Federal Ministry
of Economics
and Technology

Solving Large-Scale Railway Track Allocation Problems By Column Generation

Thomas Schlechte

Joint work with

Ralf Borndörfer **Martin Grötschel**

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SOR 2007 Saarbrücken

Overview

1. Problem Introduction
2. Model Discussion
3. Column Generation Approach
4. Computational Results

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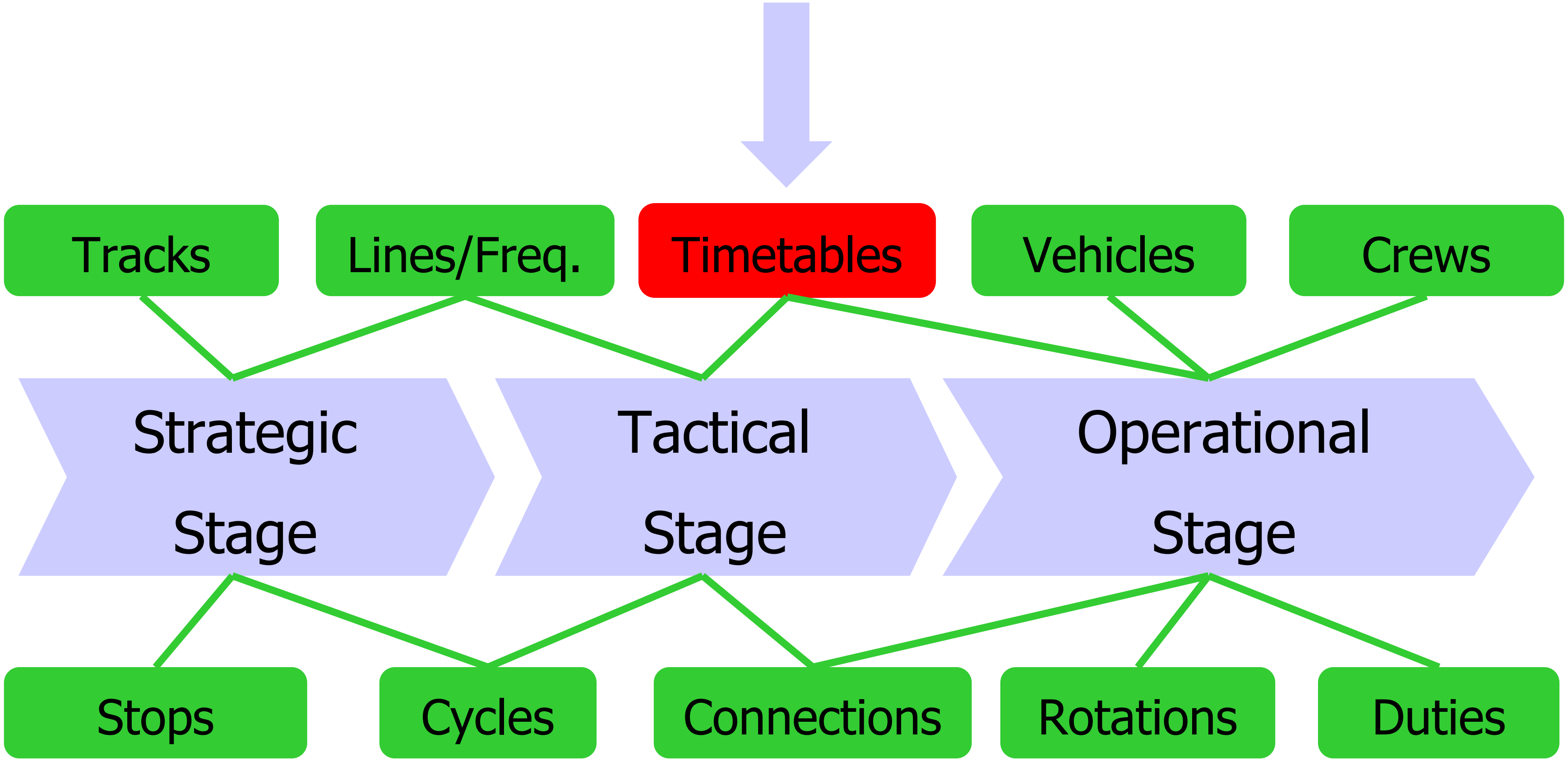
Planning in Public Transport



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Traffic Projects @ ZIB

	bmb+f / Federal Ministry of Economics and Technology	BVG	Deutsche Forschungsgemeinschaft DFG	Berlin Senatsverwaltung für Wissenschaft, Forschung und Kultur
92-94			MCF	Telebus
94-97	VS-OPT			
97-00	DS-OPT	VS: BVG		
00-03	IS-OPT	DS: BVG	Line+Price Planning	
03-07	TS-OPT			CS-OPT

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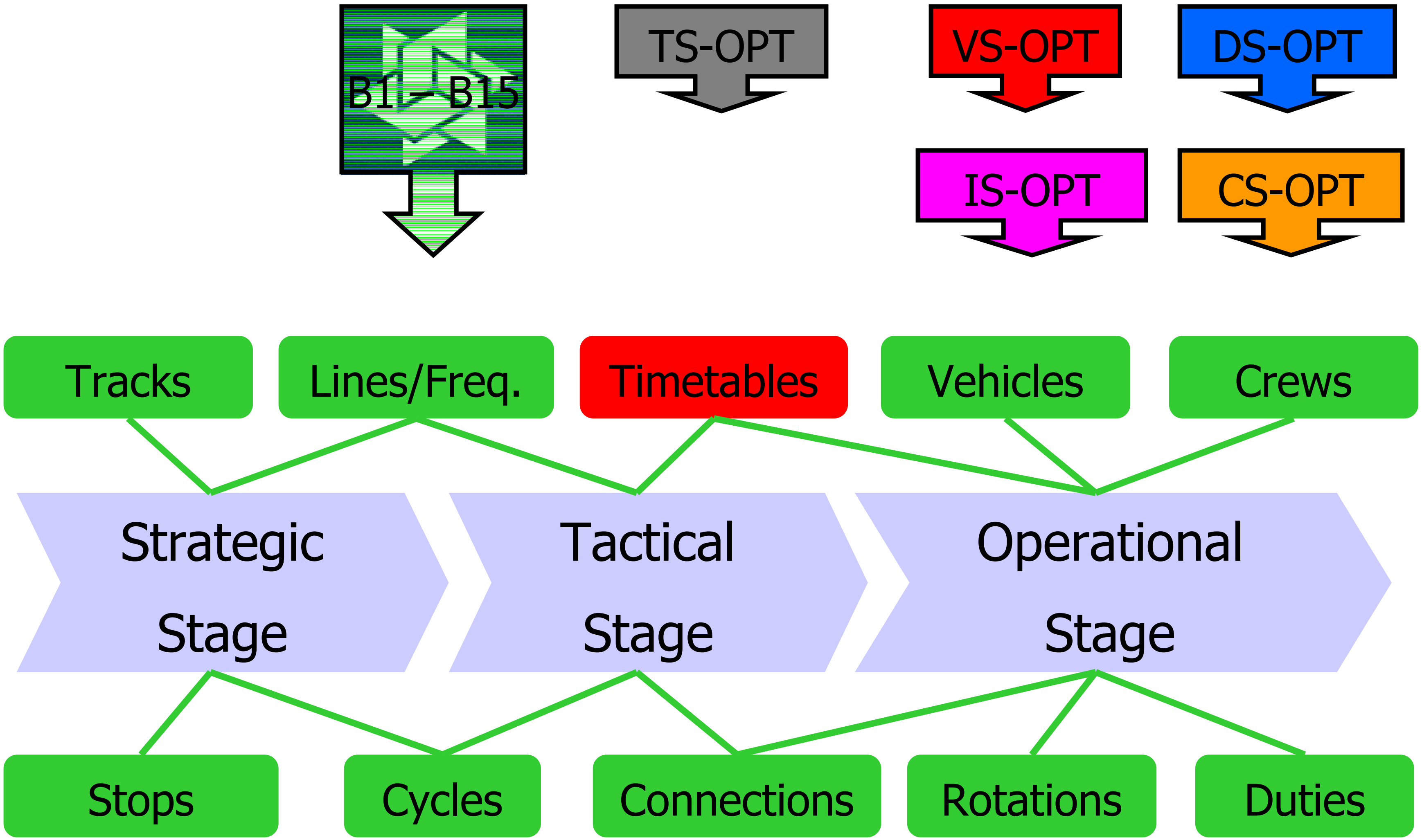
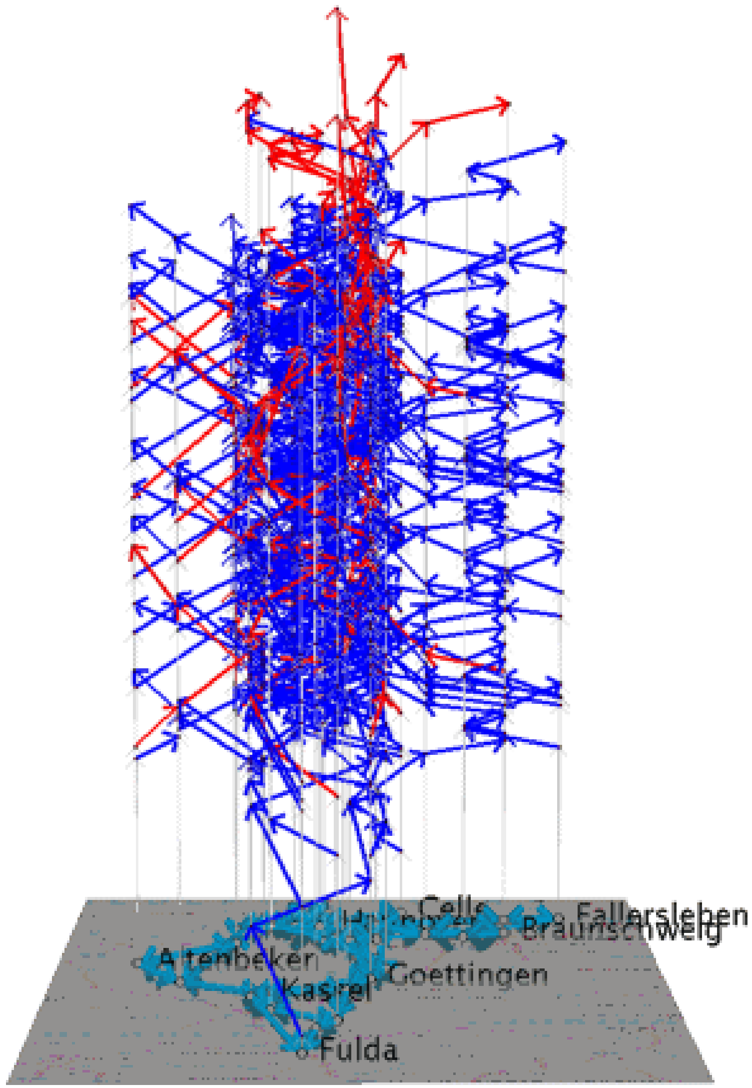
Planning in Public Transport



The Problem (TraVis by M.Kinder)

Schedule in 3d



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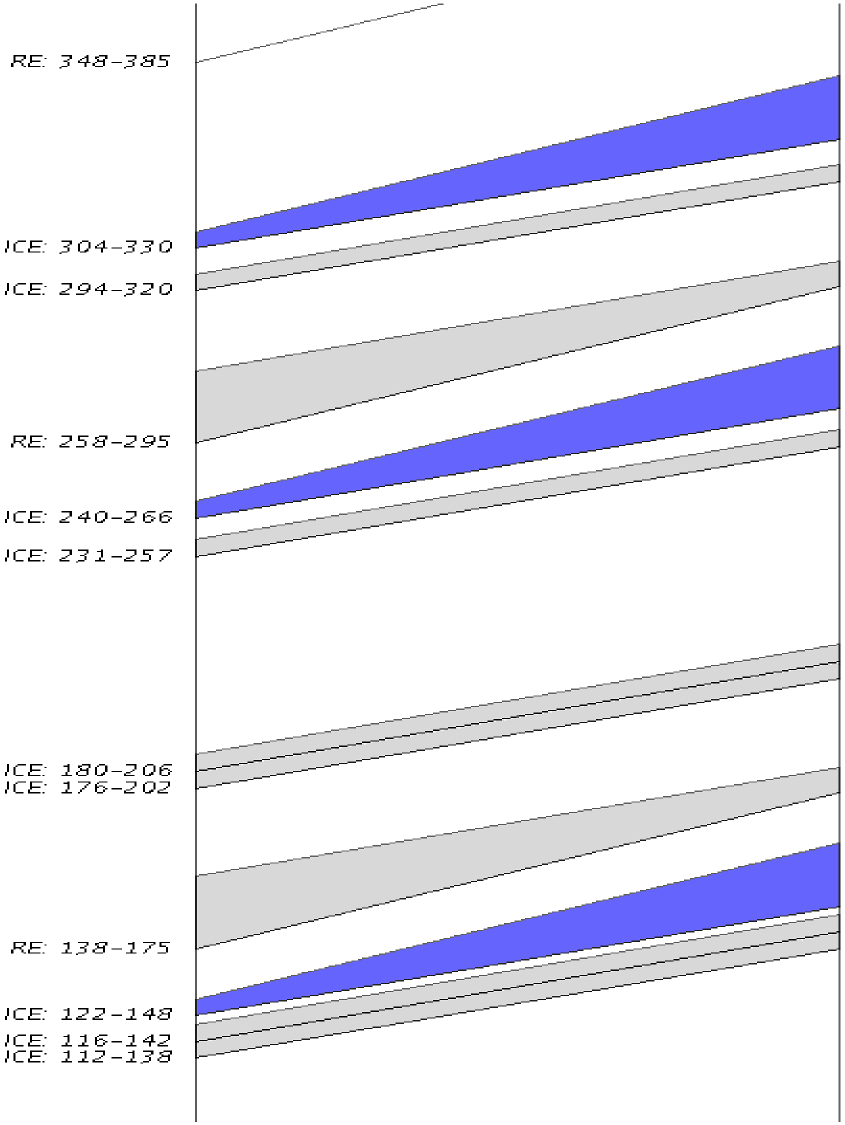
Conflict-Free-Allocation



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State-of-the-Art

- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Cai and Goh (1994), Schrijver and Steenbeck (1994), Carey and Lockwood (1995)
- Nachtigall and Voget (1996), Odijk (1996) Higgings, Kozan and Ferreira (1997)
- Brannlund, Lindberg, Nou, Nilsson (1998) Lindner (2000), Oliveira and Smith (2000)
- Caprara, Fischetti and Toth (2002), Peeters (2003)
- Kroon and Peeters (2003), Mistry and Kwan (2004)
- Barber, Salido, Ingolotti, Abril, Lova, Tormas (2004)
- Semet and Schoenauer (2005),
- Caprara, Monaci, Toth and Guida (2005)
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006),
- Cacchiani, Caprara, T. (2006)
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- and many more

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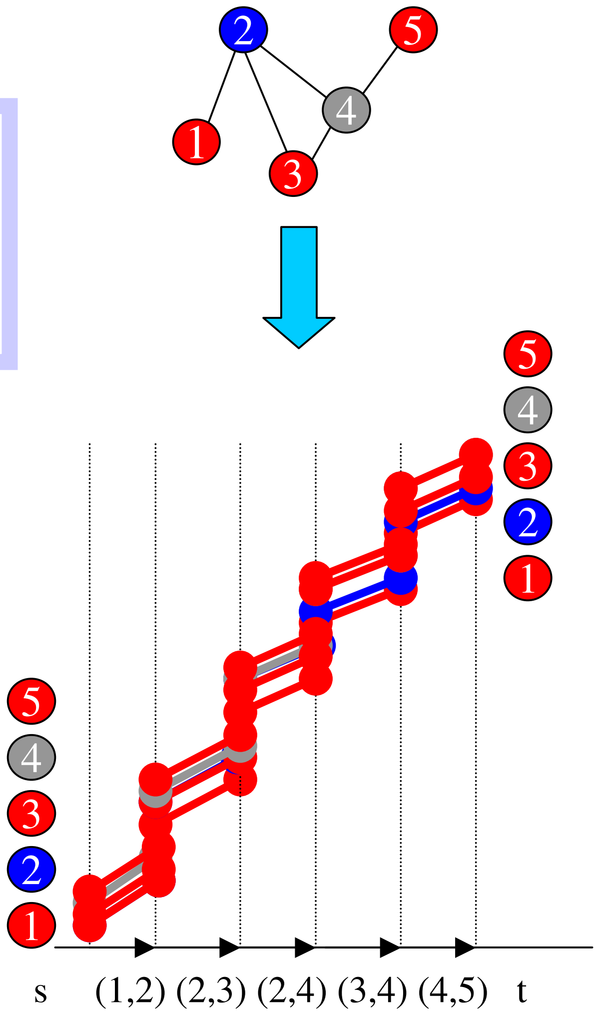
Complexity

Proposition [Caprara, Fischetti, Toth (02)]:

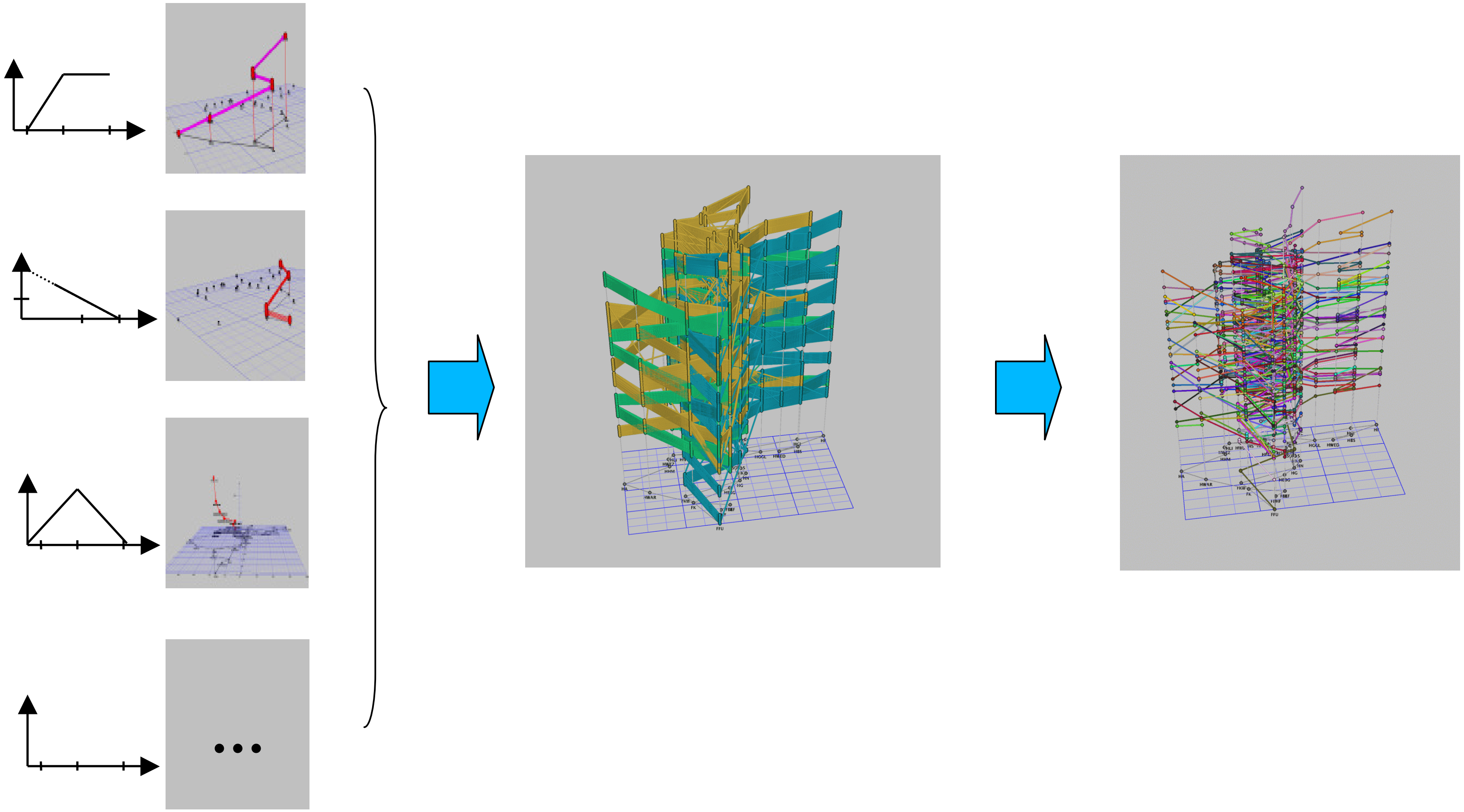
OPTRA/TTP is ***NP***-hard.

Proof:

Reduction from Independent-Set.



Track Allocation Problem



Train Requests → Scheduling Digraph → Timetable

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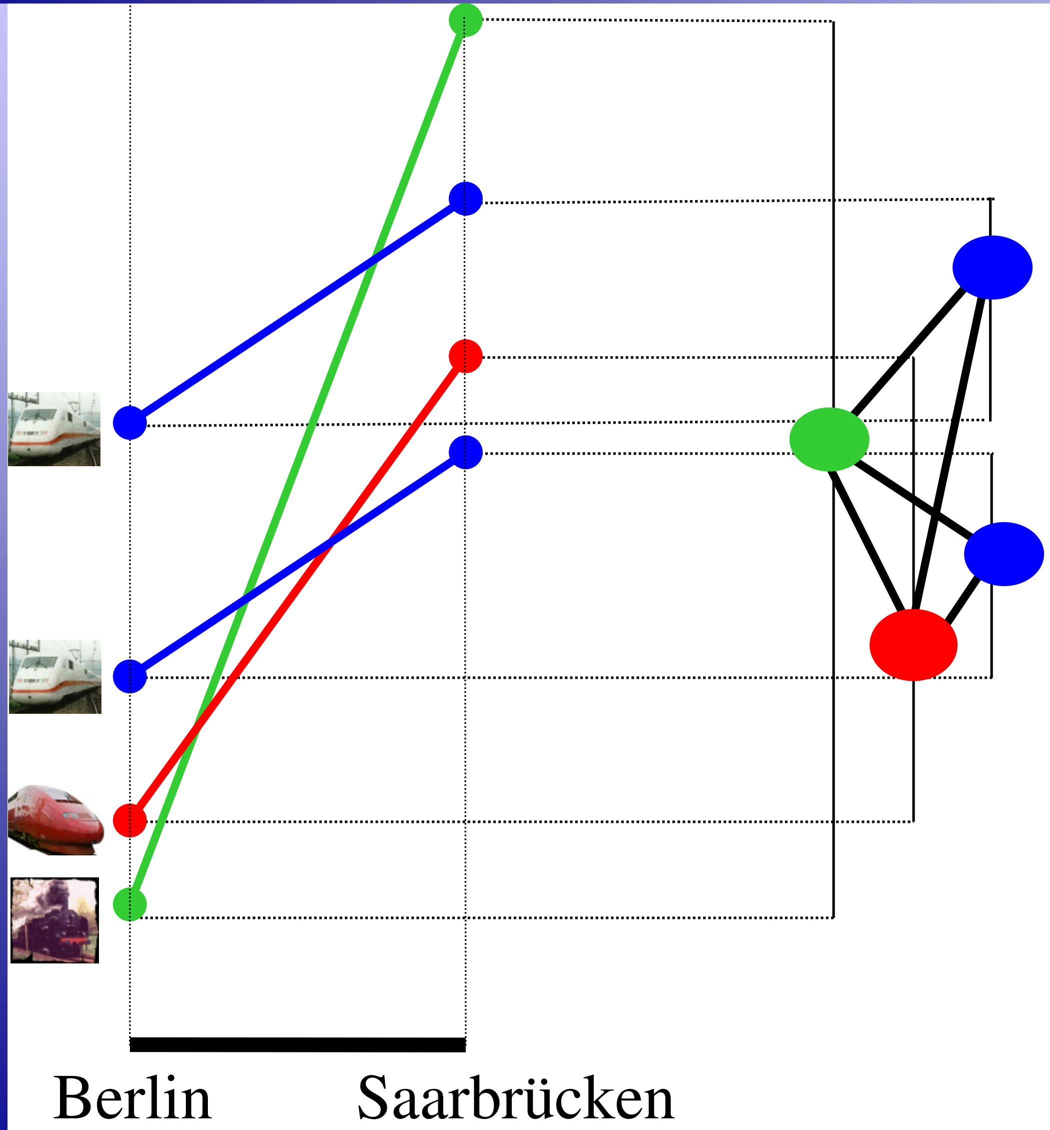
Packing Models



- **Conflict graph**
- Cliques
- Perfect

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Arc Packing Problem

$$
\begin{array}{llll}
(APP) & & & \\
\max & \displaystyle\sum_{i\in\mathcal{I}}\sum_{a\in A} p_a^i x_a^i & & \\
\text{s.t.} & \displaystyle\sum_{a\in\delta_i^{out}(v)} x_a^i - \sum_{a\in\delta_i^{in}(v)} x_a^i \le \delta_i(v) & \forall v\in V, \forall i\in\mathcal{I} & \text{(i)} \\
& \displaystyle\sum_{i\in\mathcal{I}}\sum_{a\in A} x_a^i \le 1 & \forall c\in C & \text{(ii)} \\
& x_a^i \in \{0,1\} & \forall a\in A, \forall i\in\mathcal{I} & \text{(iii)}
\end{array}
$$

Variables

- Arc occupancy (request i uses arc a)

Constraints

- Flow conservation and
- Arc conflicts (pairwise)

Objective

- Maximize proceedings

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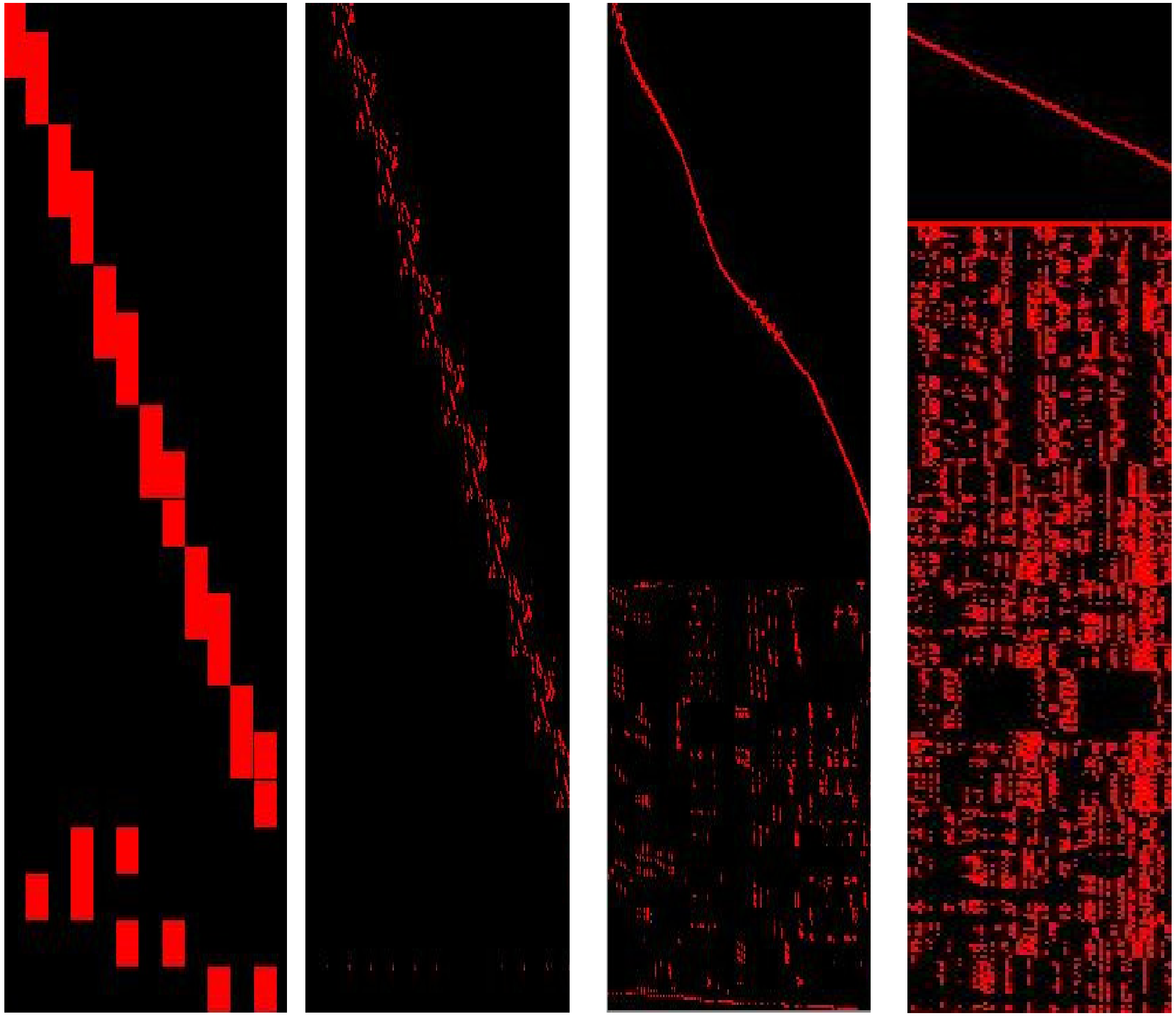
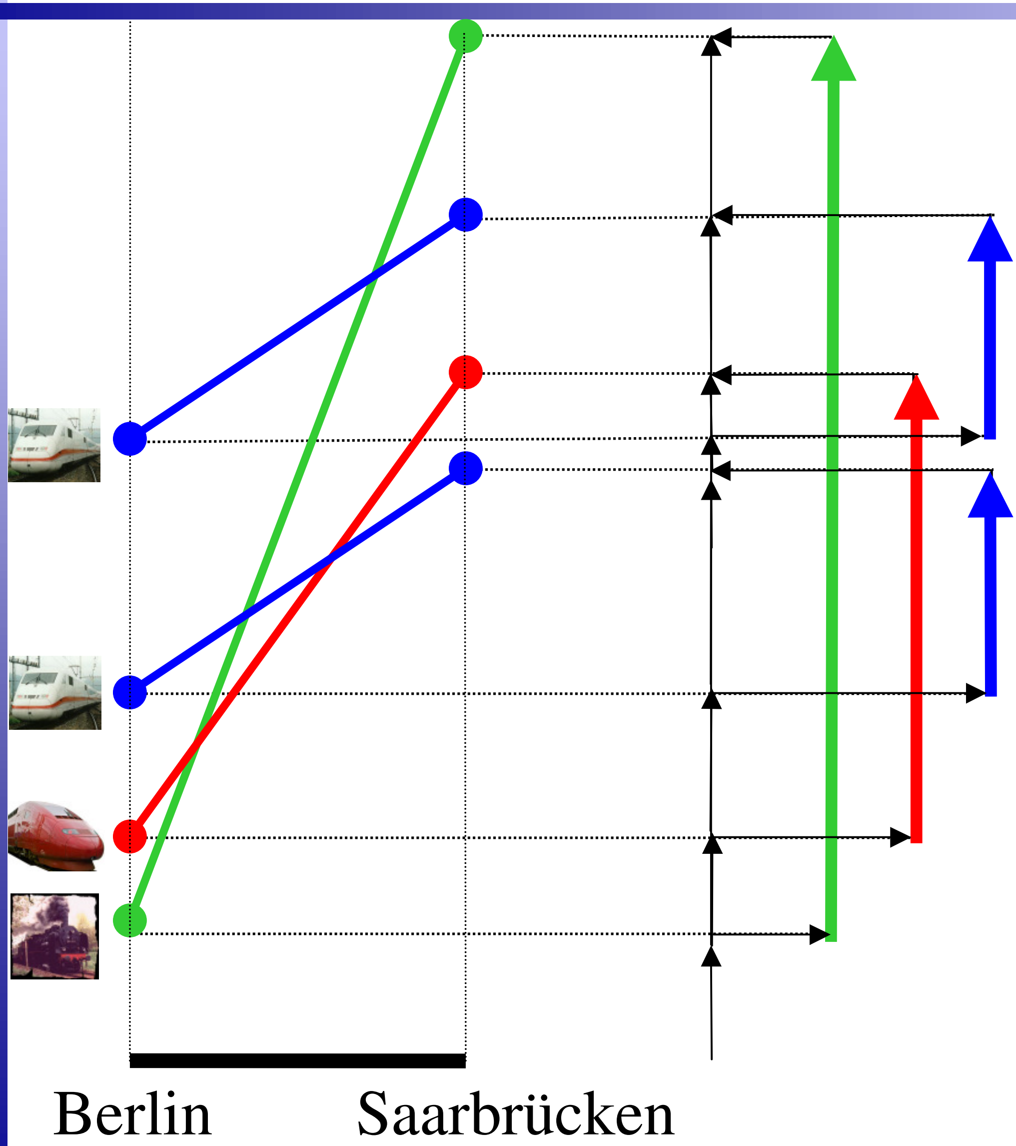
Packing Models

- **Proposition:** The LP-relaxation of APP can be solved in polynomial time.
- … and in practice.



Novel Model



- **Track Graph**
- Timeline(s)
- Config paths

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Path Coupling Problem

$$
\begin{array}{llll}
(PCP) & & & \\
\max & \sum_{p\in\mathcal{P}} \sum_{a\in p} p_a^i x_p & & \\
\text{s.t.} & \sum_{p\in\mathcal{P}_i} x_p \leq 1 & \forall i \in I & \text{(i)} \\
& \sum_{q\in\mathcal{Q}_j} y_q \leq 1 & \forall j \in J & \text{(ii)} \\
& \sum_{a\in p\in\mathcal{P}} x_p - \sum_{a\in q\in\mathcal{Q}} y_q \leq 0 & \forall a \in A_I \cap A_J & \text{(iii)} \\
& y_q \in \{0,1\} & \forall q \in \mathcal{Q}_j,\ \forall j \in J & \text{(iv)} \\
& x_p \in \{0,1\} & \forall p \in \mathcal{P}_i,\ \forall i \in I & \text{(v)}
\end{array}
$$

Variables

- Path und config usage (request i uses path p, track j uses config q)

Constraints

- Path and config choice
- Path-config-coupling (track capacity)

Objective Function

- Maximize proceedings

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Linear Relaxation of PCP

(MLP)

$$
\begin{array}{llrll}
\max & \displaystyle\sum_{p\in\mathcal{P}}\sum_{a\in p} p_a^i x_p & & & \\
\text{s.t.} & \displaystyle\sum_{p\in\mathcal{P}_i} x_p & \leq 1 & \forall i \in I & \text{(i)} \\
 & \displaystyle\sum_{q\in\mathcal{Q}_j} y_q & \leq 1 & \forall j \in J & \text{(ii)} \\
 & \displaystyle\sum_{a\in p\in\mathcal{P}} x_p - \sum_{a\in q\in\mathcal{Q}} y_q & \leq 0 & \forall a \in A_I \cup A_J & \text{(iii)} \\
 & 0 \leq y_q & \leq 1 & \forall q \in \mathcal{Q} & \text{(iii)} \\
 & 0 \leq x_p & \leq 1 & \forall p \in \mathcal{P} & \text{(iv)}
\end{array}
$$

γ_i π_j λ_a

dual variable	information about	useful to
γ_i	bundle price	analyse request
π_j	track price	analyse network
λ_a	arc price	-

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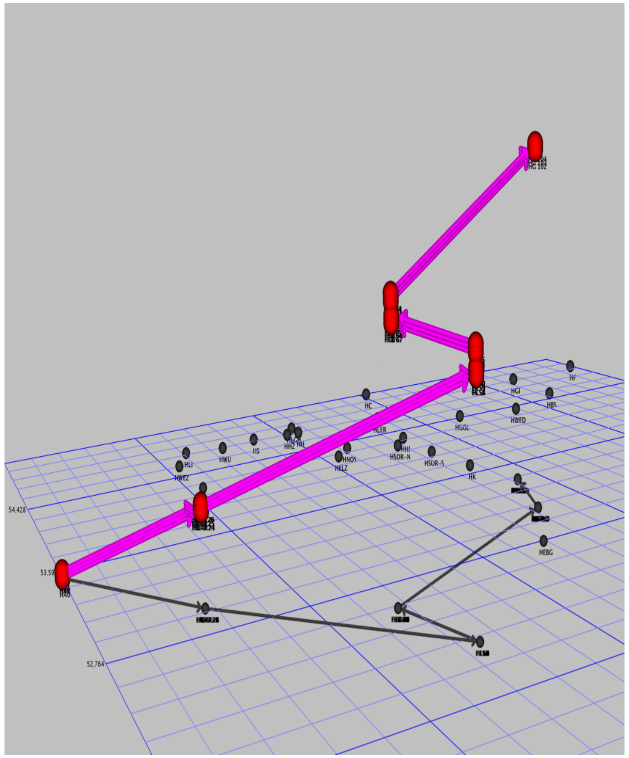
Dualization

$$
\begin{array}{llllr}
(DLP) & & & & \\
\text{min} & \sum_{j\in J} \pi_j + \sum_{i\in I} \gamma_i & & & \\
\text{s.t.} & \gamma_i + \sum_{a\in p} \lambda_a & \geq \sum_{a\in p} p_a^i & \forall p \in \mathcal{P}_i, \forall i \in I & \text{(i)} \\
 & \pi_j - \sum_{a\in q} \lambda_a & \geq 0 & \forall q \in \mathcal{Q}_j, \forall j \in J & \text{(ii)} \\
 & \gamma_i & \geq 0 & \forall i \in I & \text{(iii)} \\
 & \lambda_a & \geq 0 & \forall a \in A_I \cup A_J & \text{(iv)} \\
 & \pi_j & \geq 0 & \forall j \in J & \text{(v)}
\end{array}
$$

Pricing of x-variables



(PRICE (x)) $\exists\, \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)$

$$c_a = -p_a + \lambda_a$$

Pricing Problem(x) :
Acyclic shortest path problems for each slot request i with modified cost function c !

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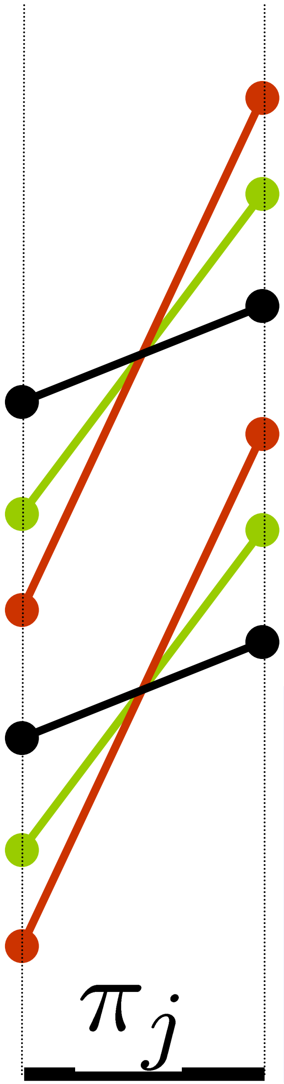
Pricing of y-variables

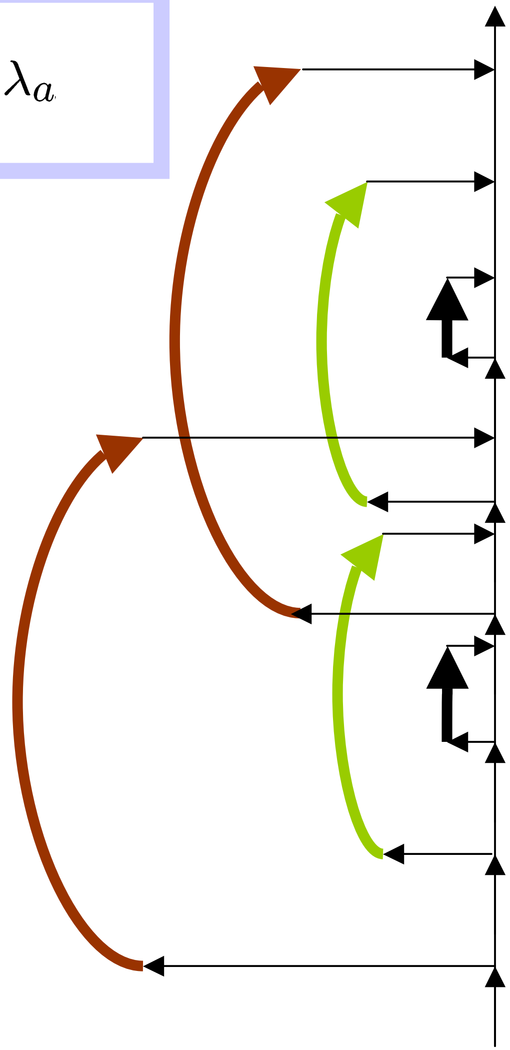


$$\text{(PRICE (y))} \quad \exists\, \overline{q} \in Q_j : \quad \pi_j < \sum_{a \in \overline{q}} \lambda_a$$

$$c_a = -\lambda_a$$

<u>Pricing Problem(y) :</u>

Acyclic shortest path problem for each track j with modified cost function c !



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Observation

$$\text{(PRICE (x))} \quad \exists\, \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)$$

$$\eta_i := \max_{p \in P_i} \sum_{a \in p} (p_a - \lambda_a) - \gamma_i, \; \forall i \in I$$

$$\eta_i + \gamma_i \geq \sum_{a \in p} (p_a - \lambda_a) \; \forall i \in I, p \in \mathcal{P}_i$$

And analogously ...

$$\text{(PRICE (y))} \quad \exists\, \overline{q} \in Q_j : \quad \pi_j < \sum_{a \in \overline{q}} \lambda_a$$

$$\theta_j := \max_{\overline{q} \in Q_j} \sum_{a \in \overline{q}} \lambda_a - \pi_j, \; \forall j \in J$$

$$\theta_j + \pi_j \geq \sum_{a \in q} \lambda_a \; \forall j \in J, q \in \mathcal{Q}_j$$

Pricing Upper Bound

$(\max\{\eta+\gamma,0\},\max\{\theta+\pi,0\},\lambda)$ is feasible for (DLP)

↳

$$\beta(\gamma,\pi,\lambda) := \sum_{i\in I}\max\{\gamma_i+\eta_i,0\}+\sum_{j\in J}\max\{\pi_j+\theta_j,0\}$$

- **Lemma** [ZR-07-02]: Given (infeasible) dual variables of PCP and let v_{LP}(PCP) be the optimum objective value of the LP-Relaxtion of PCP, then:

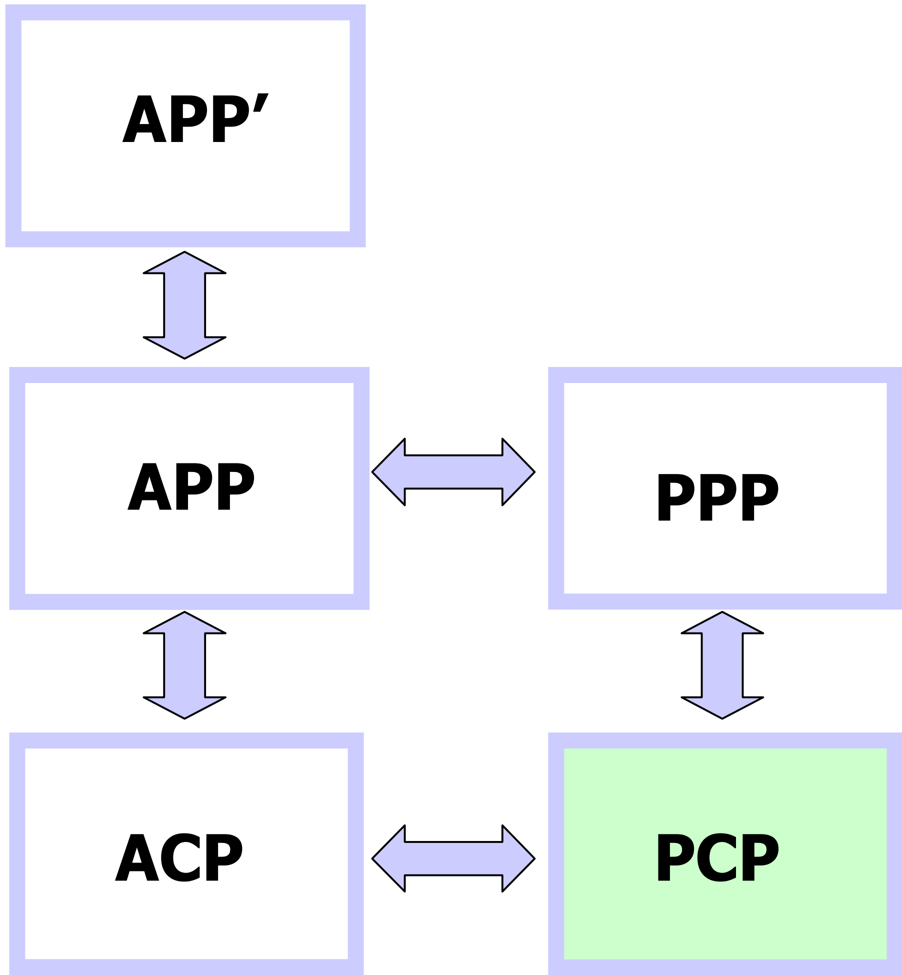
$$v_{LP}(PCP) \leq \beta(\gamma,\pi,\lambda)$$

Model Comparison

- **Theorem** [ZR-07-02]:
The LP-relaxations of ACP and PCP can be solved in polynomial time.

- **Lemma** [ZR-07-02]:

$$v_{LP}(PCP) = v_{LP}(ACP)$$
$$= \quad v_{LP}(APP) = v_{LP}(PPP)$$
$$\leq \quad v_{LP}(APP')$$



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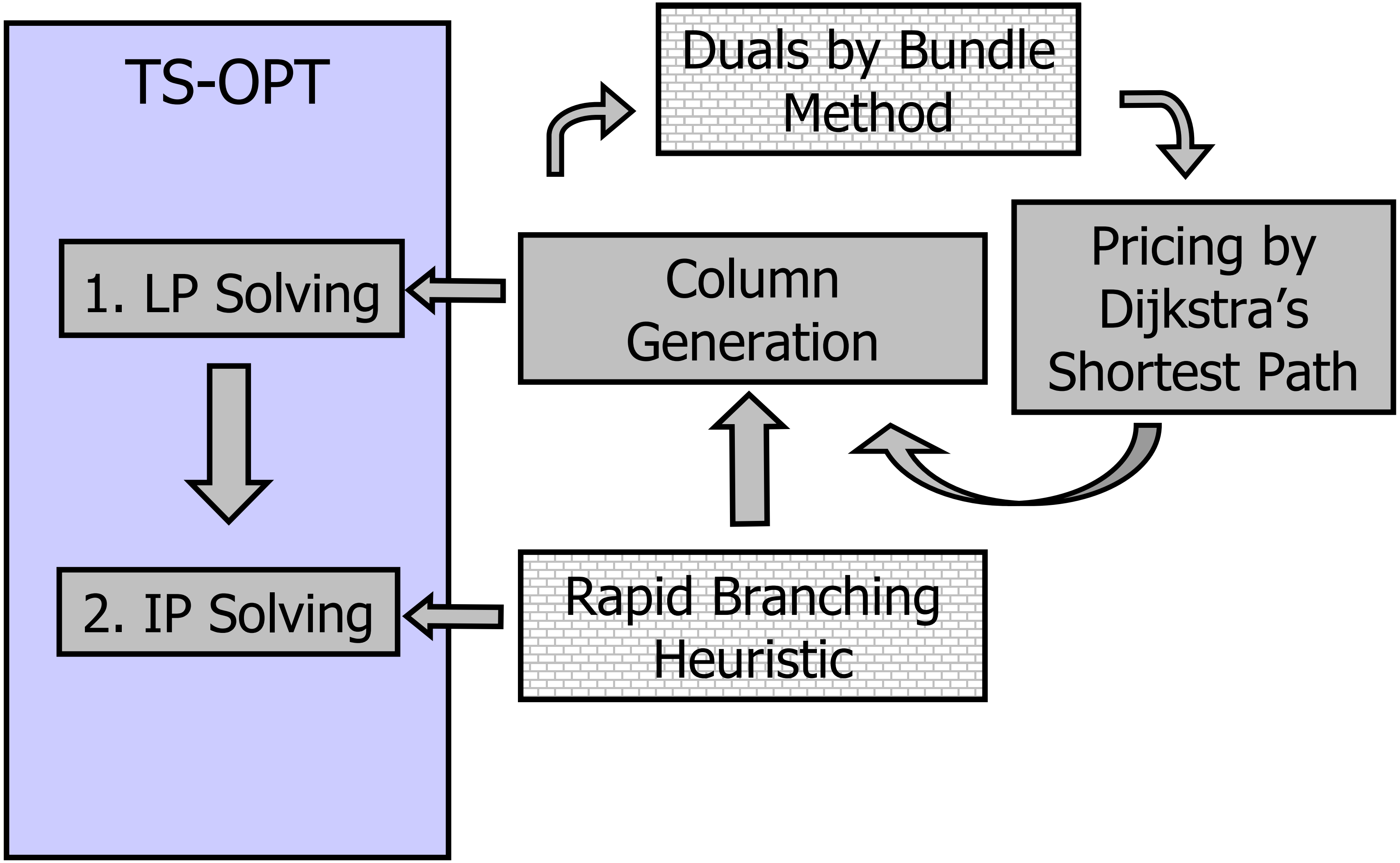
Two Step Approach



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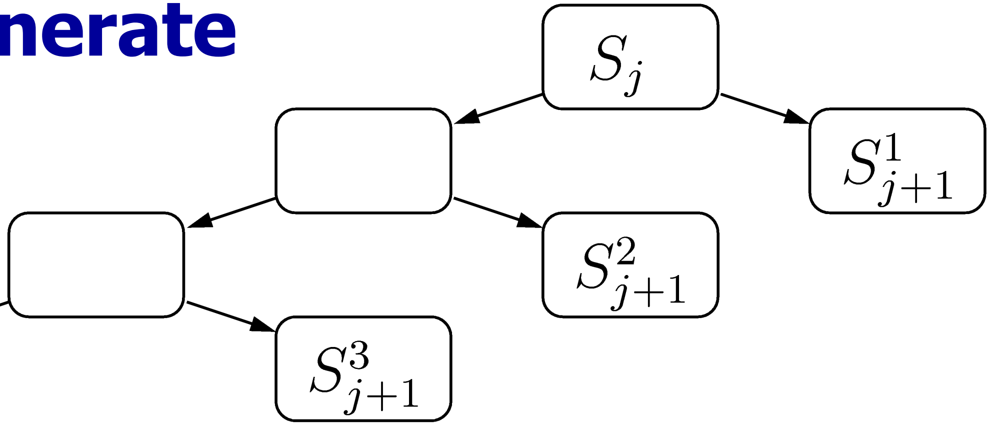
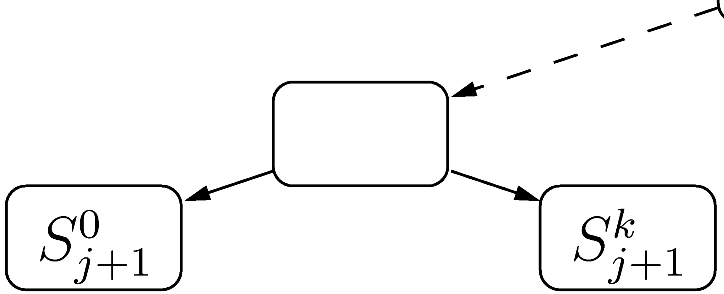
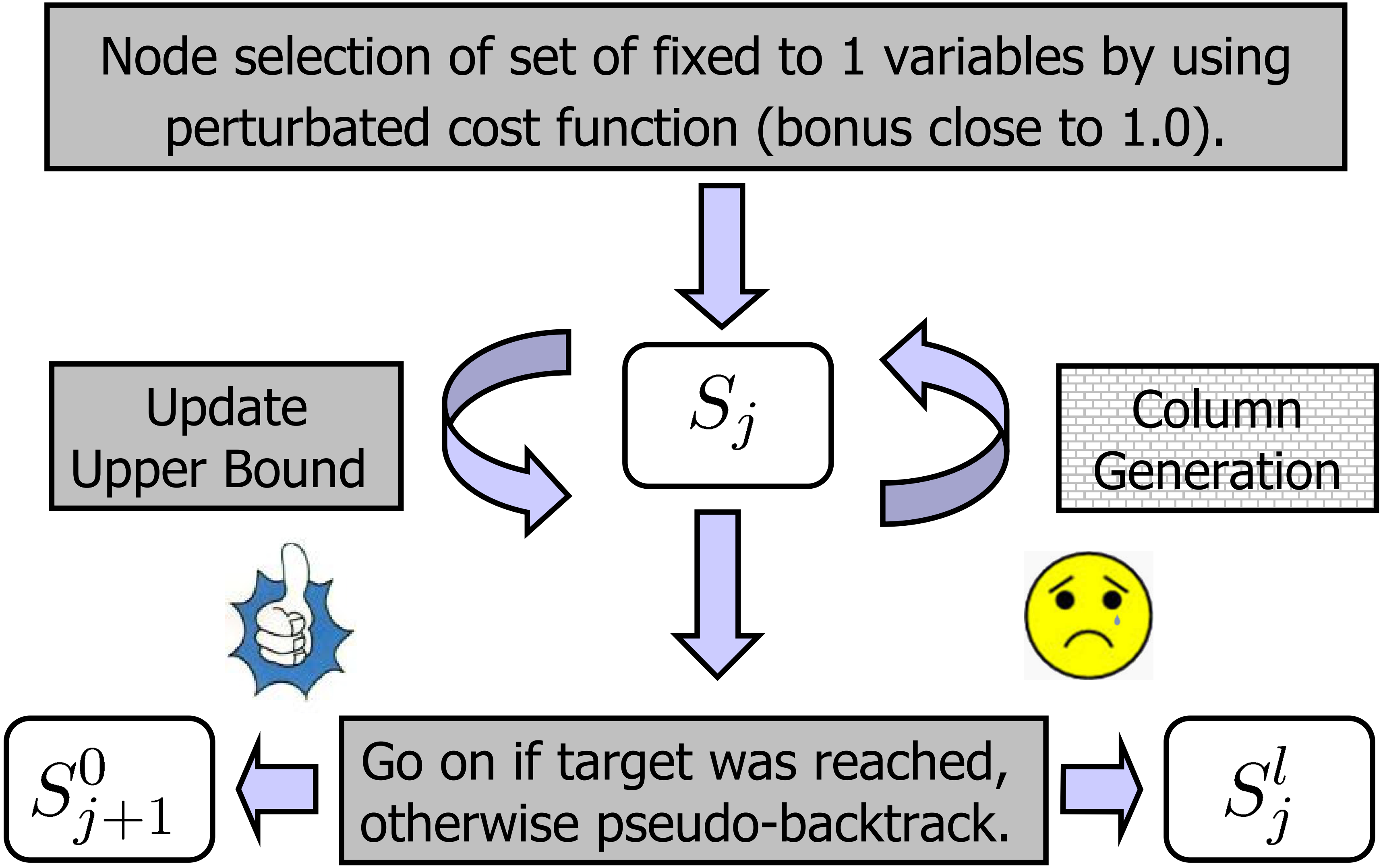
Branch-Bound-Price

or Dive-Generate

Evaluation of only few highly different sub-problems at iteration j to reach IP-Solutions fast.

Rapid Branching



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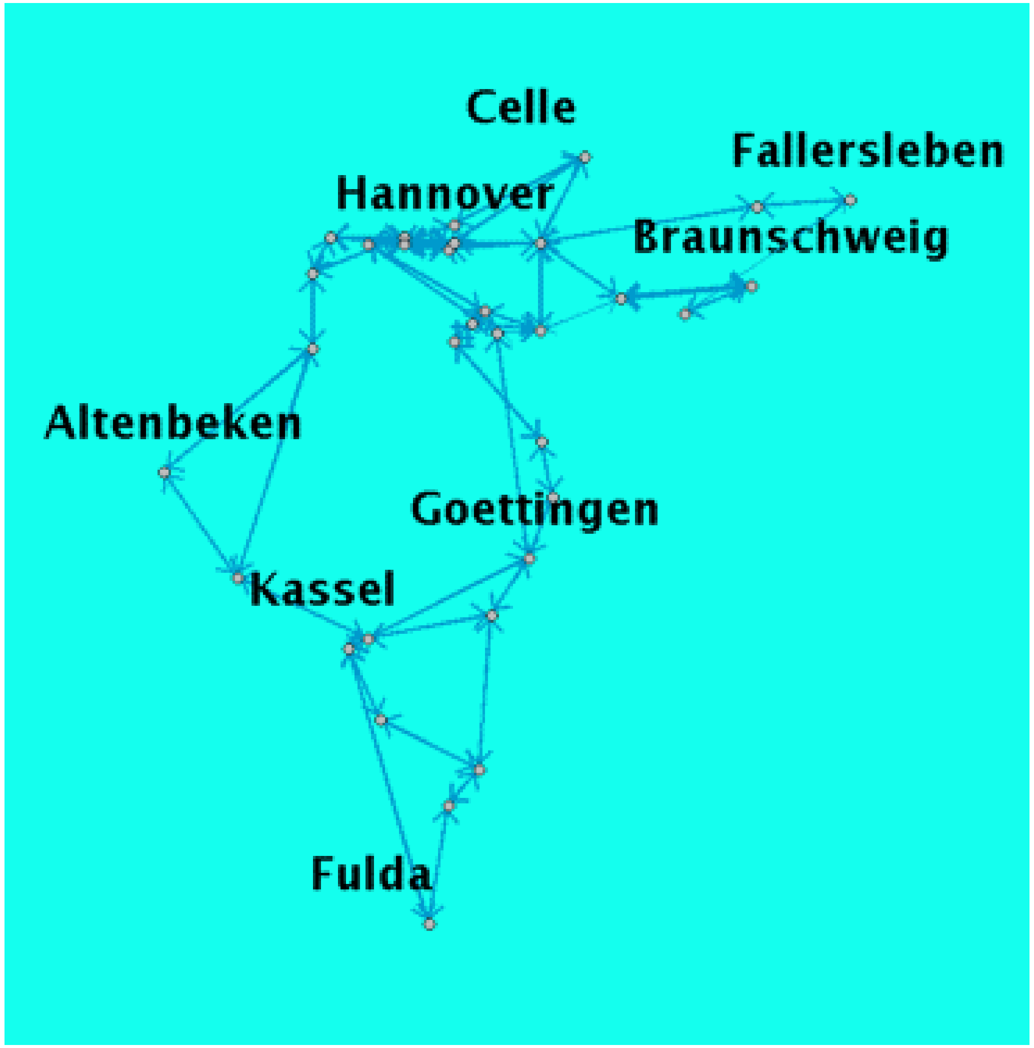
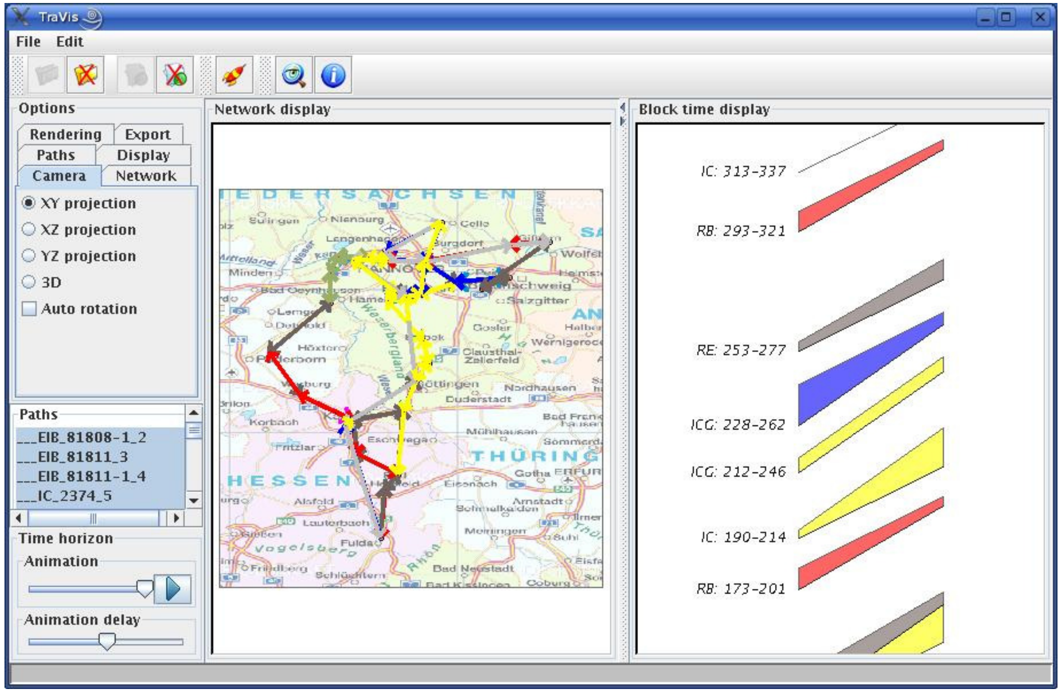
Results

- **Test Network**
  - 45 Tracks
  - 37 Stations
  - 6 Traintypes
  - 10 Trainsets
  - 146 Nodes
  - 1480 Arcs
  - 96 Station Capacities
  - 4320 Headway Times



Model Comparison

- **Test Scenarios**
  - 146 Train Requests
  - 285 Train Requests
  - 570 Train Requests



- **Flexibility**
  - 0-30 Minutes
  - earlier departure penalties
  - late arrival penalties
  - train type depending profits

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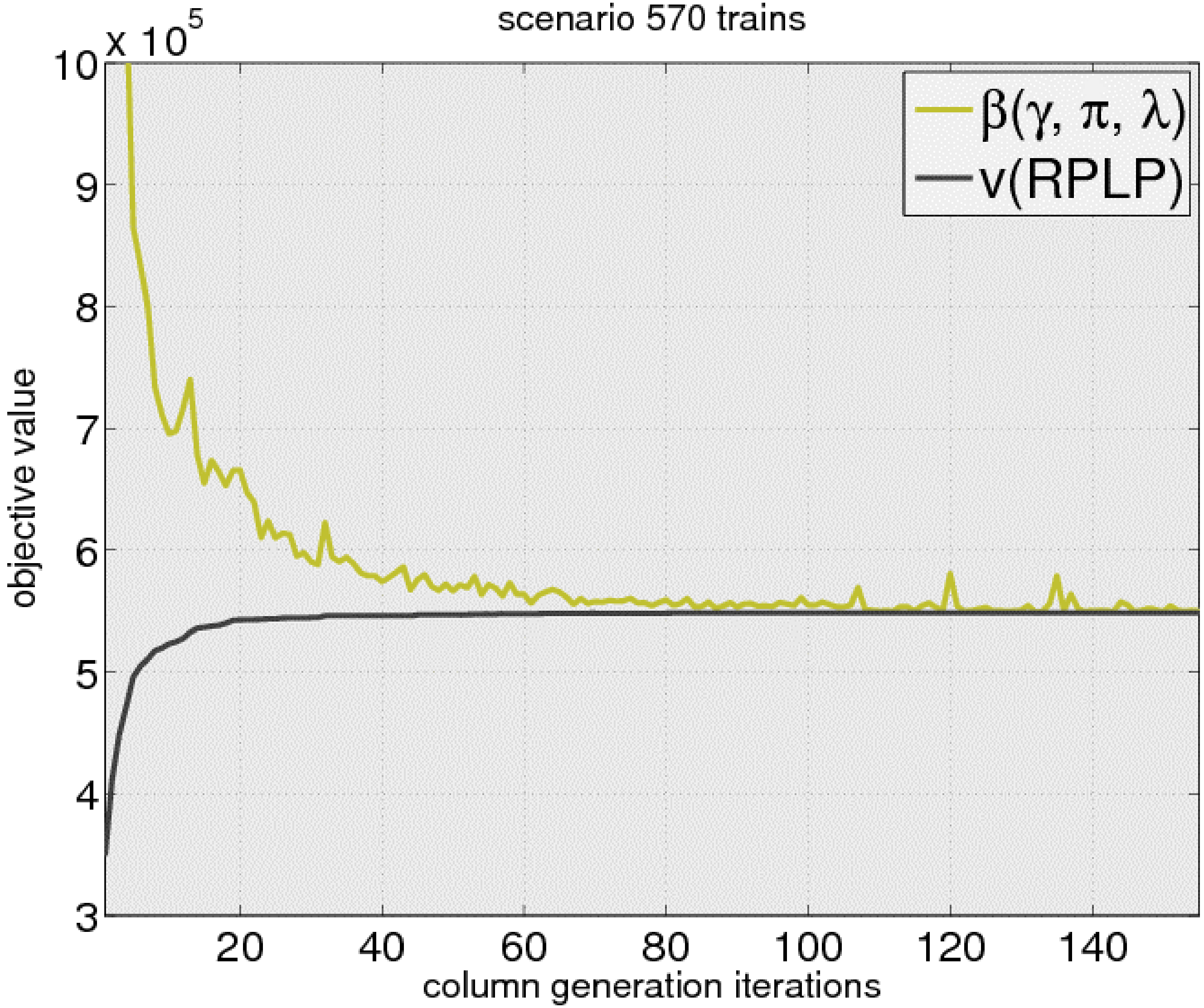
Run of TS-OPT



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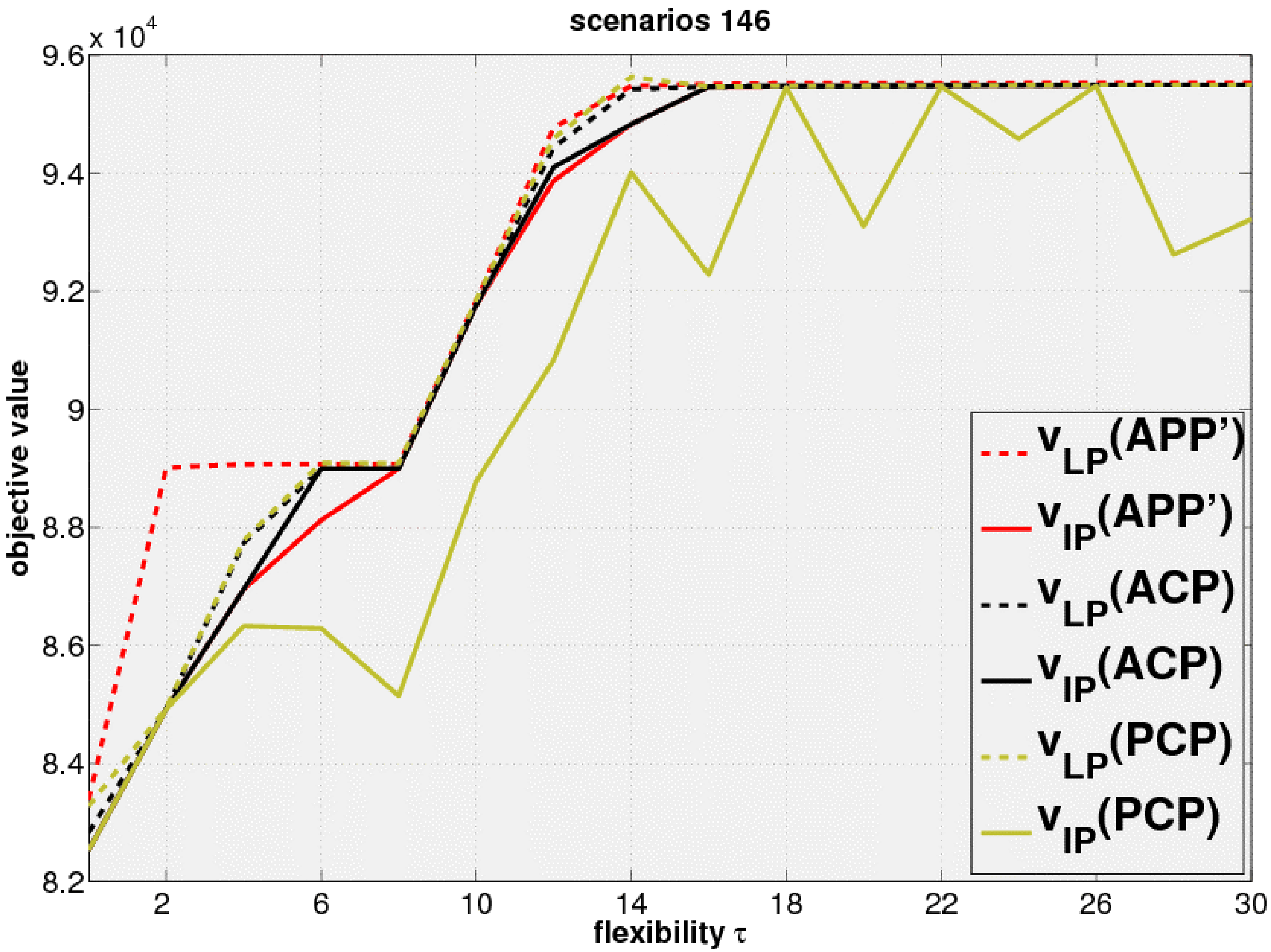
Model Comparison



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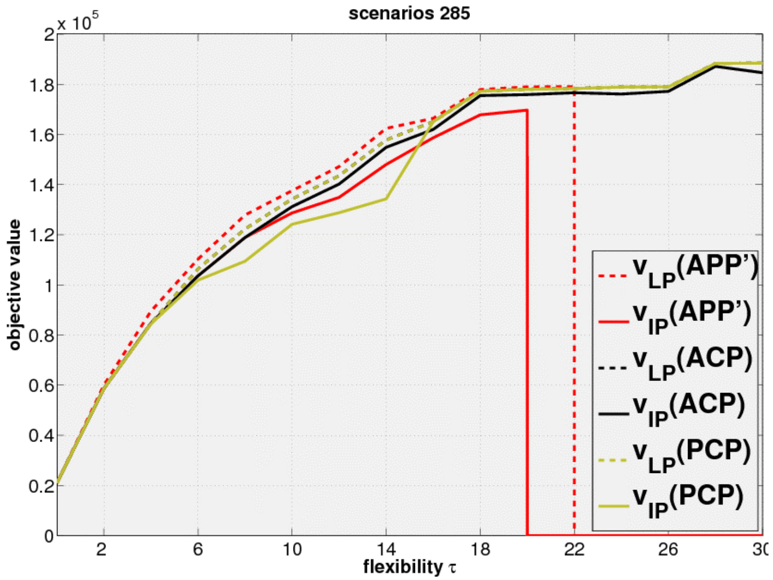
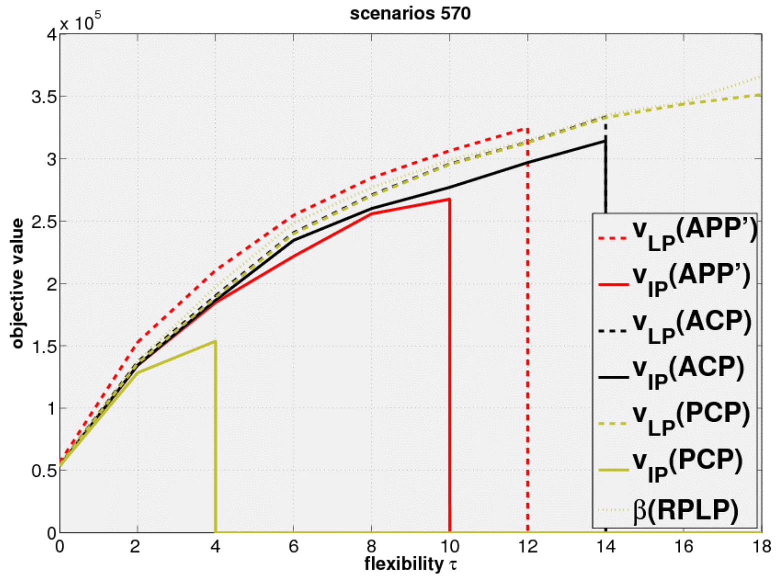
Model Comparison





For details see [ZR-07-02, ZR-07-20].

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Outlook

Algorithmic Developments

- Bundle method
- Model refinement (connections)
- Adaptive IP Heuristics
- Dynamic Discretization

Simulation of results by

Railsys®

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Thank you for your attention !

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