Fachbereich Mathematik und Informatik
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Nonlinear Optimization

http://www.zib.de/weiser/NichtlineareOptimierung/

Homework 5

Due: Friday, May 29, 2020

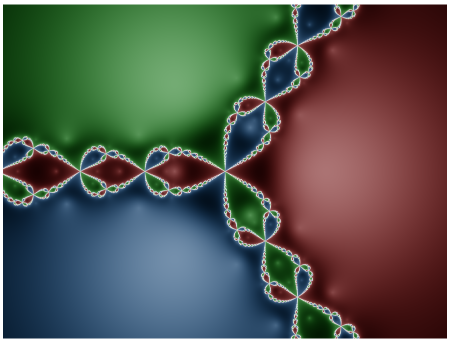
Assignment 1 (6 points)**:**
If the function to be optimized is rather complex, it might be too expensive to calculate the gradient analytically and one needs to work with a numerical estimate instead.
Show that the gradient method (with exact line search) still converges linearly for the quadratic problem, even if the estimated gradient $\tilde{g}$ deviates from the real gradient g by an angle α as long as α is sufficiently small.

Hint: First show that every accumulation point is stationary (cnf. theorem (I.3.1)). Then use theorem (I.3.2) to show that the gradient method converges linearly.

Assignment 2 (6 points, programming exercise)**:**
Newton's method can readily be applied to complex-valued functions. Use it to find the roots of a polynomial

$$f(z) = 0, \quad f : \mathbb{C} \to \mathbb{C}, \quad z \to z^3 - 1$$

This function has got 3 roots on the complex plane and depending on the starting point, Newton's method will converge to one or the other. It is not trivial to know a priori which solution the algorithm will converge to, but you can find out by running your algorithm from various starting points. Colorize the starting points depending on the found solution and you should come up with an image that looks like this



The areas that you can see here are called Newton fractals and depict the sometimes chaotic behavior of Newton's method.

If you are interested, you can find a lot of additional resources online, e.g.:
https://www.chiark.greenend.org.uk/ sgtatham/newton/

Hints: As Matlab/Octave can handle complex numbers by default, this should not require lots of changes on your code. Complex numbers are defined like $z = a + bi$; (or equivalently $z = a + bj$;) and most operators work just the same as with real numbers. Take care when comparing values and do not mix up i as variable and i as complex number. Finally, do not spend too much time on creating beautiful plots.