Freie Universität Berlin
FB Mathematik und Informatik
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Optimierung I

Exercise 1.

Consider the following LP

$$\begin{array}{rrcrcrcl} \min & x_1 & + & 2x_2 & + & 3x_3 & & \\ & x_1 & + & x_2 & + & x_3 & = & 1 \\ & 3x_1 & + & x_2 & + & 6x_3 & = & 2 \\ & & & & & x_1, x_2, x_3 & \geq & 0 \end{array}$$

a) Use the simplex algorithm to show that the optimal solution is $x_1^* = \frac{1}{2}, x_2^* = \frac{1}{2}$ and $x_3^* = 0$, by starting from the initial basic feasible solution: $x_1 = 0, x_2 = \frac{1}{2}, x_3 = \frac{1}{4}$.

b) Write down the dual problem of the given LP.

c) Use the optimal basic feasible solution from part a) to construct an optimal solution for the dual problem in part b).

Exercise 2.

a) True or False: Any LP that has an optimal solution has either a unique optimal solution, or infinitely many optimal solutions. Reformulate the

b) Give an example of an LP for which every feasible solution is optimal.

c) True or False: If an LP is unbounded, then its dual problem is infeasible.

Exercise 3.

$$\begin{array}{rrcrcrcr} \max & x_1 & & & - & 3x_3 & & \\ & x_1 & + & 2x_2 & - & 3x_3 & \leq & -1 \\ & 2x_1 & - & x_2 & - & x_3 & \leq & -2 \\ & & & & & x_1, x_2, x_3 & \geq & 0 \end{array}$$

a) Verify that $(x_1^*, x_2^*, x_3^*) = (0, 1, 1)$ is an optimal solution, and find the optimal dual values.

b) It the optimal solution unique? Is it degenerated?

c) Compute the range of all possible objective coefficients of x_3 for which $(x_1^*, x_2^*, x_3^*) = (0, 1, 1)$

d) If the right-hand side of the first constraint decreases to -2, is x^* still optimal? What is the new optimal objective function value?

Exercise 4.

Let $x_1 \in \{0, 1\}, x_2 \in \{0.1\}, x_5 \in \{0, 1\}, x_3 \geq 0, x_4 \geq 0$.

Reformulate the following constraints into linear constraints:

a) $5x_1x_2x_5 + 7x_3 + 8x_4 + |x_3 - 2x_4|$

b) $\max\left\{\frac{4+6x_3}{1+x_1}, \frac{3x_1+5x_4}{2-x_2}\right\} \leq 7$

Exercise 5.

Let $c, a, b \in \mathbb{Z}^n$ be given and consider the equality knapsack problem:

$$\max c^T x, \text{ s.t. } a^T x = b, x \geq 0$$

a) Derive conditions on c, a, and b under which the relaxation is feasible and bounded?

b) Assuming that problem is both feasible and bounded, characterize its optimal solution(s).

Exercise 6.

A primal solution x and a dual solution y are each optimal for their respective problem if

- x is feasible for the primal,
- y is feasible for the dual, and
- x and y satisfy the complementary slackness.

How many and which ones of these three properties are satisfied at every iteration of the primal(i.e.,phase two) simplex algorithm.

Homepage of the Lecture: `http://www.zib.de/ss17_Optimierung_I`
Questions?: `klug@zib.de`