Traffic Optimization: Optimal Tours in Graphs

Niels Lindner



Lecture 1 October 14, 2019

# Organization



### Lectures:

Niels Lindner (lindner@zib.de) Monday, 10-12, A6/SR 009

### Tutorials:

Pedro Maristany de las Casas (maristany@zib.de) Monday, 12-14, A6/SR 025/026 (starting from October 21)

### Homework:

50% for exam admission from Monday to Monday submission via e-mail or on paper groups of max. two students

### Website:

https://kvv.imp.fu-berlin.de/x/92kN1e (Whiteboard)

# Zuse Institute Berlin



Flight trajectory optimization





ICE vehicle rotation Integrated timetabling and routing

Moreover: Supercomputing, Energy networks, Nanooptics, Bioinformatics, ...



### §4 Integer Programming

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Linear programming, polytopes, cutting planes, IP formulations of TSP

§5 Generalized Routing Problems Directed Postman, rural Postman, generalized Postman, asymmetric TSP, clustered TSP, generalized TSP

# Course Outline

**§1** Introduction

S-Bahn challenge, mathematical optimization in public transport

- §2 Chinese Postman Problem Euler tours, shortest paths, perfect matchings, T-joins
- §3 Traveling Salesman Problem Hamiltonian paths, NP-hard optimization problems, approximation algorithms, metric TSP, spanning trees, k-opt moves, asymmetric TSP





# Chapter 1 Introduction

# $\S1.1$ S-Bahn Challenge

# Chapter 1 Introduction

# §1.2 Mathematical Optimization in Public Transport

§1.2 Mathematical Optimization in Public Transport

# **Optimal Public Transport**

Public transport should be ...





§1.2 Mathematical Optimization in Public Transport Optimal Public Transport

These goals are conflicting!

Basic Conflict: Passengers vs. Operators

### Examples

low fares	$\longleftrightarrow$	high revenues
short intervals	$\longleftrightarrow$	few vehicles
high reliability	$\longleftrightarrow$	low maintenance cost
punctuality	$\leftrightarrow \rightarrow$	small time supplements
few transfers	$\longleftrightarrow$	short lines
flexible disruption handling	$\longleftrightarrow$	simple disposition



# **Optimal Passenger Journeys**

### Which connection is the best one?

Bahnhof/Haltestelle	Zeit	~	Dauer	$^{\vee}$	Umst.	$^{\vee}$	Produk	te	Sparangebote	~	Flexpreis V	
	^	Früher							Preis für alle Reisen	den	inkl. Ermäßigungskarten*	
Berlin Hbf (tief) Frankfurt(Main)Hbf	07:04 10:56		3:52		0		ICE	ŝ	79,90 EUR		129,00 EUR	
✓ Details einblenden			<ul> <li>Rückfahrt hinzufügen</li> </ul>						Zur Ar	Zur Angebotsauswahl		
Berlin Hbf (tief) Frankfurt(Main)Hbf	07:27 11:44		4:17	-1.6	0		ICE		67,90 EUR		129,00 EUR	
Details einblenden			Kuckfahrt hinzufugen     Zur Angebotsauswahl									
Berlin Hbf (tief) Frankfurt(Main)Hbf	07:38 11:44		4:06		1		ICE		51,90 EUR		129,00 EUR	
✓ Details einblenden			<ul> <li>Rückfahrt hinzufügen</li> </ul>					Zur Angebotsauswahl				
> Details für alle	~	Später										

bahn.de

No connection is *the* best: W.r.t. the criteria *shortest travel time*, *minimum number of transfers* and *lowest price*, all connections are *Pareto optimal*.



Imagine a city without any public transportation network...





Network planning: Locate stops...











### Line planning: Lines and their frequencies





### Timetabling: Departure and arrival times







### periodic timetable with period time 10







# General Tasks

- Duty scheduling: group trips to duties
- Crew scheduling: assign workers to duties
- Re-scheduling: e.g., in case of disruptions
- Robust scheduling: for timetables, vehicles and duties
- Construction site planning
- Fare planning
- Routing: passengers, vehicles, inspectors

### Specific Tasks

. . .

Special needs of railway systems, air networks, electric vehicles, ...





Bussieck et al.: Discrete optimization in public rail transport, 1997 Liebchen: Periodic timetable optimization in public transport, 2006

# Mathematical Background

Many problems can be formulated as discrete optimization problems: Z

Public Transport	Discrete Optimization
network planning	Steiner tree, survivable network design
line planning	multi-commodity flow, Knapsack
timetabling	multi-commodity flow, periodic event scheduling
vehicle scheduling	multi-commodity flow, matching
optimal journeys	time-dependent/constrained shortest path

### Problem 1: Hardness

Most of these problems are NP-hard. Provided that  $P \neq NP$  holds, there are no polynomial-time algorithms providing exact solutions. However, this does not mean that real-world instances cannot be optimized in practice.

### Problem 2: Size

Realistic instances are large. Berlin bus scheduling, 1997: 44 bus types, 24'906 trips, 69'700'000 turnarounds. Optimization saved 20% buses.

# Success Stories



# Mathematical Optimization in Practice

- Line planning: Potsdam (2010), Istanbul (2016), Karlsruhe (2018)
- Timetabling: Germany (DB Fernverkehr, BVG U-Bahn), Netherlands (NS), Switzerland (SBB), Denmark (S-tog Copenhagen)
- Vehicle scheduling: Germany (DB Fernverkehr), France (SNCF)
- Integrated vehicle and duty scheduling: part of IVU.suite
- Disruption management: Netherlands (NS), Norway (Jernbaneverket)
- Optimal passenger journeys: standard

# Savings

- Line planning, Karlsruhe: 16% operational costs, 4% transfers
- Timetabling, Berlin U-Bahn: one vehicle
- Vehicle scheduling, Berlin-Spandau: 20% of fleet



### **Next Lecture on Optimal Tours in Graphs** No big picture, just blackboard mathematics

Next monday (no tutorial today!) Room: A6/SR 009

## Block Seminar on Graph Decompositions

Contents: treewidth, branchwidth, graph partitioning and optimization applications

First meeting: Wednesday, 10 am, ZIB, seminar room 2006

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