Freie Universität Berlin
FB Mathematik und Informatik
Sommersemester 2017

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Optimierung I

Excercise Sheet 7

Submission: until 17:00 on Monday, June 12, 2017

Exercise 7.1 — 10 Points

Proof the following alternatives of the Farkas-Lemma:

a) Let $A \in \mathbb{R}^{m\times n}, b \in \mathbb{R}^m$. Then exactly one of the following systems has a solution:

$$Ax \le b \qquad \dot{\vee} \qquad \begin{array}{rcl} y^T b & < & 0 \\ y^T A & = & 0 \\ y & \ge & 0 \end{array}$$

b) Let A, B, C, D and a, b be compatible matrices and vectors. Then exactly one of the following systems has a solution:

$$\begin{array}{rcrcl} Ax & + & By & \le & a \\ Cx & + & Dy & = & b \\ x & & & \ge & 0 \end{array} \qquad \dot{\vee} \qquad \begin{array}{rcrcl} u^T A & + & v^T C & \ge & 0 \\ u^T B & + & v^T D & = & 0 \\ u & & & \ge & 0 \\ u^T a & + & v^T b & < & 0 \end{array}$$

Exercise 7.2 — 10 Points

Given the following optimization problem:

$$\begin{array}{lrcrcrcrcrcl} \min & 4x_1 & + & 4x_2 & + & 8x_3 & + & 6x_4 & + & 6x_5 & & \\ \text{s.t.} & x_1 & + & x_2 & + & x_3 & - & 2x_4 & + & 2x_5 & \ge & 1 \\ & x_1 & - & 2x_2 & + & 2x_3 & + & x_4 & + & x_5 & \ge & 1 \\ & x_1 & , & x_2 & , & x_3 & , & x_4 & , & x_5 & \ge & 0 \end{array} \qquad \text{(LP)}$$

a) Formulate the dual (DP) of the linear program (LP). Make a sketch for DP.

b) Solve the dual DP graphically or with SCIP.

c) Construct a optimal solution of LP out of the optimal solution of DP.

Exercise 7.3 **10 Points**

Proof or counterproof the following theorem:

Let $A \in \mathbb{R}^{m \times n},\ b \in \mathbb{R}^m$. Then the vector $\bar{x}$ is an optimal solution of

$$\begin{array}{lrcl} \max & c^T x & & \\ \text{s.t.} & Ax & = & b \\ & x & \geq & 0 \end{array}$$

if and only if

$$\begin{array}{rcll} A\bar{x} & = & b & \\ \bar{x} & \geq & 0 & \\ c^T s & \leq & 0 & \forall s \in \{v \in \mathbb{R}^n \ : \ Av = 0,\ v_{\{1,\dots,n\} \setminus \mathsf{supp}(\bar{x})} \geq 0\} \end{array}$$

Homepage of the Lecture: `http://www.zib.de/ss17_Optimierung_I`
Questions?: `klug@zib.de`