Fachbereich Mathematik und Informatik
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Nonlinear Optimization

http://www.zib.de/weiser/NichtlineareOptimierung/
Homework 3

Due: Monday, May 15, 2020

Assignment 1 (6 points)**:**
Suppose that $f(x) = x^T Qx$, where Q is an $n \times n$ symmetric matrix. Using the definition from the lecture, show:

a) $f(x)$ is convex on $\mathbb{R}^n \iff Q$ is positive semidefinite;

b) $f(x)$ is strictly convex on $\mathbb{R}^n \iff Q$ is positive definite.

Hint: It may be convenient to prove the following inequality, which is equivalent to the condition of convexity:

$$f(y + \alpha(x - y)) - \alpha f(x) - (1 - \alpha)f(y) \leq 0, \quad \forall \alpha \in [0, 1] \text{ and all } x, y \in \mathbb{R}^n.$$

Assignment 2 (4 points)**:**
Specify a norm for which the gradient method converges optimally for the quadratic problem

$$f(x) = \frac{1}{2}x^T Ax - b^T x, \quad \min f(x)$$

with a symmetric positive definite matrix A. Prove the optimality.

Assignment 3 (4 points)**:**
Show (e.g. by giving a counterexample) that if $0 < c_2 < c_1 < 1$, there may be no step lengths that satisfy the Wolfe conditions.