

## Network Design and Operation (WS 2015)

### Excercise Sheet 10

Submission: Mo, 18. January 2016, tutorial session

#### Exercise 1.

**6 Points**

Let  $P$  be a polyhedron defined over  $\mathbb{R}^n$ ,  $c \in \mathbb{R}^n$  a cost vector and  $x \in \mathbb{R}^n$  a decision vector. Prove that if  $P$  is *bounded*, then the linear cost function  $c'x$  achieves its minimum on  $P$  at an extreme point of  $P$ .

#### Exercise 2.

**8 + 5 Points**

Let  $A \in \mathbb{R}^{m \times n}$  be a matrix with linearly independent columns and let  $C$  be a non-empty convex subset of  $\mathbb{R}^n$ .

- a) Show that a vector  $x \in C$  is an extreme point of  $C$  if and only if  $Ax$  is an extreme point of the image  $AC$ ;
- b) Show through an example that if the columns of  $A$  are linearly dependent, then  $Ax$  can be an extreme point of  $AC$  for some non-extreme point  $x$  of  $C$ .

#### Exercise 3.

**7+7+7 Points**

Let  $S$  be a non-empty convex subset of  $\mathbb{R}^n$ . Prove that the following statements are equivalent:

- a) all the boundary points of  $S$  are extreme points of  $S$ ;
- b) every hyperplane that supports  $S$  in some point intersects  $S$  only in that point;
- c) every line intersects the boundary of  $S$  in no more than two points.