Solving Constraint Integer Programs

Timo Berthold
Zuse Institute Berlin
Constraint Integer Programming

- Mixed Integer Programming
- Constraint Integer Programming
- Constraint Programming

\[
\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad x \in F \\
& \quad (x_I, x_C) \in \mathbb{Z}^I \times \mathbb{R}^C
\end{align*}
\]

- linear objective
- “general” constraints
- integer or real variables
Constraint Integer Programming

- Mixed Integer Programming
- Constraint Integer Programming
- Constraint Programming
- Satisfiability Testing
- Pseudo-Boolean Optimization
- Finite Domain

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\begin{align*}
\text{min} & \quad c^T x \\
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& \quad (x_I, x_C) \in \mathbb{Z}^I \times \mathbb{R}^C
\end{align*}
\]

- linear objective
- “general” constraints
- integer or real variables
Solving Techniques

Presolving

Primal heuristics

Cutting planes

Branch & Bound

Domain propagation

Conflict analysis

\[ x_1 \quad x_2 \quad x_3 \quad x_4 \quad \Rightarrow \]

Timo Berthold: SCIP – Solving Constraint Integer Programs 3 / 23
How do we solve CIPs?

**MIP**
- LP relaxation
- cutting planes

**CP**
- domain propagation

**SAT**
- conflict analysis
- periodic restarts

**MIP, CP, and SAT**
- tree search algorithms

**SCIP**
more than 300,000 lines of C code

→ 18% documentation, 20% assertions

7 examples illustrating the use of SCIP

HowTos: each plugin type, debugging, automatic testing, ...

C++ wrapper classes, python interface (beta), Java

supports 10 different input formats

7 interfaces to external LP solvers

more than 1000 parameters, 15 “emphasis” settings

active mailing list (180 members)

5000 downloads per year

free for academics, available in source code: http://scip.zib.de

runs on Linux, Windows, Mac (Darwin+PPC), SunOS, ...
New: SCIP to go
fastest non-commercial MIP solver

![Chart showing computational results for various solvers.]

- **GLPK 4.43**
- **lp_solve 5.5**
- **Symphony 5.3.0**
- **CBC 2.6.0**
- **SCIP 2.0.1 – CLP 1.13.2**
- **SCIP 2.0.1 – SoPlex 1.5**
- **Minto 3.1 – Cplex 9**
- **SCIP 2.0.1 – Cplex 12.2.0.2**
- **Cplex 12.2.0.2**
- **Gurobi 4.0.0**

Results by H. Mittelmann (16.1.2011)

Computational results

- **Fastest noncommercial solver for convex MIQCPs**

<table>
<thead>
<tr>
<th>Solver</th>
<th>Time in Seconds</th>
<th>Not Solved</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-commercial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coupene 0.3</td>
<td>8.21x</td>
<td>71%</td>
</tr>
<tr>
<td>SCIP 2.0.1 – SoPlex 1.5</td>
<td>1.00x</td>
<td>34%</td>
</tr>
<tr>
<td>LindoGlobal 6.1</td>
<td>12.08x</td>
<td>75%</td>
</tr>
<tr>
<td>BARON 9.0</td>
<td>8.03x</td>
<td>68%</td>
</tr>
<tr>
<td>CPLEX 12.2</td>
<td>0.42x</td>
<td>28%</td>
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- **Fastest MIQCP solver (non-convex)**

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<tr>
<td>BARON 9.0</td>
<td>4.64x</td>
<td>77%</td>
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<tr>
<td>Coupene 0.3</td>
<td>4.63x</td>
<td>78%</td>
</tr>
<tr>
<td>LindoGlobal 6.1</td>
<td>5.43x</td>
<td>80%</td>
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<tr>
<td>SCIP 2.0.1 – Cplex 12.2</td>
<td>0.87x</td>
<td>45%</td>
</tr>
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<td>1.00x</td>
<td>48%</td>
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Toolbox for **generating** and **solving** constraint integer programs

**ZIMPL**
- a mixed integer programming modeling language
- easily generating LPs, MIPs, and MIQCPs

**SCIP**
- a MIP and CP solver, branch-cut-and-price framework
- ZIMPL models can directly be loaded into SCIP and solved

**SoPlex**
- a linear programming solver
- SCIP uses SoPlex as underlying LP solver
Tobias Achterberg (IBM)
Thorsten Koch
Marc Pfetsch (TU Braun.)

Timo Berthold
Gerald Gamrath
Ambros Gleixner
Stefan Heinz
Matthias Miltenberger
Yuji Shinano
Stefan Vigerske
Kati Wolter

Gregor Hendel
Alexandra Kraft
Michael Winkler
1996  SoPlex – Sequential obj. simPlex (R. Wunderling [now IBM])
1998  SIP – Solving Integer Programs (A. Martin [now U Erlangen])
Siemens cooperation

Longstanding Cooperation with department
Modeling, Simulation, Optimization

- first licensee (1996) of SoPlex
- steady use in various optimization modules

placement robots in
circuit board production

optimal planning of
water networks
1996  SoPlex – Sequential obj. simPlex (R. Wunderling [now IBM])
1998  SIP – Solving Integer Programs (A. Martin [now U Erlangen])

10/2002  Start of SCIP development (T. Achterberg [now IBM])
08/2003  Chipdesign verification ⇒ Constraint Programming
Goal: (computer-)proof, that a design is free of errors

Method: property checking using CIPs

Result:

- Duration: 2003-2008

- CONSTRAINT INTEGER PROGRAMMING
  - registers: 422, 152026
  - variables: 3714, 50756

- BOOLEAN SATISFIABILITY
  - time consumption:
    - 1/2 h
    - 1 h
    - 3/2 h
    - 2 h
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Optimization of gas transport

Stochastic Mixed-Integer Nonlinear Constraint Program

Gasflow on Entry vs. Temperature

over a large network.

Goal:

- develop algorithms to solve such problems to "global optimality"!

Industry partner:

Open Grid Europe
The Gas Wheel

Start: 01/2009
Stochastic Mixed-Integer Nonlinear Constraint Program

over a large network. Start: 01/2009

Goal:

- develop algorithms to solve such problems to “global optimality”!
- rather: attempt to integrate as many aspects as possible

Industry partner:

Open Grid Europe
The Gas Wheel
History of SCIP

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09/2009  Beale-Orchard-Hays Prize (T. Achterberg)
07/2010  Supply-Chain-Management ⇒ extrem große LPs/MIPs
Huge, numerically challenging problems

Topics:
- LP solver
- presolving
- numerical stability
- multi-level objective

Start: 07/2010
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Outreach

- RWTH Aachen
- Universität Bayreuth
- FU Berlin
- TU Berlin
- HU Berlin
- WIAS Berlin
- TU Braunschweig
- TU Chemnitz
- TU Darmstadt
- TU Dortmund
- TU Dresden
- Universität Erlangen-Nürnberg
- Leibniz Universität Hannover
- Universität Heidelberg
- Fraunhofer ITWM
- Kaiserslautern
- Universität Karlsruhe
- Christian-Albrechts-Universität zu Kiel
- Hochschule Lausitz
- OvGU Magdeburg
- TU München
- Universität Osnabrück
- Universität Stuttgart
- Aarhus Universitet
- The University of Adelaide
- Università dell’Aquila
- Arizona State University
- University of Assiut
- National Technical University of Athens
- Georgia Institute of Technology
- Indian Institute of Science
- Tsinghua University
- UC Berkeley
- Lehigh University
- University of Bristol
- Eötvös Loránd Tudományegyetem
- Universidad de Buenos Aires
- Institut Français de Mécanique Avancée
- Chuo University
- Clemson University
- University College Cork
- Danmarks Tekniske Universitet
- Syddansk Universitet
- Fuzhou University
- Jinan University
- Rijksuniversiteit Groningen
- Hanoi Institute of Mathematics
- The Hong Kong Polytechnic University
- University of Hyogo
- The Irktusk Scientific Center
- University of the Witwatersrand
- Københavns Universitet
- Kunming Botany Institute
- École Poly. Fédérale de Lausanne
- Linköpings universitet
- Université catholique de Louvain
- Universidad Rey Juan Carlos
- Université de la Méditerranée
- Aix-Marseille
- University of Melbourne
- UNAM
- Politecnico di Milano
- Università degli Studi di Milano
- Monash University
- Ikerlan
- Université de Montréal
- NIISI RAS
- Université de Nantes
- The University of Newcastle
- University of Nottingham
- Universitetet i Oslo
- Università degli Studi di Padova
- L’Université Sud de Paris
- Brown University
- The University of Queensland
- IASI CNR
- Erasmus Universiteit Rotterdam
- Carnegie Mellon University
- Universidad Diego Portales
- University of Balochistan
- Universidad San Francisco de Quito
- Universidade Federal do Rio de Janeiro
- Universidade de São Paulo
- Fudan University
- University of New South Wales
- Tel Aviv University
- The University of Tokyo
- Politecnico di Torino
- University of Toronto
- NTNU i Trondheim
- The University of York
- University of Washington
- University of Waterloo
- Massey University
- Austrian Institute of Technology
- TU Wien
- Universität Wien
- Wirtschaftsuniversität Wien
- ETH Zürich
Some universities and institutes using SCIP:
Current Topics:

- non-convex constraints
- massive parallelization (→ HLRN)
- improvements in column generation
- development of a new LP solver

Research projects:

- exact integer programming
  (DFG priority program “Algorithm Engineering”)
- mixed-integer nonlinear programming
  (MATHEON project B20)
- solver technologies for supply chain management
SCIP
Solving Constraint Integer Programs

Timo Berthold
Zuse Institute Berlin

DFG Research Center MATHEON
Mathematics for key technologies

Sino-German Workshop, ZIB, 19/Aug/2011