Freie Universität Berlin
FB Mathematik und Informatik
WS 2017/18

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Optimization 2

Exercise Sheet 5

Submission: Wednesday, 29.11.2017, 12:00

Exercises:
Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two. Please put your name(s) on your exercise sheet and hand them in until 29.11 (in my office, room 3301 at ZIB) or after one of the lectures next week.

Homepage of the Lecture: `http://www.zib.de/ws17_Optimierung_II`
Questions?: `beckenbach@zib.de`

Exercise 5.1 — 4 Points

Consider the following variant of the assignment problem:

You are given n jobs and n workers such that every worker can do every job, however, each worker j needs a specific time $t_{i,j} \in \mathbb{Z}_{\geq 0}$ to complete job i. The shift is finished when the last worker has completed his job. Your task is to assign workers and jobs one to one such that the workers can finish as early as possible. Write down an integer linear program for this problem.

Exercise 5.2 — 8 Points

(a) Show that a graph $G = (V, E)$ with at least two vertices is a tree if and only if there exists a vertex v of degree one in G such that $G - v = (V \setminus v, \{e \in E : v \text{ is not an end vertex of } e\})$ is a tree.

(b) Now, let $G = (V, E)$ be a tree, and $T_i = (V_i, E_i)$, $i \in \{1, 2, 3\}$ three subtrees of T (this means that $V_i \subseteq V$ and $E_i \subseteq E$). Suppose that each pair of the three subtrees has a non-empty intersection ($V_i \cap V_j \neq \emptyset$ for all $i, j \in \{1, 2, 3\}$). Show that there exists a vertex contained in all three trees T_1, T_2, T_3.

please turn over

Exercise 5.3 **8 Points**

Let $G = (V, E)$ ba a connected graph and $T = (V, F)$ a spanning tree of G.

(a) Show that for every $e \in E \setminus F$ the graph $T + e = (V, F \cup \{e\})$ contains exactly on cycle which we denote by $C(e)$.

(b) The incidence vector χ^S of a set $S \subseteq E$ is an $|E|$-dimensional vector indexed by E such that $\chi^S_e := \begin{cases} 1 & \text{if } e \in S, \\ 0 & \text{if } e \notin S \end{cases}$.

The *cycle space* $\mathcal{C}(G)$ of G is the vector space over $\mathbb{F}_2$ generated by the incidence vectors of cycles of G, i.e.,

$$\mathcal{C}(G) = \text{span}\{\chi^C : C \text{ is a cycle in } G\} \subseteq \mathbb{F}_2^{|E|}.$$

Show that the vectors $\chi^{C(e)}$, where $C(e)$ is defined as in (a), form a basis of $\mathcal{C}(G)$.