A suitable Model for a bicriteria Optimization Approach to Railway Track Allocation

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joint work with Ralf Borndörfer
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Konrad Zuse was the creator of the first fully automatic, programm controlled and freely programmable computer working in binary floating point arithmetic. The Z3 was finished in 1941.
Overview

1. Problem Introduction
2. Bicriteria Optimization Model
3. Column Generation Approach
4. Computational Results
Overview

1. Problem Introduction
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Planning in Public Transport

- Strategic Stage
  - Tracks
  - Lines/Freq.
  - Timetables
- Tactical Stage
  - Cycles
  - Connections
  - Vehicles
  - Crews
- Operational Stage
  - Rotations
  - Duties
Traffic Projects @ ZIB

1. Traffic Projects
2. MCF
3. Telebus
4. VS-OPT
5. VS: BVG
6. DS-OPT
7. DS: BVG
8. IS-OPT
9. Line+Price Planning
10. CS-OPT

Years:
- 92-94
- 94-97
- 97-00
- 00-03
- 03-07
Planning in Public Transport

Strategic Stage
- B1 – B15
- Tracks
- Lines/Freq.
- Timetables
- Stops

Tactical Stage
- TS-OPT
- Cycles
- Connections
- Rotations

Operational Stage
- VS-OPT
- IS-OPT
- CS-OPT
- Vehicles
- Crews
- Duties

Planning in Public Transport
The Problem (TraVis by M.Kinder)
Schedule in 3d
Conflict-Free-Allocation
Railway Timetabling – State of the Art

- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Semet and Schoenauer (2005),
- Caprara, Monaci, Toth and Guida (2005)
- Kroon, Dekker and Vromans (2005),
- Vanstevenwegen and Van Oudheusden (2006),
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- Borndoerfer, Schlechte (2007)

non-cyclic timetabling literature
**Proposition** [Caprara, Fischetti, Toth (02)]:

OPTRA/TTP is \(NP\)-hard.

**Proof:**
Reduction from Independent-Set.
Track Allocation Problem

Train Requests → Scheduling Digraph → Timetable
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Single Objective Model

\[ x_p \] \hspace{1cm} \[ y_q \]

Auckland \hspace{1cm} Wellington

Auckland \hspace{1cm} Wellington
Path Coupling Problem

\[(PCP)\] (i) \[\max \sum_{p \in P} w_p x_p\]

(ii) \[\sum_{p \in P_i} x_p \leq 1, \quad \forall i \in I\]

(iii) \[\sum_{q \in Q_j} y_q \leq 1, \quad \forall j \in J\]

(iv) \[\sum_{a \in p \in P} x_p - \sum_{a \in q \in Q} y_q \leq 0, \quad \forall a \in A_{LR}\]

(v) \[x_p, y_q \in \{0, 1\} \quad \forall p \in P, q \in Q\]

**Variables**
- Path and config usage (request i uses path p, track j uses config q)

**Constraints**
- Path and config choice
- Path-config-coupling (track capacity)

**Objective Function**
- Maximize proceedings
Robust Track Allocation?
Robustness Measure

\[ r((u, v)) := \begin{cases} \frac{\sqrt{b}}{\sqrt{t(v) - t(u)}} & (u, v) \in A_{RL} \text{ and } t(v) - t(u) > b \\ \text{otherwise} \end{cases} \]
Bicriteria Optimization Model - Profit versus Robustness

\[(BI - PCP) \quad \text{(i)} \quad \max \sum_{p \in P} w_p x_p \]

\[(\text{ii}) \quad \max \sum_{q \in Q} r_q y_q \]

\[(\text{iii}) \quad \sum_{p \in P_i} x_p \leq 1, \quad \forall i \in I \]

\[(\text{iv}) \quad \sum_{q \in Q_j} y_q \leq 1, \quad \forall j \in J \]

\[(\text{v}) \quad \sum_{a \in P} x_p - \sum_{a \in Q} y_q \leq 0, \quad \forall a \in A_{LR} \]

\[(\text{vi}) \quad x_p, y_q \in \{0, 1\} \quad \forall p \in P, q \in Q \]

**Variables**
- Path und config usage (request i uses path p, track j uses config q)

**Constraints**
- Path and config choice
- Path-config-coupling (track capacity)

**Objective Function**
- Maximize proceedings and robustness
Price of Robustness (LP case)

Single-Objective Optimum

Scalarization method (see Gandibleux & Ehrgott 2002)
Price of Robustness (IP case)

Maximize Robustness

Single-Objective Optimum

2-phase method (see Gandibleux & Ehrgott 2002)

Maximize Profit

5 %
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Linear Relaxation of PCP

\[
(MLP) \quad \max \sum_{p \in \mathcal{P}} w_p x_p + \sum_{q \in \mathcal{Q}} r_q y_q \\
\text{s.t.} \quad \sum_{p \in \mathcal{P}_i} x_p \leq 1 \quad \forall i \in I \quad (i) \\
\sum_{q \in \mathcal{Q}_j} y_q \leq 1 \quad \forall j \in J \quad (ii) \\
\sum_{a \in \mathcal{P}} x_p - \sum_{a \in \mathcal{Q}} y_q \leq 0 \quad \forall a \in A_{LR} \quad (iii) \\
0 \leq y_q \leq 1 \quad \forall q \in \mathcal{Q} \quad (iii) \\
0 \leq x_p \leq 1 \quad \forall p \in \mathcal{P} \quad (iv)
\]

<table>
<thead>
<tr>
<th>dual variable</th>
<th>information about</th>
<th>useful to</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_i )</td>
<td>bundle price</td>
<td>analyse request</td>
</tr>
<tr>
<td>( \pi_j )</td>
<td>track price</td>
<td>analyse network</td>
</tr>
<tr>
<td>( \lambda_\alpha )</td>
<td>arc price</td>
<td>-</td>
</tr>
</tbody>
</table>
Dualization

\[ \text{(DLP)} \]

\[
\begin{align*}
\min & \quad \sum_{j \in J} \pi_j + \sum_{i \in I} \gamma_i \\
\text{s.t.} & \quad \gamma_i + \sum_{a \in p} \lambda_a \geq w_p \quad \forall p \in P_i, \forall i \in I \quad (i) \\
& \quad \pi_j - \sum_{a \in q} \lambda_a \geq r_q \quad \forall q \in Q_j, \forall j \in J \quad (ii) \\
& \quad \gamma_i \geq 0 \quad \forall i \in I \quad (iii) \\
& \quad \lambda_a \geq 0 \quad \forall a \in A_{LR} \quad (iv) \\
& \quad \pi_j \geq 0 \quad \forall j \in J \quad (v)
\end{align*}
\]
Pricing of x-variables

\[(\text{PRICE} (x)) \quad \exists \, \bar{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \bar{p}} (p_a - \lambda_a)\]

\[c_a = -p_a + \lambda_a\]

Pricing Problem(x):
Acyclic shortest path problems for each slot request i with modified cost function c!
Pricing of y-variables

\[(\text{PRICE (y)}) \quad \exists \ q \in Q_j : \ \pi_j < r_q + \sum_{a \in q} \lambda_a\]

\[c_a = -r_a - \lambda_a\]

Pricing Problem(y):
Acyclic shortest path problem for each track j with modified cost function c!
Observation

• **Lemma** [ZR-07-02]: The linear relaxation of PCP can be solved in polynomial time, due to the equivalence of optimization and separation (see Groetschel, Lovasz & Schrijver [88]).

• **Lemma**: The linear relaxation of PCP with an additional ε-constraint can be solved in polynomial time, due to the equivalence of optimization and separation (see Groetschel, Lovasz & Schrijver [88]).
Observation

\[(\text{PRICE } (x)) \quad \exists \overline{p} \in P_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)\]

\[\eta_i := \max_{p \in P_i} \sum_{a \in p} (p_a - \lambda_a) - \gamma_i, \quad \forall i \in I\]

\[\eta_i + \gamma_i \geq \sum_{a \in p} (p_a - \lambda_a) \quad \forall i \in I, \quad p \in P_i\]
And analogously...

\[(\text{PRICE (y)}) \quad \exists \overline{q} \in Q_j : \quad \pi_j < \sum_{a \in \overline{q}} \lambda_a\]

\[\theta_j := \max_{\overline{q} \in Q_j} \sum_{a \in \overline{q}} \lambda_a - \pi_j, \quad \forall j \in J\]

\[\theta_j + \pi_j \geq \sum_{a \in q} \lambda_a \quad \forall j \in J, q \in Q_j\]
Pricing Upper Bound

\[(\max\{\eta + \gamma, 0\}, \max\{\theta + \pi, 0\}, \lambda)\] is feasible for (DLP)

\[\beta(\gamma, \pi, \lambda) := \sum_{i \in I} \max\{\gamma_i + \eta_i, 0\} + \sum_{j \in J} \max\{\pi_j + \theta_j, 0\}\]

• **Lemma** [ZR-07-02]: Given (infeasible) dual variables of PCP and let \(v_{LP}(PCP)\) be the optimum objective value of the LP-Relaxation of PCP, then:

\[v_{LP}(PCP) \leq \beta(\gamma, \pi, \lambda)\]
Two Step Approach

1. LP Solving
2. IP Solving

TS-OPT

Duals by Bundle Method

Column Generation

Pricing by Dijkstra’s Shortest Path

Rapid Branching Heuristic
Branch-Bound-Price

or Dive-Generate

Evaluation of only few highly different sub-problems at iteration j to reach IP-Solutions fast.
Rapid Branching

Node selection of set of fixed to 1 variables by using perturbated cost function (bonus close to 1.0).

Update Upper Bound

Go on if target was reached, otherwise pseudo-backtrack.

Column Generation
Overview

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Results

- **Test Network**
  - 45 Tracks
  - 37 Stations
  - 6 Traintypes
  - 10 Trainsets
  - 146 Nodes
  - 1480 Arcs
  - 96 Station Capacities
  - 4320 Headway Times
Computational Results

- **Test Scenarios**
  - 146 Train Requests
  - 285 Train Requests
  - 570 Train Requests

- **Flexibility**
  - 0-30 Minutes
  - earlier departure penalties
  - late arrival penalties
  - train type depending profits
Run of TS-OPT/LP Stage

scenario 570 trains

objective value

column generation iterations
Model Comparison

For details see [ZR-07-02, ZR-07-20].
Outlook

Future Plans

- Efficient Set of (BI-PCP)
- Bundle method
- Model refinement (connections)
- Adaptive IP Heuristics
- Dynamic Time Discretization

Simulation of results by Railsys®
Thank you for your attention!