Federal Ministry of Economics and Technology

A suitable Model for a bicriteria Optimization Approach to Railway Track Allocation

Thomas Schlechte

joint work with Ralf Borndörfer

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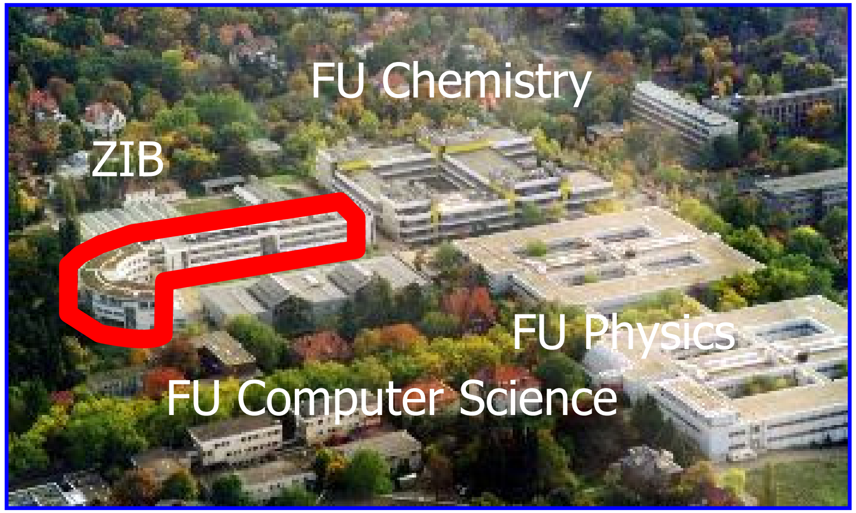

K. Zuse with Z1

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Overview

1. Problem Introduction
2. Bicriteria Optimization Model
3. Column Generation Approach
4. Computational Results

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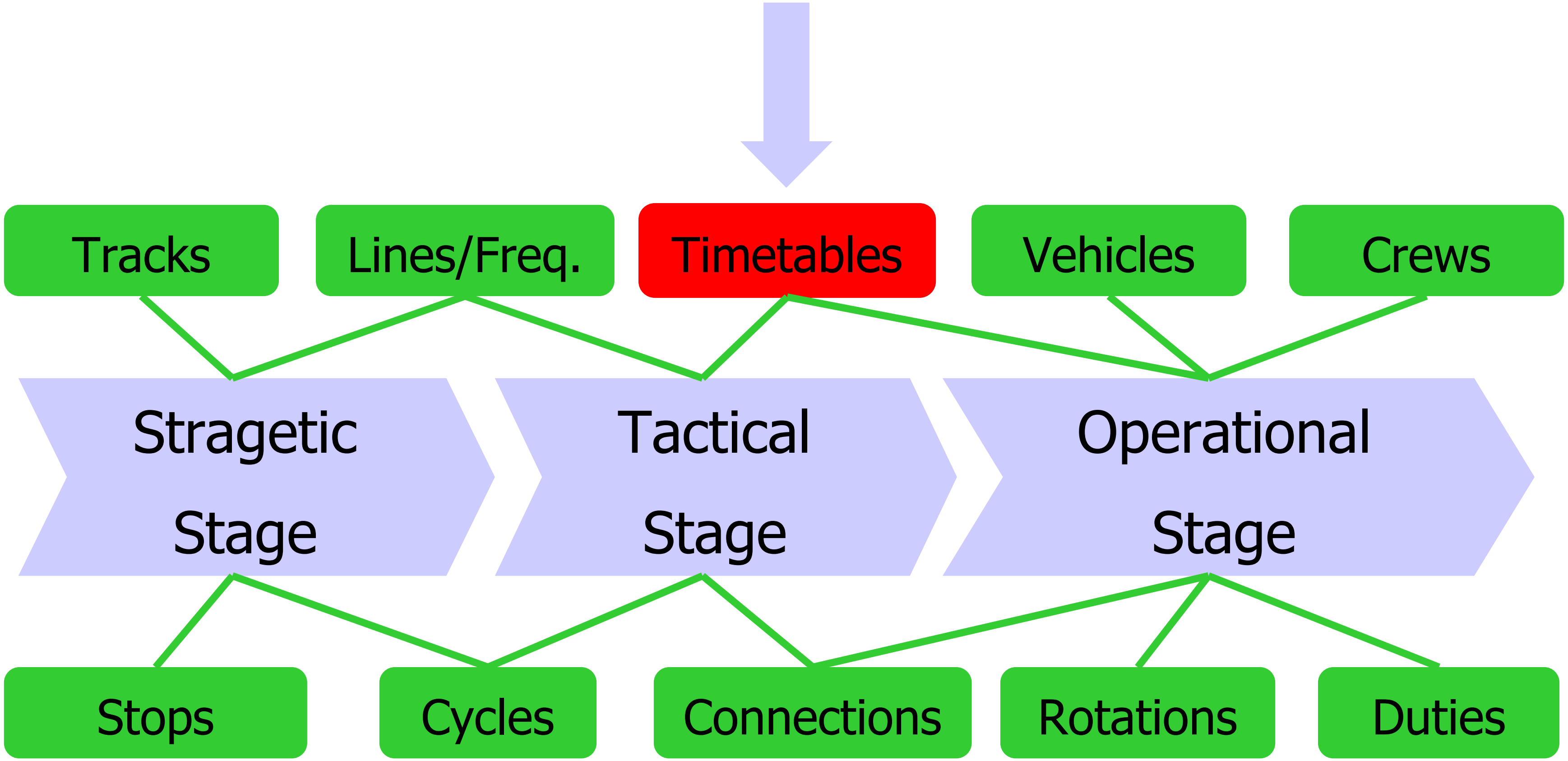
Planning in Public Transport



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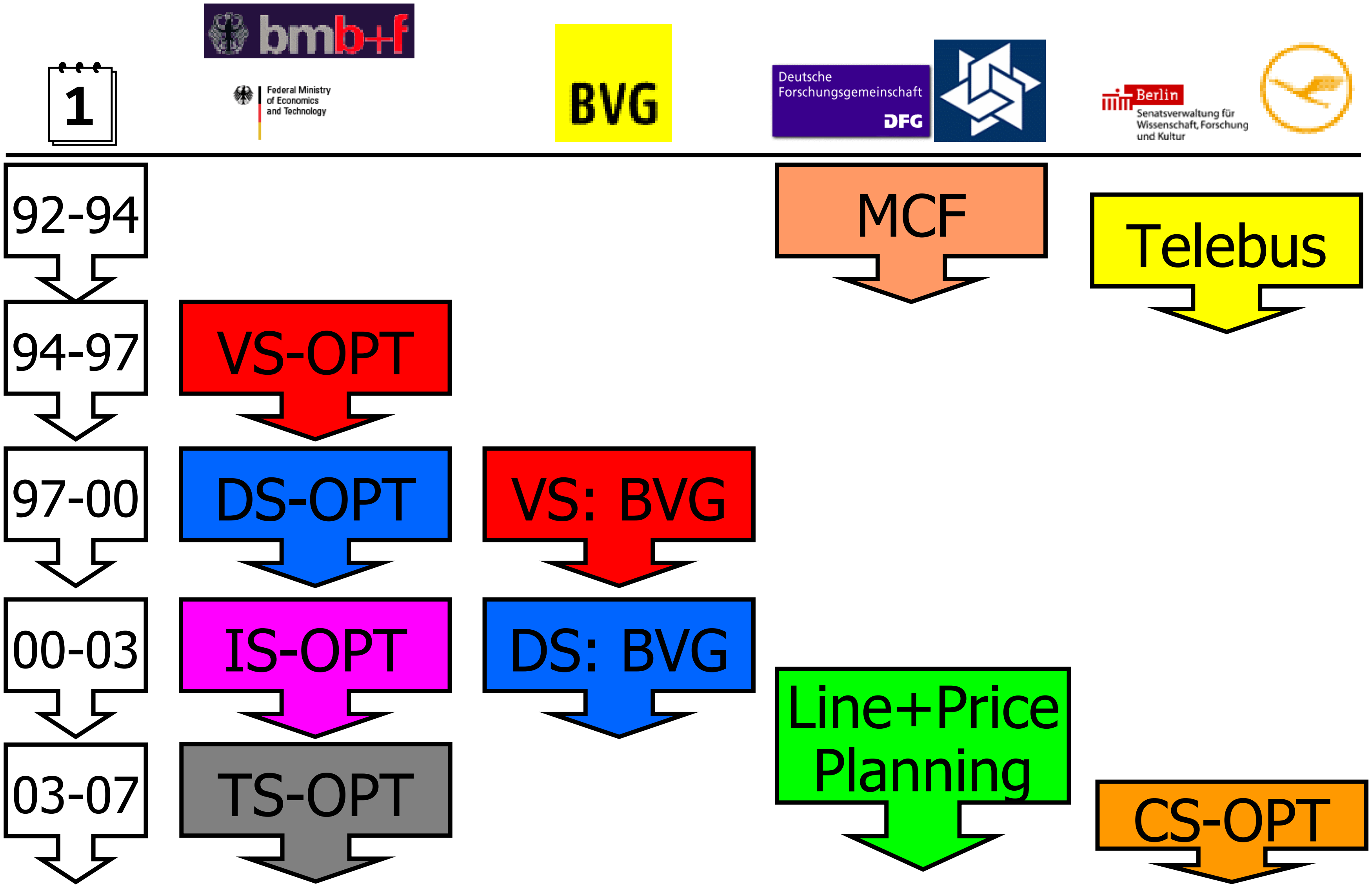
Traffic Projects @ ZIB



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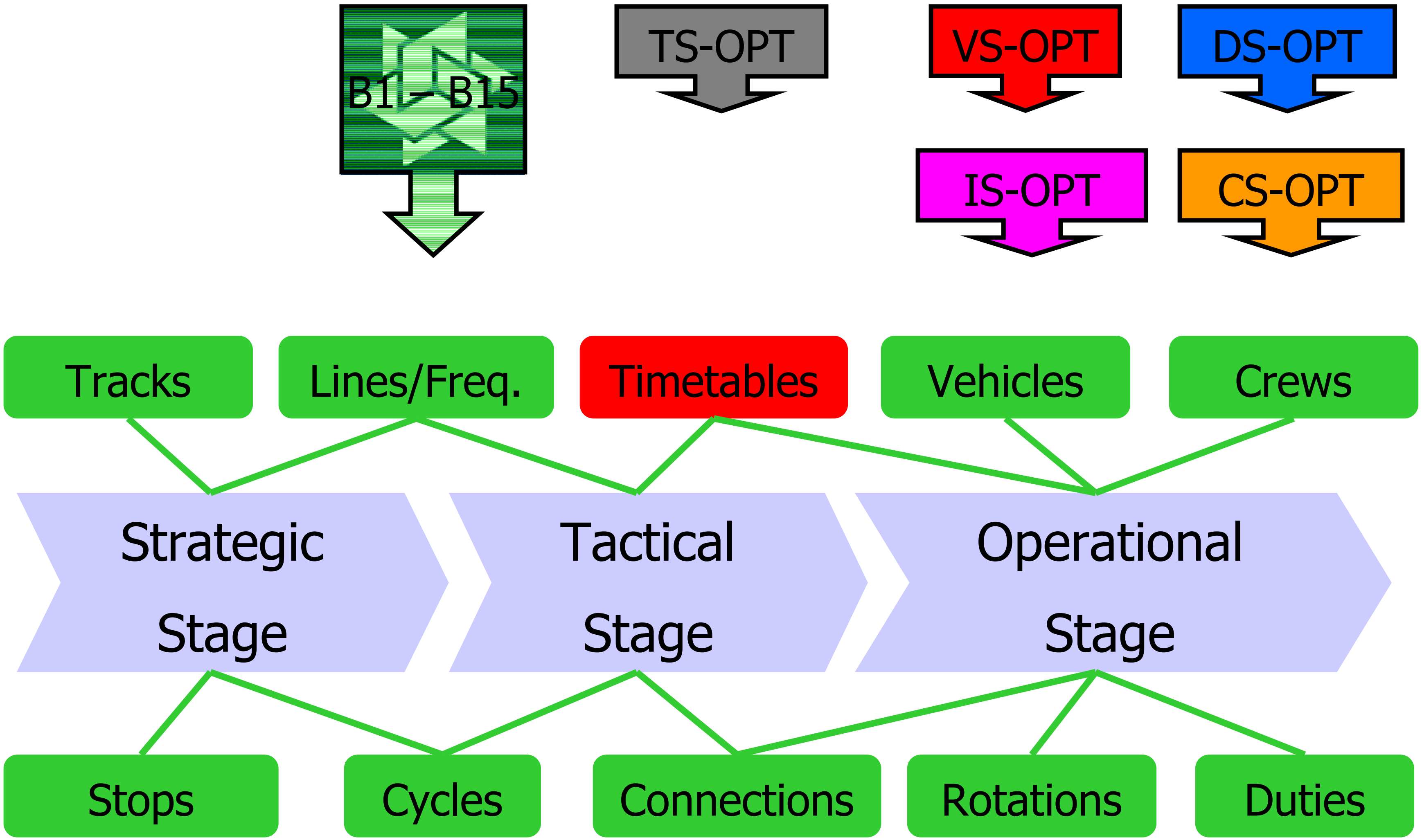
Planning in Public Transport



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The Problem (TraVis by M.Kinder)

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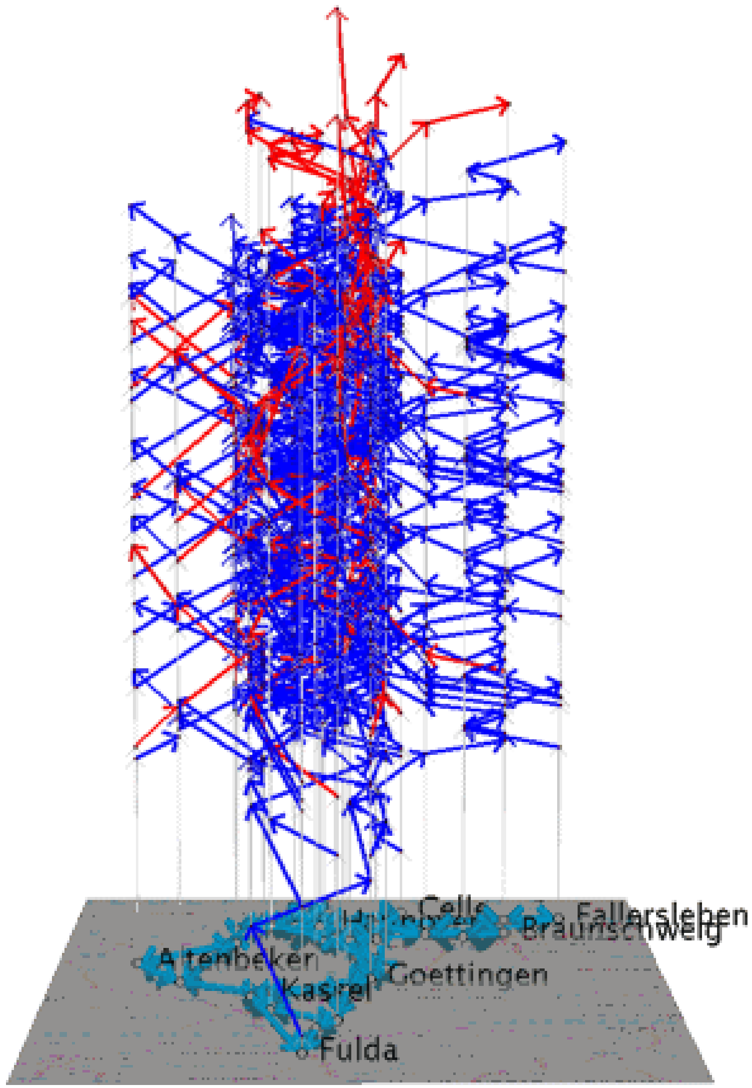
Schedule in 3d



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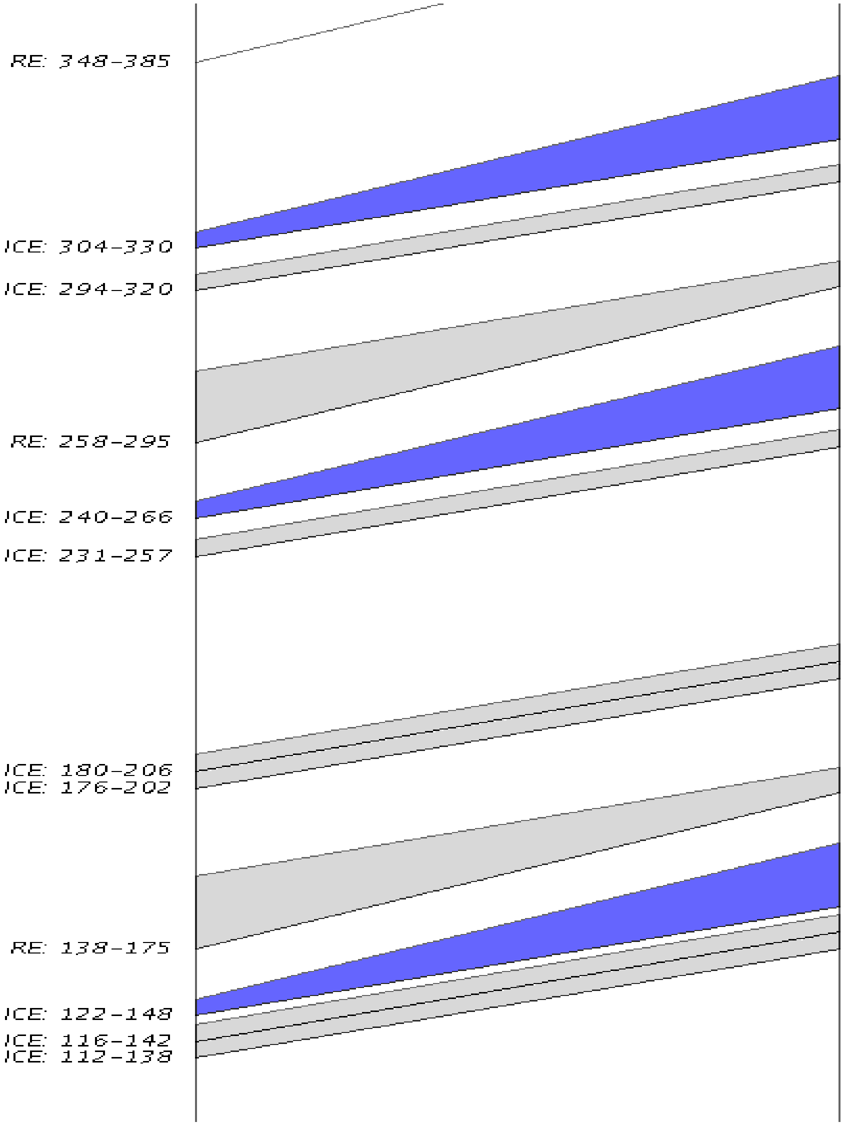
Conflict-Free-Allocation



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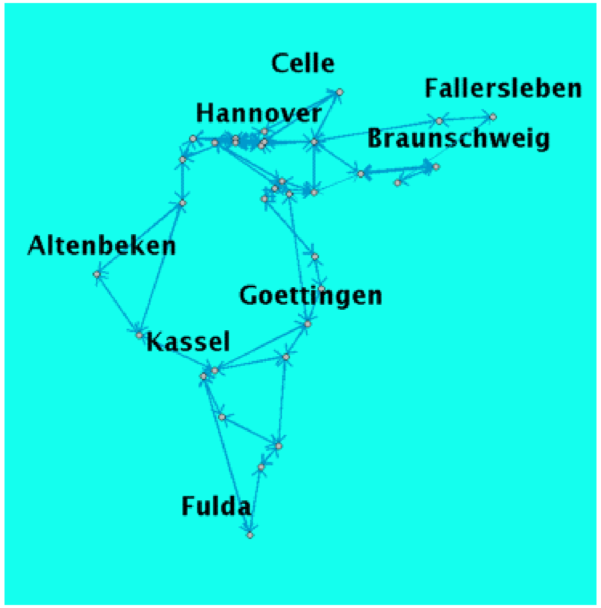
Railway Timetabling – State of the Art



- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Cai and Goh (1994), Schrijver and Steenbeck (1994), Carey and Lockwood (1995)
- Nachtigall and Voget (1996), Odijk (1996) Higgings, Kozan and Ferreira (1997)
- **Brannlund, Lindberg, Nou, Nilsson (1998)**, Lindner (2000), Oliveira and Smith (2000)
- **Caprara, Fischetti and Toth (2002)**, Peeters (2003)
- Kroon and Peeters (2003), Mistry and Kwan (2004)
- Barber, Salido, Ingolotti, Abril, Lova, Tormas (2004)
- Semet and Schoenauer (2005),
- **Caprara, Monaci, Toth and Guida (2005)**
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006),
- **Cacchiani, Caprara, T. (2006), Cachhiani (2007)**
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- **Borndoerfer, Schlechte (2007)**

non-cyclic timetabling literature

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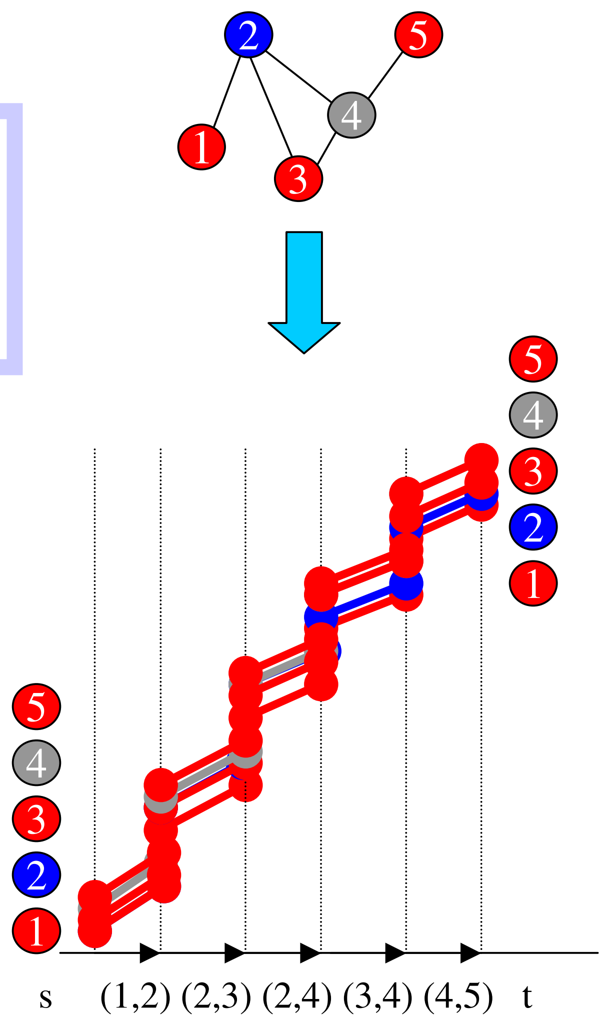
Complexity

Proposition [Caprara, Fischetti, Toth (02)]:

OPTRA/TTP is $\mathcal{NP}$-hard.

Proof:

Reduction from Independent-Set.



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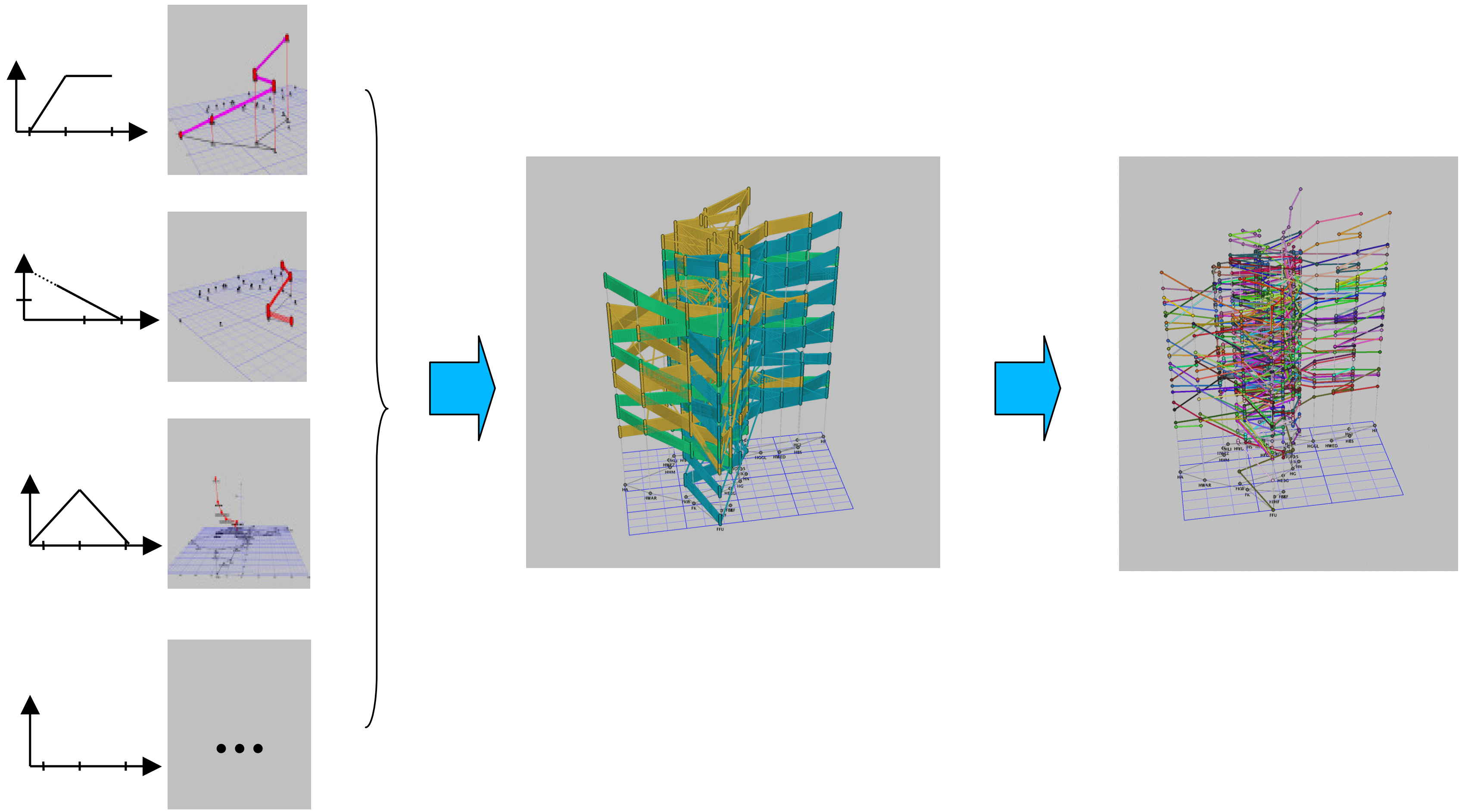
Track Allocation Problem



Train Requests → Scheduling Digraph → Timetable

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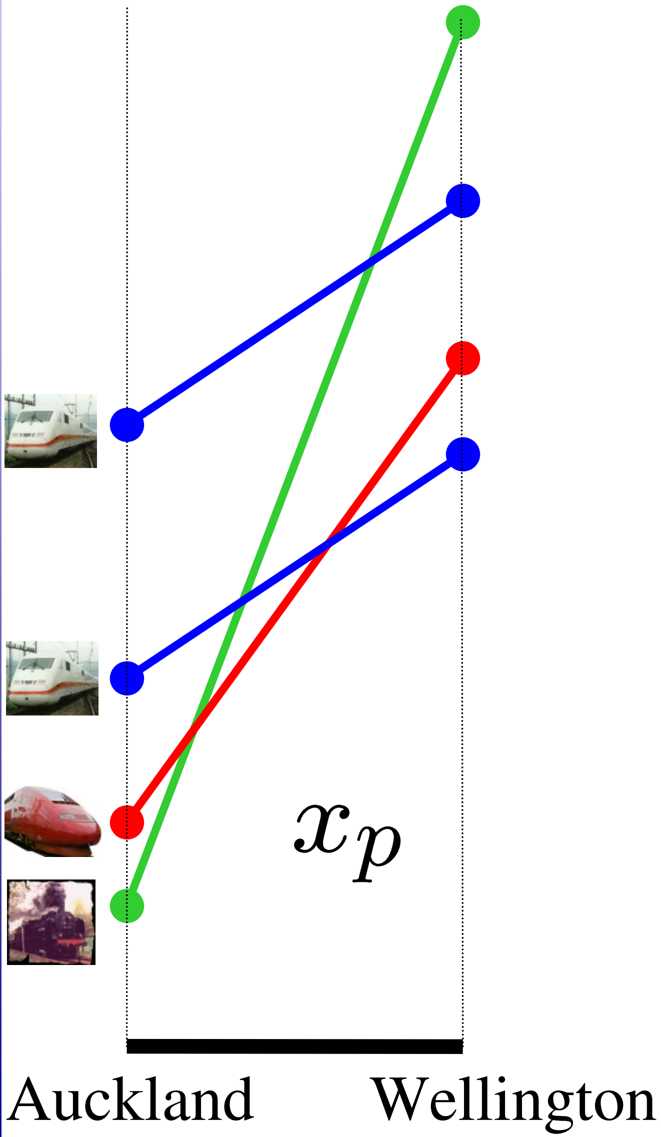
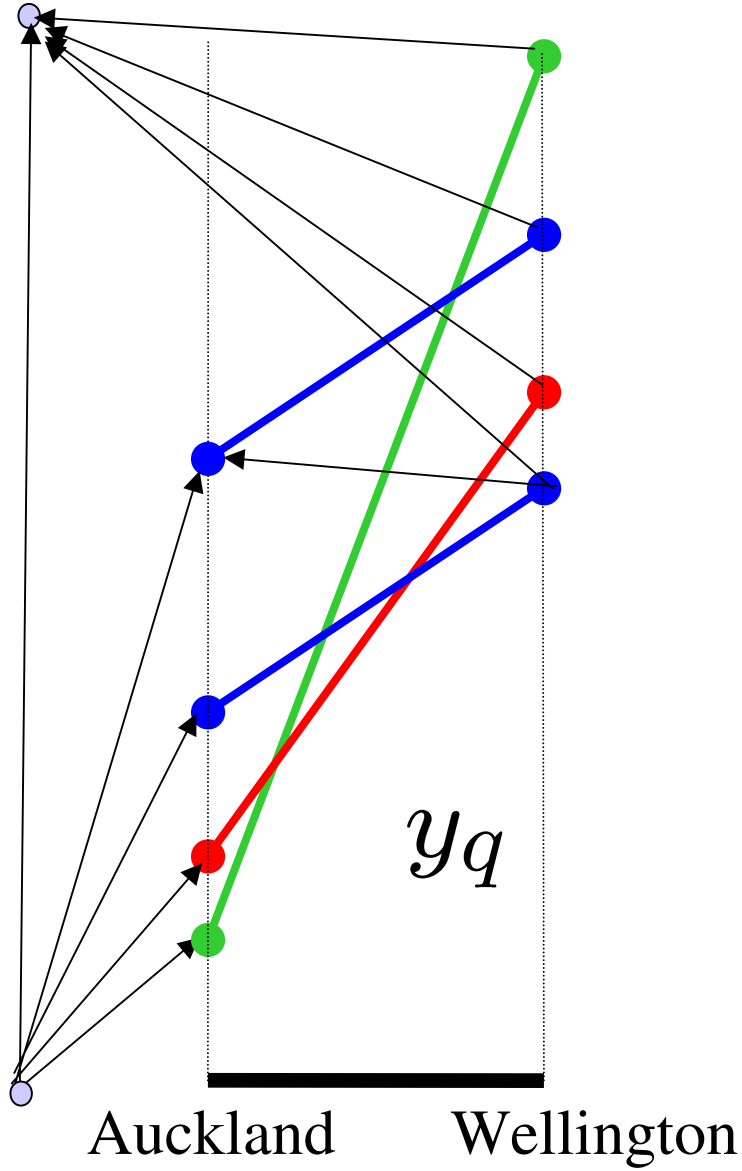
Single Objective Model





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Path Coupling Problem

$$
\begin{array}{lllrll}
(PCP) & \text{(i)} & & \max \sum_{p\in P} w_p x_p & & \\
& \text{(ii)} & & \sum_{p\in P_i} x_p & \leq 1, & \forall i \in I \\
& \text{(iii)} & & \sum_{q\in Q_j} y_q & \leq 1, & \forall j \in J \\
& \text{(iv)} & \sum_{a\in p\in P} x_p - & \sum_{a\in q\in Q} y_q & \leq 0, & \forall a \in A_{LR} \\
& \text{(v)} & & x_p, y_q & \in \{0,1\} & \forall p \in P,\ q \in Q
\end{array}
$$

Variables

- Path und config usage (request i uses path p, track j uses config q)

Constraints

- Path and config choice
- Path-config-coupling (track capacity)

Objective Function

- Maximize proceedings

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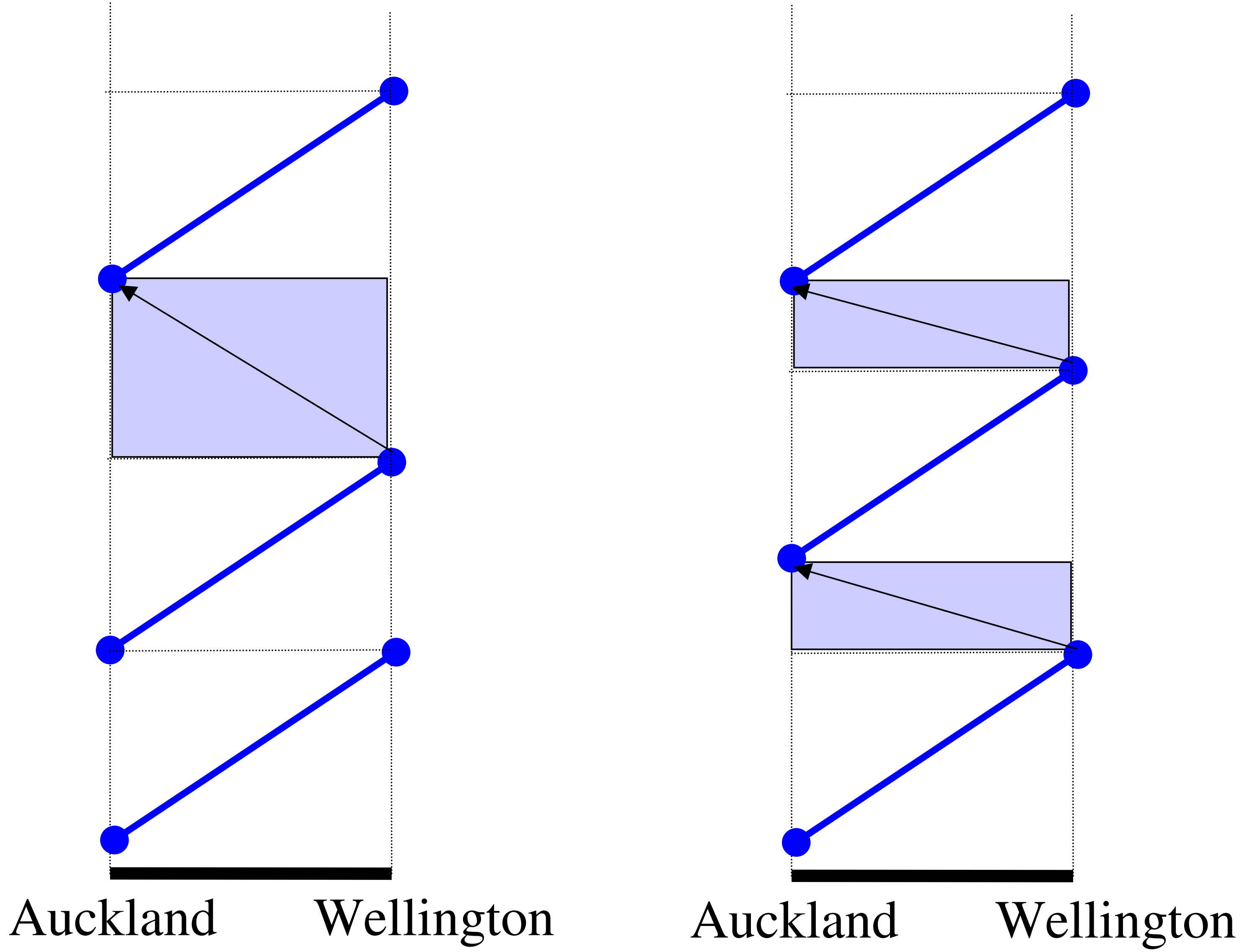
Robust Track Allocation ?



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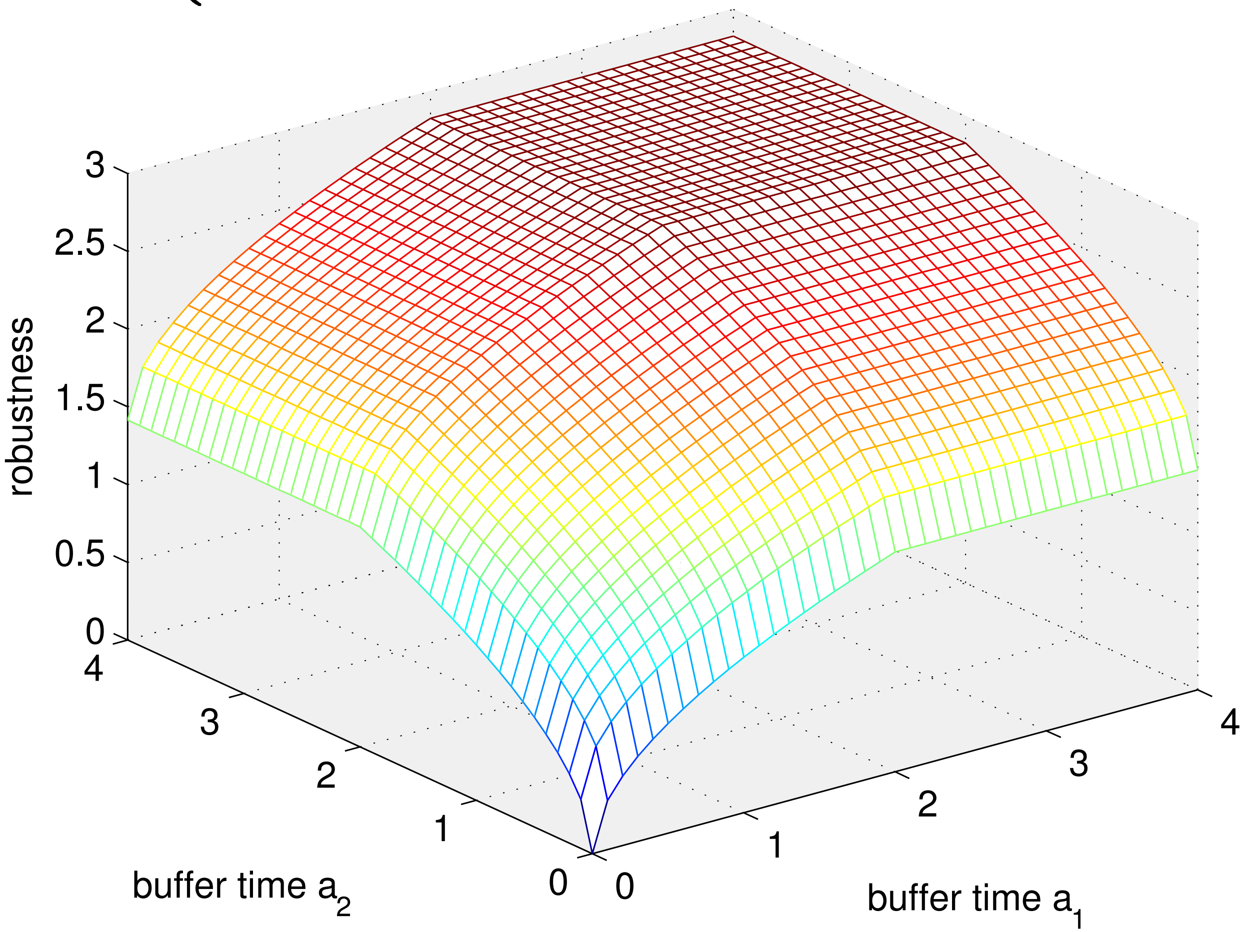
Robustness Measure

$$r((u,v)) := \begin{cases} \sqrt{b} & (u,v) \in A_{RL} \text{ and } t(v) - t(u) > b \\ \sqrt{t(v) - t(u)} & \text{otherwise} \end{cases}$$



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Bicriteria Optimization Model - Profit versus Robustness

$$
\begin{array}{llrll}
(BI-PCP) & \text{(i)} & \max \sum_{p\in P} w_p x_p & & \\
 & \text{(ii)} & \max \sum_{q\in Q} r_q y_q & & \\
 & \text{(iii)} & \sum_{p\in P_i} x_p & \leq 1, & \forall i \in I \\
 & \text{(iv)} & \sum_{q\in Q_j} y_q & \leq 1, & \forall j \in J \\
 & \text{(v)} & \sum_{a\in p\in P} x_p - \sum_{a\in q\in Q} y_q & \leq 0, & \forall a \in A_{LR} \\
 & \text{(vi)} & x_p, y_q & \in \{0,1\} & \forall p \in P,\ q \in Q
\end{array}
$$

Variables

- Path und config usage (request i uses path p, track j uses config q)

Constraints

- Path and config choice
- Path-config-coupling (track capacity)

Objective Function

- Maximize proceedings and robustness

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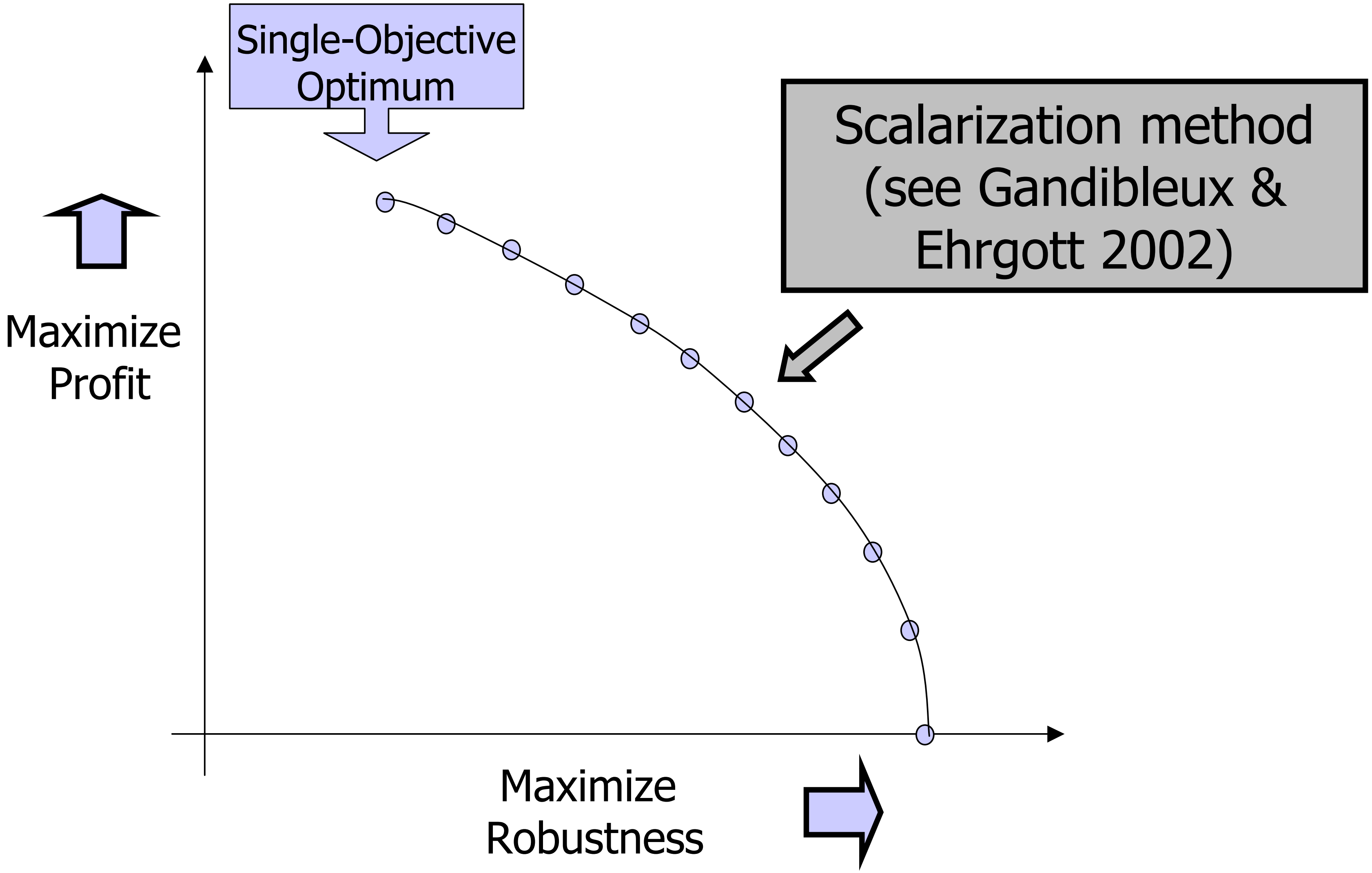
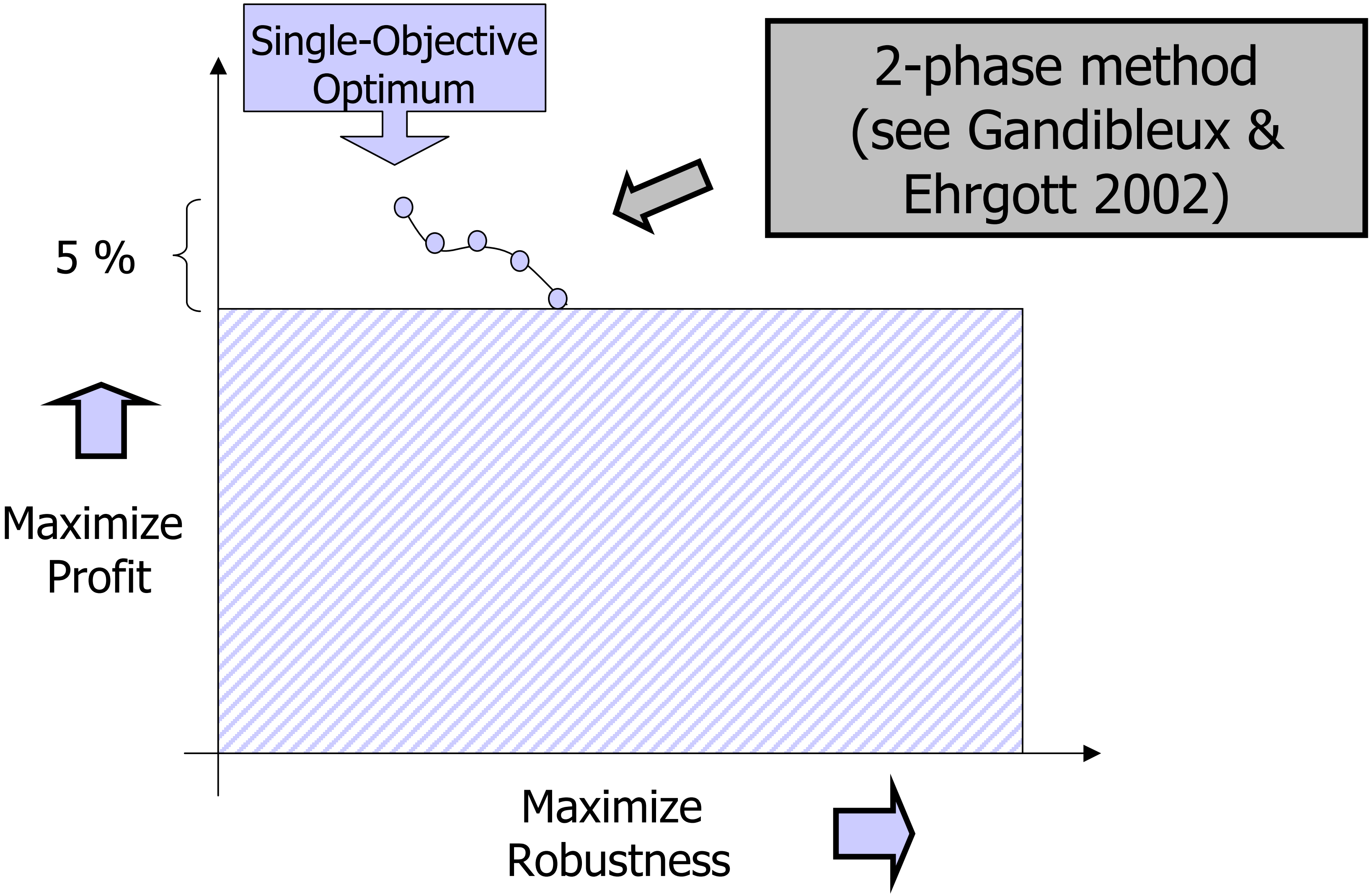
Price of Robustness (LP case)



Price of Robustness (IP case)



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Linear Relaxation of PCP

$$
\begin{array}{llllll}
(MLP) & & & & & \\
\max & \sum_{p\in\mathcal{P}} w_p x_p + \sum_{q\in\mathcal{Q}} r_q y_q & & & & \\
\text{s.t.} & \sum_{p\in\mathcal{P}_i} x_p & \le 1 & \forall i \in I & \text{(i)} & \\
 & \sum_{q\in\mathcal{Q}_j} y_q & \le 1 & \forall j \in J & \text{(ii)} & \\
 & \sum_{a\in p\in\mathcal{P}} x_p - \sum_{a\in q\in\mathcal{Q}} y_q & \le 0 & \forall a \in A_{LR} & \text{(iii)} & \\
 & 0 \le y_q & \le 1 & \forall q \in \mathcal{Q} & \text{(iii)} & \\
 & 0 \le x_p & \le 1 & \forall p \in \mathcal{P} & \text{(iv)} &
\end{array}
$$

γ_i π_j λ_a

dual variable	information about	useful to
γ_i	bundle price	analyse request
π_j	track price	analyse network
λ_a	arc price	-

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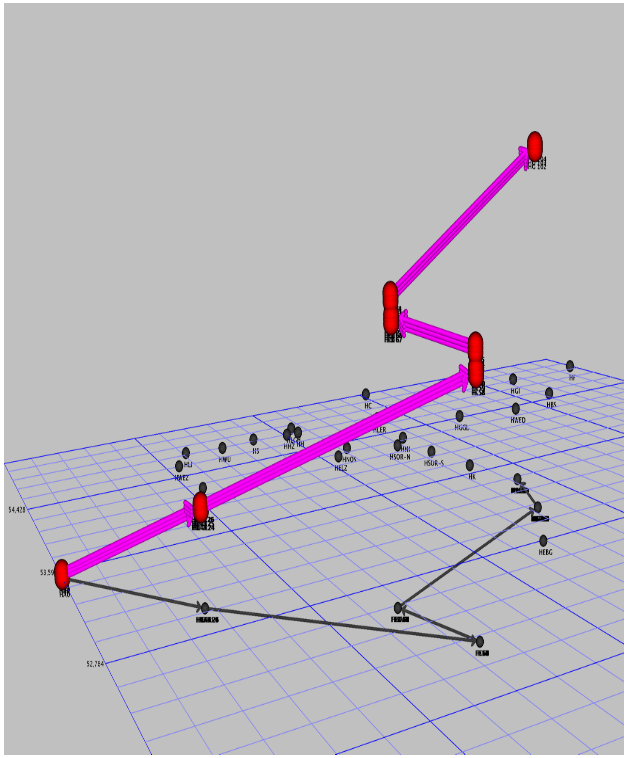
Dualization

$$
\begin{array}{llll}
(DLP) & & & \\
\text{min} & \sum_{j \in J} \pi_j + \sum_{i \in I} \gamma_i & & \\
\text{s.t.} & \gamma_i + \sum_{a \in p} \lambda_a \geq w_p & \forall p \in \mathcal{P}_i, \forall i \in I & \text{(i)} \\
& \pi_j - \sum_{a \in q} \lambda_a \geq r_q & \forall q \in \mathcal{Q}_j, \forall j \in J & \text{(ii)} \\
& \gamma_i \geq 0 & \forall i \in I & \text{(iii)} \\
& \lambda_a \geq 0 & \forall a \in A_{LR} & \text{(iv)} \\
& \pi_j \geq 0 & \forall j \in J & \text{(v)}
\end{array}
$$

Pricing of x-variables



$$\text{(PRICE (x))} \quad \exists\, \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)$$

$$c_a = -p_a + \lambda_a$$

Pricing Problem(x) :
Acyclic shortest path problems for each slot request i with modified cost function c !

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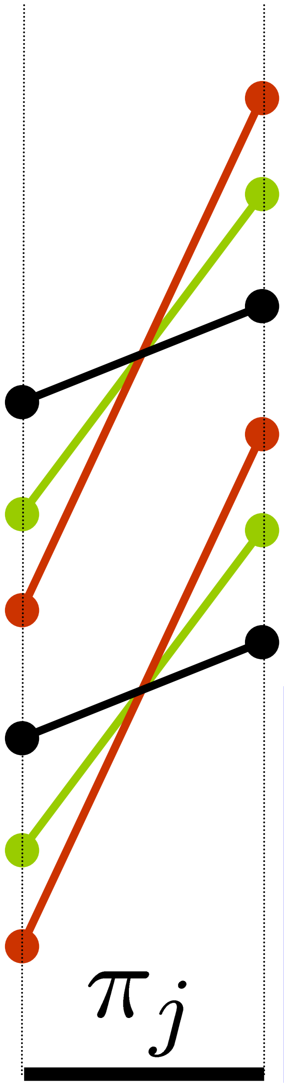
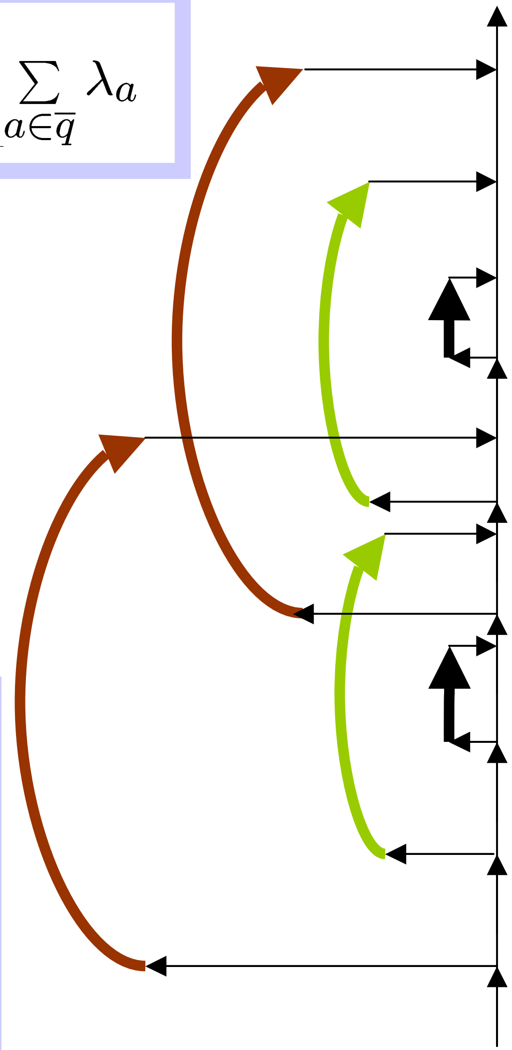
Pricing of y-variables



(PRICE (y)) $\exists\, \overline{q} \in Q_j : \quad \pi_j < r_{\overline{q}} + \sum_{a \in \overline{q}} \lambda_a$

$$c_a = -r_a - \lambda_a$$



<u>Pricing Problem(y) :</u>
Acyclic shortest path problem for each track j with modified cost function c !

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Observation

- **Lemma** [ZR-07-02]: The linear relaxation of PCP can be solved in polynomial time, due to the equivalence of optimization and separation (see Groetschel, Lovasz & Schrijver [88]).

- **Lemma**: The linear relaxation of PCP with an additional ε-constraint can be solved in polynomial time, due to the equivalence of optimization and separation (see Groetschel, Lovasz & Schrijver [88]).

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Observation

$$\text{(PRICE (x))} \quad \exists\, \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)$$

$$\eta_i := \max_{p \in P_i} \sum_{a \in p} (p_a - \lambda_a) - \gamma_i, \ \forall i \in I$$

$$\eta_i + \gamma_i \geq \sum_{a \in p} (p_a - \lambda_a) \ \forall i \in I, p \in \mathcal{P}_i$$

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And analogously ...

$$\text{(PRICE (y))} \quad \exists\, \overline{q} \in Q_j : \quad \pi_j < \sum_{a \in \overline{q}} \lambda_a$$

$$\theta_j := \max_{\overline{q} \in Q_j} \sum_{a \in \overline{q}} \lambda_a - \pi_j, \ \forall j \in J$$

$$\theta_j + \pi_j \geq \sum_{a \in q} \lambda_a \ \forall j \in J, q \in \mathcal{Q}_j$$

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Pricing Upper Bound

$(\max\{\eta+\gamma, 0\}, \max\{\theta+\pi, 0\}, \lambda)$ is feasible for (DLP)

$$\beta(\gamma,\pi,\lambda) := \sum_{i\in I} \max\{\gamma_i+\eta_i, 0\} + \sum_{j\in J} \max\{\pi_j+\theta_j, 0\}$$

- **Lemma** [ZR-07-02]: Given (infeasible) dual variables of PCP and let v_{LP}(PCP) be the optimum objective value of the LP-Relaxation of PCP, then:

$$v_{LP}(PCP) \leq \beta(\gamma,\pi,\lambda)$$

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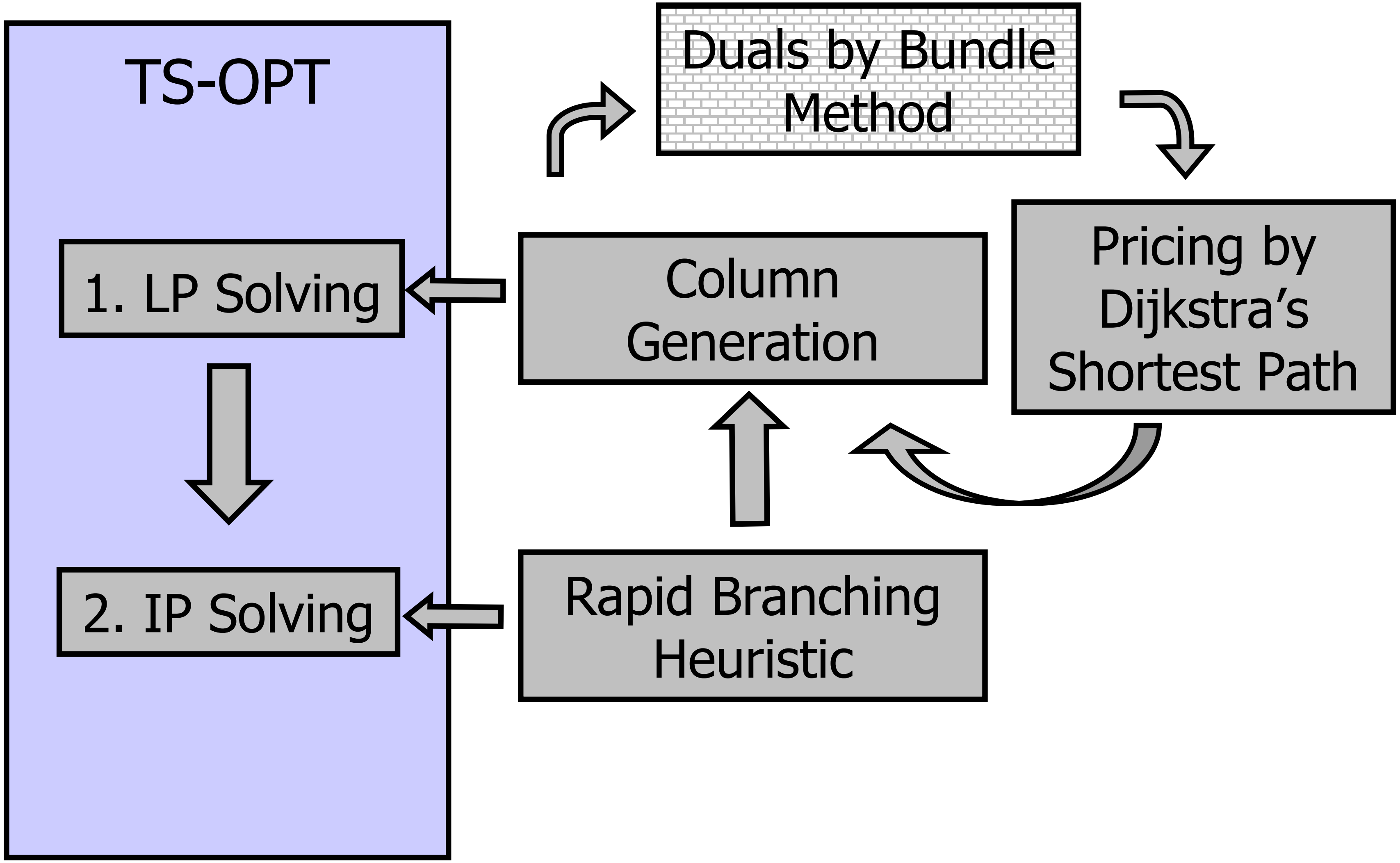
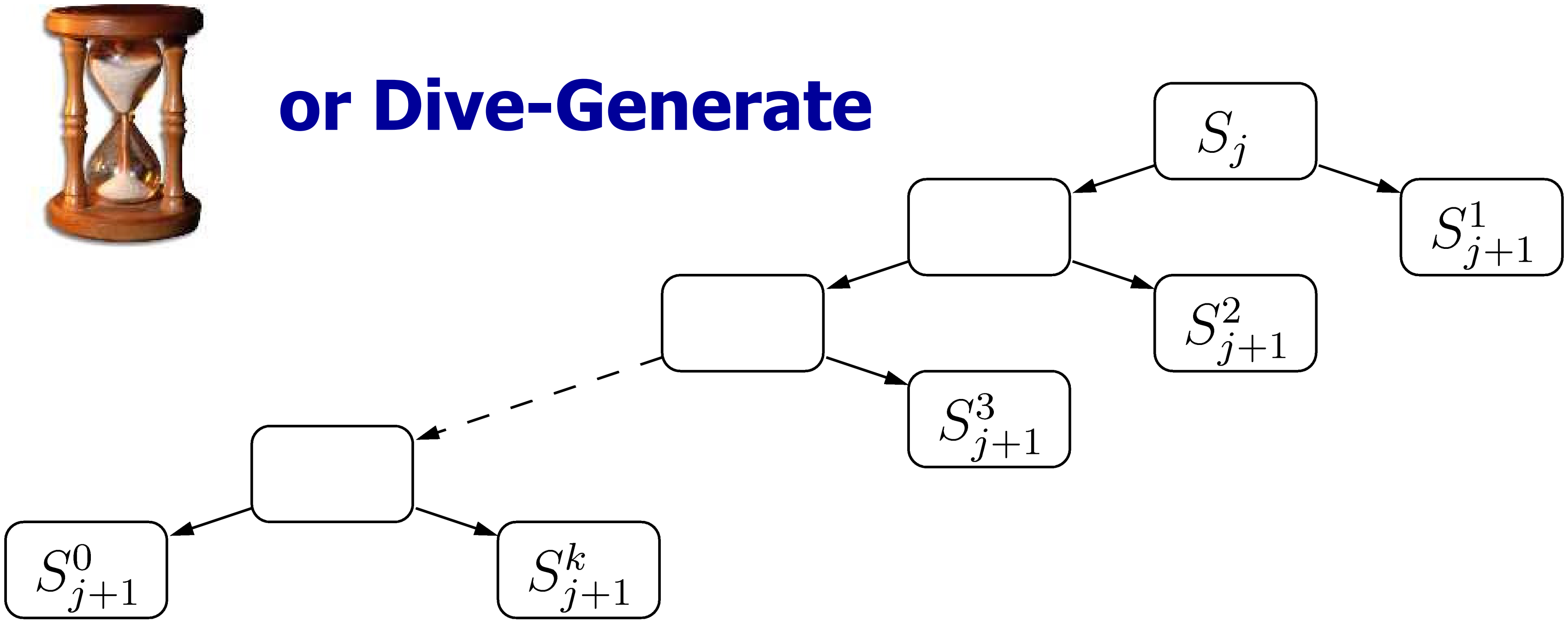
Two Step Approach



Branch-Bound-Price

or Dive-Generate



Evaluation of only few highly different sub-problems at iteration j to reach IP-Solutions fast.

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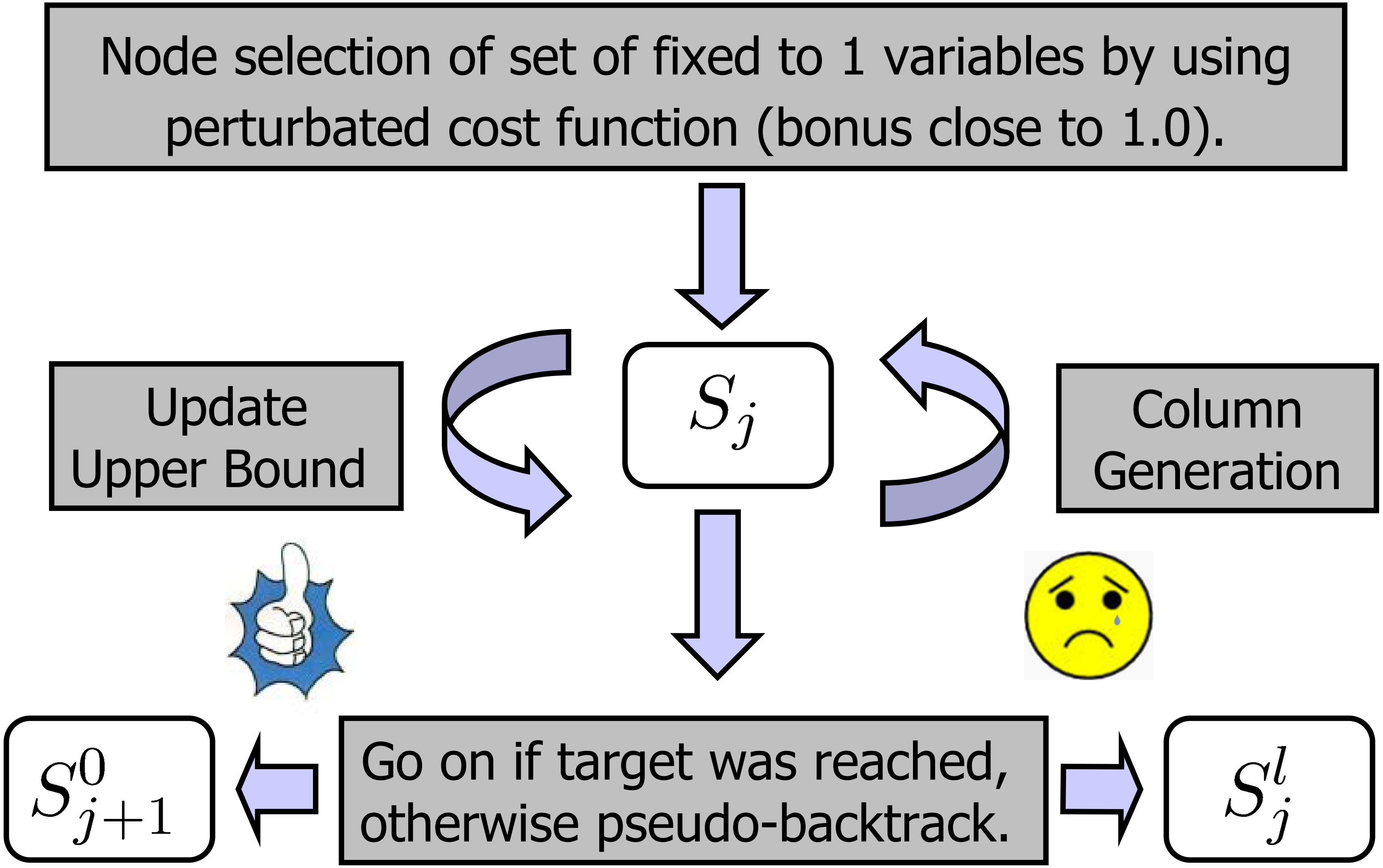
Rapid Branching



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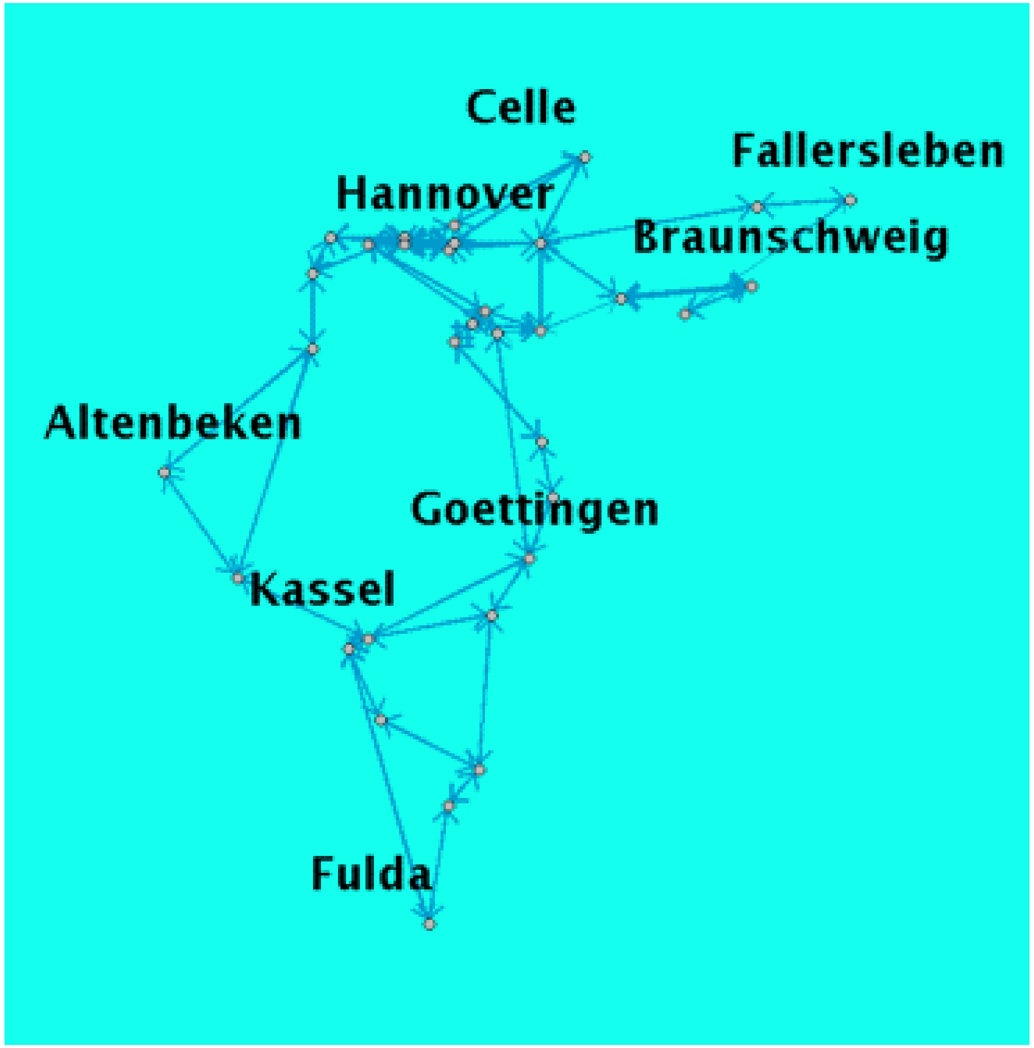
Results

- **Test Network**
  - 45 Tracks
  - 37 Stations
  - 6 Traintypes
  - 10 Trainsets
  - 146 Nodes
  - 1480 Arcs
  - 96 Station Capacities
  - 4320 Headway Times



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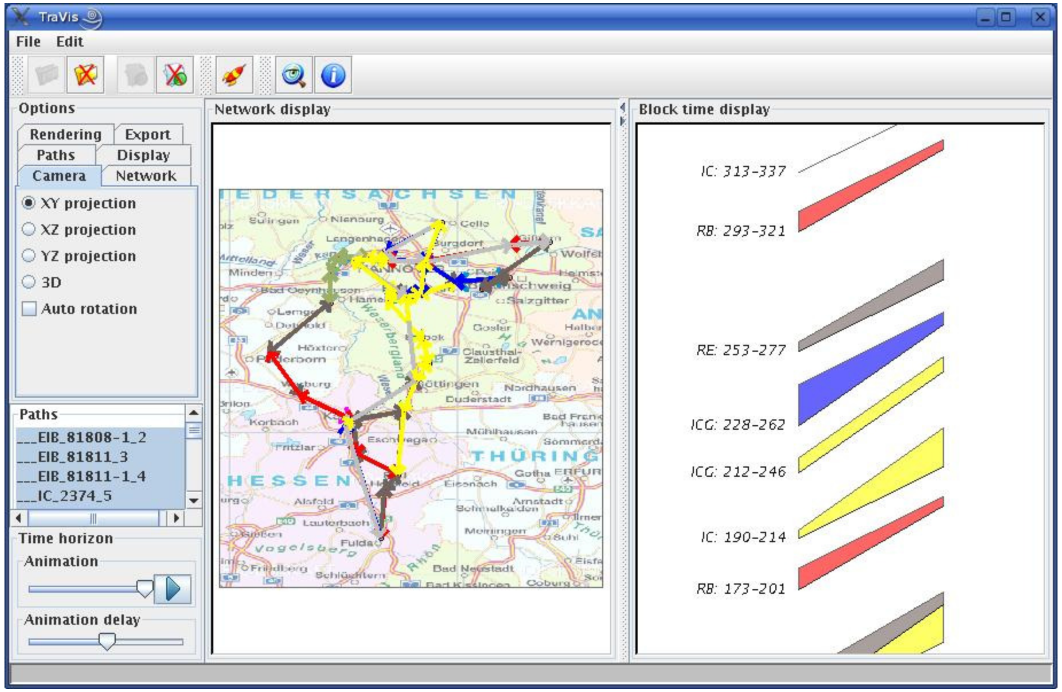
Computational Results

- **Test Scenarios**
  - 146 Train Requests
  - 285 Train Requests
  - 570 Train Requests



- **Flexibility**
  - 0-30 Minutes
  - earlier departure penalties
  - late arrival penalties
  - train type depending profits

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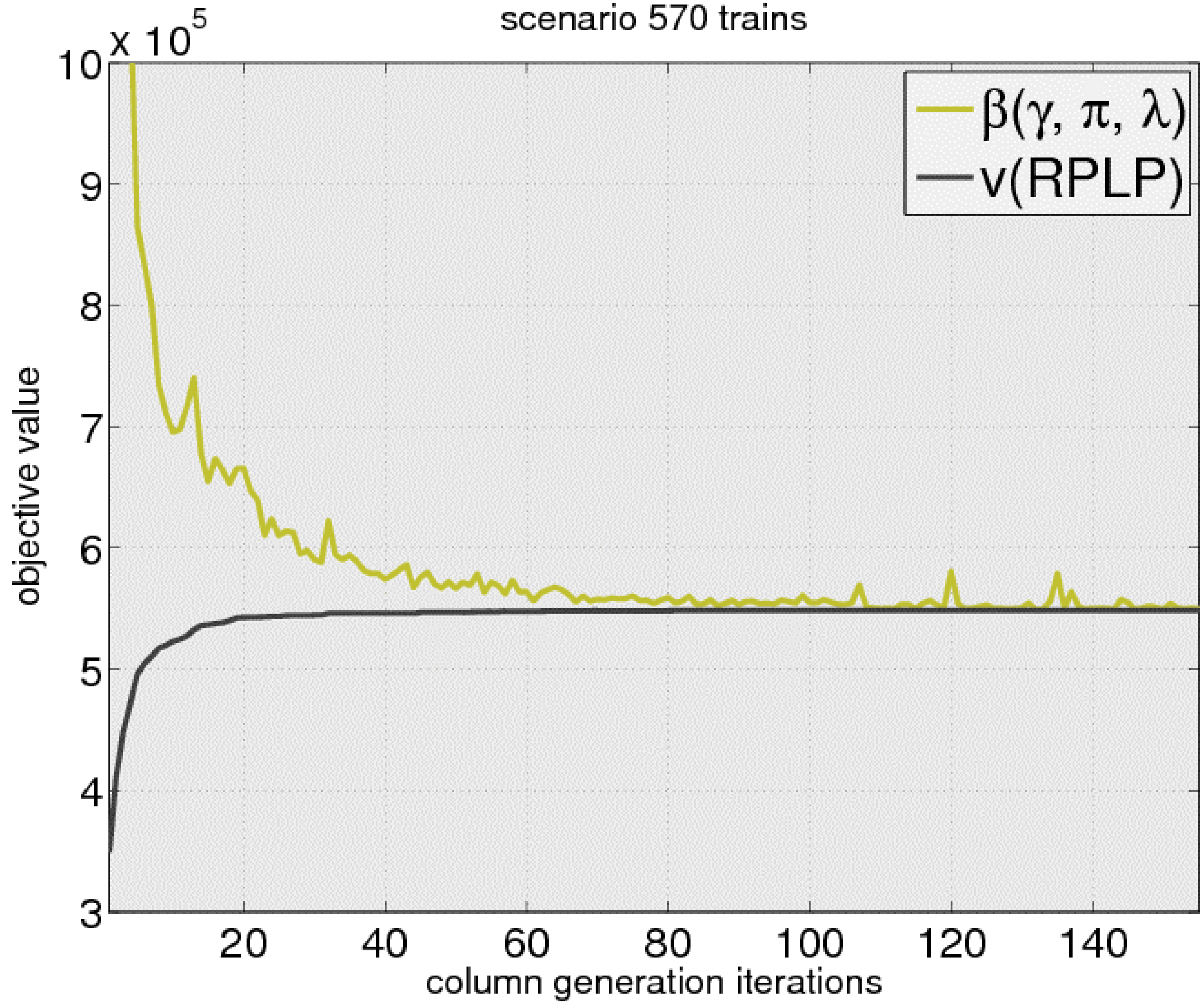
Run of TS-OPT/LP Stage



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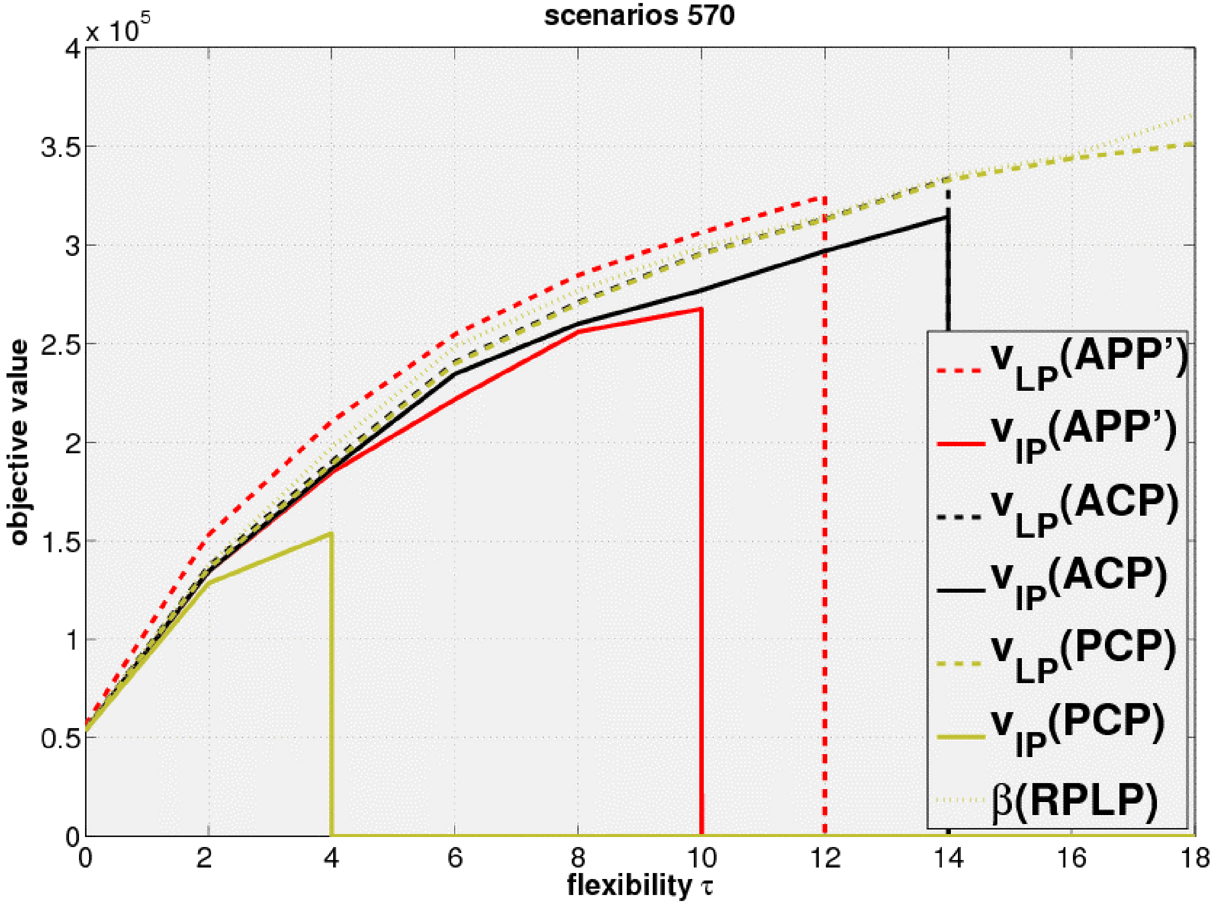
Model Comparison



For details see [ZR-07-02, ZR-07-20].

Outlook

Future Plans

- Efficient Set of (BI-PCP)
- Bundle method
- Model refinement (connections)
- Adaptive IP Heuristics
- Dynamic Time Discretization

Simulation of results by

Railsys®

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Thank you for your attention !

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