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Nonlinear Optimization

http://www.zib.de/weiser/NichtlineareOptimierung/
Exercise 1

Due: Friday, March 1, 2020

Assignment 1 (4 points)**:**
Consider the indicator function ι_U as defined in the lecture.

a) For which sets U is the indicator function ι_U lower semicontinuous (proof)?

b) For which sets U is the indicator function ι_U convex (proof)?

c) Show that level sets and epigraph of convex functions are convex. Is the converse true (proof or counter example)?

Assignment 2 (4 points)**:**
Let $U \subset \mathbb{R}^n$ be convex, closed and non-empty, and let

$$f(x) := \|x\|_2^2 = \sum_{i=1}^{n} x_i^2.$$

Examine existence and uniqueness of the minimizer.
Show the following necessary optimality condition: If $x^\star$ is a minimizer, then it holds

$$\langle x^\star, x - x^\star \rangle \geq 0 \quad \forall x \in U$$

(with $\langle u, v \rangle = \sum u_i v_i$).
Hint: Proceed as in the proof of Theorem (0.9) and use the convexity of U.

Assignment 3 (3 points):
Given $U \subset \mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}$ bounded from below. Give examples for which there is no minimizer of f in U, for:

a) f continuous, U bounded but not closed,

b) f continuous, U closed but not bounded,

c) f discontinuous, U bounded and closed.

Assignment 4 (5 points):
Compute first and second derivative of the "Rosenbrock function"

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

Show that $x^\star = (1, 1)^T$ is the unique minimizer, and that the second derivative is positive definite at $x^\star$.

Additional tasks (voluntarily, only if you're interested in functional analysis):

a) Why is the existence theorem for minimizers only valid in finite dimensional spaces? Find (in the literature) an existence theorem valid in infinite dimensional spaces. What is the role of convexity in this case?

b) With that, show existence of a solution to Assignment 2 in Hilbert spaces. What is an application example?