## Problem Set 11

due: January 13, 2020

## Exercise 1

10 points
Consider a digraph $G$, a cost function $c: E(G) \rightarrow \mathbb{R}_{\geq 0}$ and pairwise disjoint vertex clusters $V_{1}, \ldots, V_{k} \subseteq V(G)$ such that $V_{1} \cup \cdots \cup V_{k}=V(G)$. The Equality Generalized Asymmetric Traveling Salesman Problem (E-GATSP) is to find a directed circuit $C$ of length $k$ in $G$ such that $C$ visits exactly one vertex of each cluster and $C$ is of minimum cost w.r.t. $c$.

Construct explicitly the following polynomial-time reductions and prove their correctness:
(a) GATSP $\leq$ E-GATSP,
(b) E-GATSP $\leq$ ATSP.

## Exercise 2

A postal company wants to install 3 mailboxes in Dahlem. For each of the new mailboxes, 3 locations have been pre-selected:

| Mailbox 1 | Mailbox 2 | Mailbox 3 |
| :--- | :--- | :--- |
| Königin-Luise-Str. 48 | Arnimallee 3 | Ihnestr. 16 |
| Fabeckstr. 5 | Takustr. 7 | Garystr. 21 |
| Iltisstr. 4 | Königin-Luise-Str. 24 | Faradayweg 4 |

The post office is located at Habelschwerdter Allee 45. The postal company now wants to determine exactly one of the above locations for each mailbox such that the total distance the post office van needs to travel on its tour from the post office to all 3 mailboxes and back to the post office is minimized.

Solve this problem to optimality and describe your solution strategy. Use, e.g., the Google Maps route planner for cars to compute appropriate distances.

