LP Solution Polishing to improve MIP Performance

Matthias Miltenberger

Zuse Institute Berlin

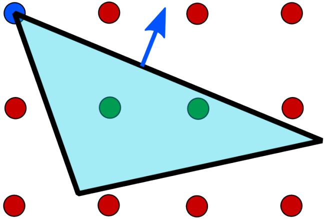
ICCOPT 2016 - Tokyo, August 9th 2016

ZIB

MODAL
Mathematical Optimization and Data Analysis Laboratories

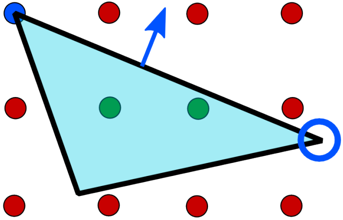
Motivation

- Solving a MIP $\min\{c^T x \mid Ax = b,\ x_i \in \mathbb{Z} \text{ for } i \in I\}$ involves solving many LPs as linear relaxations
- LP solutions are rarely unique



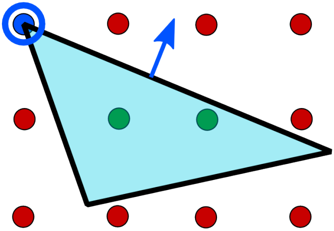
Motivation

- Solving a MIP $\min\{c^T x \mid Ax = b,\ x_i \in \mathbb{Z} \text{ for } i \in I\}$ involves solving many LPs as linear relaxations
- LP solutions are rarely unique



Motivation

- Solving a MIP $\min\{c^T x \mid Ax = b,\ x_i \in \mathbb{Z} \text{ for } i \in I\}$ involves solving many LPs as linear relaxations
- LP solutions are rarely unique



- **How to find the best one?**

Outline

1. Introduction
   - Dual Degeneracy
   - Performance Variability
2. Related Work
3. Solution Polishing
   - Integrality of Variables
4. Computational Results
   - SCIP Optimization Suite
5. Conclusion and Outlook

LP Solution Polishing to improve MIP Performance

1. Introduction
   - Dual Degeneracy
   - Performance Variability

2. Related Work

3. Solution Polishing
   - Integrality of Variables

4. Computational Results
   - SCIP Optimization Suite

5. Conclusion and Outlook

Dual Degeneracy

- Two types of degeneracy in LP:
  - primal: multiple bases defining one vertex of the polyhedron
  - dual: facet of the polyhedron parallel to the objective function
- Most (practical) problems are primal and dual degenerate
- Degeneracy is the most prominent cause of MIP performance variability

Performance Variability

- Performance of a MIP solver may vary drastically when the data changes
  - change row and column order
  - use a different random seed
  - implement a different tie breaker
  - ...
- Several causes for variability
  - different LP optima are probably most influencial
- Explained in
  - *Danna, E.*: **Performance variability in mixed integer programming** MIP Workshop (2008)
  - *Koch, T., et al.*: **MIPLIB 2010**, Math. Program. Comp. (2011)

1. Introduction
Dual Degeneracy
Performance Variability

2. Related Work

3. Solution Polishing
Integrality of Variables

4. Computational Results
SCIP Optimization Suite

5. Conclusion and Outlook

k-Sample

Improving branch-and-cut performance by random sampling
M. Fischetti, A. Lodi, M. Monaci, D. Salvagnin, A. Tramontani
Math. Program. Comp. (2016), Vol. 8

1. perform preprocessing on one core
2. solve root LP on $k-1$ cores with different random seeds
   - collect primal solutions and generated cuts
3. complete solving process on one core with yet another random seed

- previously collected information helps to improve the performance
- performance variability is reduced
- contained in the latest CPLEX release for $k = 3$

Pure Cutting Plane Algorithm

Lexicography and degeneracy: Can a pure cutting plane algorithm work?
A. Zanette, M. Fischetti, E. Balas
Math. Program. (2011) Vol. 130

- answer: Yes, it can!
- ...when choosing the correct LP basis
  - cutting plane method adds many cuts (almost) parallel to objective
  - use the lexicographic dual simplex to deal with high dual degeneracy
  - or modify the objective to mimic the lexicographic behavior
- standard cutting plane approach suffers from bad numerical stability

IBM CPLEX Patent

LP relaxation modification and cut selection in a MIP solver
T. Achterberg
US Patent (2011)

- similar to k-Sample, another optimal LP basis is constructed
  - fix some non-basic variables and modify the objective
  - use new basis to collect more information, e.g. for cuts
- implemented in CPLEX

LP Solution Polishing to improve MIP Performance

1. Introduction
   Dual Degeneracy
   Performance Variability

2. Related Work

3. Solution Polishing
   Integrality of Variables

4. Computational Results
   SCIP Optimization Suite

5. Conclusion and Outlook

General Idea of Solution Polishing

- Dual simplex algorithm terminates at first primal feasible, optimal basis
- Perform additional polishing steps altering this basis
- Reminder:
  - Basic indices: B, non-basic indices: N
  - Nonbasic variables are on their bound:
    $x_N = 0$ or $x_N = u$
  - Basic variables can be between bounds (depending on x_N)
    $x_B = A_B^{-1}(b - A_N x_N)$

- Polishing steps are primal iterations (to preserve feasibility):

1. find non-basic indices to enter the basis ($\hat{=}$ pricing step)
   - choose one with zero reduced costs to stay on optimal hyperplane
2. try pivoting and check whether leaving index is good ($\hat{=}$ ratio test)
3. repeat

Two Objectives

1. Decrease fractionality $\hat{=}$ push integer variables out of basis
   - less branching candidates
   - hopefully *closer* to an integer feasible solution

2. Increase fractionality $\hat{=}$ push integer variables into basis
   - may generate better cuts (basis matrix contains less slack)

Integrality of Variables

- Usually, LP solver has no knowledge of integrality
- Unlucky scenario:
  1. push continuous variable out of basis
  2. remaining basic integer variables are moved away from bounds
- Remedy:
  1. transfer information about integer variables to LP solver
  2. push only basic integer variables to their bounds

LP Solution Polishing to improve MIP Performance

1. Introduction
   Dual Degeneracy
   Performance Variability

2. Related Work

3. Solution Polishing
   Integrality of Variables

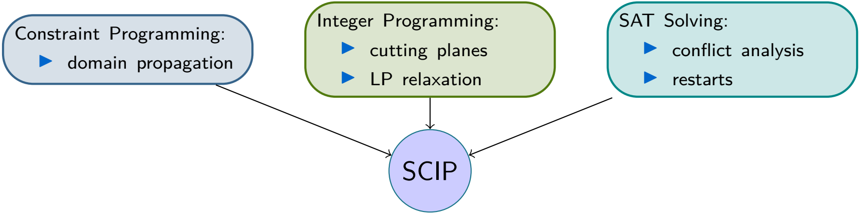
4. Computational Results
   SCIP Optimization Suite

5. Conclusion and Outlook

Setup

- Test set: MIPLIB 3 + MIPLIB 2003 + MIPLIB 2010, 168 instances
- All runs sequentially on one core
- SCIP Optimization Suite 3.2.1 with modifications

What is SCIP?

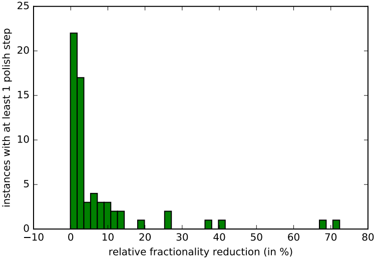


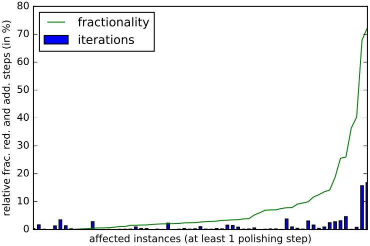
SCIP (Solving Constraint Integer Programs) ...

- has a modular structure via plugins,
- provides a full-scale global MINLP solver,
- part of the SCIP Optimization Suite (incl. SoPlex, ZIMPL, GCG, and UG),
- is free for academic purposes,
- and is available in source-code under `http://scip.zib.de`

Root Node Fractionality

- Compare fractionality before and after solution polishing
- Only root LP is solved
- With integrality information in SoPlex

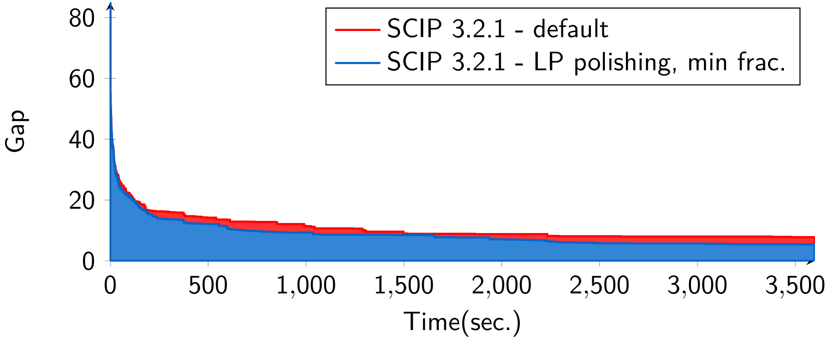




number of affected instances (of 168):	63
number of instances with a reduction of more than 5%:	22
mean percentage reduction of fractionality:	7.74
mean percentage of additional steps:	1.29

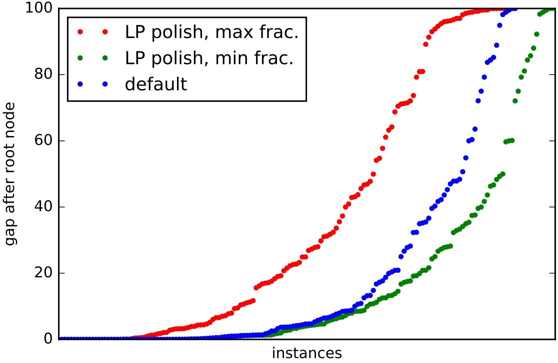
Impact on MIP Performance

- Polishing reduces number of nodes by 2-3 %
- Transferring integrality information is expensive
- Mean primal integral improvement: 38481.0 → 31316.1



Root Node Cut Performance

- Increasing fractionality leads to a high increase in nodes and deteriorates the root gap
- Reducing fractionality leads to a smaller root gap



LP Solution Polishing to improve MIP Performance

1. Introduction
   - Dual Degeneracy
   - Performance Variability

2. Related Work

3. Solution Polishing
   - Integrality of Variables

4. Computational Results
   - SCIP Optimization Suite

5. Conclusion and Outlook

Drawbacks of our Implementation

- Polished LP optimum is still not unique
  - maximum or minimum of fractionalities is not guaranteed

Possible improvements:

- Implement a (more expensive) technique to find the best basis
- Transfer of integrality information needs a more efficient implementation
  - use integrality information also in other parts of the LP solver
- Make use of several optimal bases

Conclusion and Outlook

- Solution polishing is cheap to apply
  - ...when used to reduce fractionality
  - ...when transfer of integrality information is improved
- Does not modify the LP problem data
- Already provides promising results concerning fractionality and gap reduction
- No effect on reducing performance variability observed yet

- More refinement and tuning possible
  - especially regarding fractionality increase
  - polishing could be applied more selectively
- Reduce performance variability by LP solution polishing

Conclusion and Outlook

- Solution polishing is cheap to apply
  - ...when used to reduce fractionality
  - ...when transfer of integrality information is improved
- Does not modify the LP problem data
- Already provides promising results concerning fractionality and gap reduction
- No effect on reducing performance variability observed yet

- More refinement and tuning possible
  - especially regarding fractionality increase
  - polishing could be applied more selectively
- Reduce performance variability by LP solution polishing

Thank you for your attention!
ご清聴ありがとうございました