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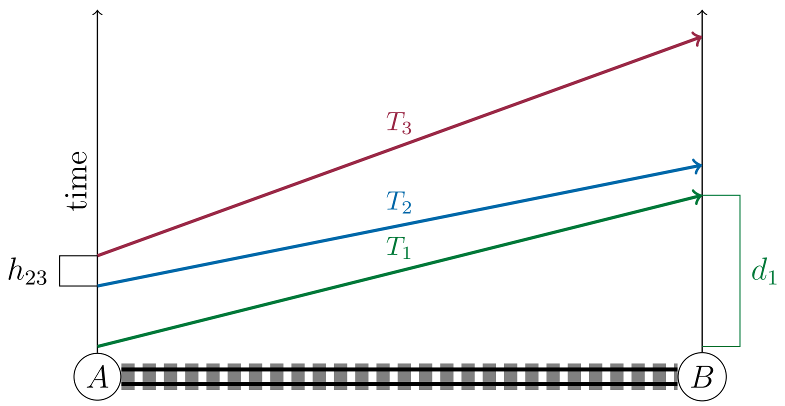
Problem Set 5

due: November 18, 2019

Exercise 1 **20 points**

On a train line, n trains $T_1, \dots, T_n$ are supposed to run from A to B. Each train T_i has a travel time $d_i \geq 0$ and moves with constant speed. However, the train line has only a single track, so that trains cannot overtake each other between A and B. For safety reasons, if train T_i runs earlier than train T_j, there should always be a minimum headway time $h_{ij} \geq 0$ between the trains. Note that h_{ij} and h_{ji} might be different.



T_i	T_1	T_2	T_3	T_4
d_i	17	38	30	22

	h_{ij} \ $j \rightarrow$	T_1	T_2	T_3	T_4
i	T_1		5	2	3
$\downarrow$	T_2	4		4	4
	T_3	3	5		3
	T_4	3	5	2	

Table 1: Travel times d_i and minimum headway times h_{ij} – only for (e) –

The infrastructure manager of the train line wants to use the maximum capacity of the track and comes up with the following optimization problem:

Train Timetabling Problem: Subject to the above conditions, find an ordering of all n trains together with departure times at A and arrival times at B such that the difference between the arrival of the last train at B and the departure of the first train at A is as minimum.

(a) Formulate the Train Timetabling Problem as an optimization problem in a mathematically precise way. Specify the input, output and the objective function.

(b) Show that the Hamiltonian Path Problem can be reduced in polynomial time to the decision version of the Train Timetabling Problem, and conclude that the latter is $\mathcal{NP}$-hard.

(c) Construct a polynomial-time reduction transforming an arbitrary instance of the Train Timetabling Problem to an instance of the Asymmetric Traveling Salesman Problem, and prove the correctness.

(d) Describe a polynomial-time algorithm that computes an optimal solution to the Train Timetabling Problem in the case that all headway times are equal.

(e) Find an optimal solution to the Train Timetabling Problem for the travel times and minimum headway times specified in Table 1.