Freie Universität Berlin
FB Mathematik und Informatik
WS 2017/18

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Optimization 2

Exercise Sheet 7

Submission: Wednesday, 13.12.2017, 12:00

Exercises:
Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two. Please put your name(s) on your exercise sheet and hand them in until 29.11 (in my office, room 3301 at ZIB) or after one of the lectures next week.

Homepage of the Lecture: `http://www.zib.de/ws17_Optimierung_II`
Questions?: `beckenbach@zib.de`

Exercise 7.1 — 1+2+3+2+2 Points

Prove or disprove the following assertions.

(a) If G is a k-connected graph, then G is k'-connected for every $k' \in \{1, \dots, k-1\}$.

(b) If G is k-connected, then every vertex has degree at least k.

(c) There exists a function $f : \mathbb{Z}_{\geq 2} \to \mathbb{Z}_{\geq 2}$ such that every graph of minimum degree at least $f(k)$ is k-connected (where $\mathbb{Z}_{\geq 2}$ is the set of all integers greater or equal to two).

(d) Every 2-connected graph contains a cycle through each pair of vertices.

(e) There exists a tree T with less than $\max_{v \in V(T)} \deg(v)$ leaves (=vertices of degree one).

PLEASE TURN OVER

Exercise 7.2 **6 Points**

Consider the following integer linear program

$$\min T$$

$$\sum_{i=1}^{n} x_{i,j} = 1 \qquad \forall j = 1, \dots, n \tag{1}$$

$$\sum_{j=1}^{n} x_{i,j} = 1 \qquad \forall i = 1, \dots, n \tag{2}$$

$$t_{i,j} x_{i,j} \leq T \qquad \forall i = 1, \dots, n, j = 1, \dots, n \tag{3}$$

$$x_{i,j} \in \{0, 1\} \qquad \forall i = 1, \dots, n, j = 1, \dots, n \tag{4}$$

where $t_{i,j} \in \mathbb{Z}_{\geq 0}$ are given for $i, j = 1, \dots, n$.

(a) Show that for fixed $T \in \mathbb{Z}_{\geq 0}$ the feasibility of (1)-(4) can be formulated as an assignment problem.

(b) If T^* is the optimal solution to the integer linear program above, then there exists $i, j \in \{1, \dots, n\}$ with $T^* = t_{i,j}$.

(c) You can use the following fact: The assignment problem can be solved in polynomial time. Show that also integer linear program above can be solved in polynomial time (Hint: use (a) and (b)).

Exercise 7.3 **4 Points**

Let $f, g : \mathbb{N} \to \mathbb{N}$ two functions with $f \in \mathcal{O}(n^k)$ and $g \in \mathcal{O}(n^l)$. Show that

(a) $f \cdot g \in \mathcal{O}(n^{k+l})$,

(b) $f + g \in \mathcal{O}(n^{\max(k,l)})$.