Cloud Branching

Timo Berthold

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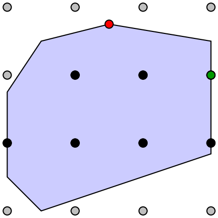
joint work with Domenico Salvagnin (Università degli Studi di Padova)

DFG Research Center MATHEON
Mathematics for key technologies

21/May/13, CPAIOR 2013, IBM T. J. Watson Research Center

$$\begin{aligned} \min \quad & c^T x \\ s.t. \quad & Ax \leq b \\ & x \in \mathbb{Z}^I_{\geq 0} \times \mathbb{R}^C_{\geq 0} \end{aligned}$$



Mixed Integer Program:

- linear objective & constraints
- integer variables
- continuous variables

Branching for MIP:

- based on LP relaxation
- fractional variables
- tries to improve dual bound

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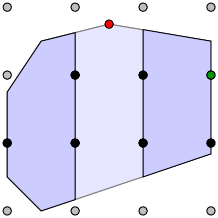

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$$x_i \leq \lfloor x_i^* \rfloor \vee x_i \geq \lceil x_i^* \rceil \text{ for } i \in I \text{ and } x_i^* \notin \mathbb{Z}$$

Most infeasible branching

- often referred to as a simple, standard rule
- computationally as bad as random branching!

Strong branching [ApplegateEtAl1995]

- solve LP relaxations for some candidates, choose best
- effective w.r.t. number of nodes, expensive w.r.t. time

Pseudocost branching [BenichouEtAl1971]

- try to estimate LP values, based on history information
- effective, cheap, but weak in the beginning
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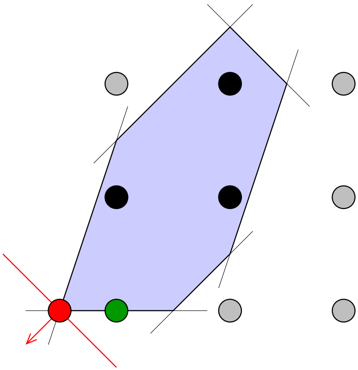
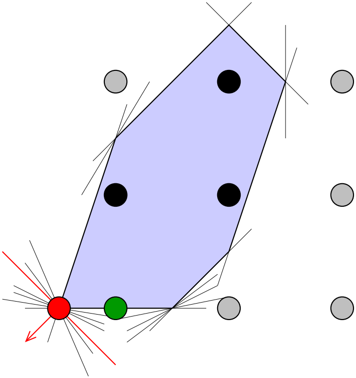


naïvely:

- 1 optimal solution, determined by n constraints

Degeneracy



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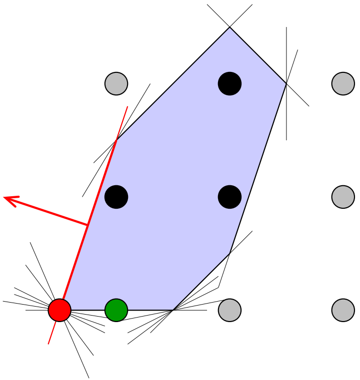
at a second thought:

▷ 1 optimal solution, $k > n$ tight constraints



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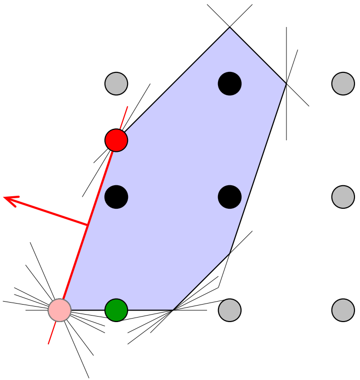
but really:

- an optimal polyhedron



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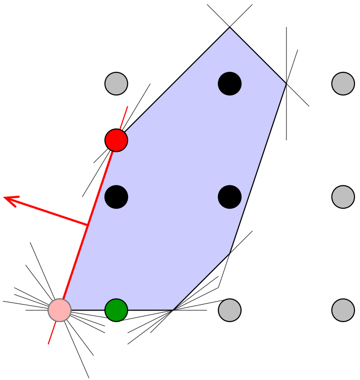

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Goal of this talk:
branch on a set (a cloud) of solutions

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2. Why should that be a good idea anyway?

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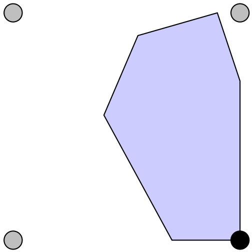
- min/max each variable (OBBT) or
- feasibility pump objective (pump&reduce [Achterberg2010])



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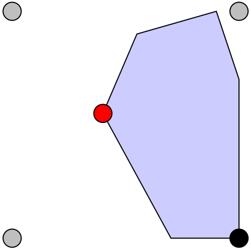
$x_2 = 0.55$



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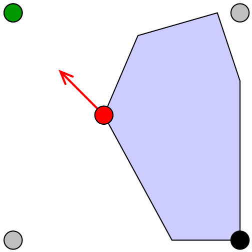

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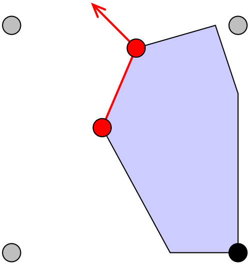

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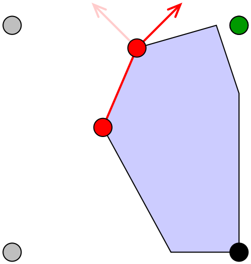

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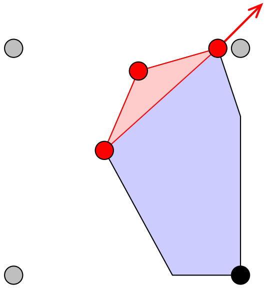

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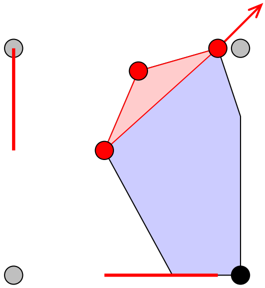
$\rightsquigarrow$ intervals instead of values



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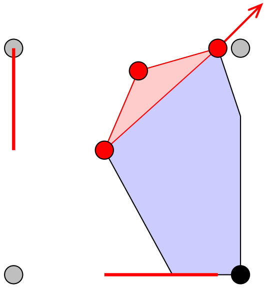

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stalling is cheap!

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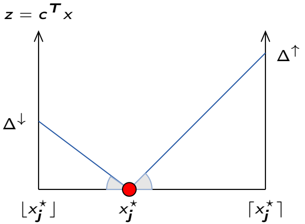

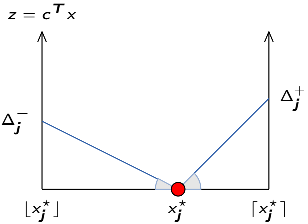

- pseudocost update
  - $\varsigma_j^+ = \frac{\Delta^\uparrow}{\lceil x_j^\star \rceil - x_j^\star}$ and $\varsigma_j^- = \frac{\Delta^\downarrow}{x_j^\star - \lfloor x_j^\star \rfloor}$
- pseudocost-based estimation
  - $\Delta_j^+ = \Psi_j^+(\lceil x_j^\star \rceil - x_j^\star)$ and $\Delta_j^- = \Psi_j^-(x_j^\star - \lfloor x_j^\star \rfloor)$

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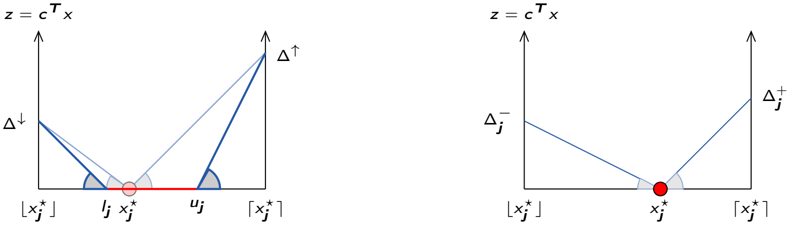

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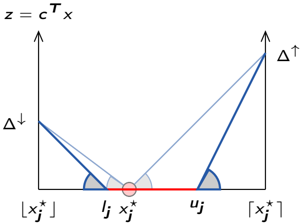

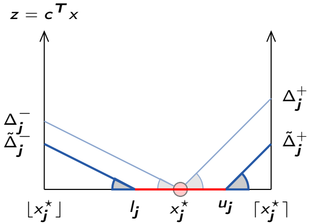

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Lemma

Let $x^\star$ be an optimal solution of the LP relaxation at a given branch-and-bound node and $\lfloor x_j^\star \rfloor \leq l_j \leq x_j^\star \leq u_j \leq \lceil x_j^\star \rceil$. Then

1. for fixed $\Delta^\uparrow$ and $\Delta^\downarrow$, it holds that $\tilde{\varsigma}_j^+ \geq \varsigma_j^+$ and $\tilde{\varsigma}_j^- \geq \varsigma_j^-$, respectively;
2. for fixed Ψ_j^+ and Ψ_j^-, it holds that $\tilde{\Delta}_j^+ \leq \Delta_j^+$ and $\tilde{\Delta}_j^- \leq \Delta_j^-$, respectively.

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Full strong branching:

- solves 2·#frac. var's many LPs
- uses product of improvement values as branching score
  - improvement on both sides better than on one

Exploiting cloud points (strong branch)

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Benefit of cloud intervals:

- frac. var. gets integral in cloud point $\rightsquigarrow$ one LP spared
- cloud branching acts as a filter
- new frac. var.'s $\rightsquigarrow$ new candidates (one side known)

Cloud strong branching algorithm

Idea: Use 3-partition F_2, F_1, F_0 of branching candidates

- if strict improvements in both directions for F_2, disregard $F_1 \cup F_0$
- if strict improvement in one direction for $F_2 \cup F_1$, disregard F_0

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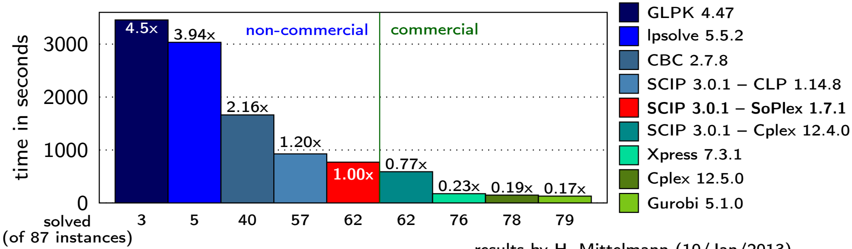
Note: In our experiments, we do not use cloud points for anything else (heuristics, cuts)

SCIP: Solving Constraint Integer Programs

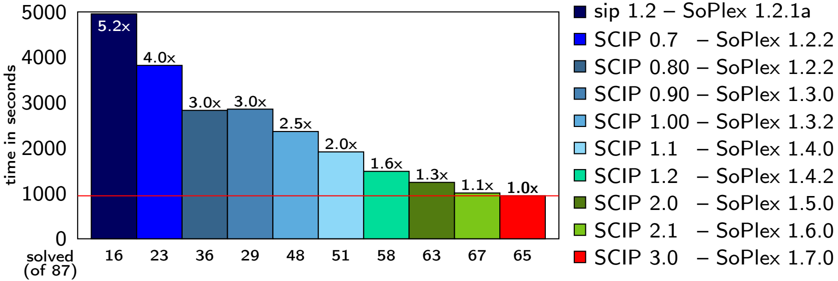
- standalone solver / branch-cut-and-price-framework
- modular structure via plugins
- free for academic use: `http://scip.zib.de`
- very fast non-commercial MIP and MINLP solver



results by H. Mittelmann (10/Jan/2013)

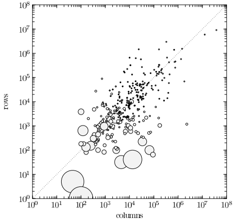
SCIP: Solving Constraint Integer Programs

- better support of MINLP
- new presolvers and propagators
- AMPL and MATLAB interface (beta)
- first releases of GCG and UG



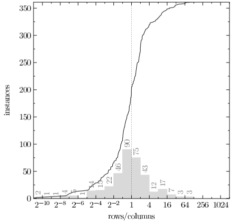
MMM

- MIPLIB 3.0, MIPLIB 2003, MIPLIB2010
- industry and academics
- 168 instances from diverse applications



Cor@l

- huge collection of 350 instances
- many combinatorial ones
- mainly collected from NEOS server



	cloud statistics			
Test set	%Succ	Pts	LPs	%Sav
MMM	12.2	2.19	74.34	21.7
COR@L	40.8	2.71	70.97	51.8

- applicable on some MMM, but many COR@L instances
- only few cloud points on average
- significants amount of LPs saved (if affected)

Test set	strong branch Nodes	strong branch Time (s)	cloud branch Nodes	cloud branch Time (s)
MMM	691	72.0	661	68.2
COR@L	593	157.3	569	118.3

- little less nodes
- 5.5% faster on MMM (few affected instances)
- 30 % faster on COR@L

Conclusion: Cloud branching...

- exploits knowledge of alternative relaxation optima
- can help to improve pseudocost predictions
- makes full strong branching up to 30% faster

Outlook

- pseudocost, reliability, hybrid branching
- cloud points from alternative relaxations (MINLP!)
- nonchimerical + cloud + propagation = ?

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