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Nonlinear Optimization

http://www.zib.de/weiser/NichtlineareOptimierung/
Homework 10

Due: Friday, July 10, 2020

Assignment 1 (4 points)**:**
Consider the following problem

$$\begin{aligned} &\min f(x) = -x_2^4 - x_2^4 \\ &\text{s.t.} \quad (x_1-1)^2 + x_2^2 - 1 = 0 \end{aligned}$$

a) Find the minimizer using the first and second order conditions.

b) Show that $L''(x^*, \lambda^*)$ is not positive definite on $\mathbb{R}^n$ but on $\ker c'(x^*)$.

c) Proceed as in theorem (III.6.1) to find μ_0 such that the Augmented Lagrangian Method is exact for $\mu \le \mu_0$.

Assignment 2 (4 points)**:**
Consider the following problem

$$\min f(x) \quad \text{s.t.} \quad c(x) = 0 \ ,$$

where $f\colon \mathbb{R}^n \to \mathbb{R}$ is convex and $c\colon \mathbb{R}^n \to \mathbb{R}^m$. $f, c \in C^1$. The minimizer of this problem can be obtained as unconstrained minimizer of the Lagrangian if the corresponding Lagrange Multipliers λ^* are known:

$$x^* = \operatorname*{argmin}_x L(x, \lambda^*) = f(x) - c(x)\lambda^* \ .$$

Now assume that the given estimate of the Lagrange Multipliers is not accurate:

$$\lambda = \lambda^* + \epsilon \Delta\lambda, \qquad \epsilon \in \mathbb{R}, \ \ \Delta\lambda \in \mathbb{R}^m \ ,$$

where ϵ is sufficiently small. How will this affect the so obtained minimizer?

Hint: Define the new optimization problem. Then apply the implicit function theorem to the first order necessary conditions in order to develop an estimate for the change of the solution $x(\epsilon) - x^*$.