Dr. Niels Lindner
ZIB 3007 (Rundbau)
lindner@zib.de

Problem Set 4

due: May 22, 2018

Exercise 1 **4 points**

Let $G = (V, E)$ be a directed graph and let $c, \ell, u \in (\mathbb{R}_{\geq 0})^E$. Transform the minimum cost circulation problem

$$\begin{array}{lll} \text{Minimize} & \displaystyle\sum_{e \in E} c_e f_e & \\ \text{s.t.} & \displaystyle\sum_{e \in \delta^+(v)} f_e - \sum_{e \in \delta^-(v)} f_e = 0, & v \in V, \\ & \ell_e \leq f_e \leq u_e, & e \in E \end{array}$$

into an equivalent minimum cost flow problem of the form

$$\begin{array}{lll} \text{Minimize} & \displaystyle\sum_{e \in E} c'_e f'_e & \\ \text{s.t.} & \displaystyle\sum_{e \in \delta^+(v)} f'_e - \sum_{e \in \delta^-(v)} f'_e = b'_v, & v \in V, \\ & 0 \leq f'_e \leq u'_e, & e \in E \end{array}$$

for suitable $c', u' \in (\mathbb{R}_{\geq 0})^E$ and $b' \in \mathbb{R}^V$ with $\sum_{v \in V} b'_v = 0$.

Exercise 2 **8 points**

Consider the sorted list of elementary connections given in `connections.csv` (available on the course homepage). Apply the *Basic Connection Scan Algorithm* from Lecture 4 with a minimum transfer time of 2 minutes at every station to solve the following problems:

(a) Compute the optimal journey for the earliest arrival query *Hamburg @ 03:00* $\to$ *Amsterdam.*

(b) Find the earliest arrival times for all reachable stations when departing in *Berlin @ 04:30.*

If you solve this exercise by programming (not required), then submit your code.

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Exercise 3 **8 points**

Let $G = (V, E)$ be a directed graph with length functions $\ell_1, \dots, \ell_k : E \to \mathbb{R}_{\geq 0}$. For a vertex $s \in V$, consider the multi-criteria single-source shortest path problem

For each $t \in V$, find the set of Pareto-minimal s-t-paths w.r.t. $(\ell_1, \dots, \ell_k)$.

(a) Let $s \in V$. Consider the following algorithm (*Multi-criteria Dijkstra algorithm*):

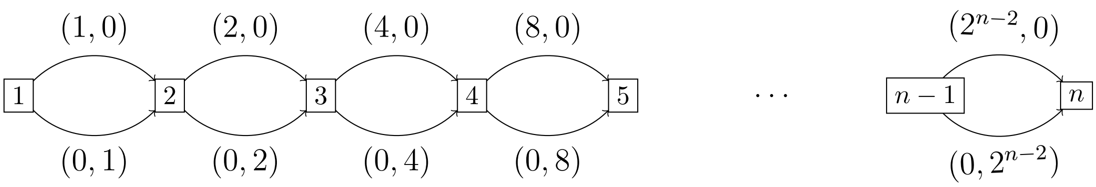
1. Set $D := \emptyset$ and $Q := \{(s, (0, \dots, 0))\}$.
2. While $Q \neq \emptyset$:
   - Pop a vector (v, a) from Q such that the sum of the entries of a is minimal.
   - Add (v, a) to D.
   - For all successors w of v:
     Add (w, b) to Q, where $b := a + (\ell_1(v, w), \dots, \ell_k(v, w))$.
     Remove all dominated elements of the form (w, c) from Q.
3. Return D.

Show that this algorithm solves the multi-criteria single-source shortest path problem:

- Prove that when (v, a) is added to D, then a is the length of a Pareto-minimal s-v-path.
- Prove that if p is a Pareto-optimal s-v-path, then $(v, (\ell_1(p), \dots, \ell_k(p))) \in D$.

Hint: Prove that if $(s, v_1, \dots, v_k)$ is a Pareto-optimal path, then so is $(s, v_1, \dots, v_{k-1})$, and use induction.

(b) For $n \geq 2$, consider the following graph:



Compute all Pareto-minimal paths from 1 to n and count them.