Fachbereich Mathematik und Informatik
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Nonlinear Optimization

http://www.zib.de/weiser/NichtlineareOptimierung/
Homework 11

Due: Friday, July 17, 2020

Assignment 1 (4 points)**:**
Consider the general equality constrained optimization problem:

$$\begin{aligned} &\min f(x) \\ &\text{s.t.} \quad c(x) = 0 \end{aligned}$$

Let x^*, λ^* satisfy the necessary and sufficient optimality conditions. Show that $\exists\ \bar{a} < \infty$ such that $\forall a \geq \bar{a}\ x^*$ is a local minimizer of the l1-merit function

$$m(x) = f(x) + a\|c(x)\|_1 \ .$$

Assignment 2 (6 points, programming exercise)**:**
Once again, revisit example (III.2.5),

$$\begin{aligned} &\min_{x \in \mathbb{R}^2} \frac{1}{2} x^T x \\ &\text{s.t.} \quad x_1 + x_2 = 1 \ . \end{aligned}$$

Implement the Augmented Lagrangian method to solve this problem with $x_0 = (2, 3)$.
Hint: The overall structure of your code might be quite similar to the penalty method.
A template for this exercise can be found on the web page.

a) Implement the objective function in `objFun()`.

b) Implement the constraints in `constrFun()`.

c) Implement the Augmented Lagrangian function in `augLagFun()`.

d) Implement a variation of Newton's method as solver for unconstrained problems in `newtonOpt()`.

e) Implement the function `augLagOpt()`, that uses your solver to find the minimum of the Augmented Lagrangian for a declining sequence of μ-values.

Assignment 3 (4 points, programming exercise)**:**
Consider the following problem involving the Rosenbrock function:

$$\begin{aligned} &\min 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \\ &\text{s.t.} \quad x_2 = 0.5\, x_1^4 \end{aligned}$$

Solve the problem with the Augmented Lagrangian and the penalty method. Evaluate which method performs better.
Hint: Use the codes that you wrote here and for H09. It should be sufficient to replace the functions `objFun()` and `constrFun()`.