Freie Universität Berlin
FB Mathematik und Informatik
WS 2017/18

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Optimization 2

Excercise Sheet 2

Submission: Friday, 10.11.2017, 12:00

Exercises:
Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two. Please put your name(s) on your exercise sheet and hand them in until 10.11, 15:00 (in my office, room 3301 at ZIB) or after one of the lectures next week.

Homepage of the Lecture: `http://www.zib.de/ws17_Optimierung_II`
Questions?: `beckenbach@zib.de`

Exercise 2.1 — 10 Points

Let L_1 and L_2 be two lines in the three-dimensional euclidean space given by

$$L_1 = \{x \in \mathbb{R}^3 : x = a + \lambda u, \lambda \in \mathbb{R}\}$$
$$L_2 = \{x \in \mathbb{R}^3 : x = b + \mu v, \mu \in \mathbb{R}\}$$

where $a, b, u, v \in \mathbb{R}^3$, the vectors u and v are not parallel, and they are normalized ($\|u\| = \|v\| = 1$). You want to find two points $x \in L_1, y \in L_2$ such that the euclidean distance between them is as small as possible.

Formulate this problem as a quadratic optimization problem in variables $\lambda, \mu \in \mathbb{R}$, argue why the resulting QP has a unique minimum, and formulate the KKT-conditions for an optimum.

Exercise 2.2 — 10 Points

Let $P : V \to V$ be a projection from a finite dimensional vector space V to itself. By $\mathrm{im}(P)$ we denote the image of P and by $\ker(P)$ the kernel of P.

(a) Show that $P(v) = v$ for all $v \in \mathrm{im}(P)$.

(b) Show that V is isomorphic to the direct sum of $\mathrm{im}(P)$ and $\ker(P)$.

please turn over

Now, let $A \in \mathbb{R}^{m \times n}$ be a matrix of full row rank.

(c) Prove that $\mathbb{R}^n$ is isomorphic to $\ker(A) \oplus \operatorname{im}(A^T)$.

(d) Show that the orthogonal projection of $\mathbb{R}^n$ onto $\operatorname{im}(A^T)$ is given by $A^T(AA^T)^{-1}A$,

(e) and the orthogonal projection of $\mathbb{R}^n$ onto $\ker(A)$ is given by $I-(A^T(AA^T)^{-1}A$.