

Optimierung I

Excercise Sheet 11

Submission: until 17:00 on Monday, Juli 10, 2017

Exercise 11.1

10 Points

Proof the following:

$$(P) \quad \begin{array}{ll} \min & c^T x \\ \text{s.t.} & Ax = b \\ & x \geq 0 \end{array} \quad (D) \quad \begin{array}{ll} \max & y^T b \\ \text{s.t.} & (A^T, I) \begin{pmatrix} y \\ z \end{pmatrix} = c \\ & z \geq 0 \end{array}$$

Let $A \in \mathbb{R}^{m \times n}$ be a matrix with $\text{rank } A = m$. Then (D) is unbounded or has an optimal basis H with $H \supseteq \{1, \dots, m\}$.

Exercise 11.2

10 Points

Solve the following linear program with the Upper-Bound-Technique.

$$\begin{array}{llll} \min & x_1 & + & x_2 & - & x_3 \\ \text{s.t.} & x_1 & - & 4x_2 & + & 3x_3 & \leq & 6 \\ & x_1 & + & 2x_2 & - & x_3 & \leq & 1 \\ & x_1 & + & 2x_2 & + & x_3 & \leq & 8 \\ & x_1 & & & & & \leq & 2 \\ & & & x_2 & & & \leq & 1 \\ & & & & & x_3 & \leq & 3 \\ & x_1 & , & x_2 & , & x_3 & \geq & 0 \end{array} \quad (\text{LP})$$

Exercise 11.3

15 Points

A furniture-manufacturing company wants to optimize their production process.

- i) A bookcase requires three hours of work, one unit of metal, and four units of wood, and it brings in a net profit of \$19.
- ii) A desk requires two hours of work, one unit of metal, and three units of wood, and it brings in a net profit of \$13.
- iii) A chair requires one hour of work, one unit of metal, and three units of wood, and it brings in a net profit of \$12.

- iv) A bedframe requires two hours of work, one unit of metal, and four units of wood, and it brings in a net profit of \$17.
- v) Only 255 hour of labor, 117 unit of metal, and 420 units of wood are available per day.

The goal is ot maximize the total net profit of the company, under the assumption that all furniture can be sold.

1. Formulate the problem as linear program.
2. Solve the linear problem by the simplex method. Any variant of the simplex method can be used.
3. Solve the following variations:
 - (a) The net profit brought in by each desk increases from \$13 to \$15.
 - (b) The availability of metal increases from 117 to 125 units per day.
 - (c) The company may also produce coffee tables, each of which requires three hours of work, one unit of metal, two units of wood, and brings in a net profit of \$14.
 - (d) The number of chairs produced must be at most five time the number of desks.
 - (e) The demand for wood per chair increases from three to four units.

Homepage of the Lecture: http://www.zib.de/ss17_Optimierung_I
Questions?: klug@zib.de