

## Optimization 2

### Excercise Sheet 1

Submission: Wednesday, 1st November 2017, 12:00

#### Exercises:

Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two. Please put your name(s) on your exercise sheet and hand them in on 1st November after the lecture or send them to me via mail.

Homepage of the Lecture: [http://www.zib.de/ws17\\_Optimierung\\_II](http://www.zib.de/ws17_Optimierung_II)  
Questions?: [beckenbach@zib.de](mailto:beckenbach@zib.de)

#### Exercise 1.1

**6 Points**

Consider the following quadratic program

$$\begin{aligned} \min \quad & \frac{1}{2}x_1^2 + \frac{3}{2}x_2^2 + \frac{1}{2}x_3^2 - 2x_1x_2 - 2x_2x_3 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 = 3 \\ & x_1 \leq x_2 \\ & x_2 \leq x_3 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

Which of the three points  $x_1 = (0, 1, 2)^T$ ,  $x_2 = (1, 1, 1)^T$ ,  $x_3 = (0, 3/2, 3/2)^T$  satisfies the KKT Conditions?

#### Exercise 1.2

**6 Points**

Solve the following quadratic program using the active set method starting at  $x^0 = (2, 0)$ . Sketch the feasible set and the iterative points  $x^k$ .

$$\begin{aligned} \min \quad & x_1^2 + x_2^2 + x_1x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 2 \\ & x_2 \geq -1 \\ & x_2 \leq x_1 - 1. \end{aligned}$$

PLEASE TURN OVER

**Exercise 1.3****4 Points**

Show that the quadratic program

$$\begin{aligned} \min \quad & \frac{1}{2}x^tQx + c^tx \\ \text{s.t.} \quad & Ax = b \end{aligned}$$

has a unique optimal solution if  $Q \in \mathbb{R}^{n \times n}$  is a symmetric matrix which is positive definite in the null space of  $A$  ( $x^tQx > 0$  for all  $x \in \ker A \setminus \{0\}$ ),  $A \in \mathbb{R}^{m \times n}$  has rank  $m$ , and  $Ax = b$  has a solution.

**Exercise 1.4****4 Points**

Give an example of a non-convex quadratic program with a local minimum which is not a global minimum.