Fachbereich Mathematik und Informatik
Freie Universität Berlin
Dr. Martin Weiser, Fabian Danecker

Nonlinear Optimization

http://www.zib.de/weiser/NichtlineareOptimierung/
Homework 4

Due: Friday, May 22, 2020

Assignment 1 (4 points)**:**
Let $B \in \mathbb{R}^{n\times n}$ be invertible. Prove that the damped affine conjugate Newton method is invariant under the transformation $x = By$. That means, that Newton's method applied to the problems

$$\min_x f(x) \quad \text{and} \quad \min_y g(y)$$

with $g(y) = f(By)$ and initial iterates x_0 and $y_0 = B^{-1}x_0$, respectively, generates equivalent iterates

$$Bx_i = y_i \quad \forall i.$$

Hint: Show that the Newton direction, the functional's values, and the energy norm is invariant under transformation.

Assignment 2 (4 points)**:**
Prove that the practical stepsize determination of the affine conjugate Newton method, based on the Lipschitz estimate

$$[\omega] = \frac{6\left|f(x+\alpha\Delta x) - f(x) + \left(\alpha - \frac{\alpha^2}{2}\right)\|\Delta x\|_x^2\right|}{\alpha^3\|\Delta x\|_x^3}$$

and the monotonicity test

$$f(x + [\alpha_{\text{opt}}]\Delta x) \le f(x) - \frac{1}{6}[\alpha_{\text{opt}}]([\alpha_{\text{opt}}]+2)\|\Delta x\|_x^2$$

terminates after finitely many steps.

Assignment 3 (6 points, programming exercise)**:**
This assignment is due on Friday, May 29.

Let

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function [y,yx,yxx] = f(x)
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define the interface for evaluating f and its first and second derivative at x. Implement the damped affine conjugate Newton method for minimizing f. Minimize the Rosenbrock function

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

starting at $(-1.9, 2)^T$. How many steps are required to achieve a tolerance of $f(x) \leq \epsilon$, for $\epsilon \in \{10^{-4}, 10^{-6}, 10^{-10}\}$?