Preface

For many years the rapid progress in the development of both *computers* and *computing* (algorithms) has led to a more and more detailed scientific and engineering modeling of reality. Mathematicians are now facing the task of solving very large ODE problems of high complexity. Accordingly, the subject *numerical analysis* must now be embedded into the larger field *scientific computing*. With this idea in mind, this book focuses on the presentation of *efficient algorithms* as well as on their mathematical theory. The book covers initial value problems (IVPs) for ordinary differential equations (ODEs) and differential-algebraic equations (DAEs) as well as boundary value problems (BVPs) for ODEs. It is a translation of the second edition of a rather successful German textbook.

The book is addressed to *students of mathematics, science*, and *engineering*. For a basic background in numerical analysis we conveniently refer to our introductory textbook [58]. From the analysis of ODEs we use only elementary existence and uniqueness theorems and some of the simple solution techniques such as "variation of constants" and "separation of variables." On rare occasions we go back to results from complex analysis such as the maximum principle and the Riemann sphere. For future scientists and engineers some passages may be somewhat hard to digest; these students are encouraged to skip the technical details and to concentrate on the mathematical substance. In order to keep the book accessible to as many readers as possible, we deliberately shifted topics, that are of interest primarily to mathematicians, to proofs or remarks.

For mathematicians who may not readily have a well-rounded knowledge of science we have included certain scientific material that we regard as important in connection with ODEs. In line with this we begin with an elementary introduction of some ODE models of time-dependent processes from science and engineering.

A theme running throughout the entire book is the close match between concepts and notation from numerical analysis and dynamical systems, for example, with respect to the *inheritance of properties* of continuous ODEs by their discretizations.

For *initial value problems* the concept of *evolution* expressing the uniqueness of a solution leads naturally to the *condition* concept of IVPs and eventually to the asymptotic stability of ODEs. In strict analogy, one-step methods can be characterized by the concept of a *discrete evolution*, which in turn leads to the *discrete condition* and to the asymptotic stability of recursions. Then a comparison of the continuous and discrete condition numbers allows for a clear distinction between *stiff* and *nonstiff* IVPs. At the same time, these concepts permit an interpretation of numerical findings obtained for Hamiltonian systems, an important topic that can only be touched upon within the scope of this book.

The presentation of *boundary value problems* also begins with local uniqueness results for BVPs. They lead directly to the *condition* of BVPs and to the *discrete condition* of the corresponding algorithms. By comparison with IVPs, a distinction between *timelike* and *spacelike* BVPs appears to be natural, which then carries over to the associated algorithms. An important source of timelike BVPs are problems of *optimal control*, a presentation of which has here been added. This presentation follows largely the lines of a course taught by R. Bulirsch first in 1971 at the University of Cologne and then, since 1973, in a further developed form at the Technical University of Munich. The first author had the pleasure of assisting several times in this interesting course. Up to now its contents have not been available on a textbook level.

The book contains a series of otherwise unpublished results by the authors. Wherever the contents had to be confined we cite references for further study. Throughout the text illustrative examples from applications are included. Numerous exercises are given for further practice and an improved understanding of the material.

We deliberately entered into certain details of the algorithmic implementation. It was our goal to equip mathematicians as well as program users with the background knowledge needed for the efficient solution of scientific problems that may be off the beaten track. Selected programs are explicitly named in their context and listed in the index. At the end of the book we include a collection of *software* packages together with web addresses from which these programs may be downloaded.

At this point it is our pleasure to thank all those who have specially helped us with the preparation of this book. Our first thanks go to Werner Rheinboldt, who did a tremendous job with his subtle translation of our original German text [55]; moreover, he gave more than one valuable piece of advice, and not only about differential equations on manifolds. Our thanks go also to Rainer Roitzsch (ZIB), without whose deep knowledge of a rich variety of intricate TEX questions this book would never have appeared.

Among many others, we are grateful to former and present coworkers: Rolf Krause for help with the graphics; Tilmann Friese, Jörg Heroth, Andreas Hohmann, and Christof Schütte for comments on the first edition; Michael Dellnitz for careful and critical reading of the manuscript from the dynamics point of view; Frank Cordes, Thorsten Hohage, Ulli Nowak, Marcus Weber, Martin Weiser, and Lin Zschiedrich for both technical and mathematical assistance; Erlinda Körnig and Sigrid Wacker for all kinds of assistance.

From outside, we gratefully acknowledge help from Martin Arnold for pointing out a well-hidden tricky error in the first German edition; Christian Lubich for valuable comments on the concept of the book; Michael Günther and Caren Tischendorf for very helpful groundwork on DAEs in electrical circuits; Georg Bock and Marc Steinbach for extensive discussions about multiple shooting; Georg Bader for extensive consultations on adaptive collocation methods; Claudia Wulff for computational assistance; and many more.

Finally, we want to thank cordially our families, who, as a consequence of our work on this book, had to do without us for many a weekend, and who put up with the situation nearly without grumbling.

Berlin, Germany P. Deuflhard
Munich, Germany F.A. Bornemann

December 2001