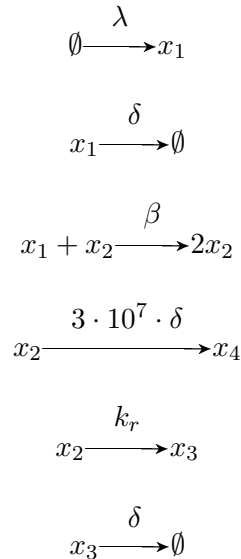


**Oberwolfach Seminar**  
**Mathematical Modelling in Systems Biology**  
**Nov 19-25, 2017**

**Task 1 (Epidemiological model)**

Consider an epidemiologic model of a virulent outbreak (also called susceptible-infected-recovered (SIR) model):



where  $x_1$  denotes the number of susceptible individuals,  $x_2$  are the number of infected individuals,  $x_3$  are the number of individuals that recovered and are subsequently resistant to infection, and  $x_4$  are the number of individuals that died from the infection. Parameter values are  $\lambda = 1 \cdot 10^{-4}$ ,  $\delta = 1 \cdot 10^{-8}$ ,  $\beta = 5 \cdot 10^{-5}$ ,  $k_r = 0.3$ , and the initial state is  $x_1(0) = \lambda/\delta$ ,  $x_2(0) = 5$ ,  $x_3(0) = 0$ ,  $x_4(0) = 0$ .

- (a) Write a program implementing this model and generate trajectories using the stochastic simulation algorithm.
- (b) Plot the trajectories for  $N = 3$  simulations up to time  $T = 10$ .
- (c) Perform  $N = 1000$  simulations, generate a histogram of the number of  $x_2$  at time  $T = 10$  and depict the probability that  $0, \dots, 20$  individuals are infected by  $T = 10$ . What is the probability that the infection is still ongoing at  $T = 10$ ?
- (d) From the  $N = 1000$  simulations, generate a histogram of the number of  $x_4$  at time  $T = 10$  (number of casualties), depicting the probability that  $0, \dots, 35$  individuals died by  $T = 10$ . How many individuals died on average ( $\pm$  standard deviation)?