Federal Ministry
of Economics
and Technology

An Auctioning Approach to Railway Track Allocation

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joint work with

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IAROR Summer Course on Railway Timetable Optimization 2008 Delft (Netherlands)

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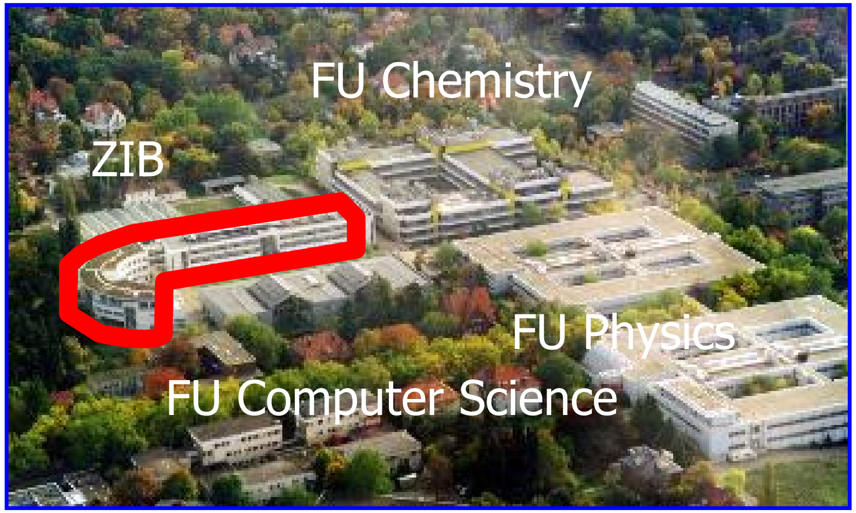

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Overview

1. Idea & Motivation
2. Train Timetabling Problem
3. Model and Algorithms
4. Computational Studies

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Overview

1. Idea & Motivation
    1. Auction Setting
    2. Planning Process
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Examples

- **In ancient times ...**
  - Auctions are known since 500 b.c.
  - March 28, 193 a.d.: The pretorians auction the Roman Emperor‘s throne to Marcus Didius Severus Iulianus, who ruled as Iulianus I. for 66 days

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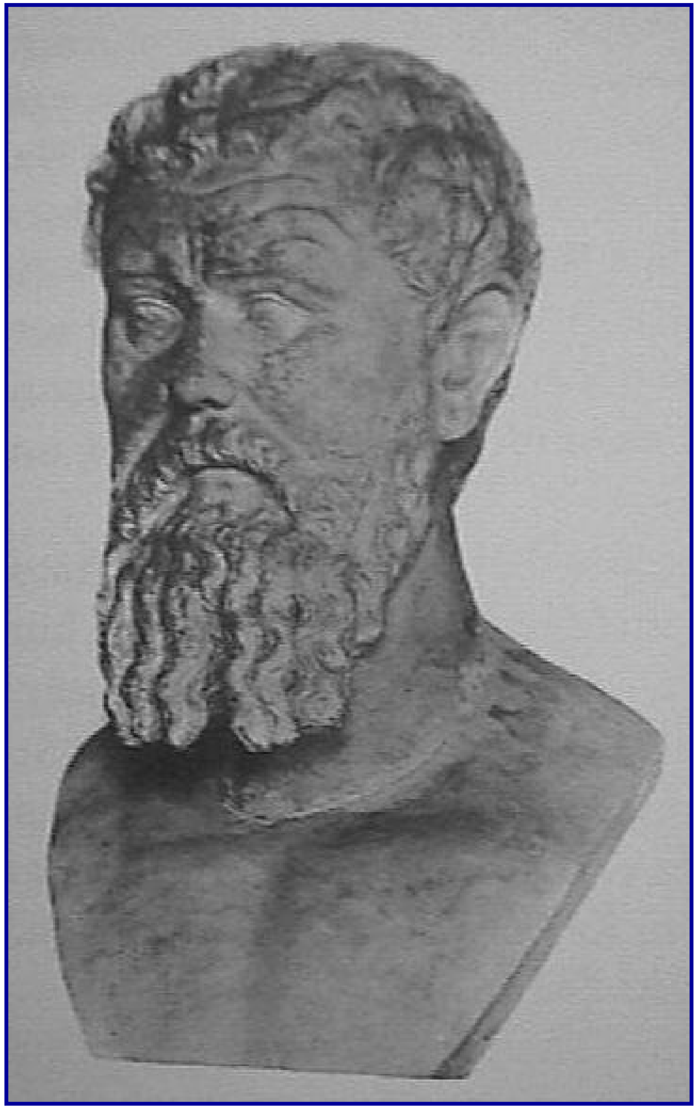
The Story of Didius Iulianus

(http://www.roman-emperors.org/didjul.htm)

[193 A.D., March 28] When the emperor **Pertinax** was killed trying to quell a mutiny, no accepted successor was at hand. **Pertinax's** father-in-law and urban prefect, Flavius Sulpicianus, entered the praetorian camp and tried to get the troops to proclaim him emperor, but he met with little enthusiasm. Other soldiers scoured the city seeking an alternative, but most senators shut themselves in their homes to wait out the crisis. **Didius Julianus**, however, allowed himself to be taken to the camp, where one of the most notorious events in Roman history was about to take place. **Didius Julianus** was prevented from entering the camp, but he began to make promises to the soldiers from outside the wall. Soon the scene became that of an auction, with Flavius Sulpicianus and **Didius Julianus** outbidding each other in the size of their donatives to the troops. The Roman empire was for sale to the highest bidder. When Flavius Sulpicianus reached the figure of 20,000 sesterces per soldier, **Didius Julianus** upped the bid by a whopping 5,000 sesterces, displaying his outstretched hand to indicate the amount. The empire was sold, **Didius Julianus** was allowed into the camp and proclaimed emperor.



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Arguments for Auctions

- Auctions can …
  - resolve user conflicts in such a way that the bidder with the highest willigness to pay receives the commodity (efficient allocation, wellfare maximization)
  - maximize the auctioneer’s earnings
  - reveal the bidders’ willigness to pay
  - reveal bottlenecks and the added value if they are removed
- Economists argue …
  - that a “working auctioning system” is usually superior to alternative methods such as bargaining, fixed prices, etc.

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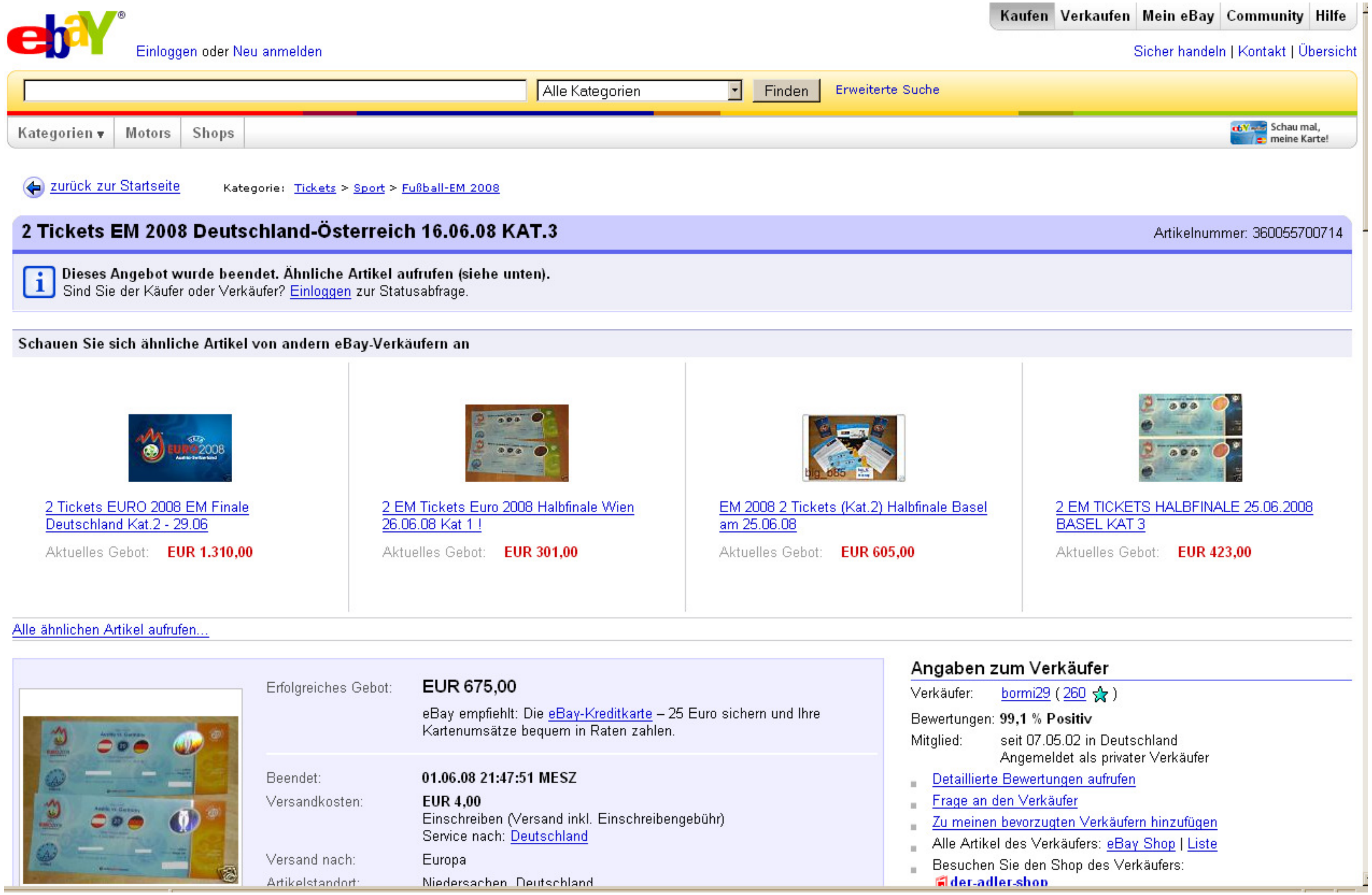
Ebay


Kaufen | Verkaufen | Mein eBay | Community | Hilfe
Einloggen oder Neu anmelden
Sicher handeln | Kontakt | Übersicht
Alle Kategorien
Finden
Erweiterte Suche
Kategorien
Motors
Shops
Schau mal, meine Karte!
zurück zur Startseite
Kategorie: Tickets > Sport > Fußball-EM 2008
2 Tickets EM 2008 Deutschland-Österreich 16.06.08 KAT.3
Artikelnummer: 360055700714
Dieses Angebot wurde beendet. Ähnliche Artikel aufrufen (siehe unten).
Sind Sie der Käufer oder Verkäufer? Einloggen zur Statusabfrage.
Schauen Sie sich ähnliche Artikel von andern eBay-Verkäufern an
2 Tickets EURO 2008 EM Finale Deutschland Kat.2 - 29.06
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EM 2008 2 Tickets (Kat.2) Halbfinale Basel am 25.06.08
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Erfolgreiches Gebot: EUR 675,00
eBay empfiehlt: Die eBay-Kreditkarte – 25 Euro sichern und Ihre Kartenumsätze bequem in Raten zahlen.
Beendet: 01.06.08 21:47:51 MESZ
Versandkosten: EUR 4,00
Einschreiben (Versand inkl. Einschreibengebühr)
Service nach: Deutschland
Versand nach: Europa
Angaben zum Verkäufer
Verkäufer: bormi29 (260)
Bewertungen: 99,1 % Positiv
Mitglied: seit 07.05.02 in Deutschland
Angemeldet als privater Verkäufer
Detaillierte Bewertungen aufrufen
Frage an den Verkäufer
Zu meinen bevorzugten Verkäufern hinzufügen
Alle Artikel des Verkäufers: eBay Shop | Liste
Besuchen Sie den Shop des Verkäufers:

William Vickrey (1914-96)

William Vickrey invented this auction desgin. As in any auction the bidder with the highest value gets the surchage, but he has only to pay the value of the second best (second-price-auction).

The advantage of this auction is that truthfull bidding is a dominant strategy for all bidders.

That means gambling makes no sense in contrast to a first price auction and therefore all bidders would prefer to bid equal to their willingness to pay.

The submission of the bids is often sealed and performed as a one shot auction.

Game Theory

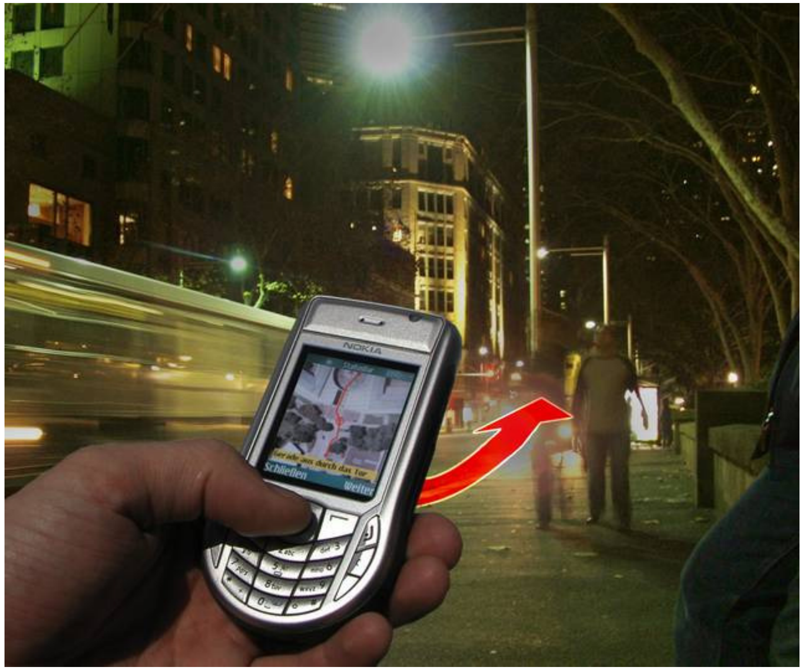
- Game (N,S,a)
  - $N=\{1,\ldots,n\}$ player
  - $S=\{(s_1,\ldots,s_n)\}$ strategies
  - $a:S\rightarrow R^n$ payoff
- Non-cooperative games
  - Dominance
  - (Nash-)Equilibrium $\hat{s}$

    $a_i(\hat{s}_1,..,s_i,..,\hat{s}_n)\leq a_i(\hat{s}_1,.., \hat{s}_n)\ \forall i$

    (i.g. no existence/uniqueness)
  - Matrix games: saddle point, minimax
- **Theorem (Nash):** Every finite non-cooperative n-person game has at least one equilibrium of mixed strategies.
- **Theorem (Nikaido, Isoda):** Generalization to auction frameworks.
- Cooperative games
  - Imputation (payoff to members of a coalition)
  - Concepts such as core, stable set, bargaining set, kernel, nucleolus, etc.

Combinatorial Auction - Easy Example

UMTS Frequencies ?



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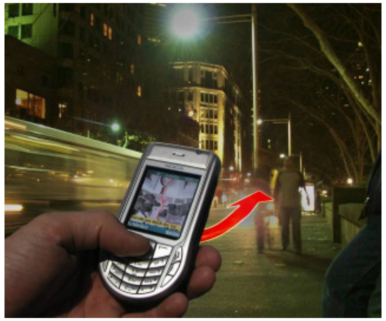
Frequency Auction



Frequency / Bidder	A	B	A and B
WodaPhon	10	10	10
Thelekom	20	20	20
F-Plus	0	0	24

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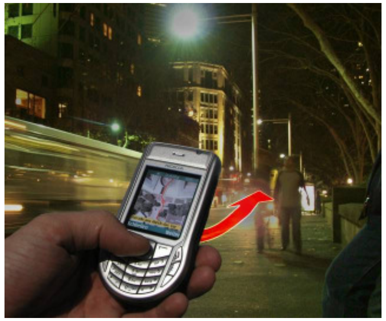
Frequency Auction

Winner



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Frequency Auction

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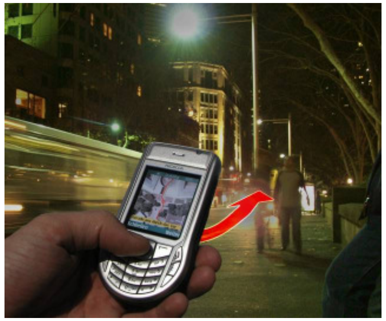
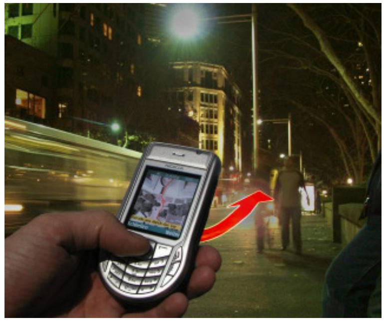


Frequency / Bidder	A	B	A and B
~~WodaPhon~~	~~10~~	~~10~~	~~10~~
Thelekom	20	20	20
F-Plus	0	0	24

Vickrey-price for WodaPhon is 24- (30 – 10) = 4

Frequency Auction

Winner



Frequency / Bidder	A	B	A and B
WodaPhon	10	10	10
~~Thelekom~~	~~20~~	~~20~~	~~20~~
F-Plus	0	0	24

Vickrey-price for Thelekom is 24 – (30 – 20) = 14

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„Collusion“

Frequency / Bidder	A	B	A and B
WodaPhon	10	10	10
Thelekom	20	20	20
F-Plus	0	0	24

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„Collusion“

Frequency / Bidder	A	B	A and B
WodaPhon	0	24	0
Thelekom	24	0	0
F-Plus	0	0	24

All vickrey-prices are 0

(= 24 – (48 -24)) !

„Shill-bidding“

Frequency / Bidder	A	B	A and B
WodaPhon	10	10	10
Thelekom	20	20	20
F-Plus	0	0	24

„Shill-bidding“

Frequency / Bidder	A	B	A and B
WodaPhon	10	10	10
Thelekom	20	20	20
F-Plus	0	0	30

Vickrey-prices are maximal

(WodaPhon 10 and Thelekom 20)

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Combinatorial Auction

- Combinatorial Auction Problem (CAP)
  - M objects, N bidders, $b^j(S)$ bid by j for $S \subseteq M$
  - y(S,j) 0/1-variable for giving S to j

$$
\begin{array}{llll}
\max & \sum_{S \subseteq M} \sum_{j \in N} b^j(S)\, y(S,j) & & \\
 & \sum_{S \ni i} \sum_{j \in N} y(S,j) & \leq 1 & \forall i \in M \\
 & y(S,j) & \in \{0,1\} & \forall S \subseteq M,\ j \in N
\end{array}
$$

  - Set Packing Problem
- Auction framework

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Vickrey Auction
(Nobel price in Economics 1996)

- Combinatorial auction

$$E(N,b) := \max \sum_{S \subseteq M} \sum_{j \in N} b^j(S)\, y(S,j)$$
$$\sum_{S \ni i} \sum_{j \in N} y(S,j) \leq 1 \quad \forall i \in M$$
$$y(S,j) \in \{0,1\} \quad \forall S \subseteq M, j \in N$$

- Private values v_j
- Mechanism
  - Bids $b_j = v_j$
  - Payments
    $z_j = E(N\backslash j, v) - E(N,v)|N\backslash j$

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Auctions

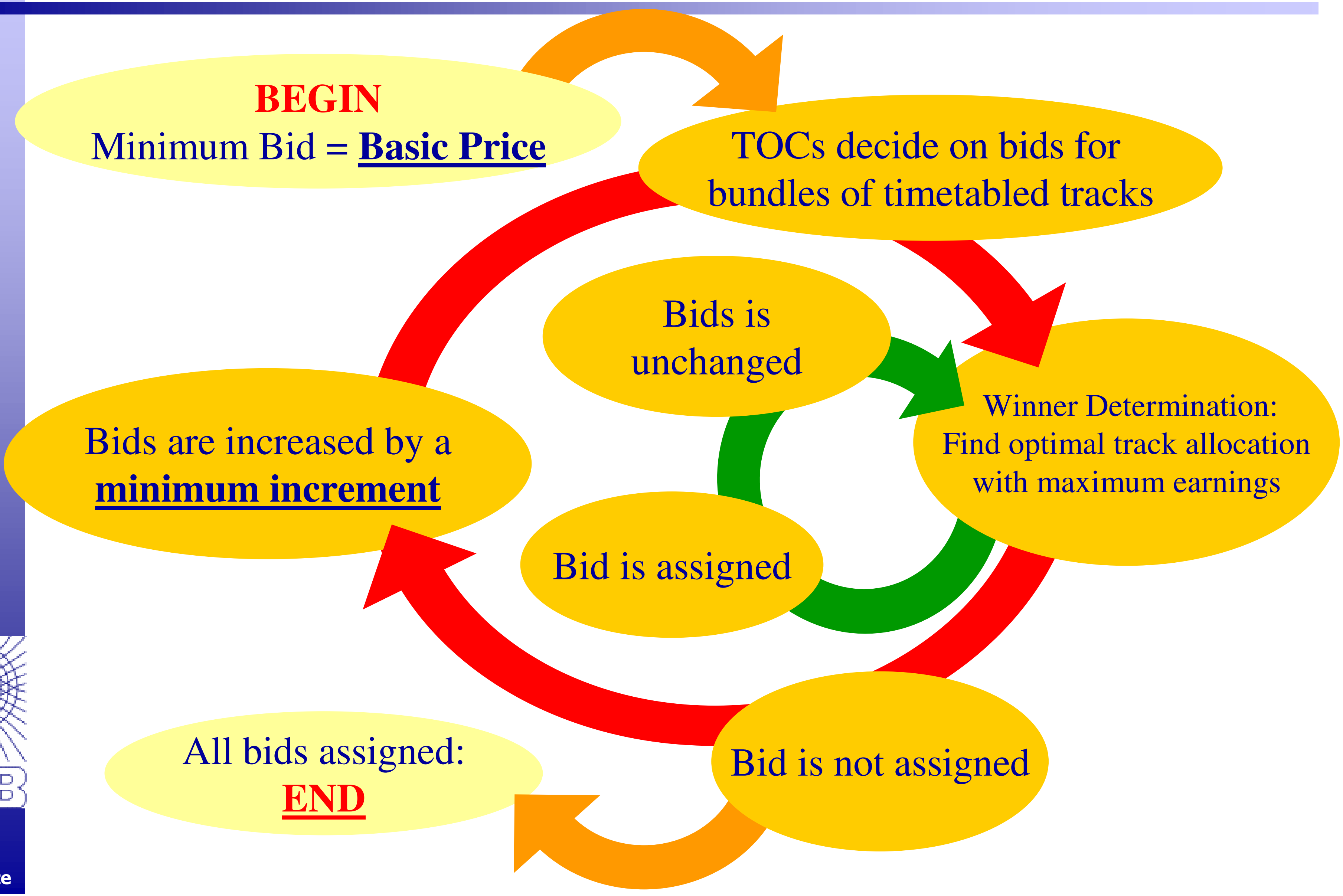
- Commodities/Bids
  - Independent commodities (classical autcion)/ commodity bundles (combinatorial auction)
  - Combinatorial bids (and/or/xor)
- Bidders
  - Cooperation forbidden/ cooperation allowed
- Payment
  - First price/second price (Vickrey) auction
- Information
  - Private Values/Common Values (winner's curse)
  - Sealed Bid/Open Bid
- Mechanism
  - English auction/dutch auction
  - Increment/number of rounds
  - Activity rules/taking bids back
  - Direct bidding/clock/proxy auction

Rail Track Auctioning

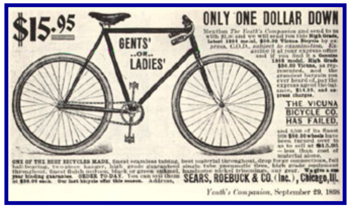
- Goals
  - More traffic at lower cost
  - Better service
- How do you measure?
  - Possible answer: in terms of willingness to pay
- What is the "commodity" of this market?
  - Possible answer: timetabled track

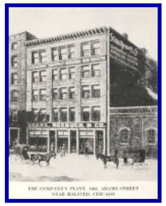
    = dedicated, timetabled track section
- How does the market work?
  - Possible answer: by auctioning timetabled tracks
- Auctions can be in-company auctions

Rail Track Auction



Sears, Roebuck & Co.





- 3-year contracts for transports on dedicated routes
- First auction in 1994 with 854 contracts
- Combinatorial auction
  - „And-" and „or-" bids allowed
  - 2^{854} ($\approx 10^{257}$) theoretically possible combinations
  - Sequential auction (5 rounds, 1 month between rounds)
- Results
  - 13% cost reduction
  - Extension to 1.400 contracts (14% cost reduction)

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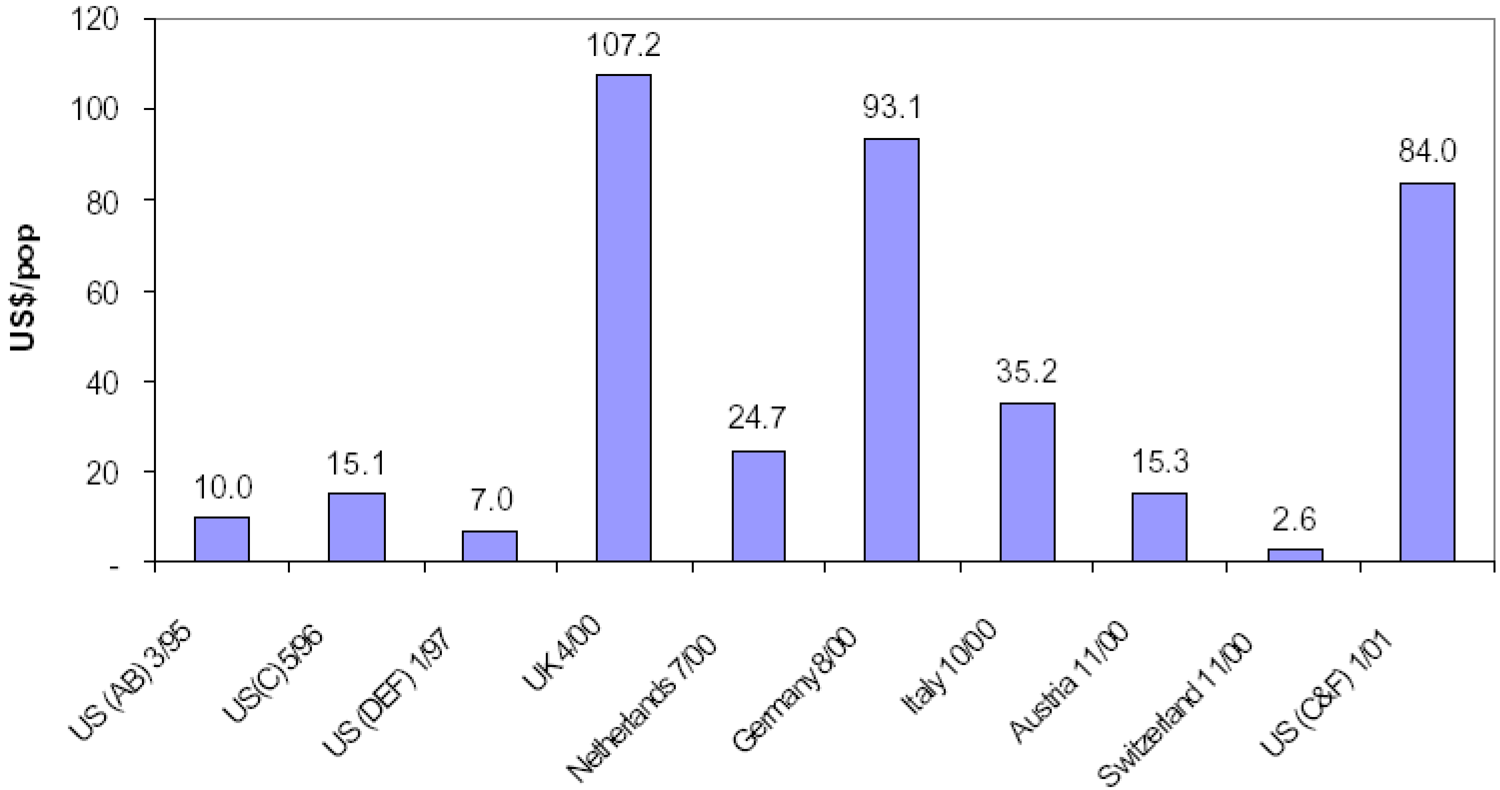
Frequency Auctions

(Cramton 2001, Spectrum Auctions, *Handbook of Telecommunications Economics*)



- Prices for mobile telecommunication frequencies (2x10 MHz+5MHz)
- Low earnings are not per se inefficient
- Only min. prices => insufficient cost recovery

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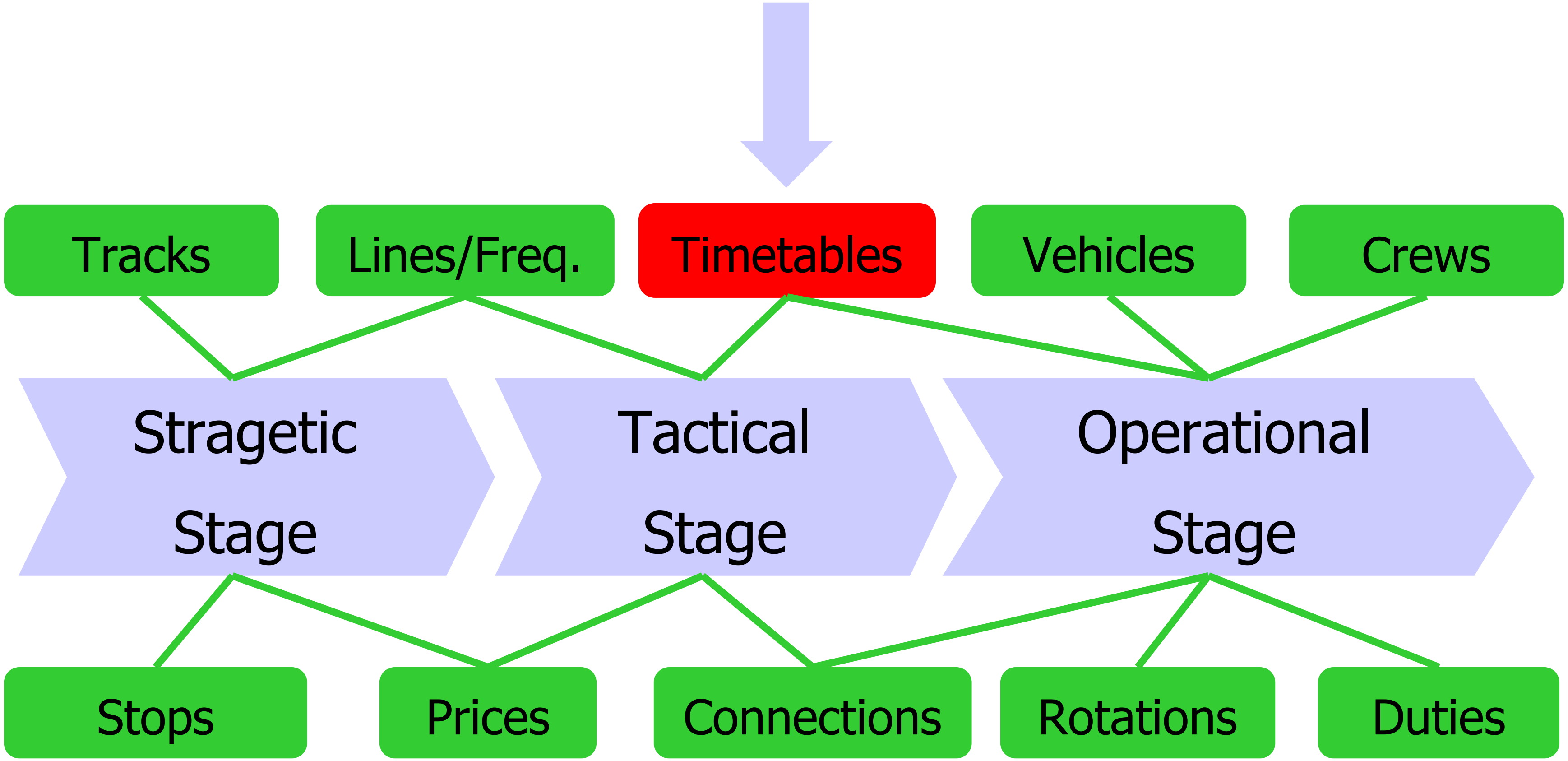
Planning in Public Transport



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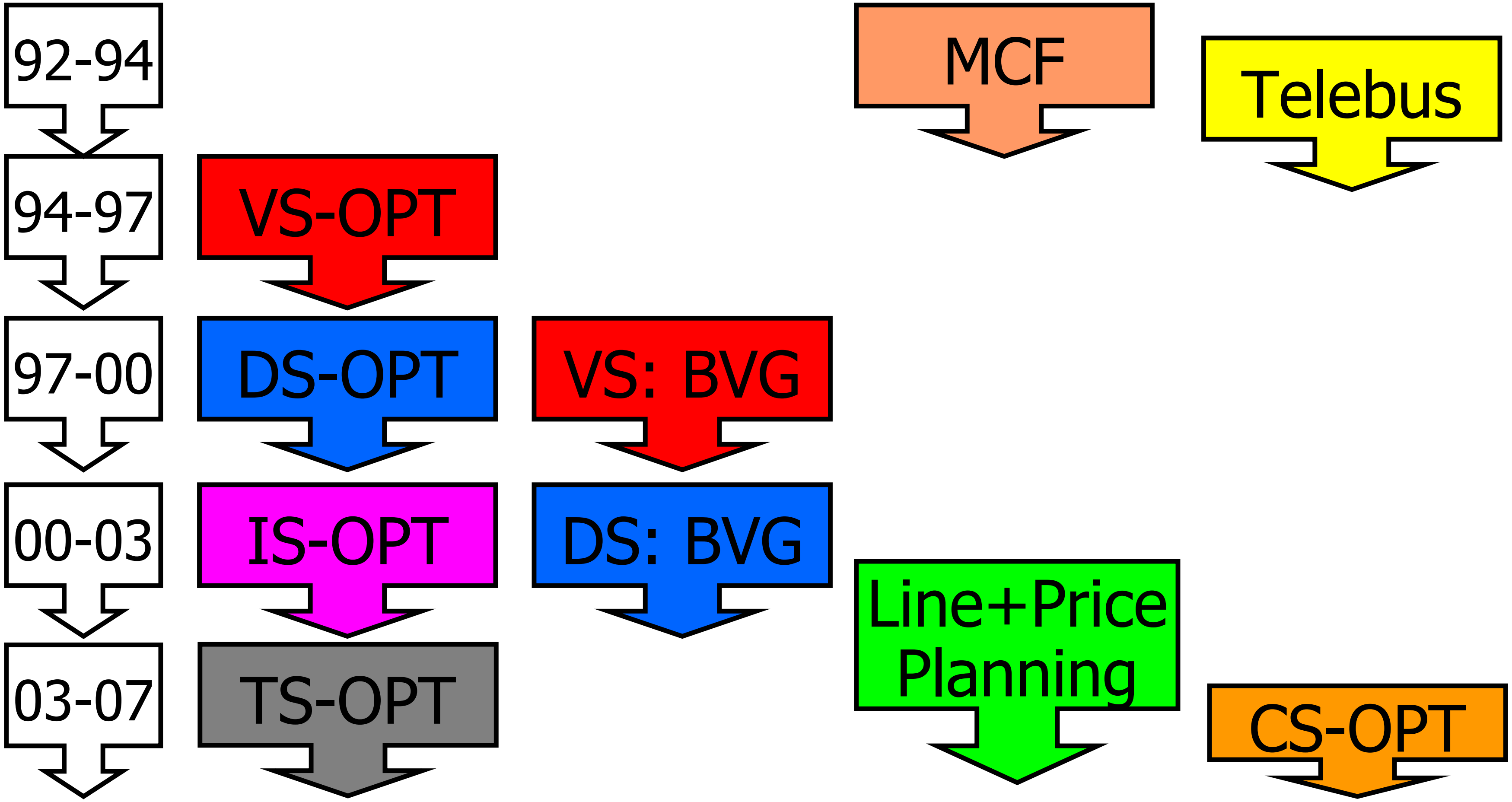
Traffic Projects @ ZIB



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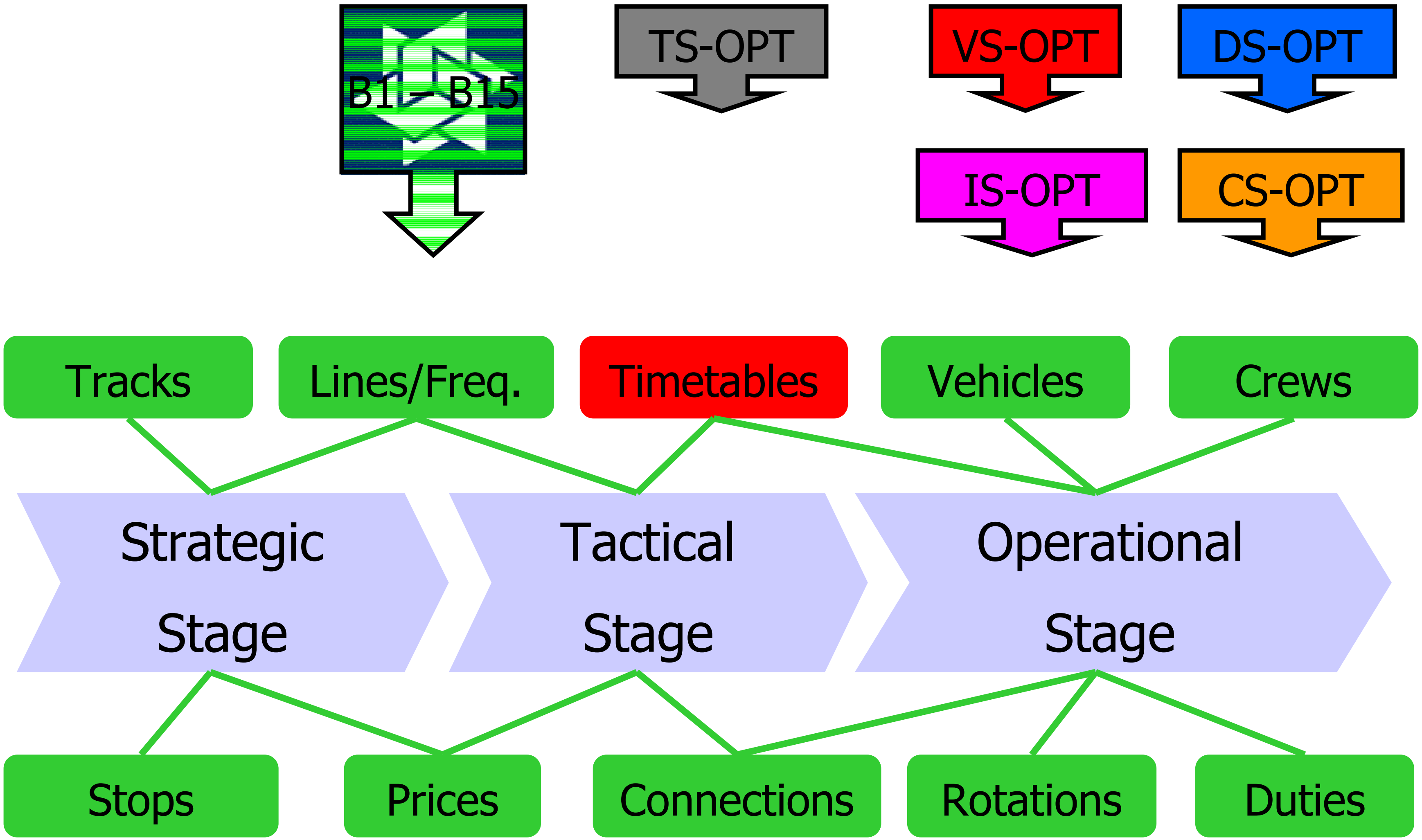
Planning in Public Transport



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1. Idea & Motivation
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Overview

1. Idea & Motivation
2. Train Timetabling Problem
   1. Problem Definition
   2. Complexity Issues
   3. Standardization
3. Models and Algorithms
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The Problem (TraVis by M.Kinder)

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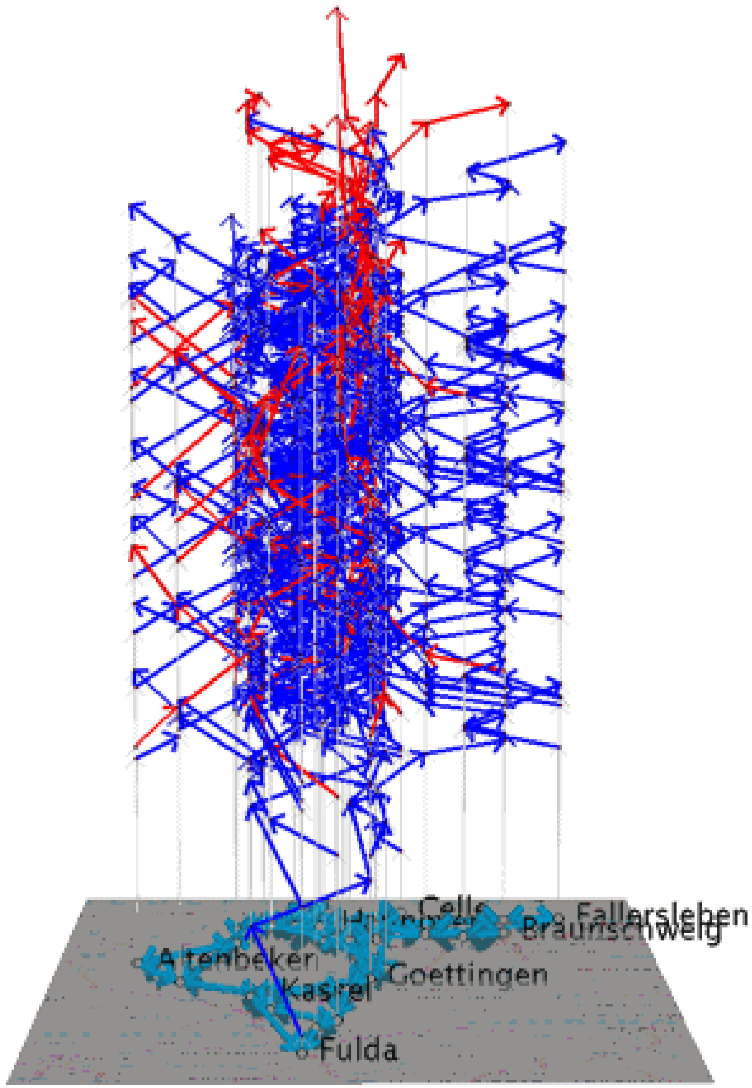
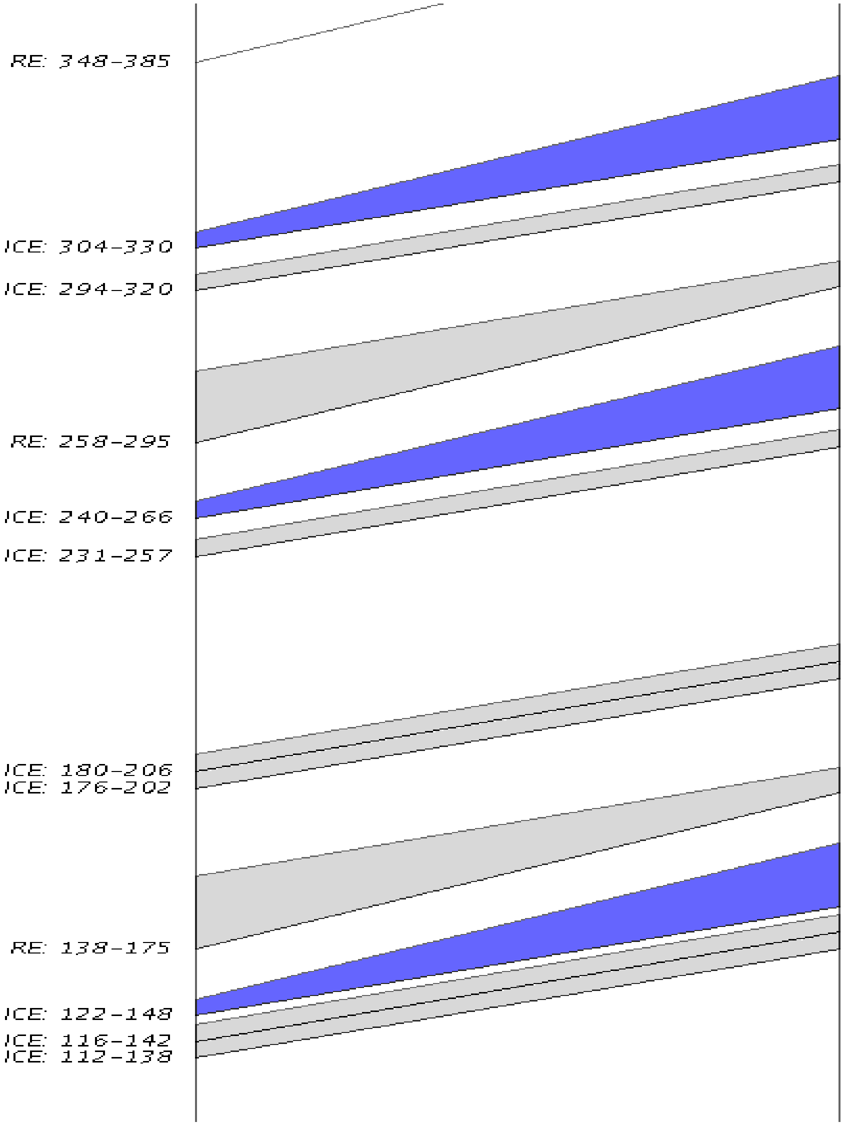
Schedule in 3d



Conflict-Free-Allocation



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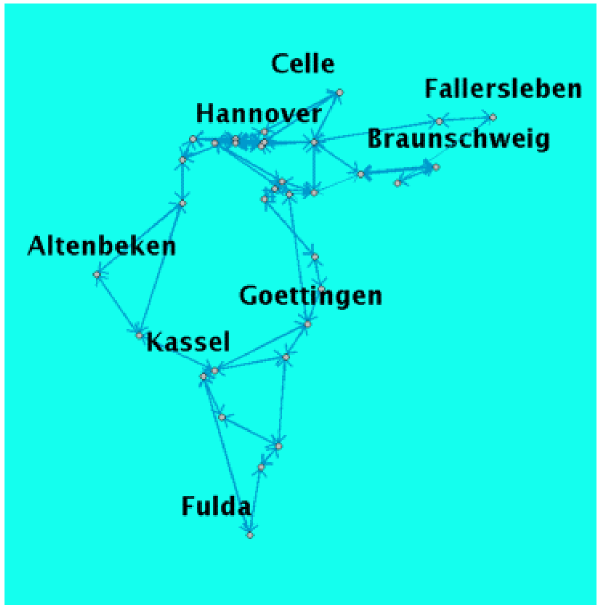
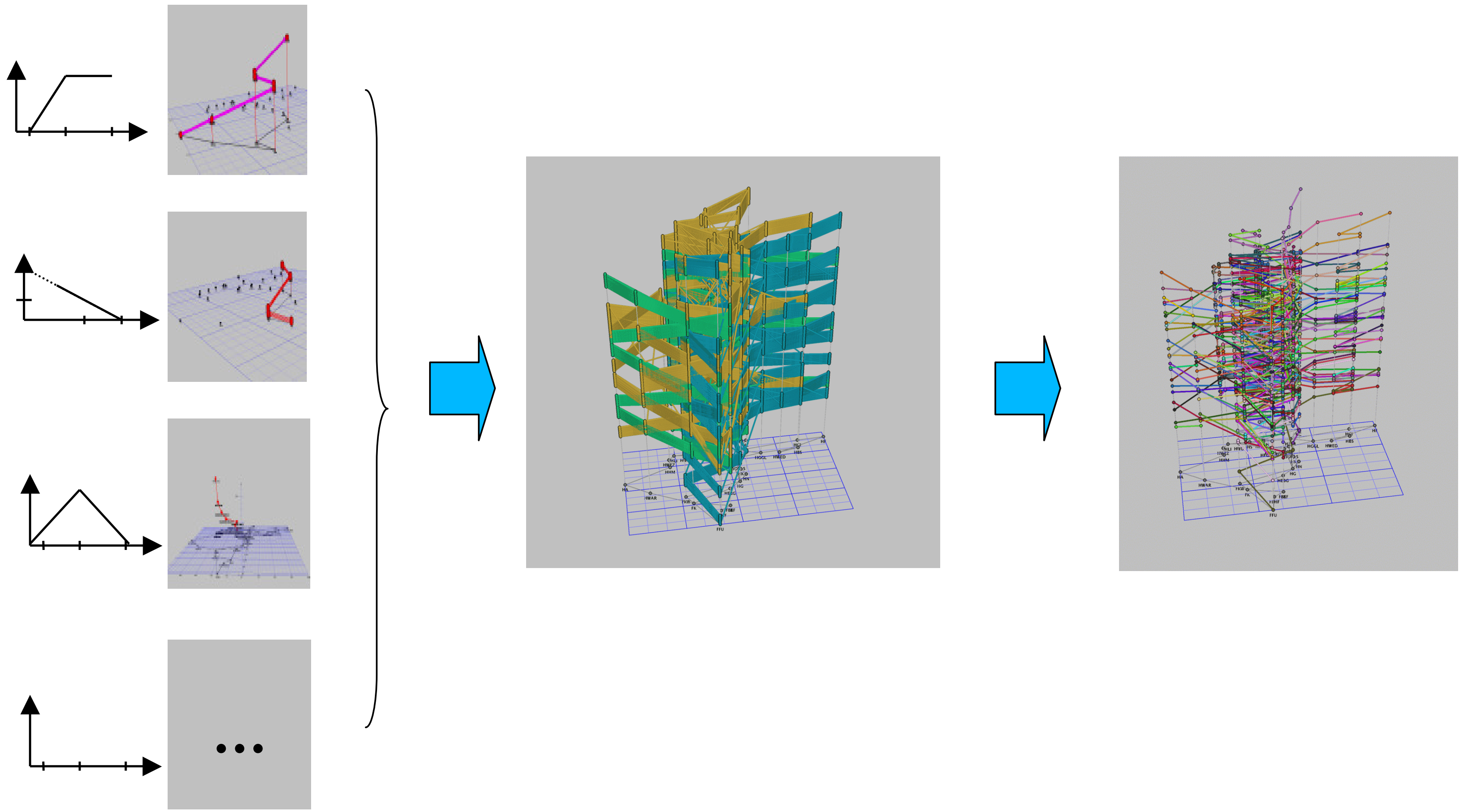
Railway Timetabling – State of the Art



- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Cai and Goh (1994), Schrijver and Steenbeck (1994), Carey and Lockwood (1995)
- Nachtigall and Voget (1996), Odijk (1996) Higgings, Kozan and Ferreira (1997)
- **Brannlund, Lindberg, Nou, Nilsson (1998)**, Lindner (2000), Oliveira and Smith (2000)
- **Caprara, Fischetti and Toth (2002)**, Peeters (2003)
- Kroon and Peeters (2003), Mistry and Kwan (2004)
- Barber, Salido, Ingolotti, Abril, Lova, Tormas (2004)
- Semet and Schoenauer (2005),
- **Caprara, Monaci, Toth and Guida (2005)**
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006),
- **Cacchiani, Caprara, T. (2006), Cachhiani (2007)**
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- **Borndoerfer, Schlechte (2007)**

non-cyclic timetabling literature

Track Allocation Problem



Train Requests → Scheduling Digraph → Timetable

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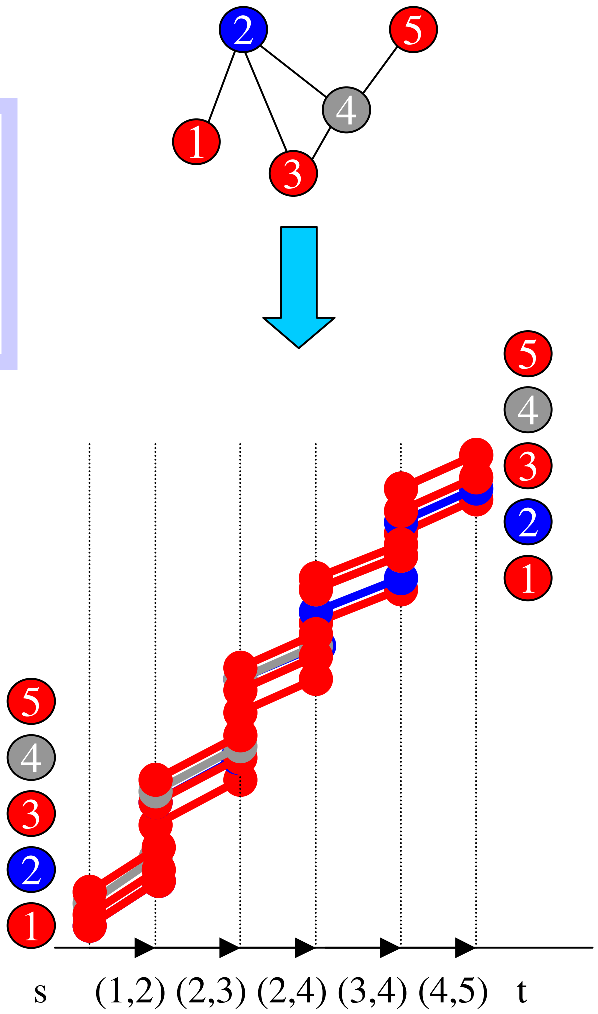
Complexity

Proposition [Caprara, Fischetti, Toth (02)]:

OPTRA/TTP is $\mathcal{NP}$-hard.

Proof:

Reduction from Independent-Set.



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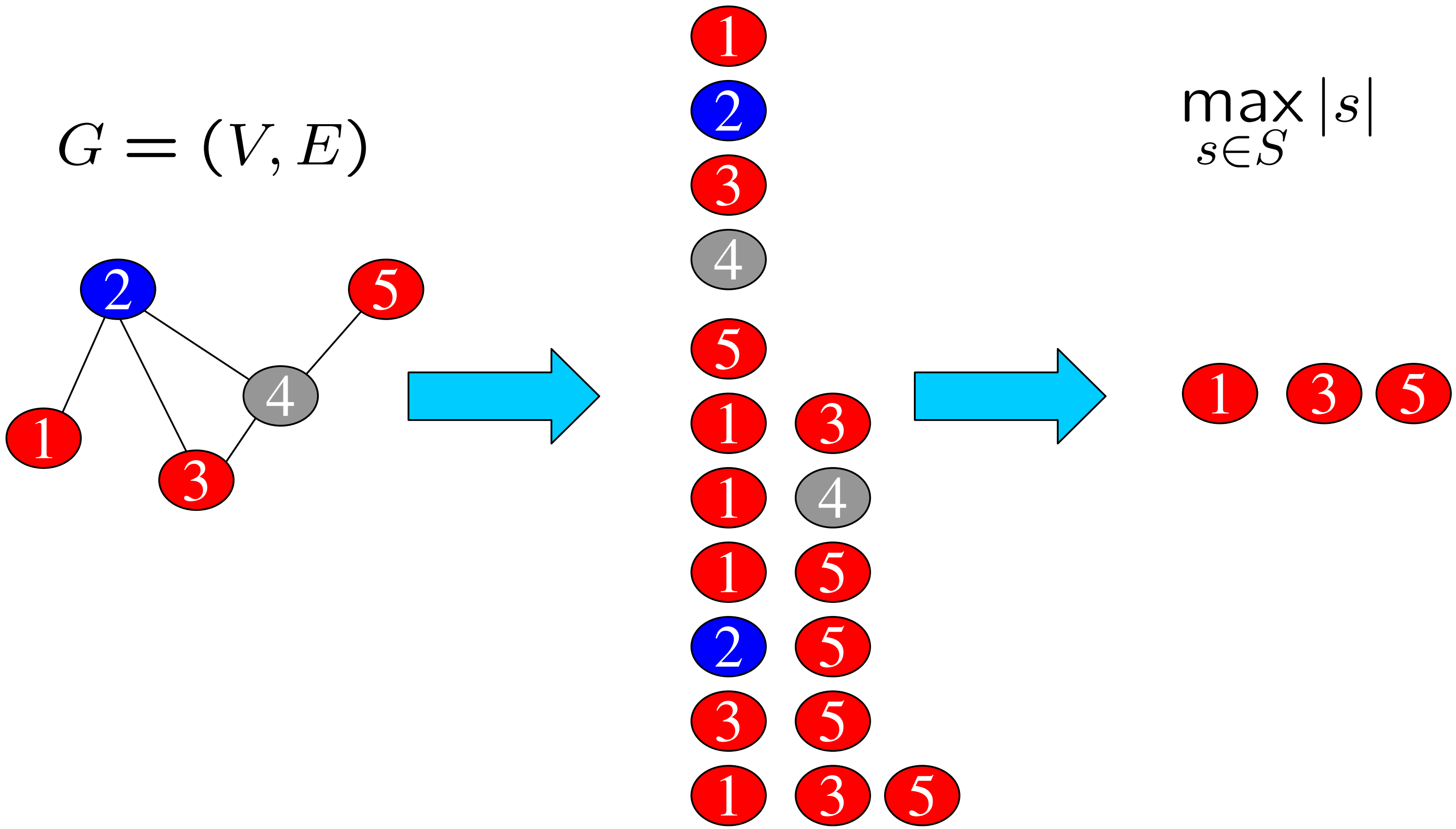
Independent/Stable Set Problem

$$S \subseteq 2^V$$

$$s \in S \Leftrightarrow \forall u, v \in s : (u,v) \notin E$$

$$G = (V, E)$$

$$\max_{s \in S} |s|$$



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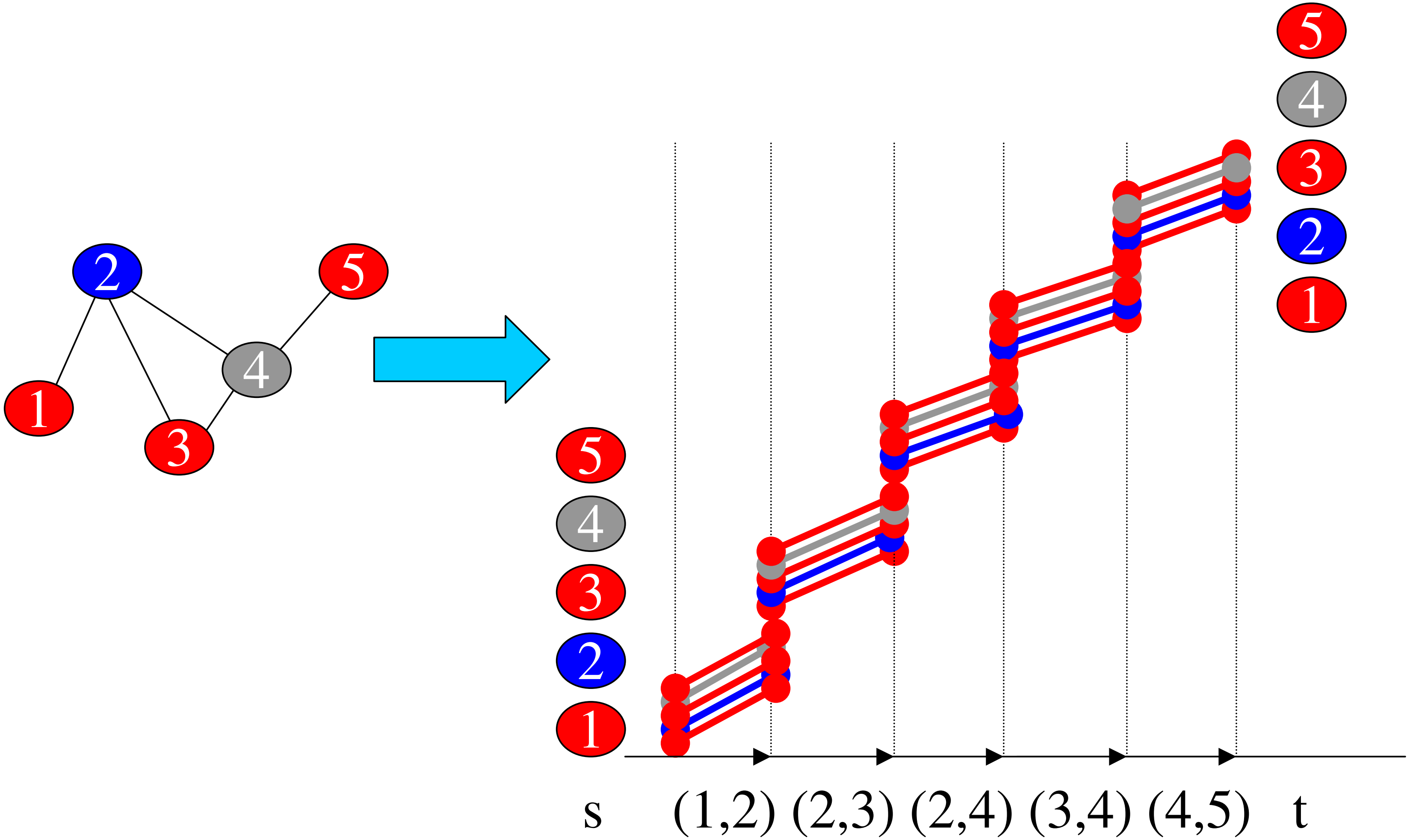
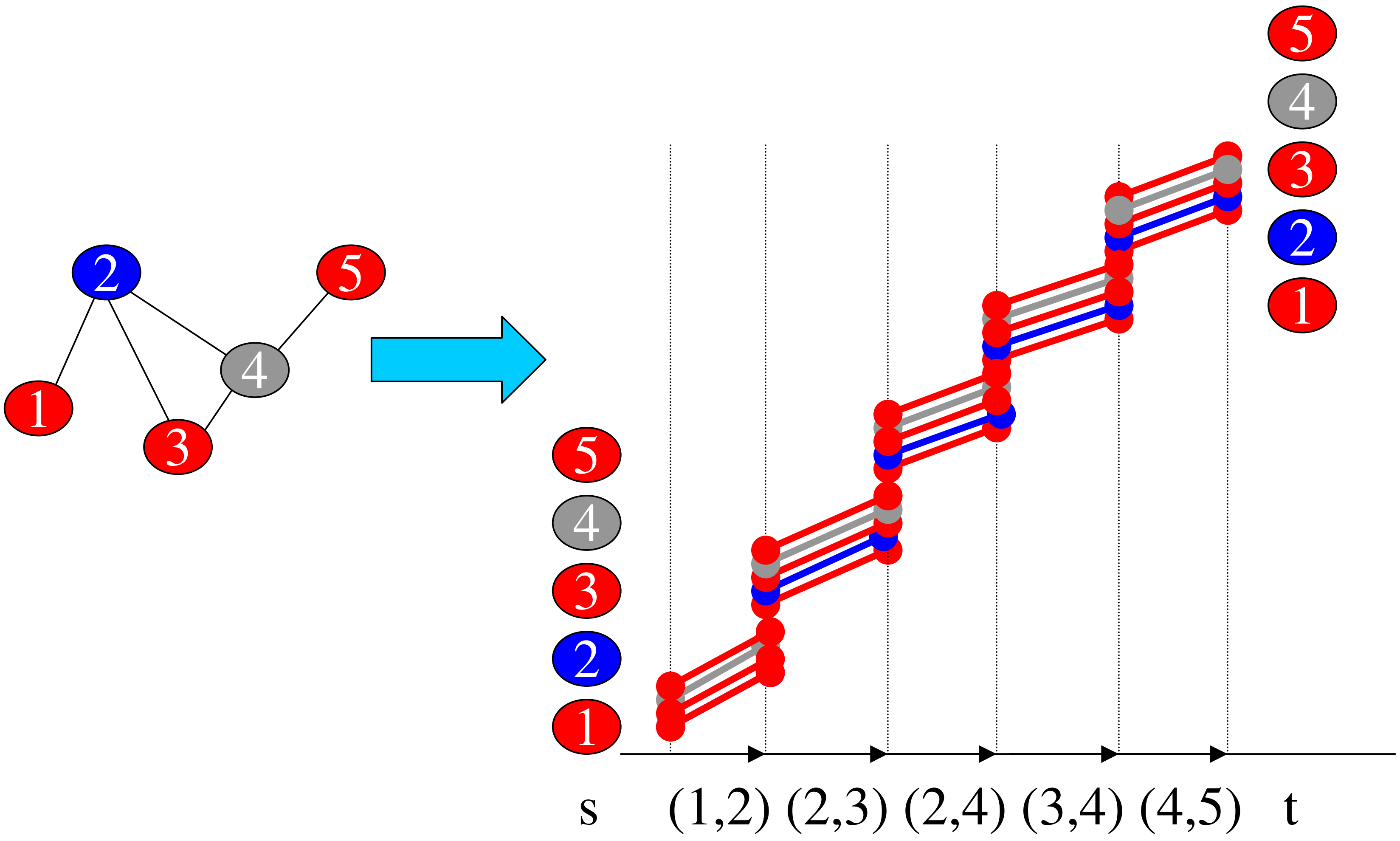
Polynomial Reduction



Conflict (1,2)



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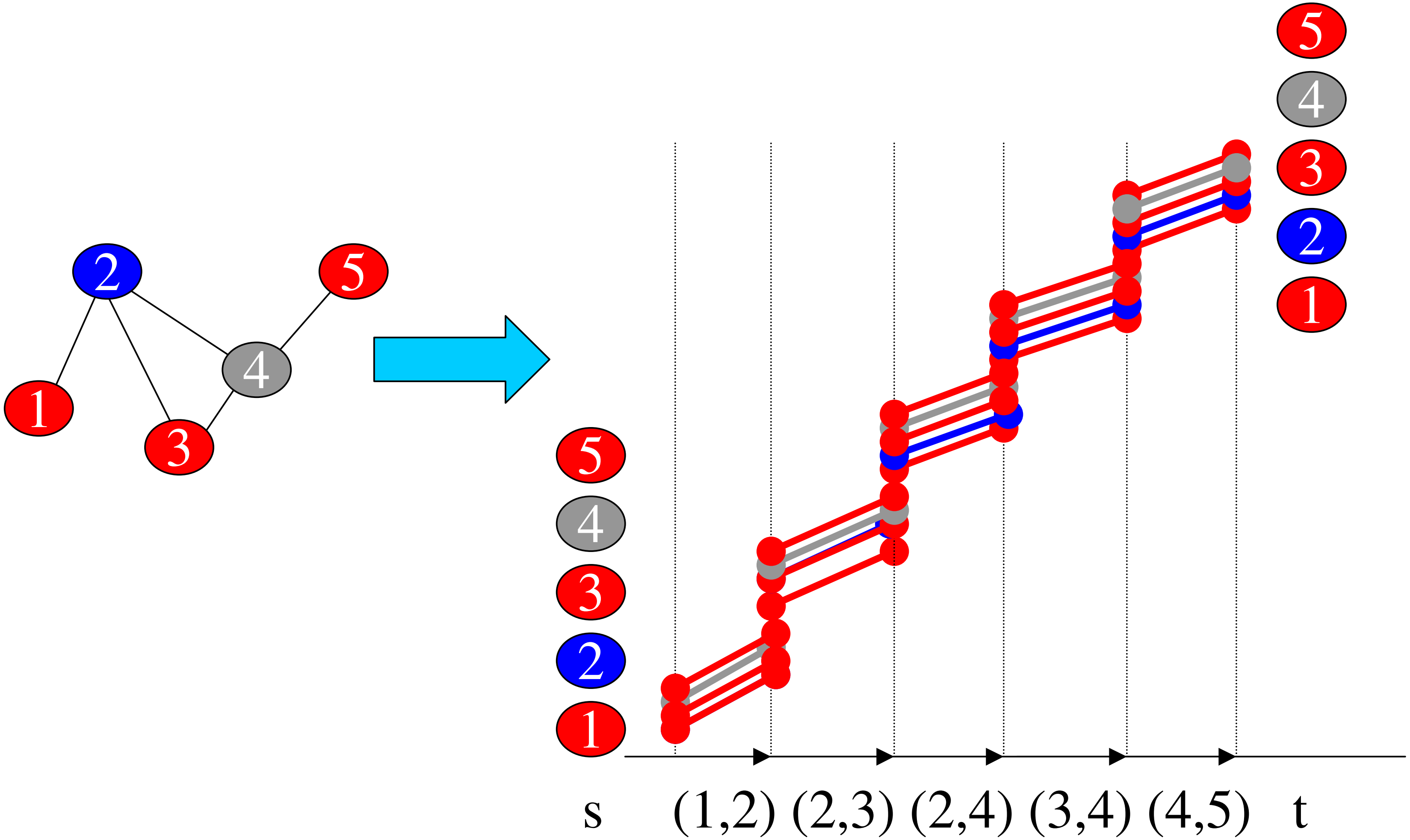
Conflict (2,3)



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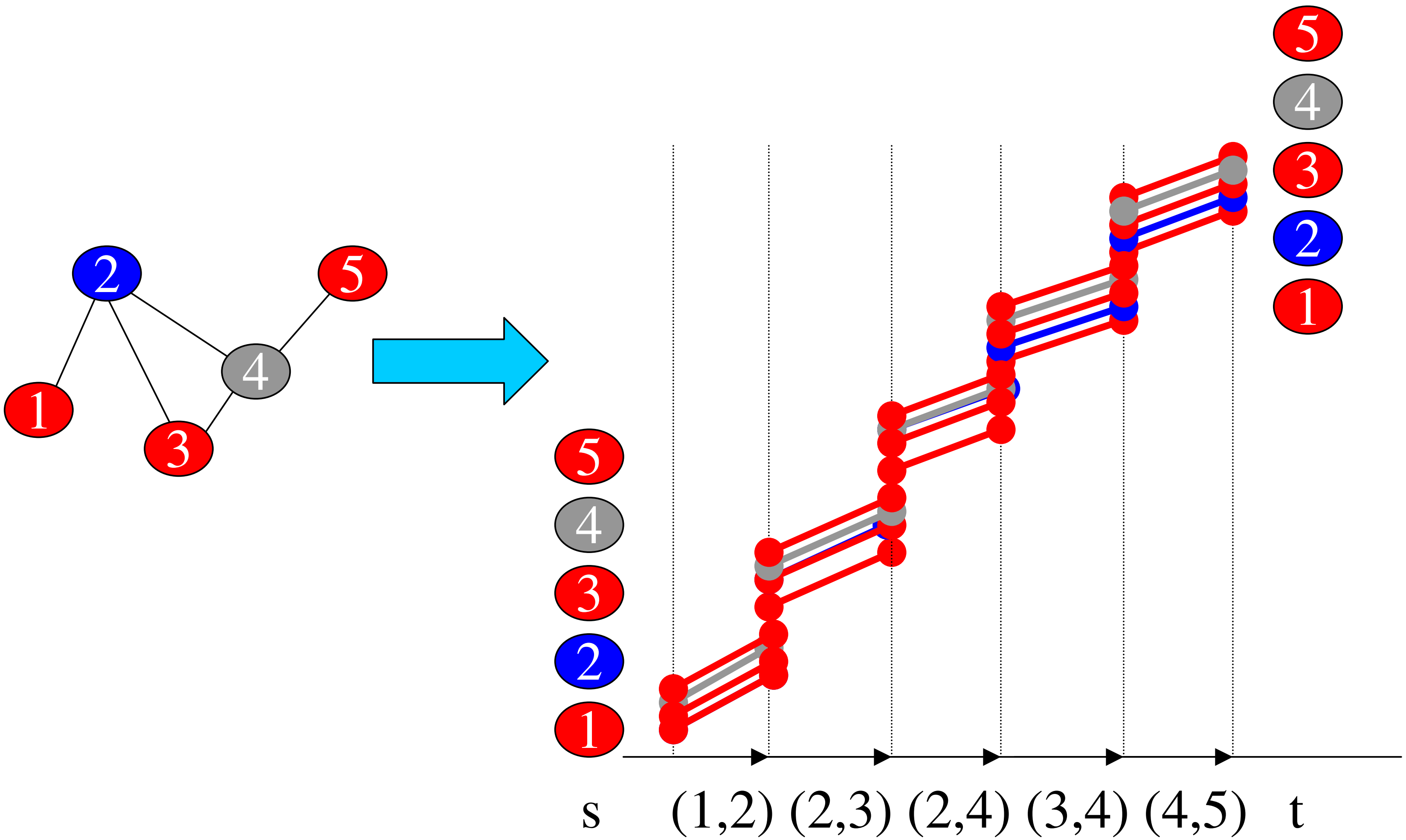
Conflict (2,4)



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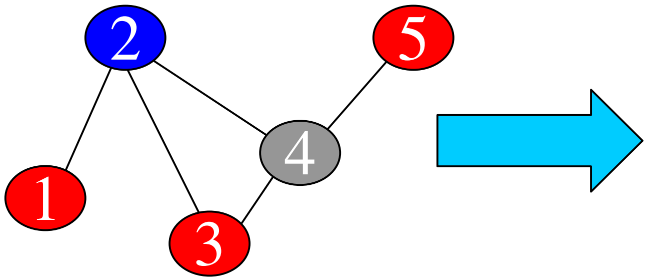
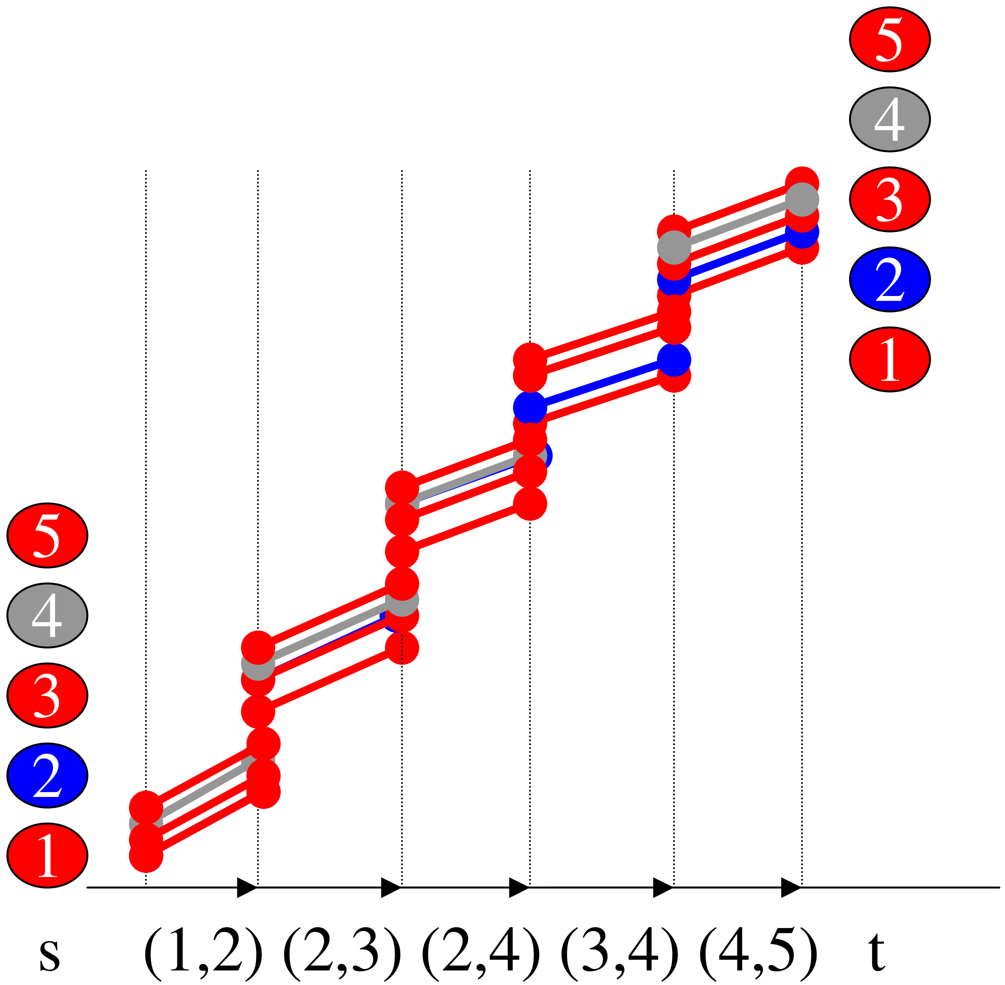
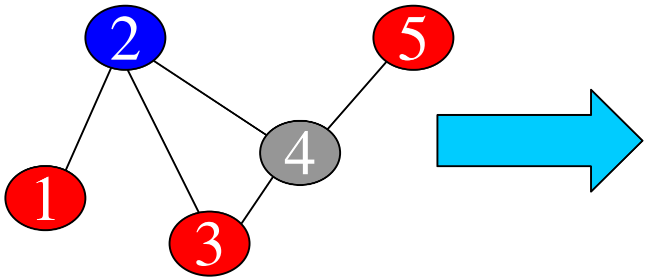
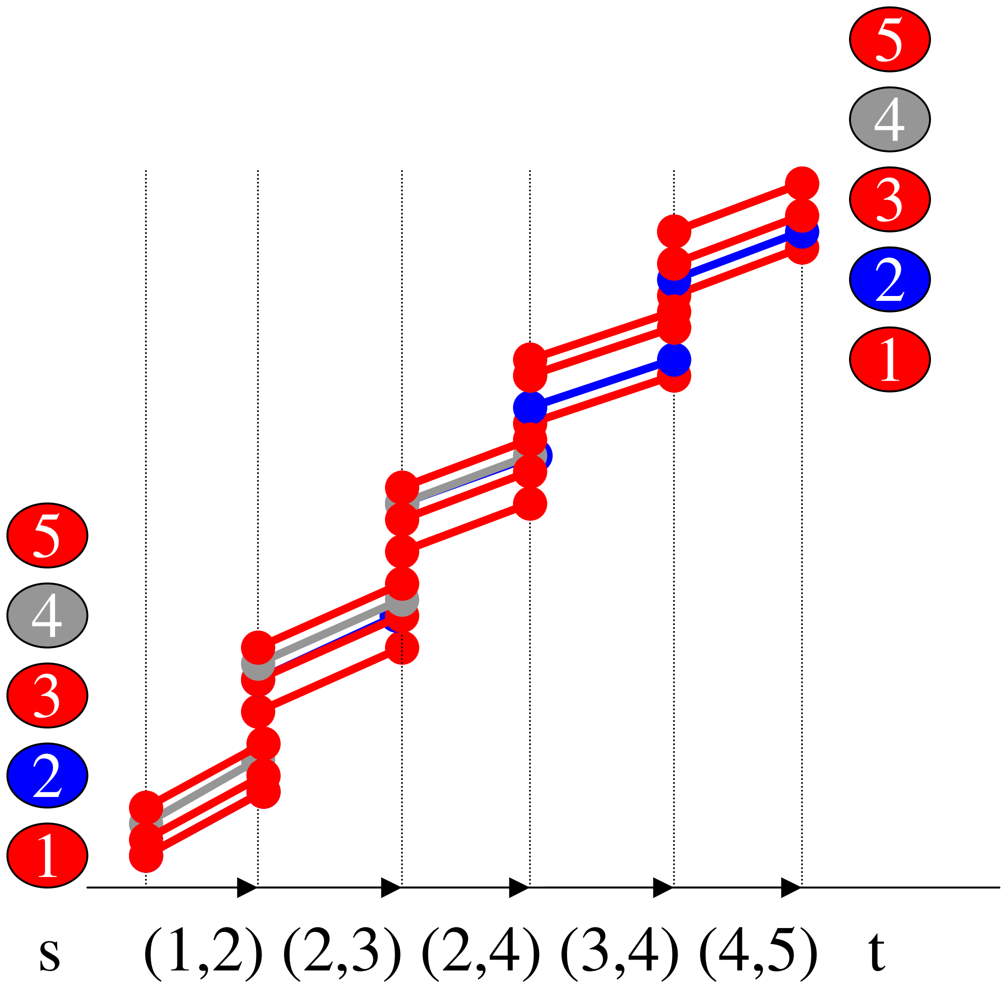
Conflict (3,4)





Conflict (4,5)





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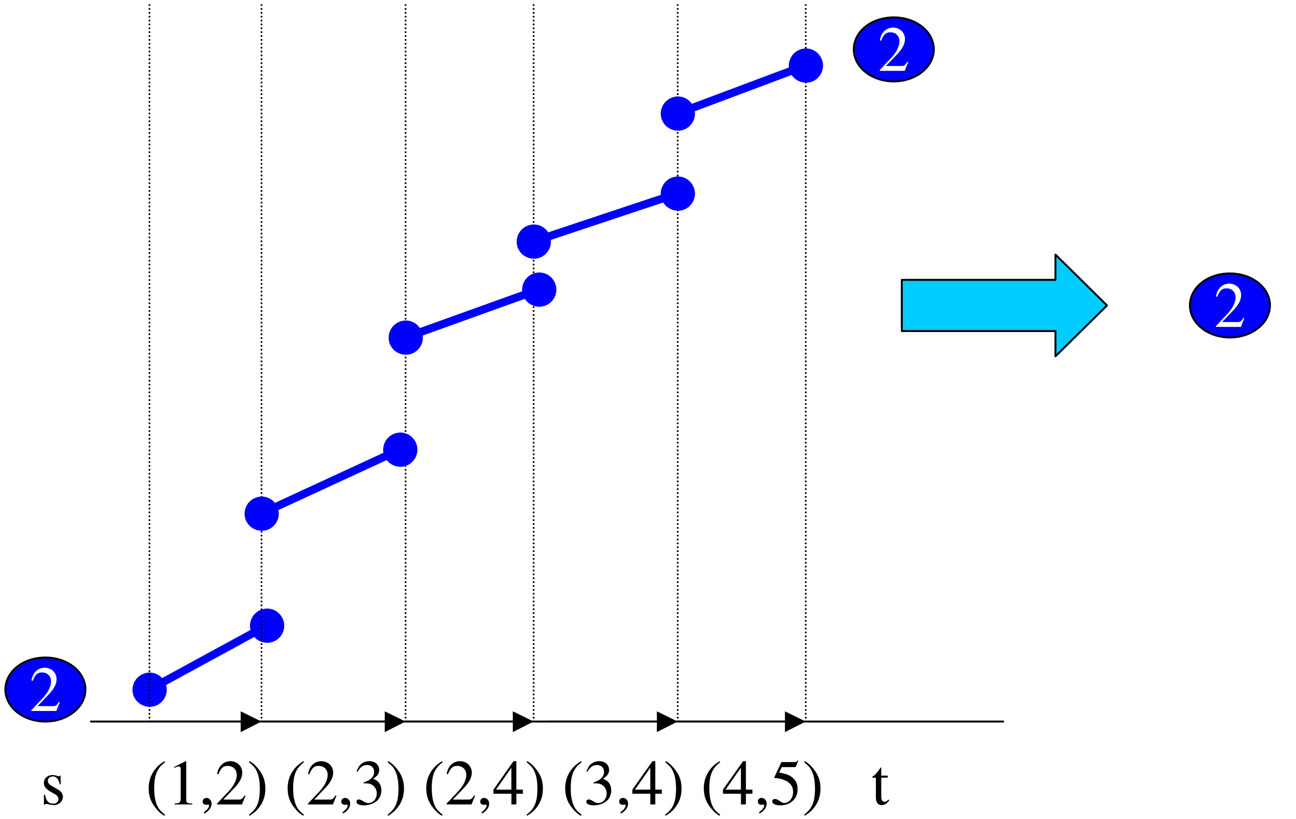
Feasible Train Set ⇔ Stable Set



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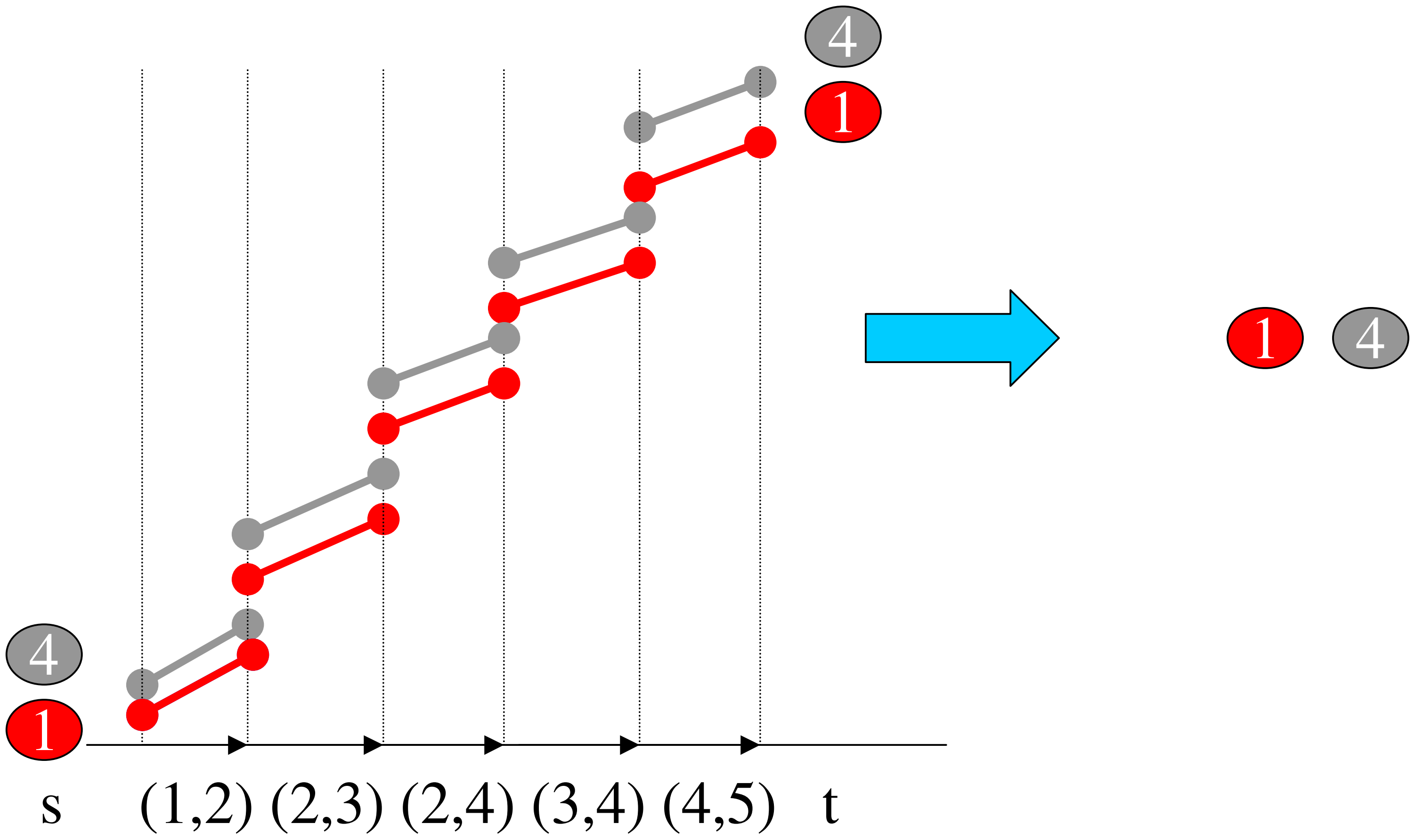
Feasible Train Set ⇔ Stable Set



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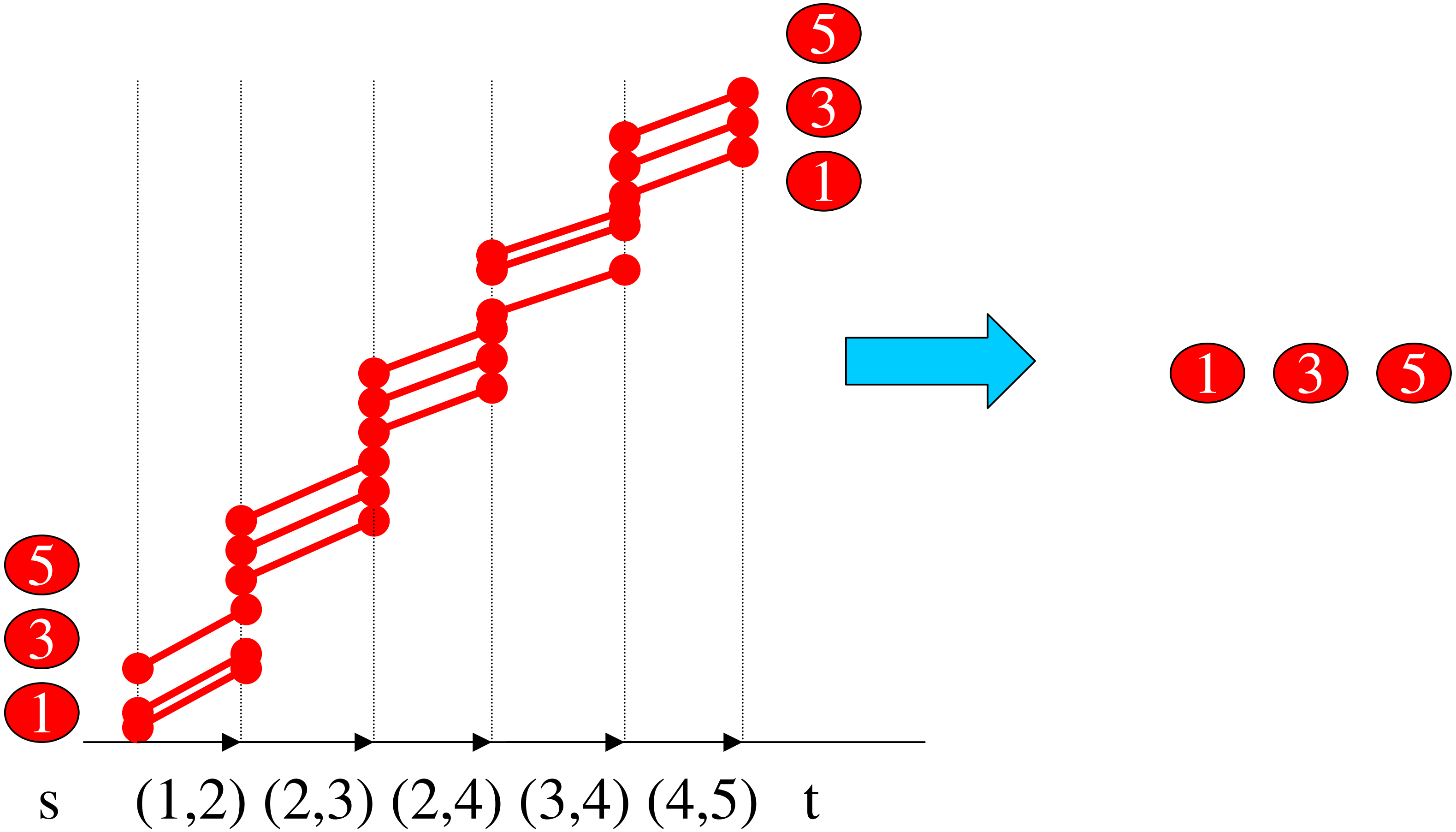

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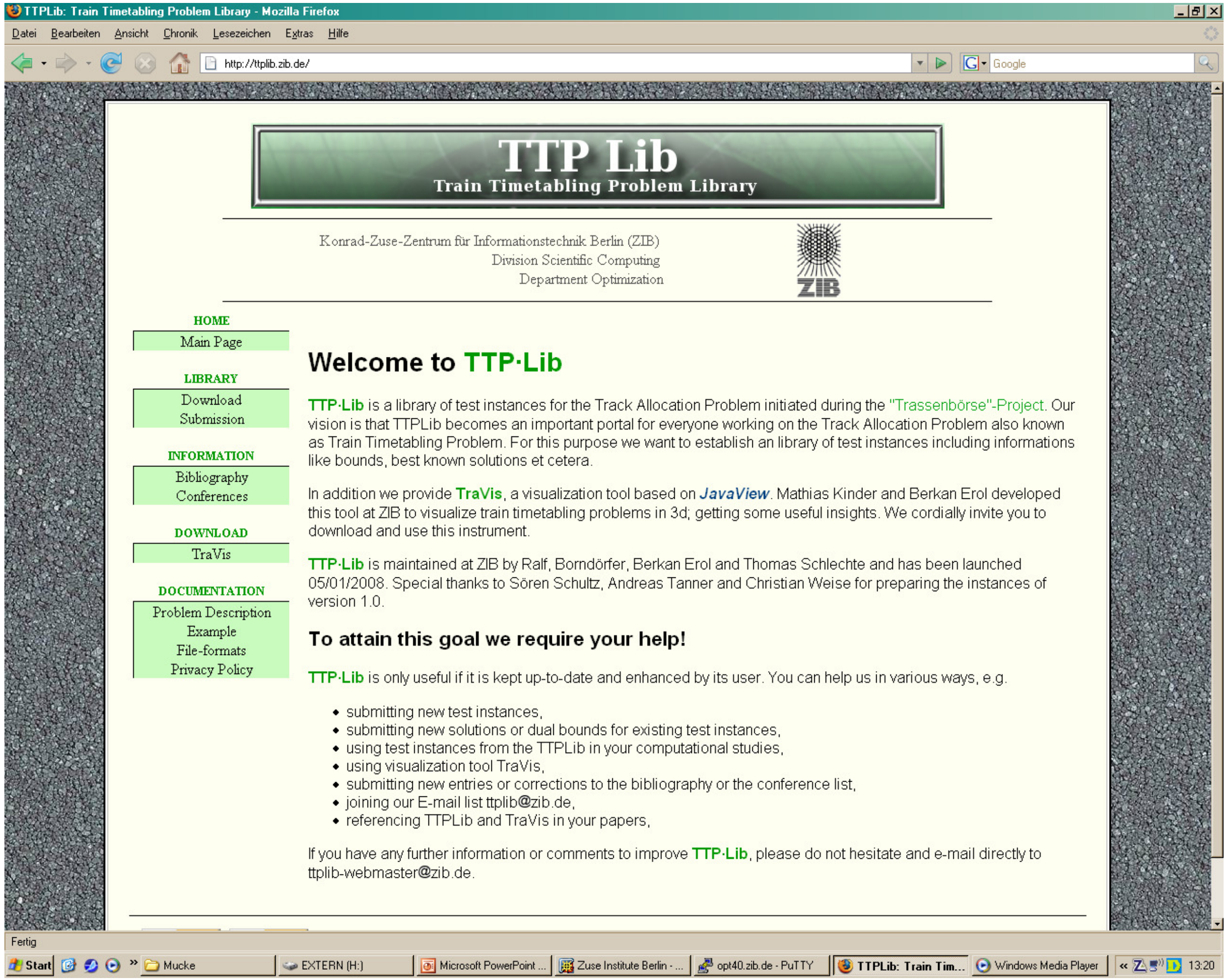
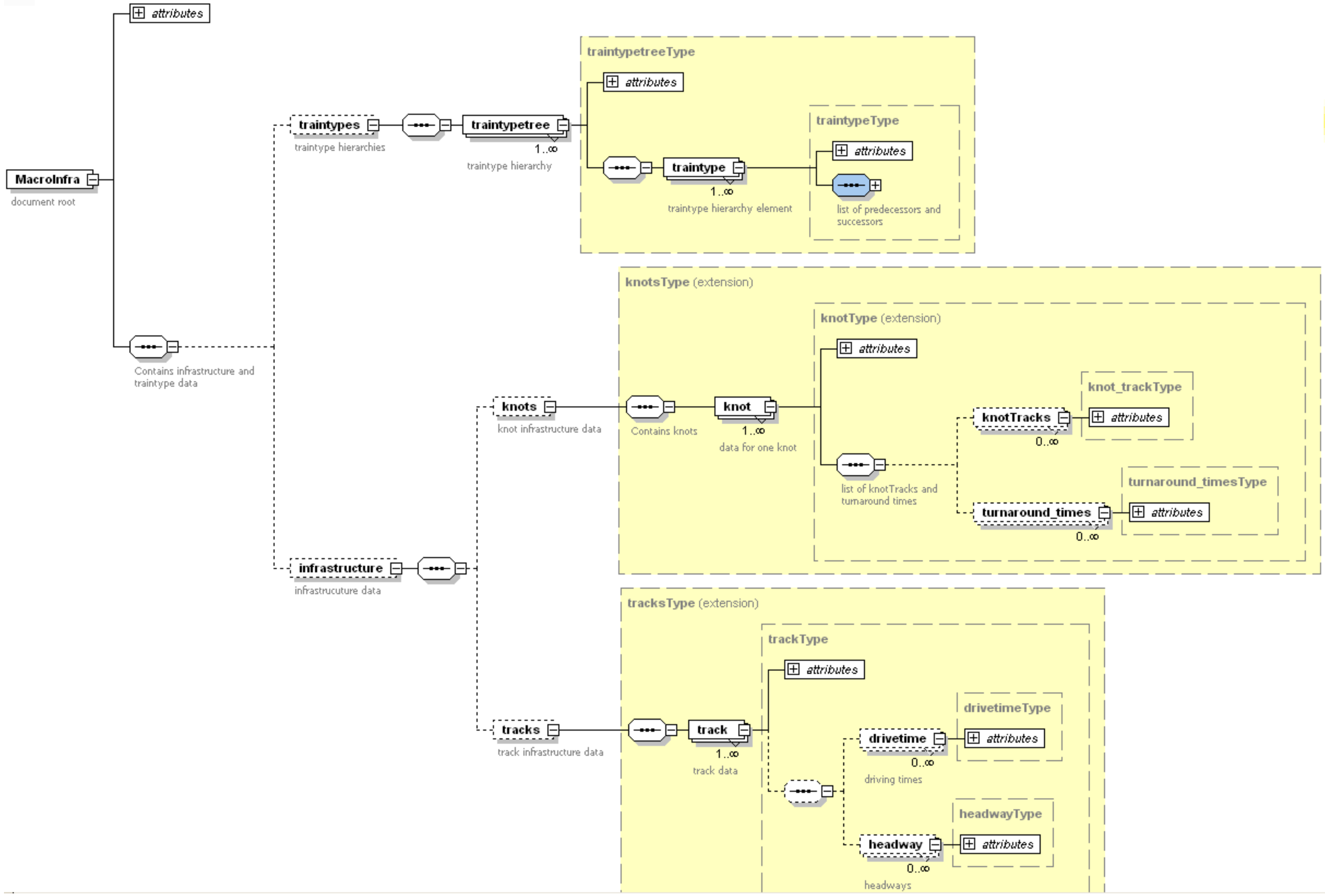
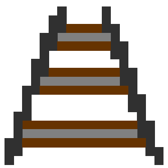
Standardization *http://ttplib.zib.de*

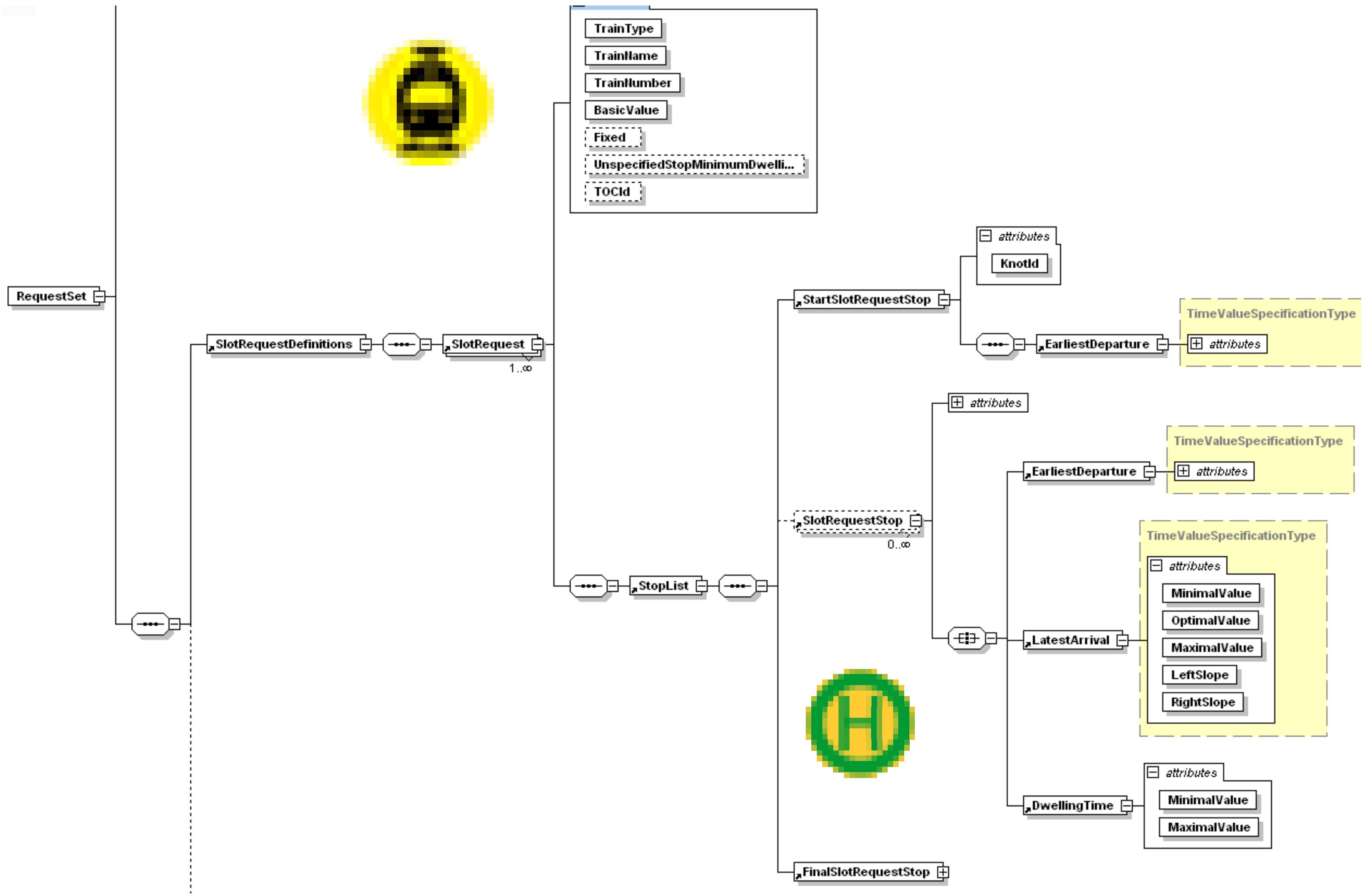


MacroInfra.xsd





RequestSet.xsd



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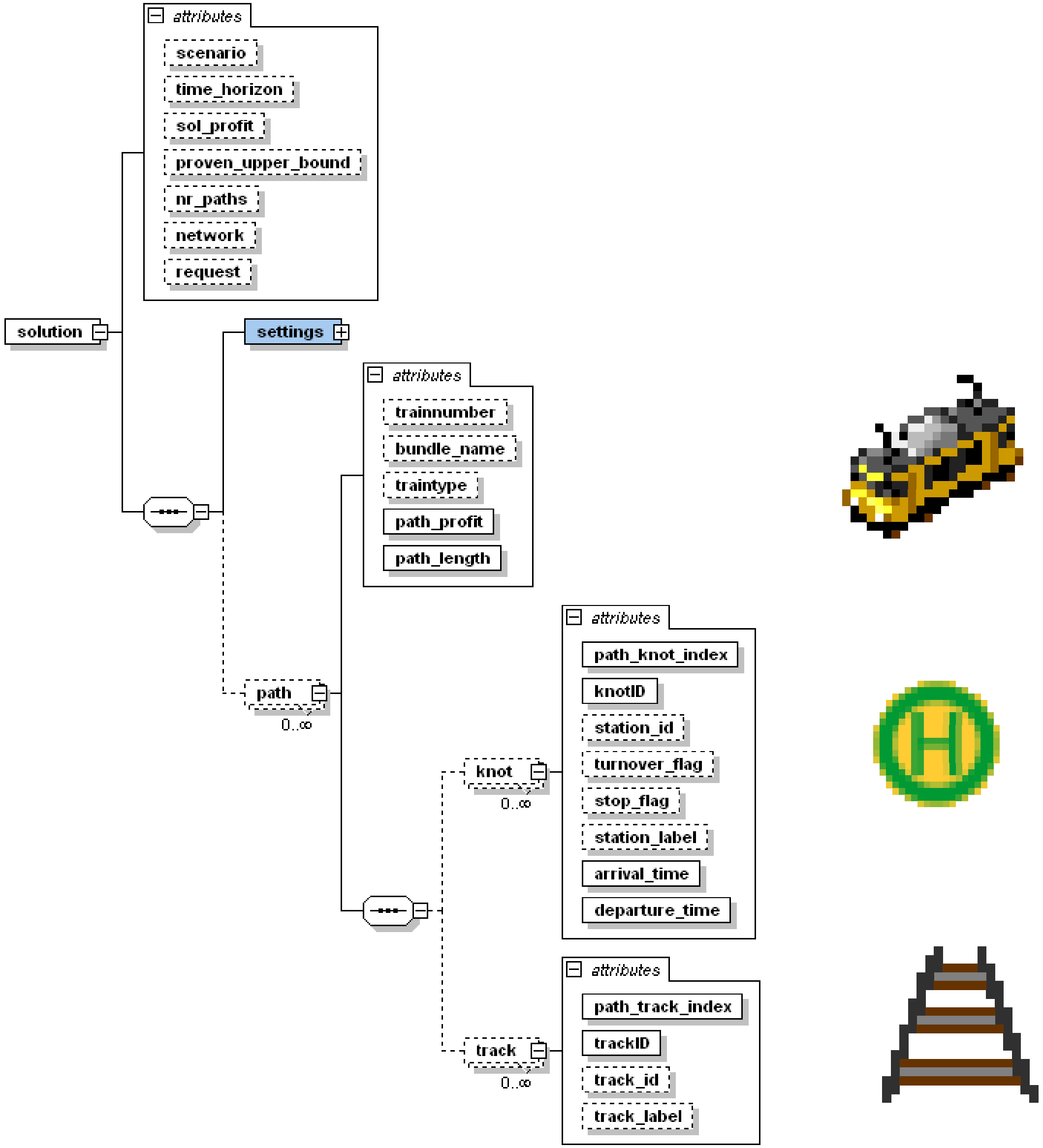
MacroTimetable.xsd



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Overview

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    1. Historical Example
    2. IP Solving
    3. IP Models for TTP
    4. Column Generation
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Management Science vol. 3, No. 1, 1956

A MODEL FOR THE OPTIMAL PROGRAMMING OF RAILWAY FREIGHT TRAIN MOVEMENTS*

A. CHARNES AND M. H. MILLER
Purdue University and Carnegie Institute of Technology

The structure shown in Table 1 can be translated into equation form by moving a row of λ's, one for each column, up through the rows and inserting the equal sign to the right of the P_0 column. The first two equations, for example, would be:

$$4 = 1\lambda_1 + 1\lambda_4 - 1\lambda_6 + 1\lambda_{12}$$

$$1 = 1\lambda_1 + 1\lambda_5 - 1\lambda_7 + 1\lambda_{13}$$

With the addition of the variables, the problem has been reduced to a standard simplex problem of the form:

$$\text{Min.} \sum_{i=1}^{n} \lambda_i c_i$$

subject to:

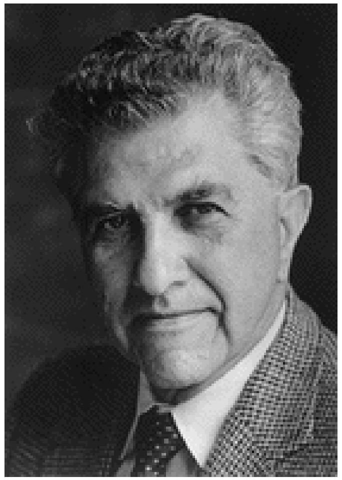
$$\sum_{i=1}^{n} \lambda_i P_i = P_0$$

$$\lambda_i \geqq 0$$

and can be solved by the simplex technique.

Abraham Charnes

Finalist for Nnobel Prize in Economic Sciences 1975



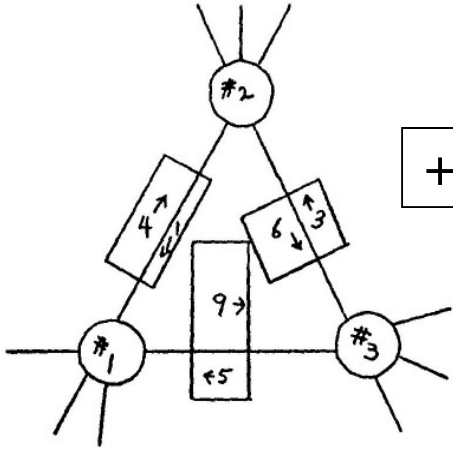
Merton H. Miller

Nobel Prize Winner in Economic Sciences 1990 with Markowitz and Sharpe

TABLE 1

Structural tableau of train-scheduling model

$c_j \rightarrow$			1.0	1.0	1.0	1.2	1.2	0	0	0	0	0	0	M	M	M	M	M	M
From	To	Shipment Requirements	Routes					Surplus Vectors (light moves)						Artificial Vectors (legs)					
			1,2	1,3	2,3	1,2,3	1,3,2	1–2	2–1	1–3	3–1	2–3	3–2	1–2	2–1	1–3	3–1	2–3	3–2
		P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}
1	2	4	1			1		−1						1					
2	1	1	1				1		−1						1				
1	3	9		1			1			−1						1			
3	1	5		1		1					−1						1		
2	3	6			1	1						−1						1	
3	2	3			1		1						−1						1



+ Rules

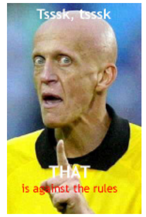


CHART 1. Simplified map of terminal switching railroad, showing connections with trunklines, major interchange and customer yard areas, and traffic requirements (in train-loads) between major points.

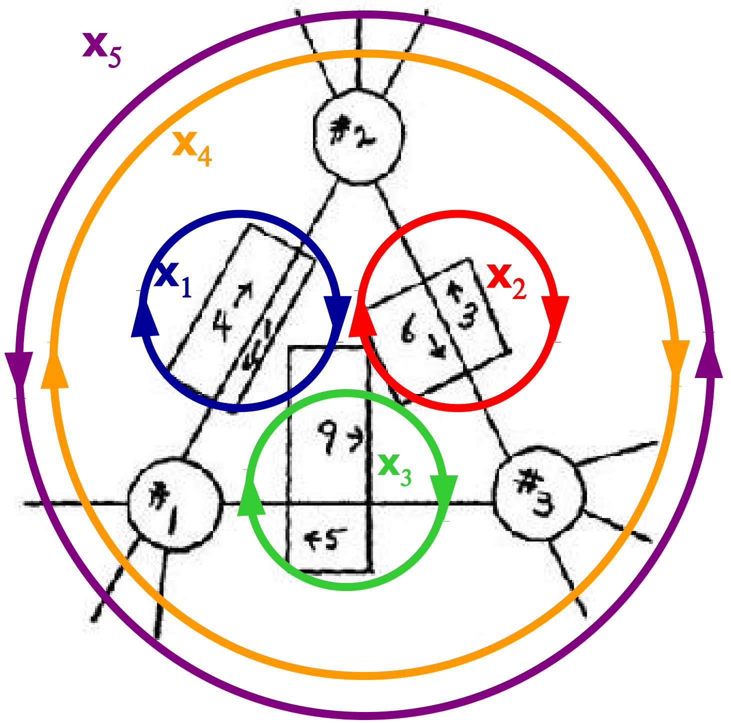
postponed until the description of the model and the computational routine has been completed.

Above the routes, in the row labeled c_j, are entered the costs of assigning a single crew and engine package to the route in question. These costs may be stated either as the standard crew and engine expense, or as the expected costs reflecting the fact that on longer runs there is a greater probability of running into overtime. We constructed working models both ways and found, that optimal programs were not particularly sensitive to variations in the cost of crews. In fact, it was usually possible to simplify the calculation by minimizing the number of crews, that is treating the cost of each crew as 1.

P_6 to P_{11} in the tableau are overfulfillment slack vectors. In the train scheduling context they correspond to "light moves", or trips by a crew and engine without cars. If, for example, four crews should be assigned to the route P_1—which runs

Integer Program

- Graph Model



- Algebraic Model

Min $x_1 + x_2 + x_3 + 1.2x_4 + 1.2x_5$

$4 \leq x_1 + x_4$

$1 \leq x_1 + x_5$

$9 \leq x_3 + x_5$

$5 \leq x_3 + x_4$

$6 \leq x_2 + x_4$

$3 \leq x_2 + x_5$

$x \geq 0$

x integer

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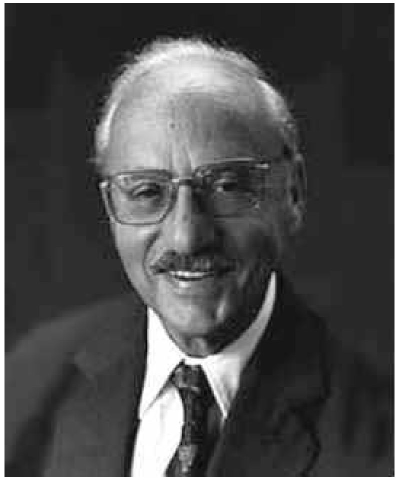
Linear Optimization

Leonid V. Kantorovich

Nobel Pirze in Economic Sciences 1975

Tjalling C. Koopmans

Nobel Pirze in Economic Sciences 1975



George W. Dantzig

Father of Linear Programming

- Algebraic Model

  (LP, Simplex algorithm)

$$\text{Min } x_1 + x_2 + x_3 + 1.2x_4 + 1.2x_5$$
$$4 \leq x_1 + x_4$$
$$1 \leq x_1 + x_5$$
$$9 \leq x_3 + x_5$$
$$5 \leq x_3 + x_4$$
$$6 \leq x_2 + x_4$$
$$3 \leq x_2 + x_5$$
$$x \geq 0$$

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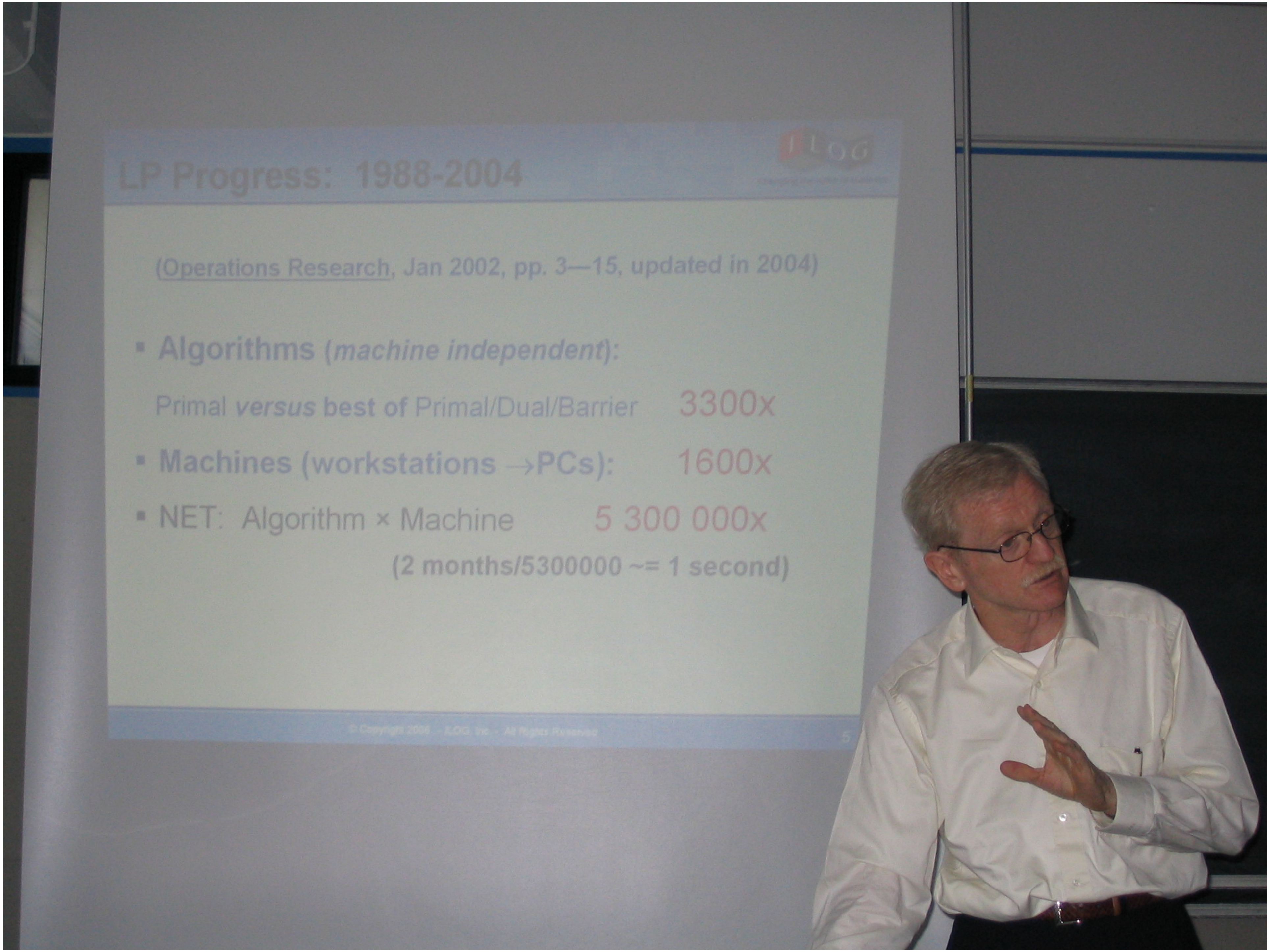
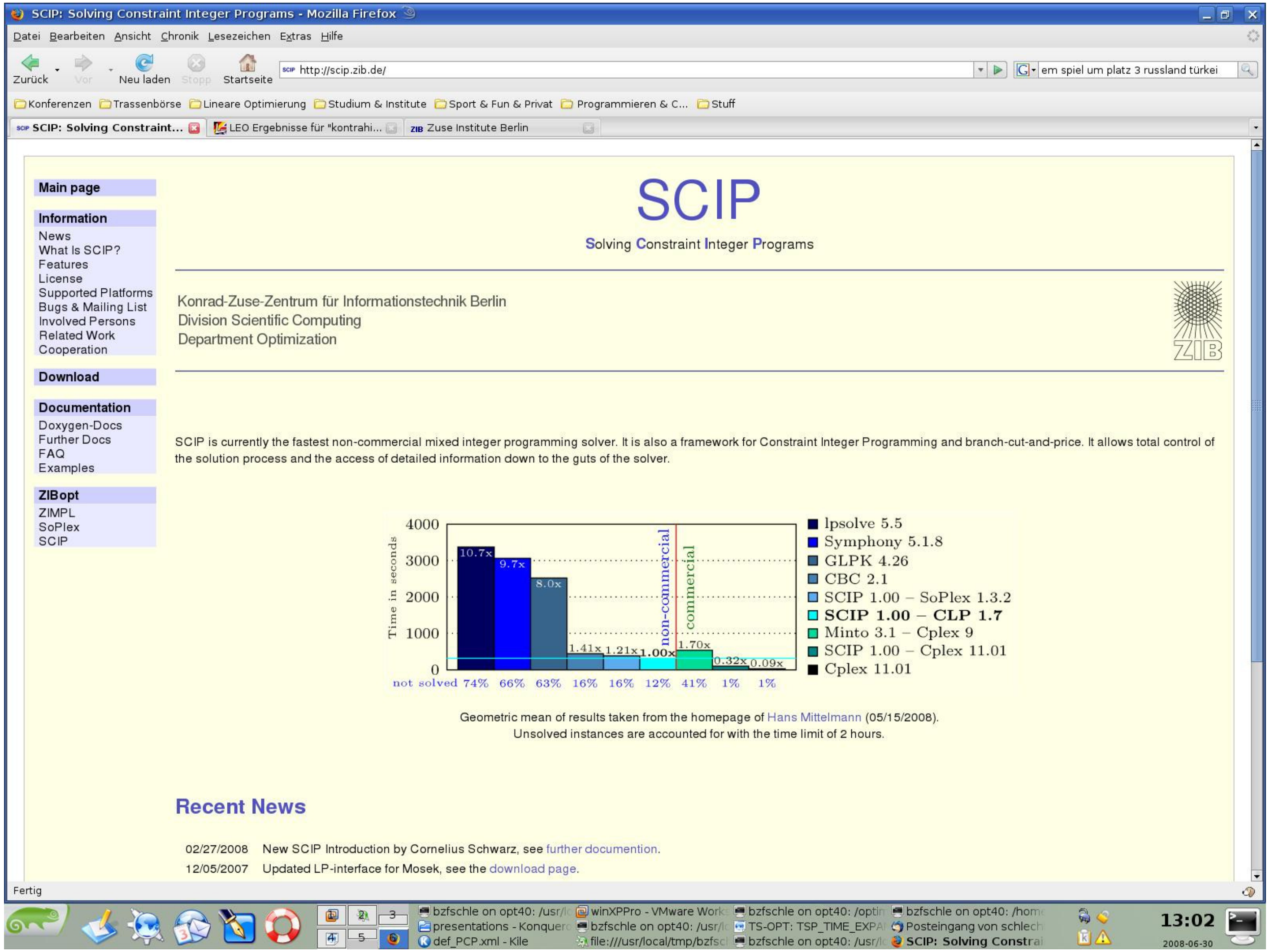

LP Progress: 1988-2004
(Operations Research, Jan 2002, pp. 3—15, updated in 2004)
Algorithms (machine independent):
Primal versus best of Primal/Dual/Barrier 3300x
Machines (workstations →PCs): 1600x
NET: Algorithm × Machine 5 300 000x
(2 months/5300000 ~= 1 second)

IP Solvers CPLEX, XPRESS, SCIP…



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Overview

1. Idea & Motivation
2. Train Timetabling Problem
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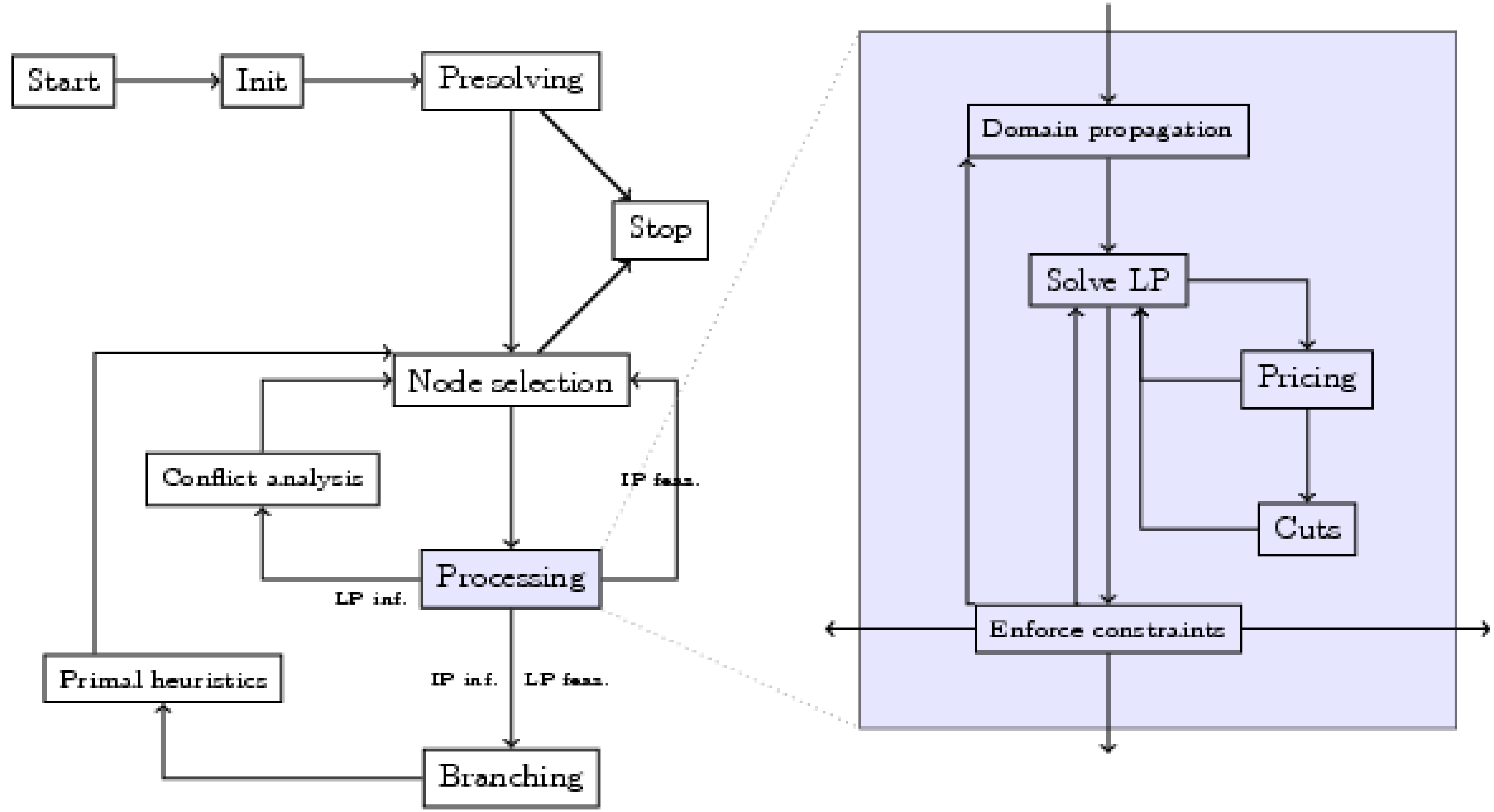
General Framework - SCIP



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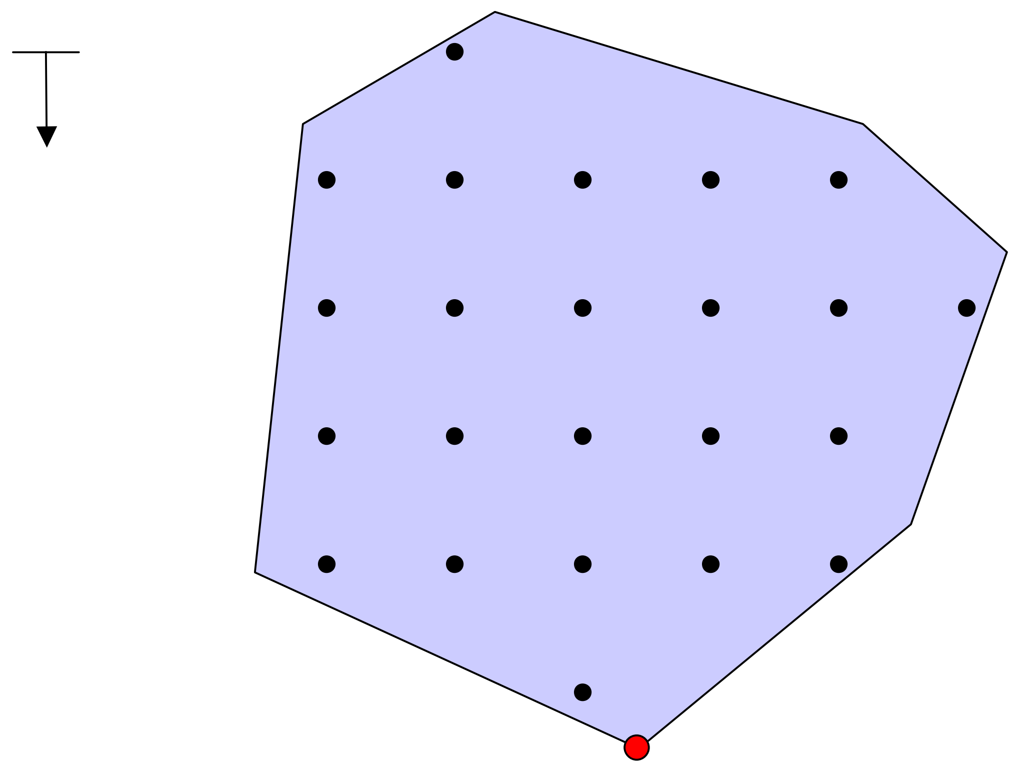
Linear Relaxation



- solve linear program by primal, dual simplex or barrier (interior point method)

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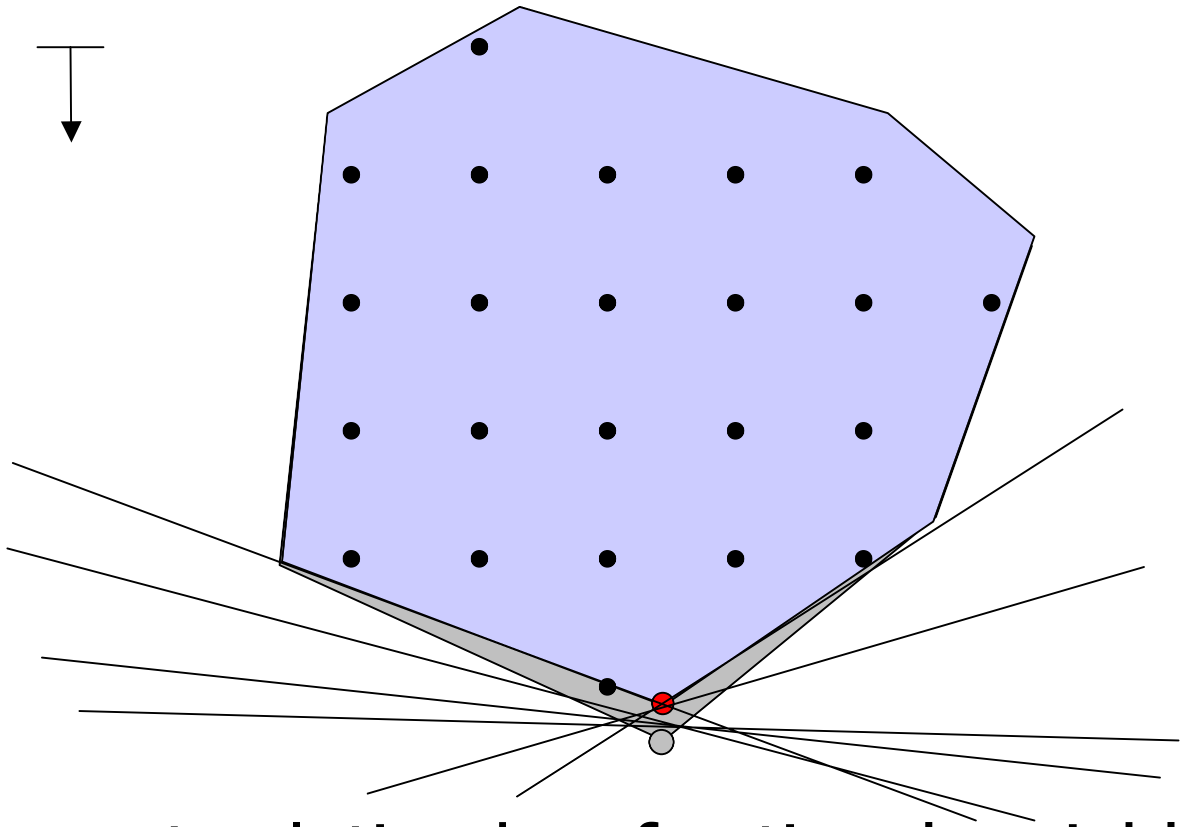
Cutting Planes



- current solution has fractional variables
- improve by valid cutting planes

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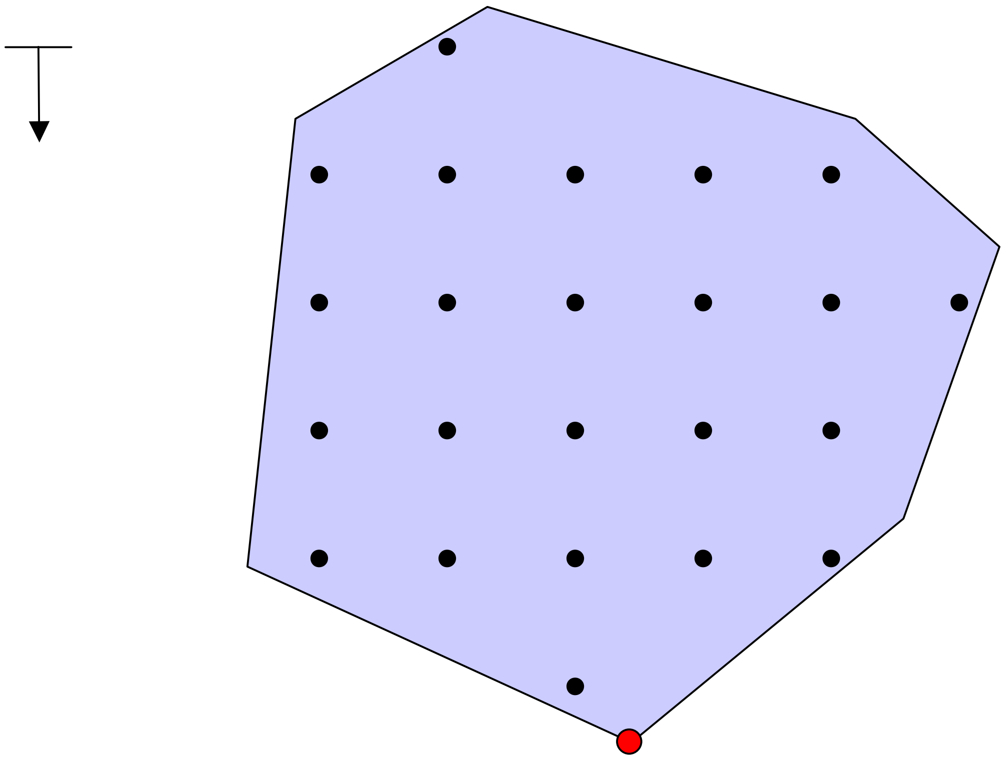
Branching



- current solution is infeasible (at least one forbidden fractional value)

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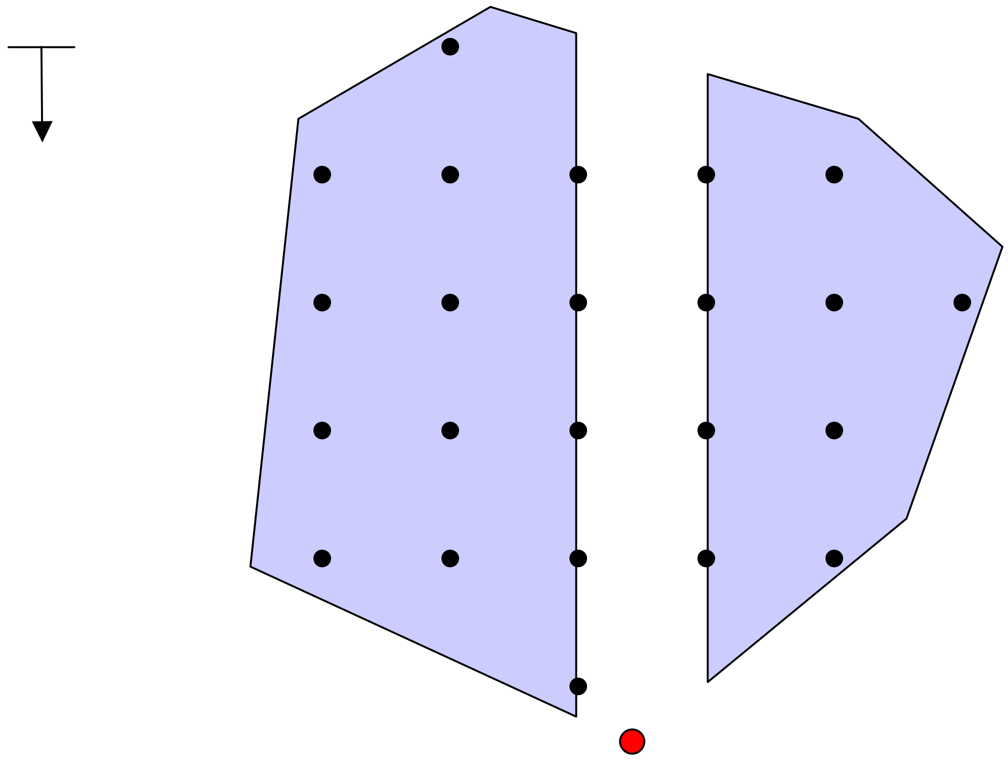
Branching on Variables



- split problems into sub problems to cut off current solution

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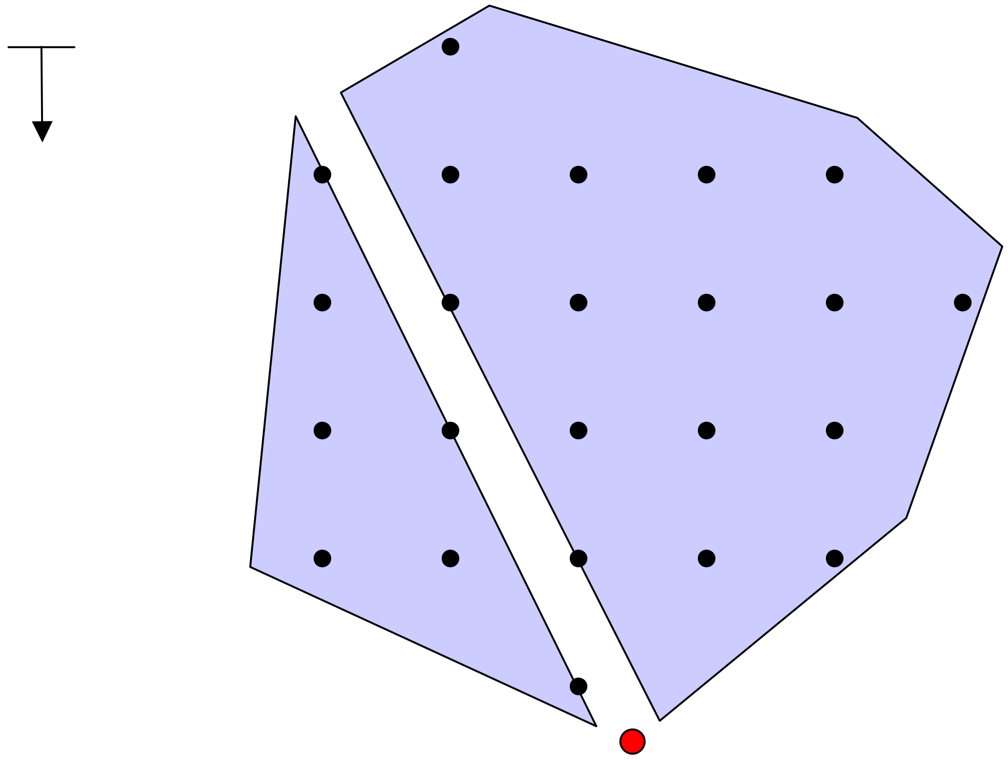
Branching on Constraints



- split problems into subproblems to cut off current solution

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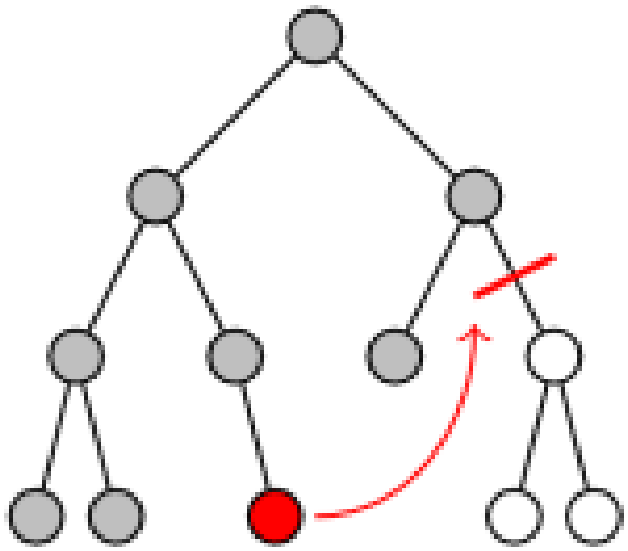
Branch & Bound Tree



- divide and conquer (split problem into smaller subproblems)
- use conflict analysis (constraint propagation)

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Arc Packing Problem

$$
\begin{array}{llll}
(APP) & & & \\
\max & \sum_{i\in\mathcal{I}}\sum_{a\in A} p_a^i x_a^i & & \\
\text{s.t.} & \sum_{a\in\delta_i^{out}(v)} x_a^i - \sum_{a\in\delta_i^{in}(v)} x_a^i \le \delta_i(v) & \forall v\in V, \forall i\in\mathcal{I} & \text{(i)} \\
 & \sum_{i\in\mathcal{I}}\sum_{a\in A} x_a^i \le 1 & \forall c\in C & \text{(ii)} \\
 & x_a^i \in \{0,1\} & \forall a\in A, \forall i\in\mathcal{I} & \text{(iii)}
\end{array}
$$

Variables

- Arc occupancy (request i uses arc a)

Constraints

- Flow conservation and
- Arc conflicts (pairwise)

Objective

- Maximize proceedings

(PPP) transformation from arc to path variables (see Cachhiani (2007))

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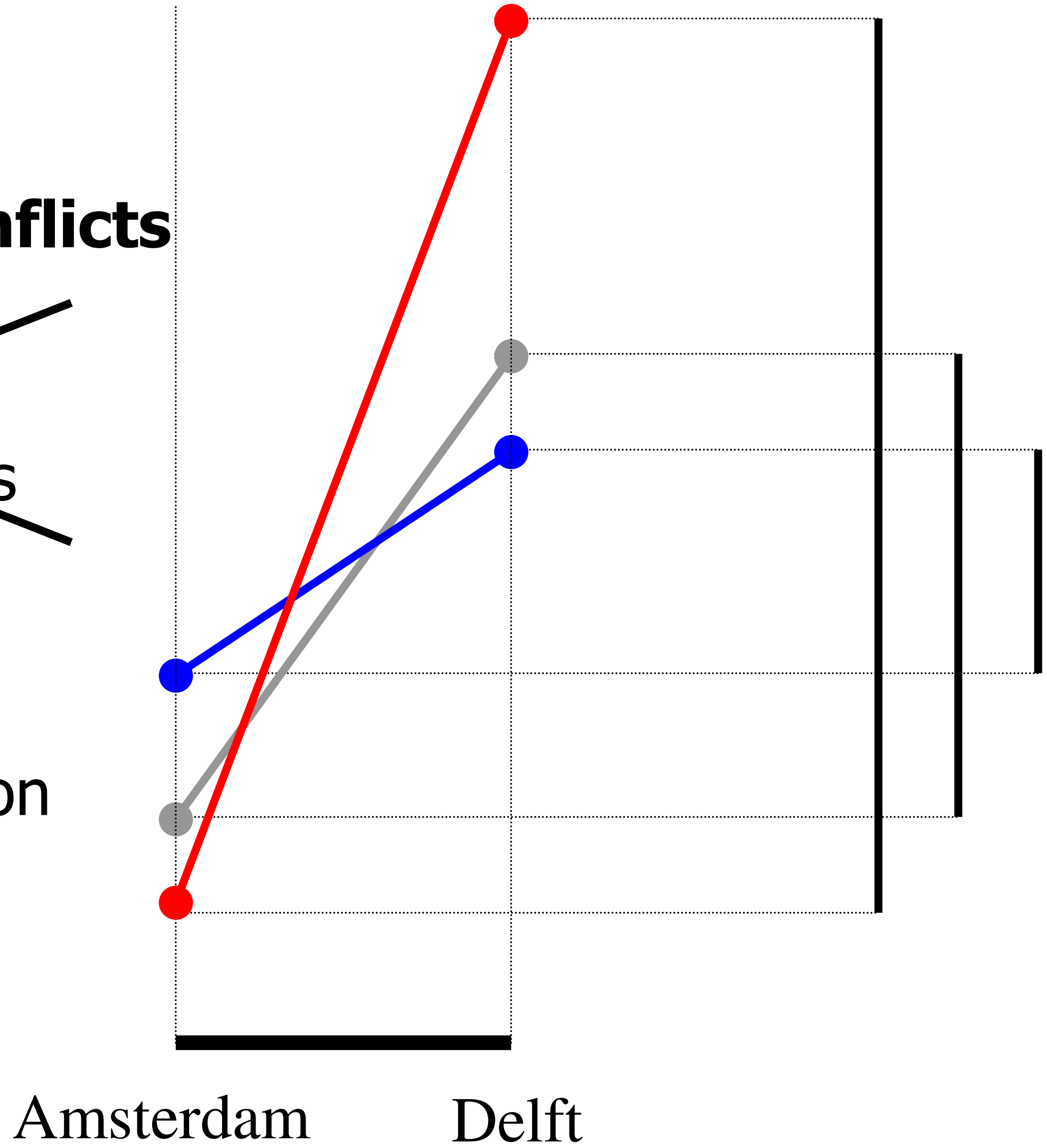
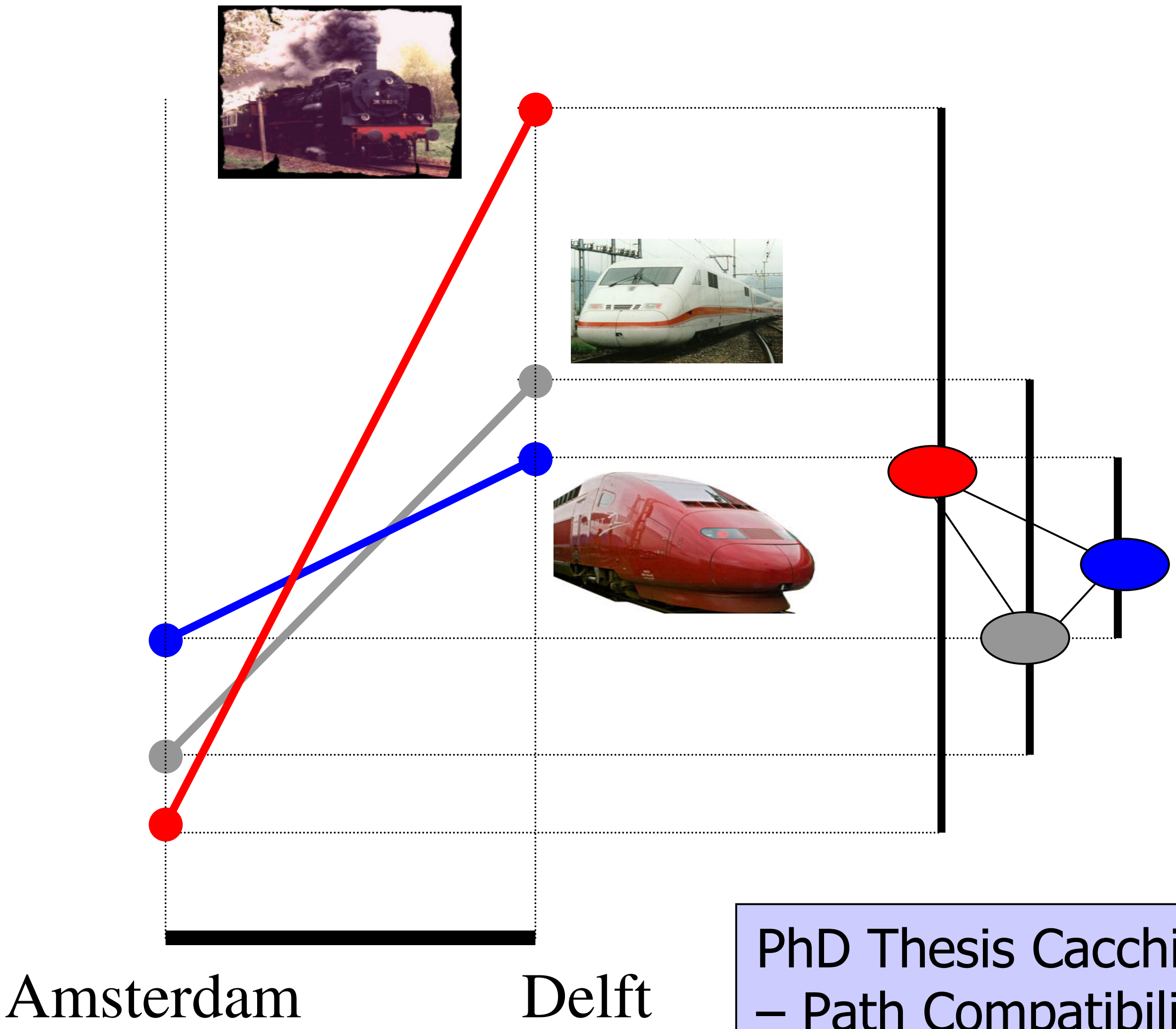
TTP as APP

- **macroscopic conflicts**
  - ~~headways~~
  - ~~station capacities~~
- **simplified**
  - block occupation



Packing Constraints



PhD Thesis Cacchiani (2007) – Path Compatibility Graphs

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Strengthen Constraints

 +  +  ≤ 1

Instead of :

 +  ≤ 1

 +  ≤ 1

 +  ≤ 1

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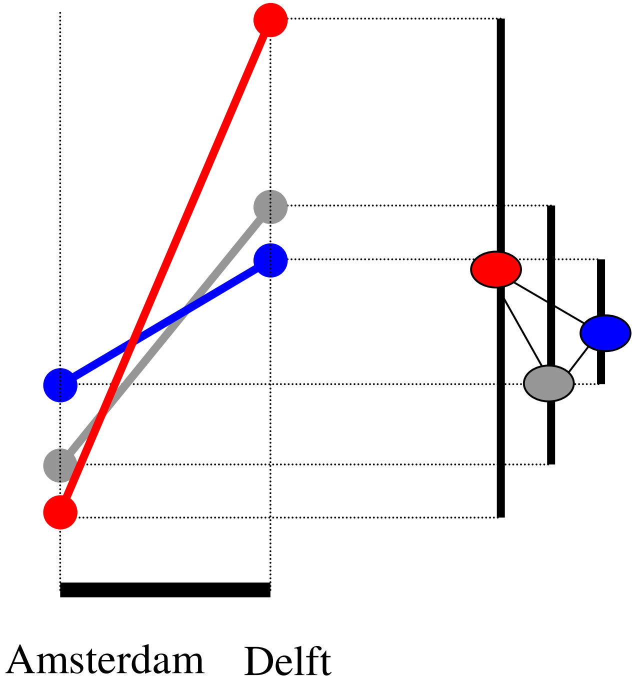
Headway conflicts

Is it possible to get all maximal (exp. ?) packing constraints in polynomial time ?



Block occupation

Intervall Graph !

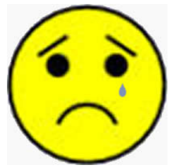
[Helly 1912, Borndörfer, Schlechte 2007]

Quadrangle-linear-headway matrices

Perfect Graph !

[Lukac 2004]

Arbitray Headway Matrices **?**



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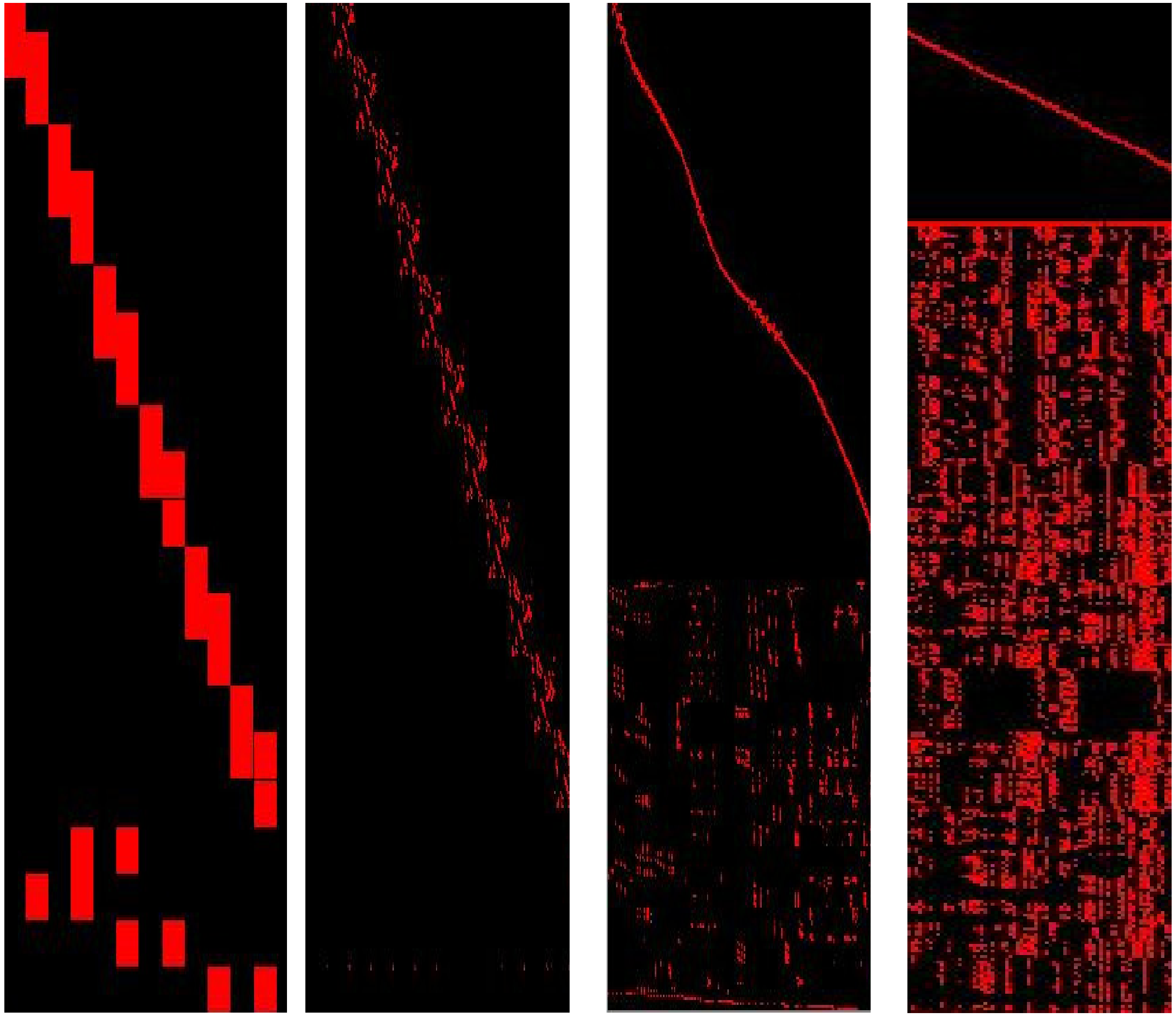
Packing Models

- **Proposition:** The LP-relaxation of APP can be solved in polynomial time.
- … and in practice.



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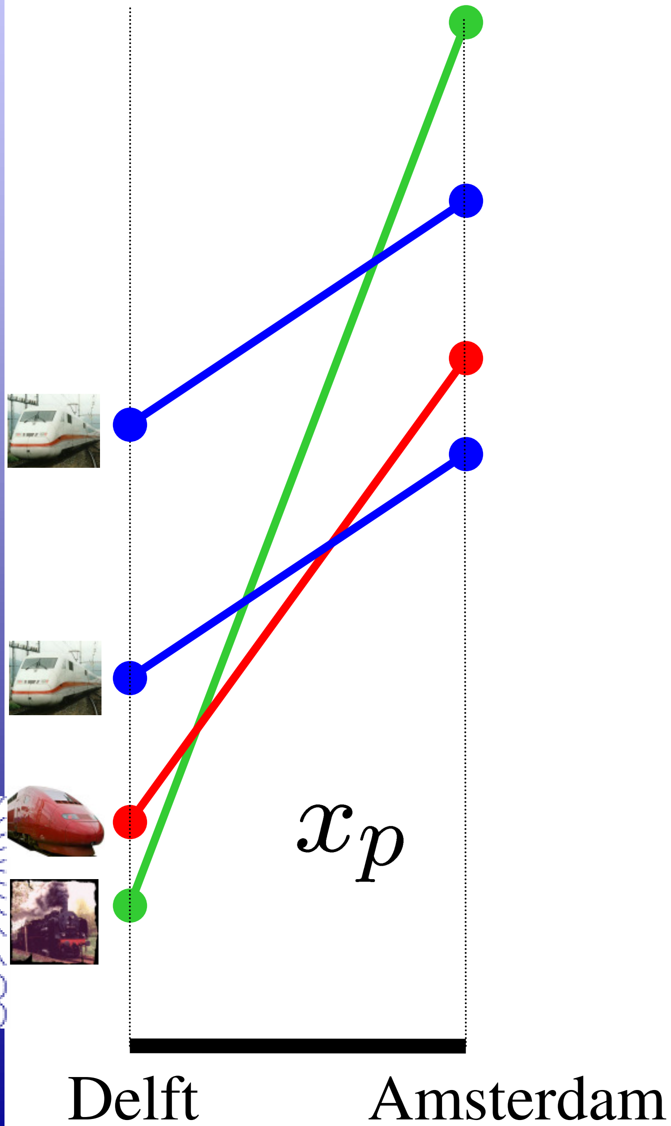
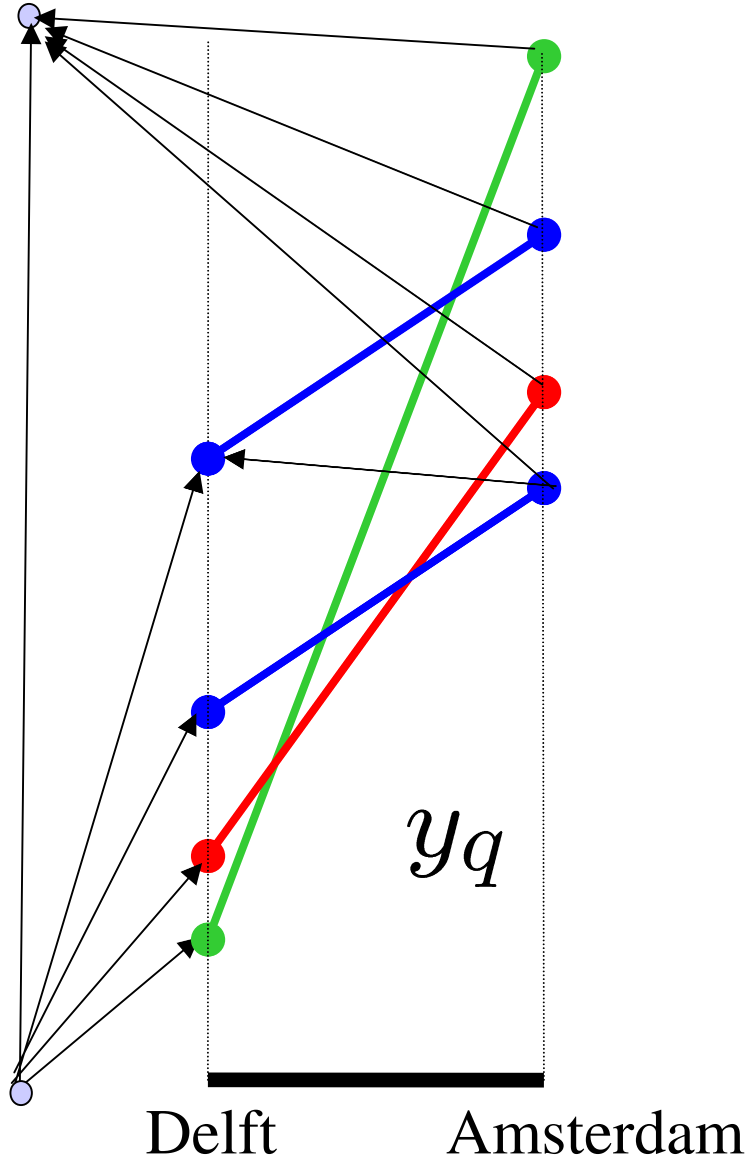
A(rc) C(oupling) P(roblem)





Path Coupling Problem

$$
\begin{array}{llrll}
(PCP) & \text{(i)} & \max \sum_{p\in P} w_p x_p & & \\
& \text{(ii)} & \sum_{p\in P_i} x_p & \leq 1, & \forall i \in I \\
& \text{(iii)} & \sum_{q\in Q_j} y_q & \leq 1, & \forall j \in J \\
& \text{(iv)} & \sum_{a\in p\in P} x_p - \sum_{a\in q\in Q} y_q & \leq 0, & \forall a \in A_{LR} \\
& \text{(v)} & x_p, y_q & \in \{0,1\} & \forall p \in P,\ q \in Q
\end{array}
$$

Variables

- Path und config usage (request i uses path p, track j uses config q)

Constraints

- Path and config choice
- Path-config-coupling (track capacity)

Objective Function

- Maximize proceedings

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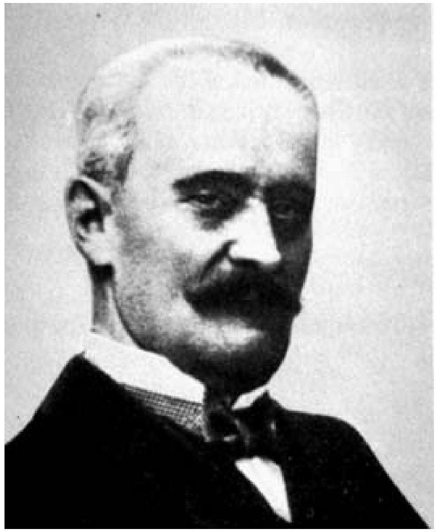
Duality of Linear Programs

Julius Farkas [1847-1930]:
Duality theorem based
Farkas Lemma „a theorem of alternatives"



(P)

minimize $\mathbf{c}^T\mathbf{x}$

subject to $\mathbf{A}\mathbf{x} \leq \mathbf{b},\ \mathbf{x} \geq 0$

(D)

maximize $\mathbf{b}^T\mathbf{y}$

subject to $\mathbf{A}^T\mathbf{y} \geq \mathbf{c},\ \mathbf{y} \geq 0$

PRICE(x) is equivalent to SEPERATION(y) !

LP with too many variables

nr	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	y
1	1						1	1											1	1	1	1							1	1	1					1		y1
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	y1
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	y1
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	y1
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	y1
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	y1
x	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18	x19	x20	x21	x22	x23	x24	x25	x26	x27	x28	x29	x30	x31	x32	x33	x34	x35	x36	x37	

- **Set Partitioning Problem**
  - minimize $c^T x$
  - row sum equal to 1

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LP Duality

nr	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	y
1	1						1	1											1	1	1	1							1	1	1					1		5
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	3
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	1
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	2
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	2
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	1
x																			1									1										

- **Strong Duality:** If there exists an optimal solution x* for (P), then there exists an optimal solution y* for (D) and their objective values are equal.

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Column Generation Example

nr	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	y
1	1						1	1											1	1	1	1							1	1	1					1		5
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	5
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	3
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	3
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	3
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	3
x	1	1	1	1	1	1																																

- $x_1=x_2=x_3=x_4=x_5=x_6=1$, primal objective $2*5+4*3=22$
- $y_1=5$

  $y_2=5$

  $y_3=3$

  $y_4=3$

  $y_5=3$

  $y_6=3$
- $c_{19}=9$, $y_1+y_2+y_3=5+5+3=13>9 \Rightarrow x_{19}=1$, $x_1=x_2=x_3=0$

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Column Generation Example

nr	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	y
1	1						1	1											1	1	1	1							1	1	1					1		1
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	5
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	3
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	3
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	3
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	3
x				1	1	1													1																			

- $x_1=x_2=x_3=0$, $x_4=x_5=x_6= x_{19}=1$, primal objective 9+3*3=18 [22]
- $y_1+y_2+y_3=11 \Rightarrow y_1=1,\ y_2=5,\ y_3=3$
- $y_4=3$
  $y_5=3$
  $y_6=3$
- $c_{34}=9,\ y_2+y_4+y_5+y_6=5+9=14>9$
  $c_{36}=12,\ y_1+y_3+y_4+y_5+y_6=1+12=13>12$

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Column Generation Example

nr	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	y
1	1						1	1											1	1	1	1							1	1	1					1		1
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	5
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	3
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	3
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	3
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	3
x				1	1	1													1																			

- $x_1=x_2=x_3=0$, $x_4=x_5=x_6= x_{19}=1$, primal objective 9+3*3=18 [22]
- $y_1+y_2+y_3=11 \Rightarrow y_1=1$, $y_2=5$, $y_3 =3$ (solution not unique)
  $y_4=3$
  $y_5=3$
  $y_6=3$
- $c_{34}=9$, $y_2+y_4+y_5+y_6=5+9=14>9$
  $c_{36}=12$, $y_1+y_3 +y_4 +y_5+y_6=1+12=13>12$
- $\Rightarrow x_{34}= x_{36}= x_{19}= 1/2$, $x_4=x_5=x_6=0$

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Column Generation Example

nr	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	y
1	1						1	1											1	1	1	1							1	1	1					1		5
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	3
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	1
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	3
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	3
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	0
x																			½															½		½		

- $x_{34} = x_{34} = x_{19} = 1/2$, primal objective $(9+9+12)/2=15$ [18]
- $y_1+y_2+y_3=9$

  $y_2+y_4+y_5+y_6 = 9$

  $y_1+y_3+y_4+y_5+y_5=12 \Rightarrow y_1=5,\ y_2 = y_4 = y_5 = 3,\ y_3 = 1$
- $c_{28}=5,\ y_4+y_5+y_6=6>5$
- $\Rightarrow x_{28} = x_{19} = 1,\ x_{34}=x_{36}=0$

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Column Generation Finished

nr	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
c	5	5	3	3	3	3	8	9	6	7	8	9	4	5	6	4	5	4	9	10	11	12	7	8	9	5	6	5	11	12	12	8	9	9	6	12	9	y
1	1						1	1											1	1	1	1							1	1	1					1		5
2		1					1		1	1	1	1							1	1	1	1	1	1	1							1	1	1			1	3
3			1					1	1				1	1	1				1				1	1	1	1	1		1	1		1	1		1	1	1	1
4				1						1			1			1	1			1			1			1	1	1	1	1	1	1	1	1	1	1	1	2
5					1						1			1		1		1			1			1		1		1	1		1	1		1	1	1	1	2
6						1						1			1		1	1				1			1		1	1		1	1		1	1	1	1	1	1
x																			1									1										

- $x_{19}= x_{28}= 1$, primal objective 9+5=14 [15]
- $y_1+y_2+y_3=9$

  $y_4+y_5+y_6 =5 \Rightarrow y_1= 5, y_2= 3, y_3= y_6= 1, y_4= y_5 = 2$
- No column exists with negative reduced cost
- $\Rightarrow$ x* and y* are optimal solutions

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History of Column Generation

Article	*Constraints*	*Variables*	*Time*
Charnes & Miller 1956	6	17	manual
Hoffman & Padberg 1993	145	1.053.137	5 min
Bixby, Gregory, Lustig, Marsten, Shanno 1992	837	12.753.313	249 sec
Barnhart, Johnson, Nemhauser, Savelsbergh, Vance 1998	>10.000	unknown	several days

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Linear Relaxation of PCP

$$
\begin{array}{lrlll}
(MLP) & & & & \\
\max & \sum_{p\in\mathcal{P}} w_p x_p + \sum_{q\in\mathcal{Q}} r_q y_q & & & \\
\text{s.t.} & \sum_{p\in\mathcal{P}_i} x_p & \leq 1 & \forall i \in I & \text{(i)} \\
 & \sum_{q\in\mathcal{Q}_j} y_q & \leq 1 & \forall j \in J & \text{(ii)} \\
 & \sum_{a\in p\in\mathcal{P}} x_p - \sum_{a\in q\in\mathcal{Q}} y_q & \leq 0 & \forall a \in A_{LR} & \text{(iii)} \\
 & 0 \leq y_q & \leq 1 & \forall q \in \mathcal{Q} & \text{(iii)} \\
 & 0 \leq x_p & \leq 1 & \forall p \in \mathcal{P} & \text{(iv)}
\end{array}
$$

γ_i
π_j
λ_a

dual variable	information about	useful to
γ_i	bundle price	analyse request
π_j	track price	analyse network
λ_a	arc price	-

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Dualization

$$
\begin{array}{llllll}
(DLP) & & & & & \\
\text{min} & \sum_{j\in J}\pi_j + \sum_{i\in I}\gamma_i & & & & \\
\text{s.t.} & \gamma_i + \sum_{a\in p}\lambda_a & \geq w_p & \forall p \in \mathcal{P}_i, \forall i \in I & & \text{(i)} \\
 & \pi_j - \sum_{a\in q}\lambda_a & \geq r_q & \forall q \in \mathcal{Q}_j, \forall j \in J & & \text{(ii)} \\
 & \gamma_i & \geq 0 & \forall i \in I & & \text{(iii)} \\
 & \lambda_a & \geq 0 & \forall a \in A_{LR} & & \text{(iv)} \\
 & \pi_j & \geq 0 & \forall j \in J & & \text{(v)}
\end{array}
$$

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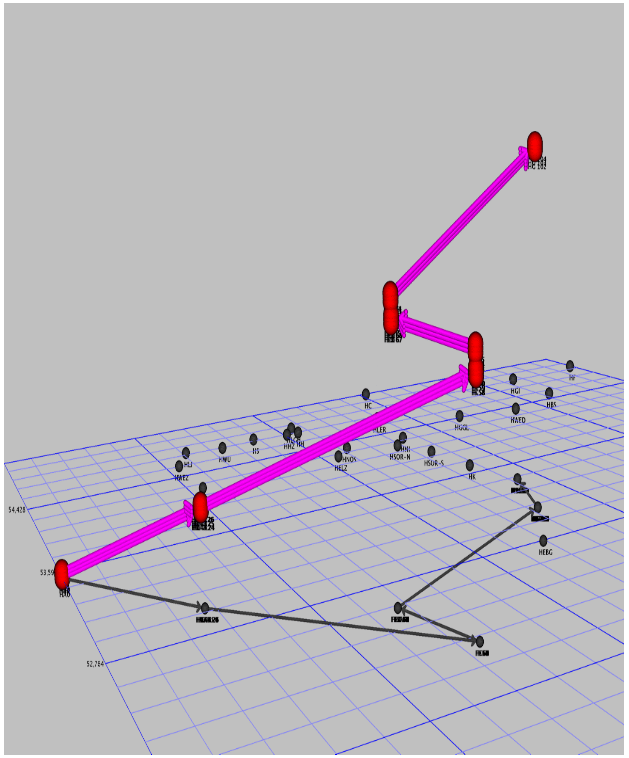
Pricing of x-variables



(PRICE (x)) $\exists\, \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)$

$$c_a = -p_a + \lambda_a$$

Pricing Problem(x) :
Acyclic shortest path problems for each slot request i with modified cost function c !

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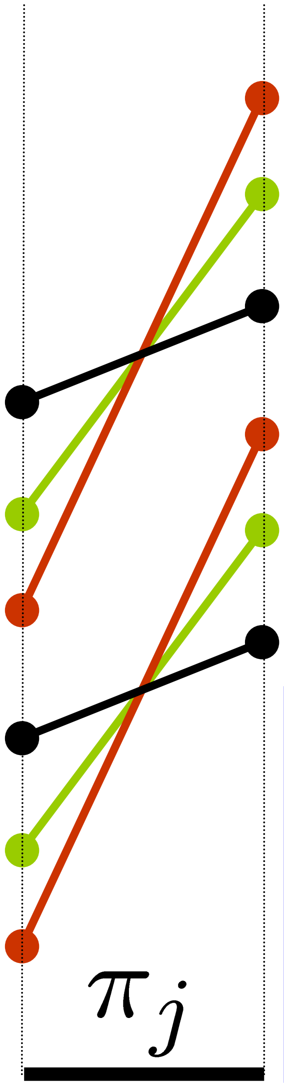
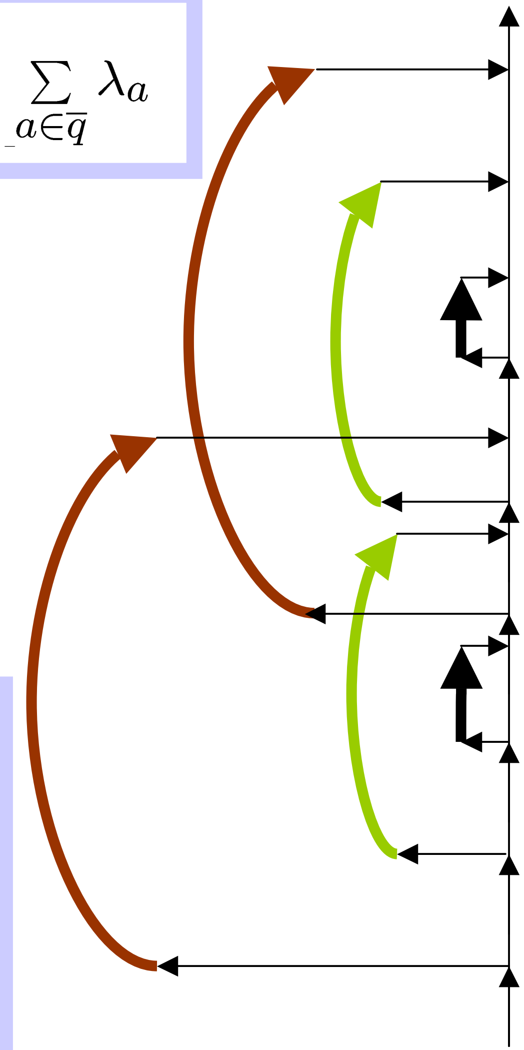
Pricing of y-variables



(PRICE (y)) $\exists\, \overline{q} \in Q_j : \quad \pi_j < r_{\overline{q}} + \sum_{a \in \overline{q}} \lambda_a$

$$c_a = -r_a - \lambda_a$$



<u>Pricing Problem(y) :</u>
Acyclic shortest path problem for each track j with modified cost function c !

Observation

- **Lemma** [ZR-07-02]: The linear relaxation of PCP can be solved in polynomial time, due to the equivalence of optimization and separation (see Groetschel, Lovasz & Schrijver [88]).

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Observation

$$\text{(PRICE (x))} \quad \exists\, \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)$$

$$\eta_i := \max_{p \in P_i} \sum_{a \in p} (p_a - \lambda_a) - \gamma_i, \ \forall i \in I$$

$$\eta_i + \gamma_i \geq \sum_{a \in p} (p_a - \lambda_a) \ \forall i \in I, p \in \mathcal{P}_i$$

And analogously ...

(PRICE (y)) $\exists\, \overline{q} \in Q_j : \quad \pi_j < \sum_{a \in \overline{q}} \lambda_a$

↳

$$\theta_j := \max_{\overline{q} \in Q_j} \sum_{a \in \overline{q}} \lambda_a - \pi_j, \ \forall j \in J$$

↳

$$\theta_j + \pi_j \geq \sum_{a \in q} \lambda_a \ \forall j \in J, q \in \mathcal{Q}_j$$

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Pricing Upper Bound

$(\max\{\eta+\gamma, 0\}, \max\{\theta+\pi, 0\}, \lambda)$ is feasible for (DLP)

$$\beta(\gamma,\pi,\lambda) := \sum_{i\in I} \max\{\gamma_i+\eta_i, 0\} + \sum_{j\in J} \max\{\pi_j+\theta_j, 0\}$$

- **Lemma** [ZR-07-02]: Given (infeasible) dual variables of PCP and let v_{LP}(PCP) be the optimum objective value of the LP-Relaxation of PCP, then:

$$v_{LP}(PCP) \leq \beta(\gamma,\pi,\lambda)$$

Overview

1. Idea & Motivation
2. Train Timetabling Problem
3. Models and Algorithms
    1. Historical Example
    2. IP Solving
    3. IP Models for TTP
    4. Column Generation
    5. Branch & Bound & Price
4. Computational Studies

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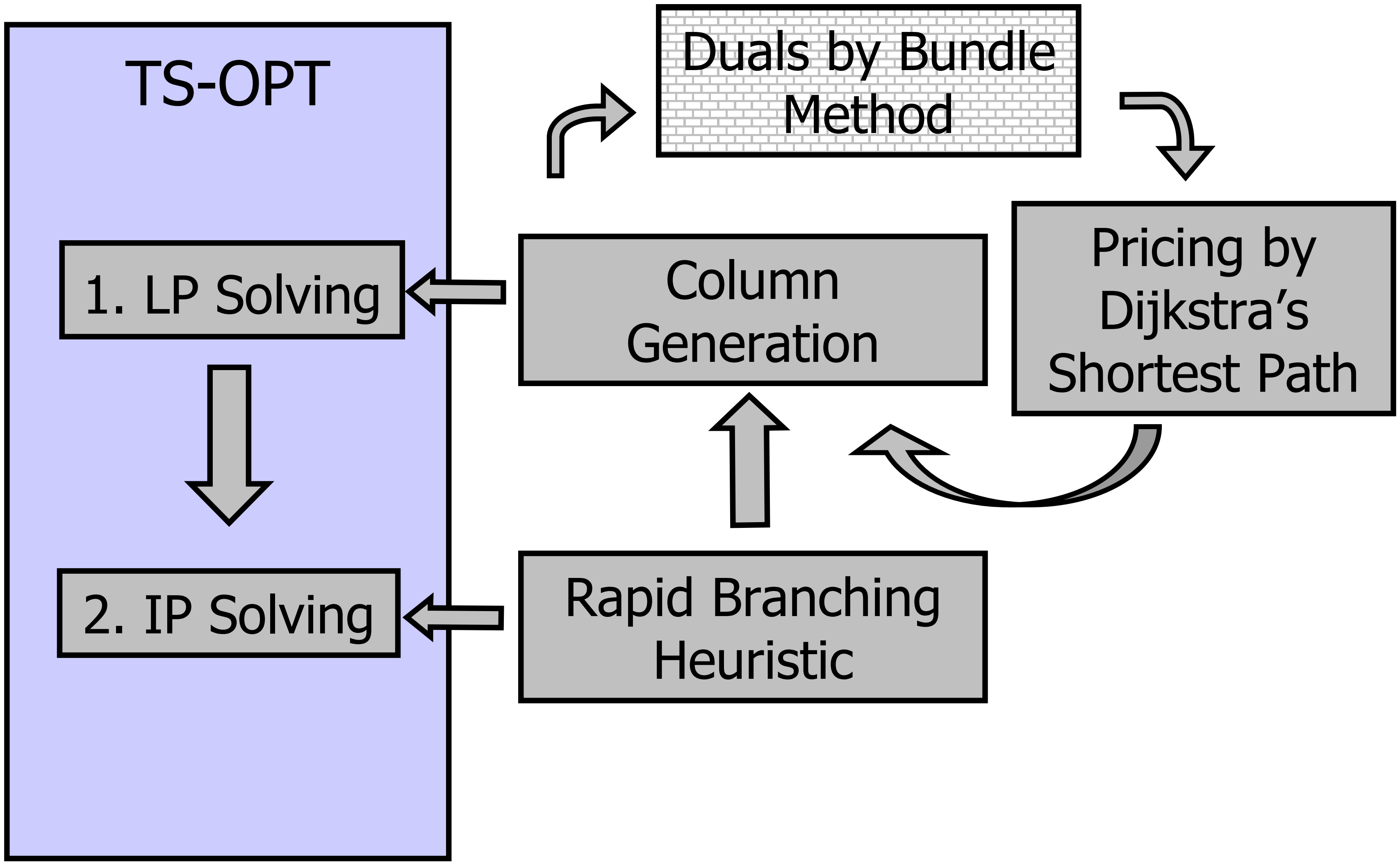
Two Step Approach



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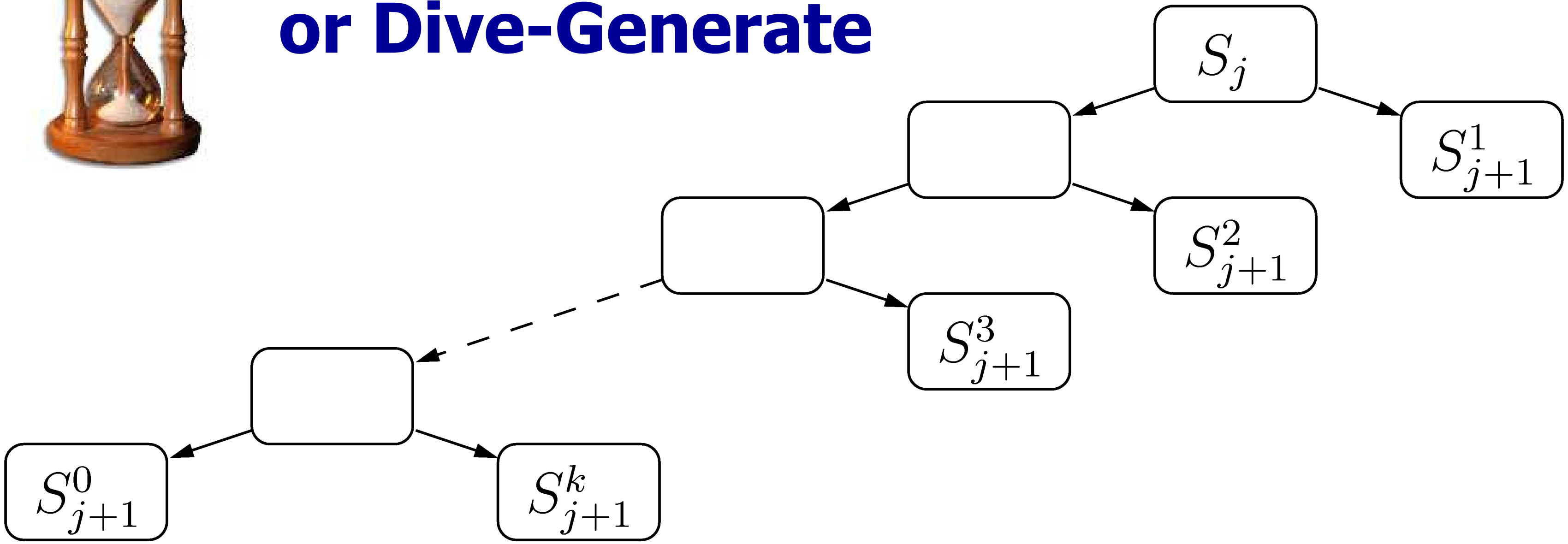
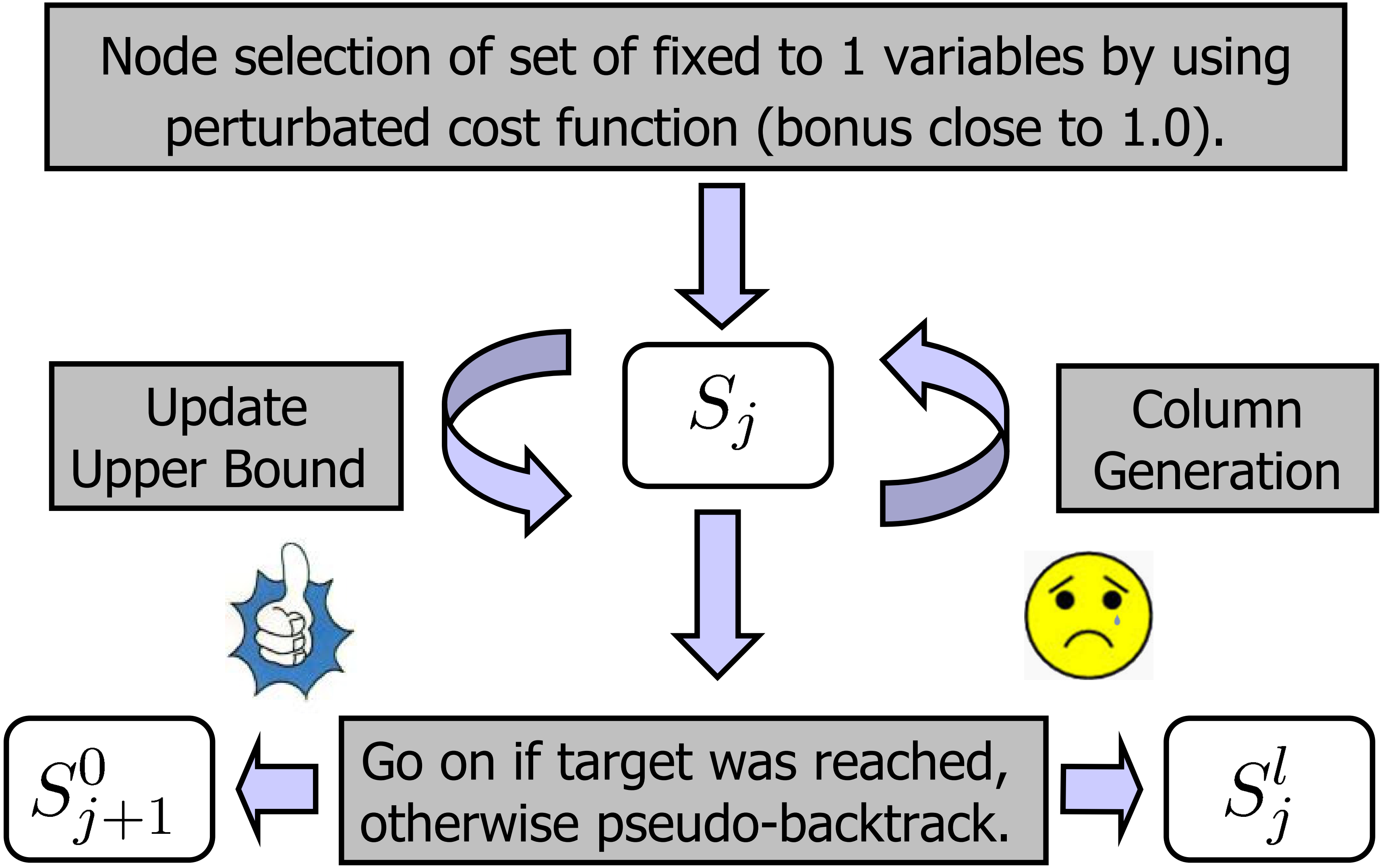
Branch-Bound-Price

or Dive-Generate



Evaluation of only few highly different sub-problems at iteration j to reach IP-Solutions fast.

Rapid Branching



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Overview

1. Idea & Motivation
2. Train Timetabling Problem
3. Models and Algorithms
4. Computational Studies

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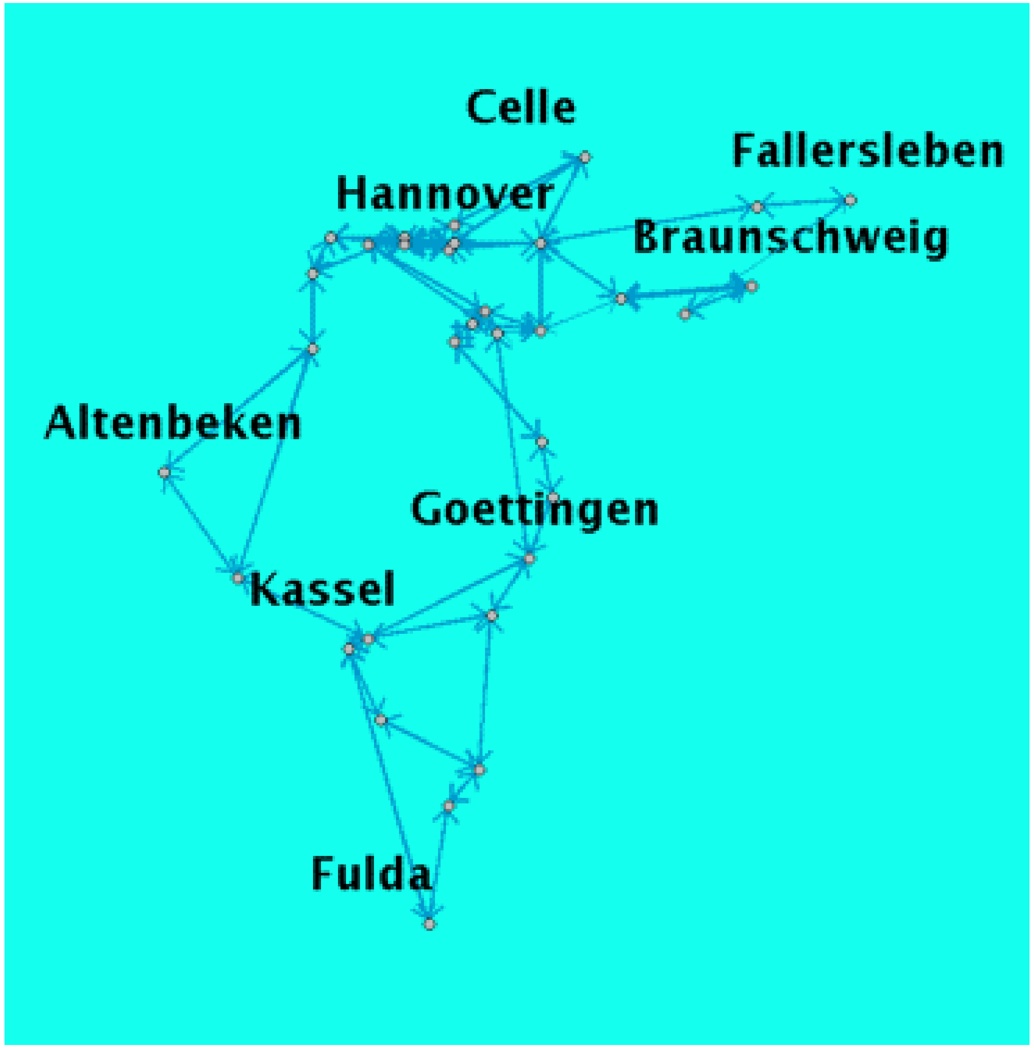
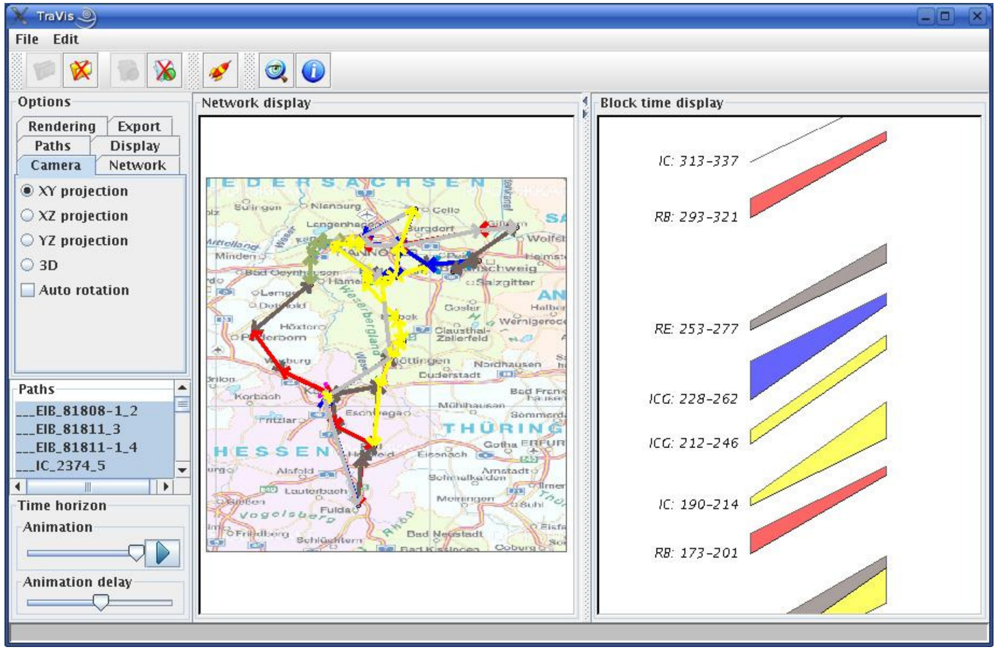
Results

- **Test Network**
  - **45 Tracks**
  - **37 Stations**
  - **6 Traintypes**
  - **10 Trainsets**
  - **146 Nodes**
  - **1480 Arcs**
  - **96 Station Capacities**
  - **4320 Headway Times**



Computational Results

- **Test Scenarios**
  - **146 Train Requests**
  - **285 Train Requests**
  - **570 Train Requests**



- **Flexibility**
  - **0-30 Minutes**
  - **earlier departure penalties**
  - **late arrival penalties**
  - **train type depending profits**

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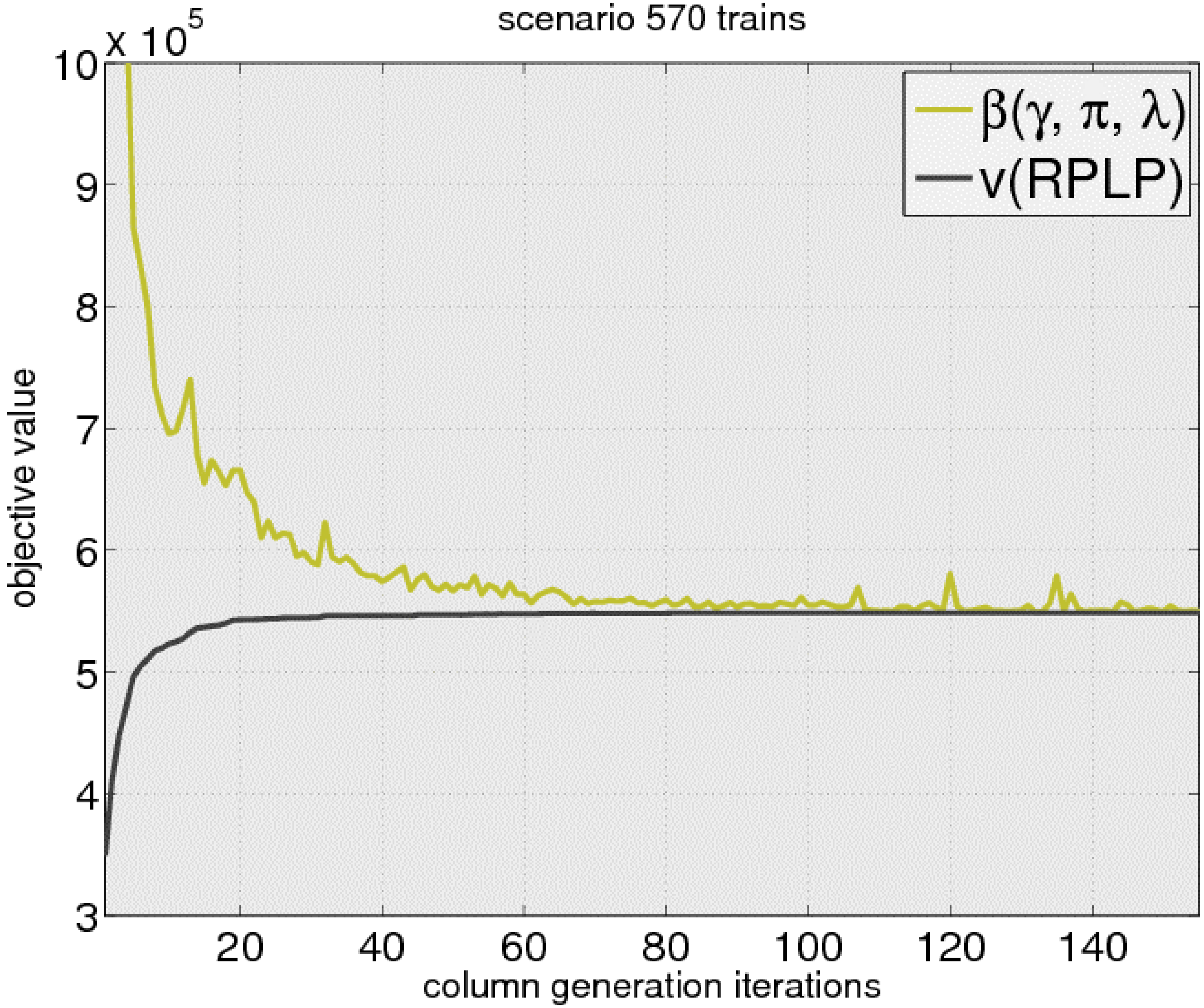
Run of TS-OPT/LP Stage



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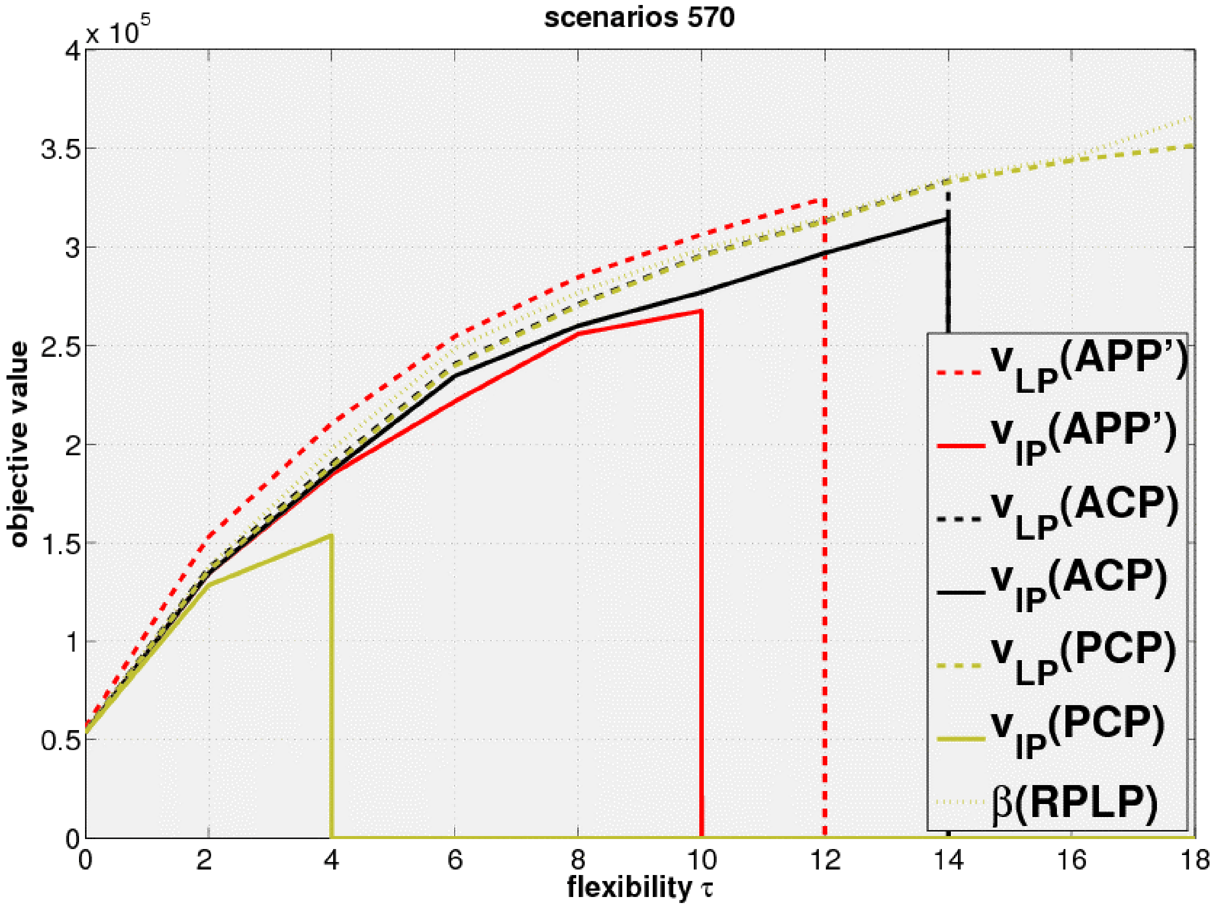
Model Comparison



For details see [ZR-07-02, ZR-07-20].

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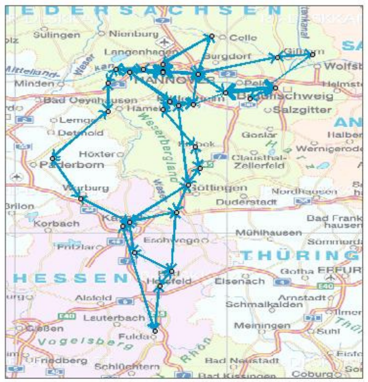
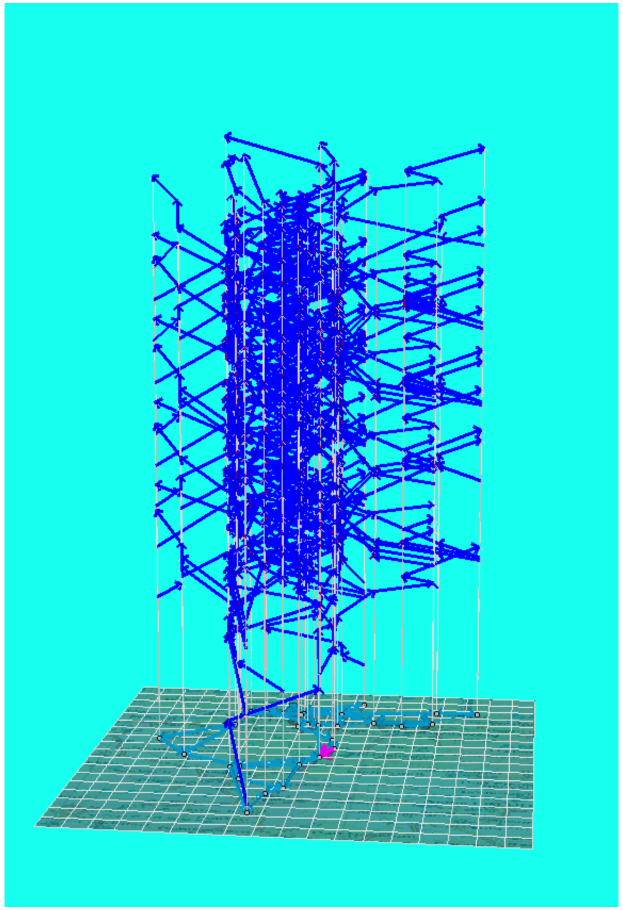
Scenario: Hannover-Kassel-Fulda

- **Situation**
  - ~ 8h time window
  - 250 trains (ICE,IC,RE,RB,S)
  - ~ original public timetable 2002
  - utility function 156468.0
    (~track prices)





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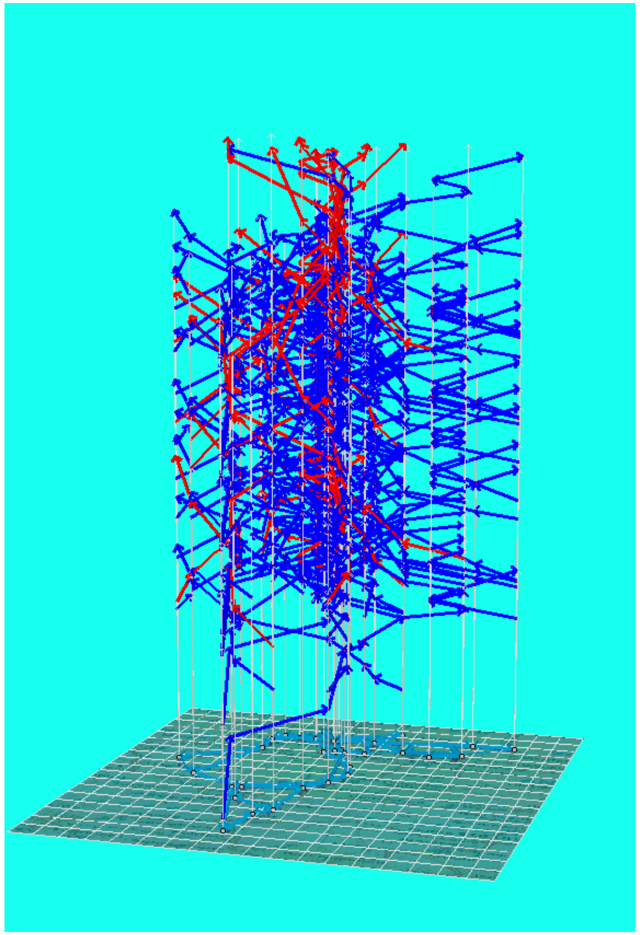
Optimzation problem with thousands of possible paths

- **additional demand of 35 cargo trains**

- **265 trains scheduled**
  - 237 „old“ passenger trains
  - 28 „new“ cargo trains
  - utility function 174628.0 (ca. + 12%)



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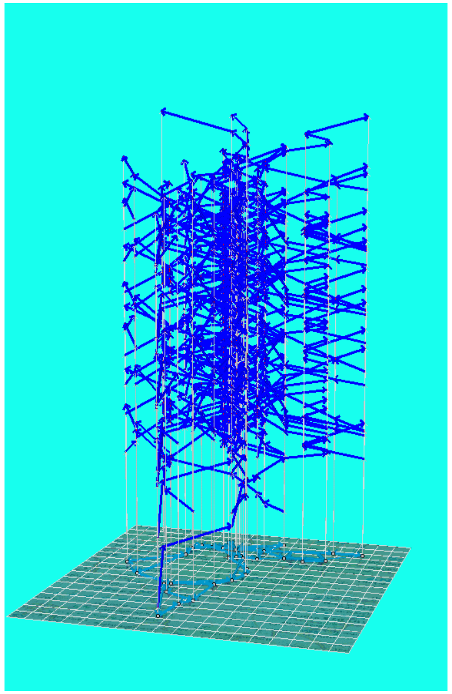
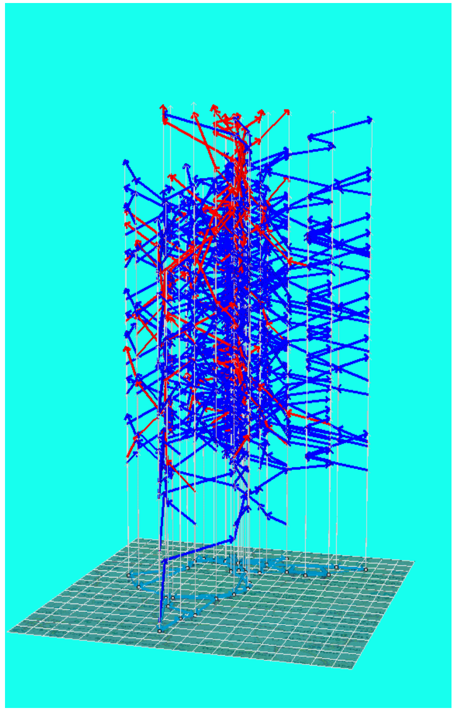
Don‘t try this by hand !

- **237 “old” passenger trains**
  - 13 deleted
  - more than 100 trains were shifted (at least one difference in departure or arrival at some station)
  - computation time: several minutes





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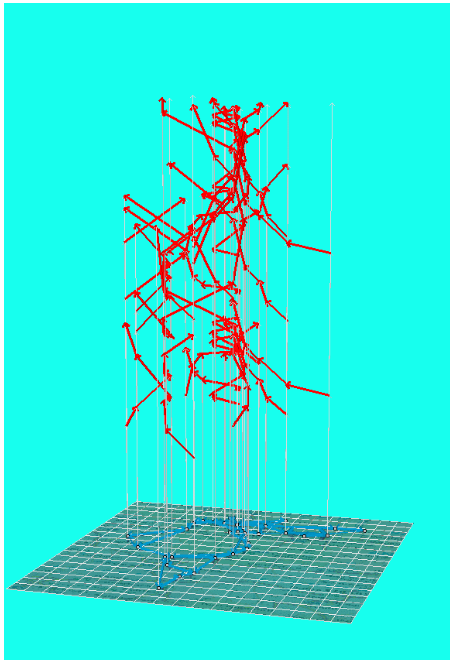
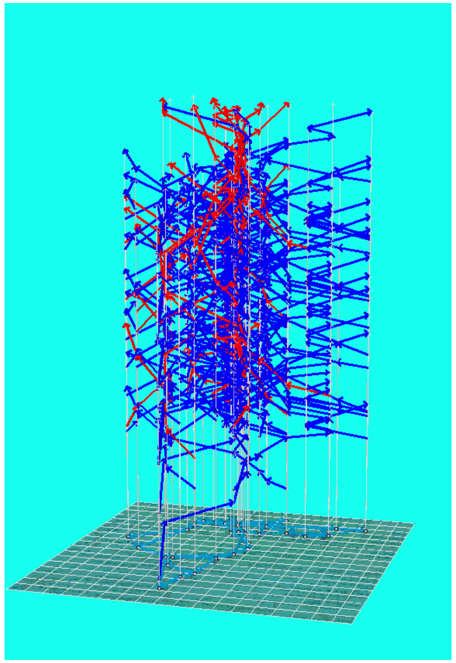
Scheduling with guarantee !

- **28 "new" cargo trains**
  - scheduling per hand is to complex, time-consuming and not resistant against mistakes of planners
  - there is a need to have decision support for macroscopic timetabling to support crucial strategic decisions
  - there is a need to evaluate macroscopic timetables on "real-world" microscopic stage





Outlook

Future Plans

- Model refinement (robustness)
- Model refinement (connections)
- Solver speedup by bundle method
- Adaptive IP heuristics
- Transformation Micro->Macro->Micro

Simulation of results by

OPEN TRACK

Railsys®

Thank you for your attention !

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