Freie Universität Berlin
FB Mathematik und Informatik
WS 2017/18

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Optimization 2

Exercise Sheet 10

Submission: Wednesday, 17.01.2018, 12:00

Exercises:
Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two.

Homepage of the Lecture: `http://www.zib.de/ws17_Optimierung_II`
Questions?: `beckenbach@zib.de`

Exercise 10.1 — 2+4+2 Points

Let $G = (V, E)$ be a connected graph. The graphic matroid $\mathcal{M}_G$ associated to G has E as its ground set and $F \subseteq E$ is independent if and only if $G[F]$ contains no cycle. Denote the set of independent sets of $\mathcal{M}_G$ by $\mathcal{I}_G$.

(a) Show that $\mathcal{M}_G$ is indeed a matroid.

(b) How do the bases and circuits of $\mathcal{M}_G$ look like?

(c) Give a formula for the rank function of $\mathcal{M}_G$.

Exercise 10.2 — 6 Points

Give a closed formula for the rank functions of the following matroids.

(a) The free matroid $(E, 2^E)$.

(b) The k-uniform matroid $U_{k,n}$ on a ground set of size n, i.e.

$$U_{k,n} := (\{1, \dots, n\}, \{I \subseteq \{1, \dots, n\} : |I| \leq k\}).$$

(c) The trivial matroid $(E, \{\emptyset\})$.

Exercise 10.3 **3+3 Points**

Let $G = (V, E)$ be a connected graph. Look at the following independence systems and prove or disprove that they are matroids.

(a) $\mathcal{M}_1 := (E, \mathcal{I}_1)$ where $\mathcal{I}_1$ is the set of all matchings in G, i.e.,

$$\mathcal{I}_1 = \{M \subseteq E : \deg_M(v) \leq 1 \ \forall v \in V\}.$$

(b) $\mathcal{M}_2 := (V, \mathcal{I}_2)$ where $\mathcal{I}_2$ is the set of all vertex subsets covered by some matching of G, i.e.,

$$\mathcal{I}_2 = \{U \subseteq V : \exists \text{ maching M with } U \subseteq V(M)\}.$$