Problem Set 6

due: November 25, 2019

Exercise 1

5 points

Consider the instance of the symmetric TSP instance shown in Figure 1.

- (a) Starting at node A, use the Nearest Neighbor Heuristic to find a feasible tour. Specify the tour and its total cost.
- (b) Improve the tour from (a) using the *k*-opt algorithm presented in the lecture with k = 2, until no more improving 2-opt move can be found.



Figure 1: TSP instance for Exercise 1

Exercise 2

Given an instance of the Traveling Salesman Problem (TSP), the *Nearest Neighbor Heuristic* (NN) presented in the lecture may produce solutions that are arbitrarily bad:

Show that for every $k \ge 1$, there exists a symmetric TSP instance $\mathcal{I}_k = (K_4, c_k)$ with cost function $c_k : E(K_4) \to \mathbb{R}_{\ge 0}$ s.t. independent from the starting vertex for NN,

$$\operatorname{NN}(\mathcal{I}_k) \ge k \cdot \operatorname{OPT}(\mathcal{I}_k).$$

Exercise 3

Prove or disprove: Any optimal TSP tour on an undirected complete graph K_n with edge costs $c: E(K_n) \to \mathbb{R}_{\geq 0}$ contains a Hamiltonian path of minimum cost w.r.t. c.

Exercise 4

Do you remember Odysseus? Compute a Hamiltonian path from Troy to Ithaca of minimum length w.r.t. the distance table given in Problem Set 1. The path is supposed to visit all 16 places exactly once. Proceed as follows:

5 points

5 points

5 points

- (a) Transform this problem into a standard symmetric TSP instance.
- (b) Encode the TSP instance in a TSPLIB format file: http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/tsp95.pdf
- (c) Solve the TSP instance to optimality using concorde on the NEOS server: https://neos-server.org/neos/solvers/co:concorde/TSP.html Alternatively, you may also download and install concorde: http://www.math.uwaterloo.ca/tsp/concorde/index.html
- (d) Write down an optimal Hamiltonian path and its total length.

Please submit your TSPLIB file as well.