Optimisation of Public Transport Systems

Executive summary

The decision, planning, and optimisation problems arising in public transport are highly complex and often of gigantic size. In the last twenty years, mathematical theory and algorithmic methods have been developed that make it possible to provide provably optimal solutions to many of the basic problems coming up here. These have made their way into the industry, and thus, mathematics helps to keep public transport systems affordable. Having the basic problems "under control" allows more advanced questions to be addressed like intermodal and integrated service design, disruption handling, and market aspects such as competition and user behaviour.

Challenge overview

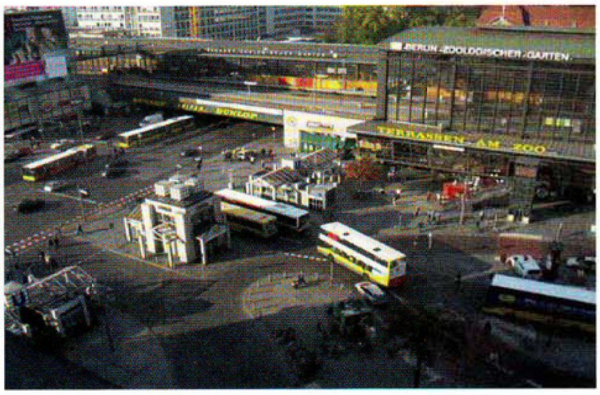
In the early 1990s the Zuse Institute Berlin (ZIB) was approached by public transportation companies in Germany - in some cases by the request of politicians who provide significant subsidies to keep the buses, etc. running - to help them plan the operation of their vehicle fleets and schedule their work forces. Among our main partners were the Hamburger Hochbahn and Berliner Verkehrsbetriebe (BVG), two of Europe's largest public transport companies. At the time most of the planning steps were carried out manually. We initially addressed the bus circulation problem; many projects followed including driver scheduling, time tabling, network, line and fare planning. Some of the latter projects were carried out together with colleagues within the DFG Research Center MATHEON.

Implementation of the Initiative

The research took place in a series of projects funded by a variety of sources including the public transport industry, the Bundesministerium für Bildung und Forschung, and the Deutsche Forschungsgemeinschaft. All initial research activities were carried out at ZIB, mainly by PhD students and post-docs, in close cooperation with the industry partners who also provided staff support and data. At the end of each individual project the industry partners and ZIB spin-off companies turned the prototype codes into commercial software products.

The problem

Optimisation problems in public transport can, in general, be modelled in terms of network flow problems in suitable planning graphs (of very large scale). The whole range of methodology from integer programming (LP relaxation, cutting planes, branch & bound, primal heuristics, etc) was applied. Particularly important tools are column generation and multi-commodity flow techniques. Major challenges arise from the enormous sizes coming up. Bus circulation at BVG, for example, leads to integer programs with about 100 million variables, and driver scheduling problems may go even beyond this size.



Results and Achievements

The projects carried out have led to a mathematical understanding of the basic problems of public transport. They have given rise to significant mathematical challenges in the fields of combinatorial optimisation and mixed-integer linear programming. They have inspired new algorithmic developments coping with huge optimisation problems. Public transport questions have also resulted in a fresh look at research fields such as robust and online optimisation. We now have mathematical models for most of the basic problems arising; many of these problems can be solved to the satisfaction of the customers. The projects have resulted in spin-off companies (such as LBW GbR) providing today mathematical tools that are employed worldwide. Hundreds of millions of Euros are saved in this way annually. These tools have become best practice industry standards.

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