The $\underline{x}$
$\Rightarrow$ Telecommunication access networks: "last mile" of connection between customer homes (or business units) and telecommunication central offices
$\Rightarrow$ Fiber optic technology: much higher transmission rates, lower energy consumption
$\triangleright$ Multitude of choices in the planning of FTTx networks

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Architecture:


PON


Point-to-point
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$\triangleright$ Multitude of choices in the planning of FTTx networks

Target coverage rate:

$$
80 \%
$$



capacity restrictions!
$\diamond$
CO (central office): connection to backbone networkBTP ("customer" location): target point of a connection
$\triangle$ DP (distribution point): passive optical switching elements
$\Rightarrow$ splitters, closures with capacities


capacity restrictions!
length restrictions!

$\diamond$
CO (central office): connection to backbone networkBTP ("customer" location): target point of a connection
$\triangle$ DP (distribution point): passive optical switching elements
$\Rightarrow$ splitters, closures with capacities

- Links: fibers in cables (in micro-ducts) (in ducts) in the ground

- Given a trail network with
- special locations: potential $\mathrm{COs}, \mathrm{DPs}$, and BTPs,
- trails with trenching costs, possibly with existing infrastructure (empty ducts, dark fibers)
- catalogue of installable components with cost values
- further planning parameters (target coverage rate, max. number of residents/fibers per CO/DP, etc)

$\Rightarrow$ Find a valid, cost-optimal FTTx network!
$\triangleright$ BMBF funded project 2009-2011

- Partners:
- Industry Partners: R•KOM Draka

Breitband Kompetenz Zentrum Niedersachsen
$\triangleright$ Compute FTTx network in several steps:

1. step: network topology
a) connect BTPs to DPs
b) connect DPs to COs
$\Rightarrow$ integer linear program: concentrator-location
2. step: cable \& component installation
3. step: duct installation $\Rightarrow$ integer linear program: cable-duct-installation
$\triangleright$ Given: undirected graph with

- client nodes: fiber demand, number of residents, revenue (for optional clients)
- concentrator nodes: capacities for components, fibers, cables, ..., cost values
- edges: capacity in fibers or cables (possibly 0 ), cost values for trenching
$\triangleright$ Task: compute a cost-optimal network such that
- each mandatory client is connected to one concentrator
- various capacities at concentrators and edges are respected
- Integer program:
- select paths that connect clients
- capacity constraints on edges
- capacity constraints for fibers, cables, closures, (cassette trays), (splitter) ports at concentrators
- constraints for coverage rate, limit on the number of concentrators
minimize $\sum_{i \in V_{D}} c_{i} x_{i}+\sum_{t \in T} c_{t} y_{t}+\sum_{e \in E} c_{e} w_{e}+\sum_{p \in P \cup \hat{P}} c_{p} f_{p}-\sum_{v \in V_{B}} r_{v} q_{v}$
s.t.

$$
\begin{aligned}
& \sum_{p \in P_{v}} f_{p}=1 \quad \forall v \in V_{A} \\
& \sum_{p \in P_{v}} f_{p}=q_{v} \quad \forall v \in V_{B} \\
& f_{p} \leq f_{p^{\prime}} \quad \forall p \in P^{\prime} \\
& \sum_{p \in P_{e} \cup \hat{P}_{e}} f_{p} \leq\left|P_{e} \cup \hat{P}_{e}\right| w_{e} \quad \forall e \in E_{0} \\
& \sum_{p \in P_{e} \cup \hat{P}_{e}} d_{p}^{e} f_{p} \leq u_{e}+u_{e}^{\prime} w_{e} \quad \forall e \in E_{>0} \\
& x_{i} \leq \sum_{p \in \hat{P}_{i}} f_{p} \leq 1 \quad \forall i \in \hat{V}_{D} \\
& \sum_{t \in T_{i}} y_{t}=x_{i} \quad \forall i \in V_{D} \\
& \sum_{v \in V_{B} \cap V_{k}} n_{k, v} q_{v} \geq\left\lceil\chi_{k} n_{k}\right\rceil-n_{k}^{A} \quad \forall k \in C \\
& \sum_{i \in V_{D}} x_{i} \leq m \\
& \sum_{p \in P_{i}} d_{p}^{f} f_{p} \leq \sum_{t \in T_{i}} u_{t}^{f} y_{t} \quad \forall i \in V_{D} \\
& \sum_{p \in P_{i}} d_{p}^{c} f_{p} \leq \sum_{t \in T_{i}} u_{t}^{c} y_{t} \quad \forall i \in V_{D} \\
& \sum_{p \in P_{i}} d_{p}^{r} f_{p} \leq \sum_{t \in T_{i}} u_{t}^{r} y_{t} \quad \forall i \in V_{D} \\
& \sum_{p \in P_{e}} d_{p}^{f} f_{p} \leq \sum_{l \in L_{e}} u_{l}^{f} z_{l} \quad \forall e \in E_{D} \\
& \sum_{l \in L_{i}} d_{l}^{c} z_{l} \leq \sum_{t \in T_{i}} u_{t}^{c} y_{t} \quad \forall i \in V_{D} \\
& \sum_{l \in L_{i}} d_{l}^{r} z_{l} \leq \sum_{t \in T_{i}} u_{t}^{r} y_{t} \quad \forall i \in V_{D} \\
& \sum_{p \in P_{i}} d_{p}^{s} f_{p} \leq \sum_{t \in T_{i}} u_{t}^{s} y_{t} \quad \forall i \in V_{D} \\
& \sum_{p \in P_{i}} n_{p} f_{p} \leq n_{i} x_{i} \quad \forall i \in V_{D}
\end{aligned}
$$



CATEGORY
Served residents

| Served residents | BUSINESS |
| :--- | :--- |
| Setup cost | HOUSEHO |
|  | BTP |
|  | DP |
|  | CO |
|  | Total |

BTP terminations
BTP splitters BTP ONUs

## DP closures

DP splitters
CO ODF

CO switches
CO eards

CO ports
CO trans ceivers
Cables
COMPONENT

DP

Total

Total oost
SP-FOC-FIC-32
TYFIST-OR2

OUSEHOLDS
-

TRX-OPON-ClassC +-2488-1244 ONU ONU-ALU-OPON-ONT-1

MUFF ETY-OCOR-64 MUFFE TY-OCO2-9E

OLTSW-ALU-7342-ISAMFTTU-2500B
CARD-ALU-OLT-4
PORT-OPON-2488-1244

TRX OPON-ClassC +-2488-1244 OLT
CABLE-DRA-Nano-6×12. 12
CABLE-DRA-Nano-6×12-24
CABLE-DRA-Nano-6x12-4B

COUNT METER COST
PERCENT

| 60 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 803 |  |  |  |  |  |
|  |  |  |  |  | CABLE-DRA-Nano-6x12-60 |
| 225 |  | 0,00 | 0.00 |  | CABLE-DRA-URX-10 |
| $\theta$ |  | 10800,00 | 0,87 |  | CABLE-DRA-URX 12 |
| 1 |  | 470447,00 | 37,90 |  | CABLE-DRA-URX-2 |
| 1 |  | 481247,00 | 38,77 |  | CABLE-DRA-URX 4 |
|  |  |  |  |  | CABLE-DRA-URX 6 |
|  |  |  |  |  | CABLE-DRA-URX-8 |
| - |  |  |  |  | Total |
|  |  |  |  | Ducts |  |
|  |  |  | 3,61 |  | DUCT-110 |
| 280 |  | 44800 |  |  | DUCT-60 |
|  |  |  |  |  | DUCT-MF R-2 $\times 40-2 \times 30$ |
| 280 |  | 44800 | 3,81 |  | DUCT-SPQ-1×10×10 |
|  |  |  |  |  | DUCT-SP O-1×10 $\times 10-1 \times 6 \times 10$ |
| 7 |  | 13868 | 1,10 |  | DUCT-SPO- $1 \times 18 \times 10$ |
| 2 |  | 4036 | 0,33 |  | DUCT-SP Q $1 \times 18 \times 10-1 \times 10 \times 10$ |
| $\theta$ |  | 17701 | 1,43 |  | DUCT-SPQ-1×1× 10 |
|  |  |  |  |  | DUCT-SP0-1.24×10 |
| 10 |  | 17480 | 1,41 |  | DUCT-SPQ-1 $2 \times 10$ |
|  |  |  |  |  | DUCT-SPQ-1x6x 10 |
| 1 |  | 300 | 0,02 |  | DUCT-SPO- $2 \times 10 \times 10$ |
|  |  |  |  |  | DUCT-SP $0.2 \times 6 \times 10$ |
| 1 |  | 6025 | 0,40 | Fibers | Total |
|  |  |  |  |  |  |
| 3 |  | 7660 | 0,81 | Trails | FIBER-COR-LWP |
|  |  |  |  |  |  |
| 11 |  | 0 | 0.00 |  | Digging |
|  |  |  |  |  | Digging Drop-Area |
| 11 |  | 110 | 0.01 |  | Digging Non-Drop-Area |
|  |  |  |  |  | Digging DP.Area ex clusively |
| 3 | 360 | 334 | 0.03 |  | Digging CO-Area exolusively |
| 1 | 675 | 738 | 0.06 |  | Digging DP. and CO-Area inters eoting |
| 1 | 1082 | 1473 | 0,12 |  | Existing ducts |
|  |  |  |  | Total |  |
|  |  |  |  | Cost per resident |  |
|  |  |  |  | Cost per building |  |


$\triangleright$ How much trenching cost is unavoidable?

- All (mandatory) customer locations have to be connected to a CO
$\Rightarrow$ More COs have to be opened if the capacities are exceeded
$\triangleright$ Steiner tree approach:
- Construct a directed graph $G$ with:
- all trail network locations, BTPs and COs, plus an artificial root node, as node set
- forward- and backward-arcs for each trail, plus capacitated artificial arcs connecting the root to each CO

$\Rightarrow$ Compute a Steiner tree in $G$ with:
- all BTPs, plus the artificial root node, as terminals
- capacity restrictions on the artificial arcs

$$
\begin{array}{rlrl}
\operatorname{minimize} & \sum_{e \in E} c_{e} w_{e}+\sum_{a \in A_{0}} c_{a} x_{a} & \\
\text { s.t. } \quad \sum_{a \in \delta^{-}(v)} f_{a}-\sum_{a \in \delta^{+}(v)} f_{a} & =\left\{\begin{array}{cc}
N_{v} & \text { if } v \in V_{B} \\
0 & \text { otherwise }
\end{array}\right. & & \forall v \in V \\
f_{a} & \leq\left|N_{B}\right| x_{a} & & \forall a \in A \\
x_{e^{+}}+x_{e^{-}} & =w_{e} & & \forall e \in E \\
\sum_{a \in \delta^{-}(v)} x_{a} & =1 & \forall v \in V_{B} \\
\sum_{a \in \delta^{-}(v)} x_{a} & \leq 1 & \forall v \in V \backslash V_{B} \\
\sum_{a \in \delta^{-}(v)} x_{a} & \leq \sum_{a \in \delta^{+}(v)} x_{a} & \forall v \in V \backslash V_{B} \\
\sum_{a \in \delta^{-}(v)} x_{a} & \geq x_{a^{\prime}} & \forall v \in V \backslash V_{B}, a^{\prime} \in \delta^{+}(v) \\
f_{a} & \leq k_{a} x_{a} & \forall a \in A_{0} \\
\sum_{a \in A_{0}} x_{a} & \leq N_{C} & & \\
f_{a} \geq 0, x_{a} & \in\{0,1\} & & \forall a \in A \\
w_{e} & \in\{0,1\} & \forall e \in E
\end{array}
$$

- Instances:
- a*: artificially generated, based on GIS information from www.openstreetmap.org
- $\mathrm{c} *$ : real-world studies, based on information from industry partners

| Instance: | a1 | a2 | a3 | c1 | c2 | c3 | c4 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| \# nodes | 637 | 1229 | 4110 | 1051 | 1151 | 2264 | 6532 |
| \# edges | 826 | 1356 | 4350 | 1079 | 1199 | 2380 | 7350 |
| \# BTPs | 39 | 238 | 1670 | 345 | 315 | 475 | 1947 |
| \# potential COs | 4 | 5 | 6 | 4 | 5 | 1 | 1 |
| network trenching cost | 235640 | 598750 | 2114690 | 322252 | 1073784 | 2788439 | 4408460 |
| lower bound | 224750 | 575110 | 2066190 | 312399 | 1063896 | 2743952 | 4323196 |
| relative gap | $4.8 \%$ | $4.1 \%$ | $2.3 \%$ | $3.2 \%$ | $0.9 \%$ | $1.6 \%$ | $2.0 \%$ |

$\Rightarrow$ Trenching costs in the computed FTTx networks are quite close to the lower bound
$\triangleright$ BMBF funded project 2009-2011

- Partners:

Fraunhofer
Heinrich-Hertz-Institut

Partners:

- Industry Partners: R•KOM: Draka

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- Given
- network topology
- a fiber demand at every connected BTP
- restrictions on cable and duct installations:

Example: Micro-ducts
Every customer gets their own cable(s), each in a separate micro-duct within a micro-duct bundle

$\triangleright$ Task: compute cost-optimal cable and duct installations that meet the restrictions such that all fiber demands at customer locations are met

$\Rightarrow$ DPs and COs are roots of undirected trees

- Given
- an undirected rooted tree with
- one concentrator (root)
- client locations and
- other locations
- set $C$ of cable installations to embed with
- path in the tree
- number of cables

- Task: compute cost-optimal duct installations, such that every cable is embedded in a micro-duct on every edge of its path

s.t.
\# pipes of type $p$ provided
by duct installation $d$
\# cables in installation $c \quad k_{c}=\sum_{p \in P_{c}^{O}} \sum_{\substack{d \in D_{p}: \\ e \in q_{d}}} x_{c, d}^{p} \quad \forall c \in C, e \in q_{c}$

$$
\begin{array}{cc}
x_{d} \in \mathbb{Z}_{\geq 0} & \longleftarrow \quad \# \text { ducts of duct installation } d \text { used } \\
x_{c, d}^{p} \in \mathbb{Z}_{\geq 0} & \longleftarrow \quad \# \text { cables for } c \text { embedded in pipes of type } p \\
\text { provided by duct installation } d
\end{array}
$$


(a)

(b)

(c)

Possible duct sizes 6, 12 and 24

- Trail network
- Client
- Cable installation
- Duct installation

1 Number of cables/ducts used in installation
maximal direction:
downward direction at an intersection with maximal number of cables on it
(a) Given cable installations
(b) Cost optimal installations with downgrading at intersections
(c) Installations used in practice (downgrading in maximal direction not allowed)
minimize $\sum_{d \in D} c_{d} x_{d}+\sum_{e \in E} c_{e} z_{e}$
s.t. $\quad k_{p}^{d} x_{d} \geq \sum_{\tilde{d} \in D} x_{\tilde{d}, d}^{p}+\sum_{c \in C} x_{c, d}^{p} \quad \forall d \in D, p \in P^{d}$

$$
\begin{array}{ll}
k_{c} \leq \sum_{p \in P_{c}^{O}} \sum_{\substack{d \in D_{p}: \\
e \in q_{d}}} x_{c, d}^{p}+z_{e} k_{c} & \forall c \in C_{G}, e \in q_{c} \bar{v} \\
k_{c}=\sum_{p \in P_{c}^{O}} \sum_{\substack{d \in D_{p}: \\
e \in q_{d}}} x_{c, d}^{p} & \forall c \in C \backslash C_{G}, e \in q_{c}
\end{array}
$$


$\forall \tilde{d} \in D_{G}, e \in q_{\tilde{d}}$ either embed

$$
x_{\tilde{d}} \leq \sum_{p \in P_{\tilde{d}}^{O}} \sum_{\substack{d \in D_{p}: \\ e \in q_{d}}} x_{\tilde{d}, d}^{p}+z_{e} M_{\tilde{d}} \quad \forall \tilde{d} \in D_{G}, e \in q_{\tilde{d}}
$$ or trench

$$
x_{\tilde{d}}=\sum_{p \in P_{\tilde{d}}^{O}} \sum_{\substack{d \in D_{p}: \\ e \in q_{d}}} x_{\tilde{d}, d}^{p}
$$

$$
\forall \tilde{d} \in D \backslash D_{G}, e \in q_{\tilde{d}}
$$

$$
k_{c} \geq \sum_{p \in P_{c}^{O}} \sum_{\substack{d \in D_{p}: \\ e \in q_{d}}} x_{c, d}^{p} \quad \forall c \in C_{G}, e \in q_{c}
$$

$$
x_{\tilde{d}} \geq \sum_{p \in P_{\tilde{d}}^{O}} \sum_{\substack{d \in D_{p}: \\ e \in q_{d}}} x_{\tilde{d}, d}^{p}
$$

one cable/duct embedded in at most one duct
$z_{e} \in\{0,1\} \quad \longleftarrow$ trenching trail $e$ (or not)

