Oberwolfach Seminar
Mathematical Modelling in Systems Biology
Nov 19-25, 2017

Task 1 (Predator-prey model)
Consider the following predator-prey-model (also known as Lotka-Volterra-model),

$$\emptyset \xrightarrow{\lambda} x_1$$

$$x_1 + x_2 \xrightarrow{k_1} x_2$$

$$x_1 + x_2 \xrightarrow{k_2} 2x_2$$

$$x_2 \xrightarrow{\delta_2} \emptyset$$

where x_1 and x_2 denote the population of prey and predators (x_2 eats x_1). Parameters are $\lambda = 0.3$, $k_1 = k_2 = 0.01$, $\delta_2 = 0.3$, and initial conditions are $x_1(0) = 32$ and $x_2(0) = 16$.

(a) Implement this model, and generate 3 trajectories using the stochastic simulation algorithm until $t_{\text{final}} = 50$. Generate one plot per species with the 3 trajectories in it.

(b) Store the population of x_1 and x_2 every 0.1 time units until you reach $t_{\text{final}} = 50$. Compute the sample mean and its standard deviation for x_1 and x_2 for each time instance (every 0.1 time units) and plot them.

(c) Generate 300 trajectories and plot the sample mean and standard deviation for x_1 and x_2. Why is the sample mean $\overline{x}_1$ increasing?

(d) From the 300 trajectories, compute the the probability that x_2 died out by t_{final}.

(e) Consider a modified Lotka-Volterra model that includes more realistic logistic (rather than simply exponential) growth of the prey and predator species:

$$\begin{aligned} y_1' &= y_1 \left(1 - y_1 - \frac{a y_2 (1 - \exp(-b y_1))}{y_1}\right) \\ y_2' &= r y_2 \left(1 - \frac{y_2}{y_1 + c}\right) \end{aligned}$$

Determine the stability of the fixed points for $a = 2, b = 1, c = 0.1$ and $r = 0.5$. Check the results of the numerical analysis by solving the ODE with initial conditions in the vicinity of the fixed points.