Technische Universität Berlin SS 1994
Fachbereich 3
Prof. M. Grötschel/ Kamin

Linear Programming
4. Set of Exercises

13. Exercise **(3 Points)**

Check whether the point $x^* = (0, \frac{4}{3}, \frac{2}{3}, \frac{5}{3}, 0)^T$ is optimal for the following LP:

$$\begin{array}{rrrrrrl}
\max & 7x_1 + 6x_2 + 5x_3 & - 2x_4 & + 3x_5 & \\
\text{s.t.} & x_1 + 3x_2 + 5x_3 & - 2x_4 & + 2x_5 & \leq 4 \\
& 4x_1 + 2x_2 - 2x_3 & + x_4 & + x_5 & \leq 3 \\
& 2x_1 + 4x_2 + 4x_3 & - 2x_4 & + 5x_5 & \leq 5 \\
& 3x_1 + x_2 + 2x_3 & - x_4 & - 2x_5 & \leq 1 \\
& x_1, x_2, x_3, & x_4, & x_5 & \geq 0
\end{array}$$

14. Exercise **(4 Points)**

Solve the following LP graphically:

$$\begin{array}{rl}
\min & -3y_1 + 5y_2 + 40y_3 + 6y_4 + 4y_5 - 2y_6 \\
\text{s.t.} & -3y_1 + y_2 + y_3 + y_4 + y_5 - y_6 \geq -1 \\
& -y_1 + y_2 + 5y_3 - y_5 - y_6 \geq -2 \\
& y_1, y_2, y_3, y_4, y_5, y_6 \geq 0
\end{array}$$

15. Exercise **(4 Points)**

State and prove the **Strong Complementary Slackness Theorem** for the following pair of dual linear programs:

$$\begin{array}{rl}
\max & c^T x + d^T y \\
& Ax + By \leq a \\
& Cx + Dy = b \\
& x \geq 0
\end{array}
\quad \text{and} \quad
\begin{array}{rl}
\min & u^T a + v^T b \\
& u^T A + v^T C \geq c^T \\
& u^T B + v^T D = d^T \\
& u \geq 0
\end{array}$$

16. Exercise **(5 Points)**

Prove that, for $S, S_i \subseteq \mathbb{K}^n$, $i = 1, \ldots, k$, the following hold:

(a) $S_i \subseteq S_j \Rightarrow S_j^o \subseteq S_i^o$

(b) $S \subseteq S^{oo}$

(c) $(\bigcup_{i=1}^{k} S_i)^o = \bigcap_{i=1}^{k} S_i^o$

(d) $S^o = \text{cone}(S^o) = (\text{cone}(S))^o$

(e) $S = \text{lin}(S) \Rightarrow S^o = S^{\perp}$ (Is the converse true?)

Due date: 13.5.1994 (13.00-14.00 , MA 623)

Solutions have to be delivered in English!!!