Models for Railway Track Allocation

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Overview

1. Problem Introduction
2. Model Discussion
3. Column Generation Approach
4. Computational Results
Planning in Public Transport

- Tracks
- Lines/Freq.
- Timetables
- Vehicles
- Crews

- Strategic Stage
  - Stops
  - Cycles

- Tactical Stage
  - Connections

- Operational Stage
  - Rotations
  - Duties
Traffic Projects @ ZIB

- VS-OPT
- DS-OPT
- IS-OPT
- TS-OPT
- MCF
- Telebus
- VS: BVG
- DS: BVG
- Line+Price Planning
- CS-OPT

Years:
- 92-94
- 94-97
- 97-00
- 00-03
- 03-07

Institutes:
- bmb+f
- BVG
- DFG
- Berlin
- ZIB
Planning in Public Transport

- Strategic Stage
  - B1 – B15
- Tactical Stage
  - TS-OPT
- Operational Stage
  - VS-OPT
  - B1 – B15
  - DS-OPT
  - IS-OPT
  - CS-OPT

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- Planning in Public Transport

- CS-OPT
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The Problem (TraVis by M.Kinder)
Schedule in 3d
Conflict-Free-Allocation
Railway Timetabling – State of the Art

- Charnes and Miller (1956), Szpiegal (1973), Jovanovic and Harker (1991),
- **Caprara, Fischetti and Toth (2002)**, Peeters (2003)
- Semet and Schoenauer (2005),
- **Caprara, Monaci, Toth and Guida (2005)**
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006),
- Caprara, Kroon, Monaci, Peeters, Toth (2006)

**non-cyclic timetabling literature**
Complexity

**Proposition** [Caprara, Fischetti, Toth (02)]: OPTRA/TTP is *NP*-hard.

**Proof:**
Reduction from Independent-Set.
Track Allocation Problem

Train Requests  →  Scheduling Digraph  →  Timetable
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Packing Models

- Conflict graph
- Cliques
- Perfect

Cacchiani (2007) – Path Compatibility Graphs
Arc Packing Problem

\[(APP)\]

\[\begin{align*}
\text{max} & \quad \sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}} p_a^i x_a^i \\
\text{s.t.} & \quad \sum_{a \in \delta_i^{\text{out}}(v)} x_a^i - \sum_{a \in \delta_i^{\text{in}}(v)} x_a^i \leq \delta_i(v) \quad \forall v \in V, \forall i \in \mathcal{I} \quad (i) \\
& \quad \sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}} x_a^i \leq 1 \quad \forall c \in \mathcal{C} \quad (ii) \\
& \quad x_a^i \in \{0, 1\} \quad \forall a \in \mathcal{A}, \forall i \in \mathcal{I} \quad (iii)
\end{align*}\]

**Variables**
- Arc occupancy (request i uses arc a)

**Constraints**
- Flow conservation and
- Arc conflicts (pairwise)

**Objective**
- Maximize proceedings

(PPP) transformation from arc to path variables (see Cachhiani (2007))
Packing Models

- **Proposition:** The LP-relaxation of APP can be solved in polynomial time.
- ... and in practice.
Novel Model

- Track Digraph
- Timeline(s)
- Config paths

Artificial arcs represent valid successors!
Path Coupling Problem

\[(PCP)\]
\[\max \quad \sum_{p \in \mathcal{P}} \sum_{a \in p} p_a^i x_p\]
\[\text{s.t.} \quad \sum_{p \in \mathcal{P}_i} x_p \leq 1 \quad \forall i \in I \quad (i)\]
\[\quad \sum_{q \in \mathcal{Q}_j} y_q \leq 1 \quad \forall j \in J \quad (ii)\]
\[\quad \sum_{a \in p \in \mathcal{P}} x_p - \sum_{a \in q \in \mathcal{Q}} y_q \leq 0 \quad \forall a \in \mathcal{A}_I \cap \mathcal{A}_J \quad (iii)\]
\[\quad y_q \in \{0, 1\} \quad \forall q \in \mathcal{Q}_j, \forall j \in J \quad (iv)\]
\[\quad x_p \in \{0, 1\} \quad \forall p \in \mathcal{P}_i, \forall i \in I \quad (v)\]

Variables
- Path und config usage (request i uses path p, track j uses config q)

Constraints
- Path and config choice
- Path-config-coupling (track capacity)

Objective Function
- Maximize proceedings

(ACP) transformation from path to arc variables (see Borndörfer, S. (2007))
Linear Relaxation of PCP

\[(MLP)\]
\[
\max \sum_{p \in \mathcal{P}} \sum_{a \in p} p_a^i x_p
\]
\[\text{s.t.} \quad \sum_{p \in \mathcal{P}_i} x_p \leq 1 \quad \forall i \in I \quad (i)\]
\[\sum_{q \in \mathcal{Q}_j} y_q \leq 1 \quad \forall j \in J \quad (ii)\]
\[\sum_{a \in \mathcal{P}} x_p - \sum_{a \in \mathcal{Q}} y_q \leq 0 \quad \forall a \in A_I \cup A_J \quad (iii)\]
\[0 \leq y_q \leq 1 \quad \forall q \in \mathcal{Q} \quad (iii)\]
\[0 \leq x_p \leq 1 \quad \forall p \in \mathcal{P} \quad (iv)\]

<table>
<thead>
<tr>
<th>dual variable</th>
<th>information about</th>
<th>useful to</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma_i)</td>
<td>bundle price</td>
<td>analyse request</td>
</tr>
<tr>
<td>(\pi_j)</td>
<td>track price</td>
<td>analyse network</td>
</tr>
<tr>
<td>(\lambda_a)</td>
<td>arc price</td>
<td>-</td>
</tr>
</tbody>
</table>
Dualization

\[ (DLP) \]
\[
\begin{align*}
\text{min} & \quad \sum_{j \in J} \pi_j + \sum_{i \in I} \gamma_i \\
\text{s.t.} & \quad \gamma_i + \sum_{a \in p} \lambda_a \geq \sum_{a \in p} p^i_a \quad \forall p \in \mathcal{P}_i, \forall i \in I \quad (i) \\
& \quad \pi_j - \sum_{a \in q} \lambda_a \geq 0 \quad \forall q \in \mathcal{Q}_j, \forall j \in J \quad (ii) \\
& \quad \gamma_i \geq 0 \quad \forall i \in I \quad (iii) \\
& \quad \lambda_a \geq 0 \quad \forall a \in A_{I \cup J} \quad (iv) \\
& \quad \pi_j \geq 0 \quad \forall j \in J \quad (v)
\end{align*}
\]
Pricing of x-variables

\[(\text{PRICE}(x)) \exists p \in P_i: \gamma_i < \sum_{a \in p} (p_a - \lambda_a)\]

\[c_a = -p_a + \lambda_a\]

Pricing Problem(x):
Acyclic shortest path problems for each slot request i with modified cost function c!
Pricing of $y$-variables

(PRICING (y)) \( \exists q \in Q_j : \pi_j < \sum_{a \in q} \lambda_a \)

\[ c_a = -\lambda_a \]

Pricing Problem (y):
Acyclic shortest path problem for each track j with modified cost function $c$!
Observation

\[(\text{PRICE } (x)) \quad \exists \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)\]

\[\eta_i := \max_{p \in \mathcal{P}_i} \sum_{a \in p} (p_a - \lambda_a) - \gamma_i, \quad \forall i \in I\]

\[\eta_i + \gamma_i \geq \sum_{a \in p} (p_a - \lambda_a) \quad \forall i \in I, p \in \mathcal{P}_i\]

\[\eta_i + \gamma_i \text{ satisfies } (DLP)(i)\]
And analogously ...

(PRICE (y)) \( \exists \overline{q} \in Q_j : \pi_j < \sum_{a \in \overline{q}} \lambda_a \)

\( \theta_j := \max_{\overline{q} \in Q_j} \sum_{a \in \overline{q}} \lambda_a - \pi_j, \forall j \in J \)

\( \theta_j + \pi_j \geq \sum_{a \in q} \lambda_a \forall j \in J, q \in Q_j \)

\( \theta_j + \pi_j \) satisfies \((DLP)(ii)\)
Pricing Upper Bound

\[(\max\{\eta+\gamma, 0\}, \max\{\theta+\pi, 0\}, \lambda) \text{ is feasible for (DLP)}\]

\[\beta(\gamma, \pi, \lambda) := \sum_{i \in I} \max\{\gamma_i + \eta_i, 0\} + \sum_{j \in J} \max\{\pi_j + \theta_j, 0\}\]

- **Lemma [ZR-07-02]**: Given (infeasible) dual variables of PCP and let \(v_{LP}(PCP)\) be the optimum objective value of the LP-Relaxtion of PCP, then:

\[v_{LP}(PCP) \leq \beta(\gamma, \pi, \lambda)\]
Model Comparison

- **Theorem [ZR-07-02]:** The LP-relaxations of ACP and PCP can be solved in polynomial time.

- **Lemma [ZR-07-02]:**
  \[
  v_{LP}(PCP) = v_{LP}(ACP) = v_{LP}(APP) = v_{LP}(PPP) 
  \leq v_{LP}(APP')
  \]
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Two Step Approach

TS-OPT

1. LP Solving

2. IP Solving

Duals by Bundle Method

Column Generation

Pricing by Dijkstra’s Shortest Path

Rapid Branching Heuristic
Branch-Bound-Price

or Dive-Generate

Evaluation of only few highly different sub-problems at iteration j to reach IP-Solutions fast.
Rapid Branching

Node selection of set of fixed to 1 variables by using perturbated cost function (bonus close to 1.0).

- Update Upper Bound
- Column Generation
- Go on if target was reached, otherwise pseudo-backtrack.
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Results

- **Test Network**
  - 45 Tracks
  - 37 Stations
  - 6 Traintypes
  - 10 Trainsets
  - 146 Nodes
  - 1480 Arcs
  - 96 Station Capacities
  - 4320 Headway Times
Model Comparison

- **Test Scenarios**
  - 146 Train Requests
  - 285 Train Requests
  - 570 Train Requests

- **Flexibility**
  - 0-30 Minutes
  - earlier departure penalties
  - late arrival penalties
  - train type depending profits
Run of TS-OPT / LP Stage

scenario 570 trains

objective value

column generation iterations

$\beta(\gamma, \pi, \lambda)$

$\nu(\text{RPLP})$
Model Comparison

The graph shows the comparison of different model scenarios for flexibility $\tau$. The x-axis represents the flexibility $\tau$, while the y-axis shows the objective value. Various models are compared, indicated by different line styles and colors:
- $v_{LP}(APP')$
- $v_{IP}(APP')$
- $v_{LP}(ACP)$
- $v_{IP}(ACP)$
- $v_{LP}(PCP)$
- $v_{IP}(PCP)$

Scenarios 146 are plotted, with the objective value ranging from $8.2 \times 10^4$ to $9.6 \times 10^4$. The graph illustrates how each model performs under varying levels of flexibility.
Model Comparison

For details see [ZR-07-02, ZR-07-20].
Outlook

Algorithmic Developments

- Bundle method
- Model refinement (connections)
- Adaptive IP Heuristics
- Dynamic Discretization

Simulation of results by

Railsys®
Thank you for your attention!

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