Freie Universität Berlin
FB Mathematik und Informatik
Sommersemester 2017

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Optimierung I

Excercise Sheet 9

Submission: until 17:00 on Monday, June 26, 2017

Exercise 9.1 **10 Points**

Unless otherwise stated, we consider a linear program in standard form

$$\begin{array}{rrcl} \max & c^T x & & \\ \text{s.t.} & Ax & = & b \\ & x & \geq & 0 \end{array}$$

with $A \in \mathbb{R}^{m\times n},\ b \in \mathbb{R}^m,\ c \in \mathbb{R}^n, m < n,\ \mathsf{rank}(A) = m,\ P^=(A, b) \neq \emptyset$.

Prove or disprove the following statements.

a) If x is a non-feasible basic solution with corresponding reduced costs $\overline{c} \leq 0$, then $c^\top x \geq c^\top y$ for all feasible solutions y.

b) If the optimal value of the linear program

$$\begin{array}{rl} \text{maximize} & c^\top x \\ \text{subject to} & A\,x = b \\ & x \geq 0 \end{array}$$

is finite, then the linear program

$$\begin{array}{rl} \text{maximize} & c^\top x \\ \text{subject to} & A\,x = b' \\ & x \geq 0 \end{array}$$

is bounded for all b'.

c) The number of positive x_j in a feasible basic solution does not exceed the rank of the matrix A.

d) For every linear program in n unbounded variables there exists an equivalent linear program in $n + 1$ nonnegative variables.

e) The both LP's, max $c^T x$, s. t. $Ax \leq b$, and max $-c^T x$, s. t. $Ax \leq b$, may have feasible solutions with arbitrary large objective function values.

Exercise 9.2 **10 Points**

Given the following LP:

$$
\begin{array}{lrcrcrcrcl}
\min & -10x_1 & + & 57x_2 & + & 9x_3 & + & 24x_4 & & \\
s.t. & \frac{1}{2}x_1 & - & \frac{11}{2}x_2 & - & \frac{5}{2}x_3 & + & 9x_4 & \leq & 0 \\
 & \frac{1}{2}x_1 & - & \frac{3}{2}x_2 & - & \frac{1}{2}x_3 & + & x_4 & \leq & 0 \\
 & x_1 & & & & & & & \leq & 1
\end{array}
$$

The slack variables are a feasible basis. Do six iteration of the simplex algorithm. Therefor, choose the entering variable with the smallest reduced cost. If there are more than one possible leaving variable then choose the one with smallest index. What is your observation after six iterations.

Exercise 9.3 **10 Points**

Solve the following LP problem by using the two-phase simplex method:

$$
\begin{array}{lrcrcrcrcl}
\max & x_1 & + & 2x_2 & + & 3x_3 & & & & \\
 & x_1 & + & 2x_2 & + & 3x_3 & & & = & 15 \\
 & 2x_1 & + & x_2 & + & 5x_3 & & & = & 20 \\
 & x_1 & + & 2x_2 & + & x_3 & + & x_4 & = & 10 \\
 & x_1 & , & 2x_2 & , & x_3 & , & x_4 & \geq & 0
\end{array}
$$

Homepage of the Lecture: `http://www.zib.de/ss17_Optimierung_I`
Questions?: `klug@zib.de`