Freie Universität Berlin
FB Mathematik und Informatik
Sommersemester 2017

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Optimierung I

Excercise Sheet 8

Submission: until 17:00 on Monday, June 19, 2017

Exercise 8.1 — 10 Points

Unless otherwise stated, we consider a linear program in standard form

$$\begin{array}{rrcl} \max & c^T x & & \\ \text{s.t.} & Ax & = & b \\ & x & \geq & 0 \end{array}$$

with $A \in \mathbb{R}^{m \times n},\ b \in \mathbb{R}^m,\ c \in \mathbb{R}^n,\ m < n,\ \mathsf{rank}(A) = m,\ P^=(A, b) \neq \emptyset$.

Prove or disprove the following statements.

a) A non-basic variable that enters the basis in any step of the simplex algorithm cannot leave the basis in the next step.

b) A basic variable that just left the basis in any step of the simplex algorithm cannot enter the basis in the next step.

c) If $A = A^\top$ then each feasible solution of the linear program

$$\begin{array}{rl} \max & c^T x \\ \text{subject to} & A\,x = c \end{array}$$

is optimal.

d) If none of the basic solutions is degenerate and the LP is bounded, then the optimal solution is unique.

e) If an unbounded variable x_j were substituted by $x_j^+ - x_j^-$ $(x_j^+, x_j^- \geq 0)$, then in each step of the simplex method at most one of the variables x_j^+, x_j^- is not equal to zero.

Exercise 8.2 — 10 Points

Solve the following problem by the simplex method:

$$\begin{array}{lrcrcrcl} \max & 3x_1 & + & 2x_2 & + & 4x_3 & & \\ \text{s.t.} & x_1 & + & x_2 & + & 2x_3 & \leq & 4 \\ & 2x_1 & & & + & 3x_3 & \leq & 5 \\ & 2x_1 & + & x_2 & + & 3x_3 & \leq & 7 \\ & x_1, & & x_2, & & x_3 & \geq & 0 \end{array}$$

Emphasis for each iteration which variable leaves and which variable enters the basis.

Exercise 8.3 **10 Points**

Consider the linear Programm (P)

$$\begin{array}{rrcl} \max & c^T x & & \\ \text{s.t.} & Ax & = & b \end{array}$$

with $P(A, b) \neq \emptyset$.

Show that the following statements are equivalent:

1. (P) has an optimal solution.
2. All feasible solutions of (P) are optimal.
3. c is a linear combination of rows of A.

Homepage of the Lecture: `http://www.zib.de/ss17_Optimierung_I`
Questions?: `klug@zib.de`