Solving Railway Track Allocation Problems

By Column Generation

Thomas Schlechte
Joint work with
Ralf Borndörfer
Martin Grötschel

05.09.2007
SOR 2007 Saarbrücken

Federal Ministry of Economics and Technology

Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)

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Overview

1. Problem Introduction
2. Model Discussion
3. Column Generation Approach
4. Computational Results
Overview

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Planning in Public Transport

Strategic Stage
- Tracks
- Lines/Freq.
- Stops

Tactical Stage
- Timetables
- Cycles
- Connections

Operational Stage
- Vehicles
- Crews
- Rotations
- Duties
Traffic Projects @ ZIB

- BS-OPT
- VS-OPT
- DS-OPT
- VS: BVG
- IS-OPT
- DS: BVG
- Line+Price Planning
- TS-OPT
- MCF
- Telebus
- CS-OPT

Years:
- 92-94
- 94-97
- 97-00
- 00-03
- 03-07
The Problem (TraVis by M.Kinder)
Schedule in 3d
Conflict-Free-Allocation
State-of-the-Art

- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Semet and Schoenauer (2005),
- Caprara, Monaci, Toth and Guida (2005)
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006),
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- and many more
**Proposition** [Caprara, Fischetti, Toth (02)]:
OPTRA/TTP is \textit{NP}-hard.

**Proof:**
Reduction from Independent-Set.
Track Allocation Problem

Train Requests → Scheduling Digraph → Timetable
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Packing Models

- Conflict graph
- Cliques
- Perfect

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Arc Packing Problem

\[(APP)\]
\[
\max \sum_{i \in I} \sum_{a \in A} p^{i}_a x^{i}_a \\
\text{s.t. } \sum_{a \in \delta^{\text{out}}_i(v)} x^{i}_a - \sum_{a \in \delta^{\text{in}}_i(v)} x^{i}_a \leq \delta_i(v) \quad \forall v \in V, \forall i \in I \quad (i) \\
\sum_{i \in I} \sum_{a \in A} x^{i}_a \leq 1 \quad \forall c \in C \quad (ii) \\
x^{i}_a \in \{0, 1\} \quad \forall a \in A, \forall i \in I \quad (iii)
\]

Variables
- Arc occupancy (request i uses arc a)

Constraints
- Flow conservation and
- Arc conflicts (pairwise)

Objective
- Maximize proceedings
Packing Models

- **Proposition:**
  The LP-relaxation of APP can be solved in polynomial time.
- ... and in practice.
Novel Model

- Track Graph
- Timeline(s)
- Config paths
Path Coupling Problem

\[(PCP)\]

\[
\begin{align*}
\text{max} & \quad \sum_{p \in P} \sum_{a \in p} p_a^i x_p \\
\text{s.t.} & \quad \sum_{p \in P_i} x_p \leq 1 & \forall i \in I \\
& \quad \sum_{q \in Q_j} y_q \leq 1 & \forall j \in J \\
& \quad \sum_{a \in p \in P} x_p - \sum_{a \in q \in Q} y_q \leq 0 & \forall a \in A_I \cap A_J \\
& \quad y_q \in \{0, 1\} & \forall q \in Q_j, \forall j \in J \\
& \quad x_p \in \{0, 1\} & \forall p \in P_i, \forall i \in I
\end{align*}
\]

**Variables**
- Path and config usage (request i uses path p, track j uses config q)

**Constraints**
- Path and config choice
- Path-config-coupling (track capacity)

**Objective Function**
- Maximize proceedings
Linear Relaxation of PCP

\[(MLP)\]

\[
\begin{align*}
\max & \sum_{p \in \mathcal{P}} \sum_{a \in \mathcal{P}_i} p_a x_p \\
\text{s.t.} & \sum_{p \in \mathcal{P}_i} x_p \leq 1 \quad \forall i \in I \quad (i) \\
& \sum_{q \in \mathcal{Q}_j} y_q \leq 1 \quad \forall j \in J \quad (ii) \\
& \sum_{a \in \mathcal{P}} x_p - \sum_{a \in \mathcal{Q}} y_q \leq 0 \quad \forall a \in A_I \cup A_J \quad (iii) \\
& 0 \leq y_q \leq 1 \quad \forall q \in \mathcal{Q} \quad (iv) \\
& 0 \leq x_p \leq 1 \quad \forall p \in \mathcal{P}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Dual Variable</th>
<th>Information About</th>
<th>Useful To</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma_i)</td>
<td>bundle price</td>
<td>analyse request</td>
</tr>
<tr>
<td>(\pi_j)</td>
<td>track price</td>
<td>analyse network</td>
</tr>
<tr>
<td>(\lambda_a)</td>
<td>arc price</td>
<td>-</td>
</tr>
</tbody>
</table>
(DLP)
\[
\begin{align*}
\min & \quad \sum_{j \in J} \pi_j + \sum_{i \in I} \gamma_i \\
\text{s.t.} & \quad \gamma_i + \sum_{a \in p} \lambda_a \geq \sum_{a \in p} p_a^i \quad \forall p \in \mathcal{P}_i, \forall i \in I \quad (i) \\
& \quad \pi_j - \sum_{a \in q} \lambda_a \geq 0 \quad \forall q \in \mathcal{Q}_j, \forall j \in J \quad (ii) \\
& \quad \gamma_i \geq 0 \quad \forall i \in I \quad (iii) \\
& \quad \lambda_a \geq 0 \quad \forall a \in A_I \cup A_J \quad (iv) \\
& \quad \pi_j \geq 0 \quad \forall j \in J \quad (v)
\end{align*}
\]
Pricing of $x$-variables

\[(\text{PRICE}(x)) \ \exists \bar{p} \in \mathcal{P}_i : \ \gamma_i < \sum_{a \in \bar{p}} (p_a - \lambda_a)\]

\[c_a = -p_a + \lambda_a\]

**Pricing Problem(x)**: Acyclic shortest path problems for each slot request $i$ with modified cost function $c$. 
Pricing of $y$-variables

(PRICING \(y\)) \(\exists \bar{q} \in Q_j : \pi_j < \sum_{a \in \bar{q}} \lambda_a\)

\[ c_a = -\lambda_a \]

Pricing Problem\((y)\):
Acyclic shortest path problem for each track \(j\) with modified cost function \(c\)!
Observation

\[(\text{PRICE (x)}) \exists \overline{p} \in \mathcal{P}_i : \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)\]

\[\eta_i := \max_{p \in \mathcal{P}_i} \sum_{a \in p} (p_a - \lambda_a) - \gamma_i, \ \forall i \in I\]

\[\eta_i + \gamma_i \geq \sum_{a \in p} (p_a - \lambda_a) \ \forall i \in I, p \in \mathcal{P}_i\]
And analogously ...

(PRICE (y)) \[ \exists \overline{q} \in Q_j : \pi_j < \sum_{a \in \overline{q}} \lambda_a \]

\[ \theta_j := \max_{\overline{q} \in Q_j} \left( \sum_{a \in \overline{q}} \lambda_a - \pi_j \right), \forall j \in J \]

\[ \theta_j + \pi_j \geq \sum_{a \in \overline{q}} \lambda_a \forall j \in J, q \in Q_j \]
Pricing Upper Bound

\[(\max\{\eta + \gamma, 0\}, \max\{\theta + \pi, 0\}, \lambda)\] is feasible for \((DLP)\)

\[\beta(\gamma, \pi, \lambda) := \sum_{i \in I} \max\{\gamma_i + \eta_i, 0\} + \sum_{j \in J} \max\{\pi_j + \theta_j, 0\}\]

- **Lemma [ZR-07-02]**: Given (infeasible) dual variables of PCP and let \(v_{LP}(PCP)\) be the optimum objective value of the LP-Relaxtion of PCP, then:

\[v_{LP}(PCP) \leq \beta(\gamma, \pi, \lambda)\]
Model Comparison

- **Theorem [ZR-07-02]:** The LP-relaxations of ACP and PCP can be solved in polynomial time.

- **Lemma [ZR-07-02]:**
  
  \[ v_{LP}(PCP) = v_{LP}(ACP) = v_{LP}(APP) = v_{LP}(PPP) \leq v_{LP}(APP') \]
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Two Step Approach

TS-OPT

1. LP Solving
2. IP Solving

Duals by Bundle Method

Column Generation

Pricing by Dijkstra’s Shortest Path

Rapid Branching Heuristic
Branch-Bound-Price

or Dive-Generate

Evaluation of only few highly different sub-problems at iteration $j$ to reach IP-Solutions fast.
Rapid Branching

Node selection of set of fixed to 1 variables by using perturbated cost function (bonus close to 1.0).

\[ S_j \]

Go on if target was reached, otherwise pseudo-backtrack.

\[ S_{j+1}^0 \]

\[ S_j^l \]

Update Upper Bound

Column Generation
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Results

• Test Network

• 45 Tracks
• 37 Stations
• 6 Traintypes
• 10 Trainsets
• 146 Nodes
• 1480 Arcs
• 96 Station Capacities
• 4320 Headway Times
Model Comparison

- **Test Scenarios**
  - 146 Train Requests
  - 285 Train Requests
  - 570 Train Requests

- **Flexibility**
  - 0-30 Minutes
  - earlier departure penalties
  - late arrival penalties
  - train type depending profits
Run of TS-OPT
Model Comparison

![Graph showing model comparison with various lines representing different models and scenarios. The x-axis represents flexibility τ, and the y-axis shows the objective value. Different models are labeled with notations such as $v_{\text{LP}}(\text{ACP})$ and $v_{\text{IP}}(\text{PCP})$. There are 146 scenarios depicted in the graph.]
Model Comparison

For details see [ZR-07-02, ZR-07-20].
Outlook

Algorithmic Developments

- Bundle method
- Model refinement (connections)
- Adaptive IP Heuristics
- Dynamic Discretization

Simulation of results by Railsys®
Thank you for your attention!

Thomas Schlechte  
Zuse-Institut Berlin (ZIB) 
Takustr. 7, 14195 Berlin  
Deutschland

Fon (+49 30) 84185-317  
Fax (+49 30) 84185-269  
schlechte@zib.de  
www.zib.de/schlechte