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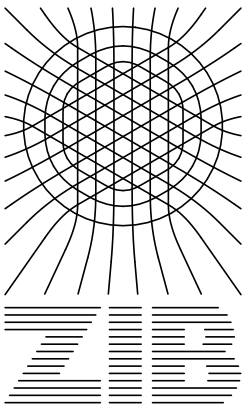

Survivable Mobile Phone Network Architectures : Models and Solution Methods

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Survivable Mobile Phone Network Architectures:

Models and Solution Methods

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1. General

The design, dimensioning and administration of survivable telecommunications networks, i.e., networks that survive the failure of certain components, is getting more and more important. This is because the overall service quality has become a major competition criterion for telecommunications services. End-to-end survivability is not only a subject of broadband networks, as, e.g., in the EC-sponsored IMMUNE project which is part of the RACE program, but also for smaller telecommunications networks like mobile networks where overall service quality is eminent and vital in a highly competitive environment. However, the right balance between costs and quality has to be determined by the design engineers.

Certain protection mechanisms have been developed and applied in SDH technology and in Digital Cross Connect (DCC) systems (SDH or PDH technology). Diverse Routed Protection, 1+1 Automatic Protection Rings (APR), Path Protected Switched Routing (PPS), Multiplexer Section Protected Rings (MSP-Rings), Restoration on DCC level are a few among others. Together with component redundancy and dynamic restoration methods these are applied to help specific parts of specific networks to survive failures of one or more of their components.

What is still missing is an integrated approach to the network design problem where, at the same time, cost effectiveness, survivability and network management aspects are taken into account to achieve a solution that appears economically efficient from various points of view.

Figure 1 shows the architecture of a typical mobile telecommunications network including a Multiplexer Network level. Each level is equipped with a certain functionality which describes the behavior of the network. The Switching Level takes control of the GOS (Grade Of Service), which normally is defined as the number of blocked (lost) calls in the network. The planning result of this level, which takes into account the number of switching nodes and the selected routing schemes, will be used as input for the Transport Level. Using multiplexing systems is the key for optimizing the allocation of physical resources when mapping the demand from the Switching Level to physical resources. This is because, economies of scale are realized by multiplexing of 64kbit/s or even less (16 or 8 kbit/s) channels for different applications and services (e.g. mobile communication, office data communication, corporate networks, etc.). Using SDH technology the lowest multiplexing level is 2.048 Mbit/s which will be mapped into virtual containers. Quality Of Service (QOS), in terms of availability of used physical transmission systems (e.g. leased lines or microwave), is considered on this level.

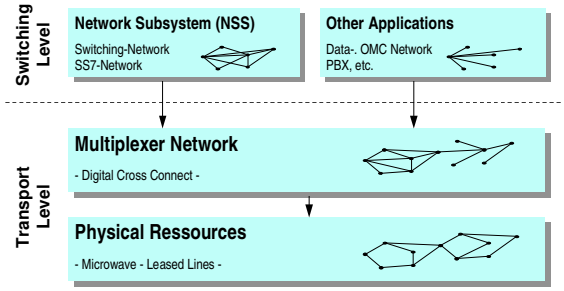

Figure 1: Typical mobile telecommunications network architecture

Survivability is considered during the design process by using the mentioned protection mechanisms at the Transport Level. Survivability in this context is the fraction of the demand that is satisfiable in a failure case, e.g., if a physical link or node fails.

In this paper, we consider the problem of designing a survivable telecommunications network, i.e., the problem of selecting from a discrete set of capacities which one to install on each link of the physical network

and deciding how to route each demand (even in the case of a single node or single edge failure) at a minimum installation cost. Additional restrictions to the percentage of a demand routed through a particular node or edge of the network, and the length of the paths between two demand nodes are considered. We model the problem as a mixed-integer programming problem and present a cutting plane algorithm to solve it. Due to the complexity of the problem, we do not expect to get optimal solutions. Instead, we get low cost solutions with a quality guarantee, which is an upper bound to the gap between the solution value and the (unknown) optimum.

Variations of our problem can be found in the literature. Most of these models consider either non-discrete capacities and survivability (see e.g. Minoux [Min81] and Lisser, Sarkissian and Vial [LSV95]), or they consider discrete capacities without survivability (see e.g. Bienstock and Günlück [BG95], Magnanti, Mirchandani, and Vachani [MMV93, MMV95]). These later studies, however, restrict the possible capacities to multiples of two base-capacities. Dahl and Stoer [DS92, DS93] study a problem similar to ours but without imposing length restrictions on the paths between demand nodes.

Here we present a network dimensioning tool, which we call DISCNET (Dimensioning of Survivable Cellular phone NETworks), that was developed for E-Plus Mobilfunk GmbH, Germany. The rest of the paper is organized as follows. In the next section we formally define the problem and present the model. A high-level description of the algorithm is given in Section 3, while in Section 4, we describe the input data and present a typical output of the tool. In Section 5 we discuss three methods of setting parameters of the model, to realize survivability in the network.

2. Formulation of the problem

Formally, the problem is defined as follows. The input consists of two graphs, the **supply graph** $G = (V, E)$ and the **demand graph** $H = (V, D)$. Both graphs share the same node-set *V*. The edge-set *E* of the supply graph consists of all the possible links to be considered. Different types of links (e.g., microwave, leased lines etc.) are modeled using **parallel edges**. An edge between two nodes in the demand graph denotes an existing demand between the two nodes.

With each edge $e \in E$ of the supply graph we associate a **cost type** that provides information about the possible capacities available for the particular link, as well as the related costs. We refer to each possible capacity for a particular link as a **breakpoint capacity**. The number T_e of possible capacities is also referred to as the number of **breakpoints**. The cost type also includes a "free" capacity which is the capacity already installed on a link (and comes at no cost), and the incremental capacity m_e^t and incremental cost k_e^t for each breakpoint t $(0 \le t \le T_e)$, which correspond to the additional capacity and the additional cost, respectively, resulting from going from breakpoint *t-1* to breakpoint *t*.

With each edge $uv \in D$ of the demand graph, we associate four parameters:

- d_{uv} which is the demand between nodes *u* and *v*
- δ_{uv} which is the **diversification** parameter, i.e., the maximum fraction of the demand d_{uv} allowed to flow through any edge or node (other than nodes *u* and *v*), see Figure 2.
- r_{uv} which is the **reservation** parameter, i.e., the minimum fraction of the demand d_{uv} that must be satisfied in a single node or a singe edge failure
- ℓ_{uv} which is the path **length restriction**, i.e., the maximum number of edges allowed in any path on which the demand d_{uv} is routed.

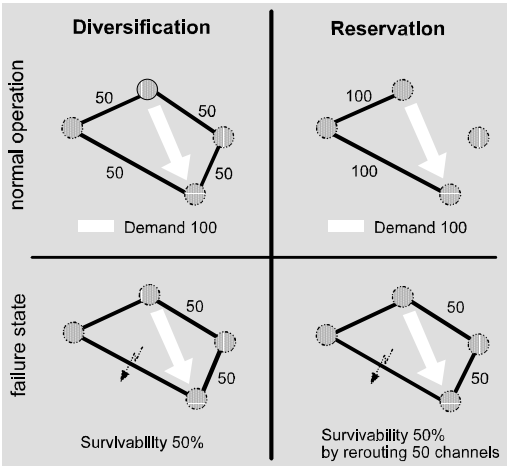

Figure 2: Diversification and reservation

All these parameters correspond to restrictions that a **feasible network** must satisfy. Eventually, we wish to determine the capacity to install on each link (edge of the supply graph) to have a low-cost or minimal-cost feasible network. In addition, we wish to have the routings of the demands for each **operating state** of the network. We use the index *s* to denote an operating state. The operating states of the network are:

- the **normal** state $(s = 0)$, which is the state with all nodes and all edges operational, and
- the **failure** states, which are the states with a single node *u* $(s = u)$ or a single edge *e* $(s = e)$ non-operational.

We denote by $G_s = (V_s, E_s)$ the supply graph for the operating state *s*, where V_s is the set of nodes that are still operational in operating state *s*, and similarly E_s is the set of the operational edges in operating state *s*. Similar notational conventions apply to the demand graph.

We model the problem as a **mixed-integer linear programming problem.** There are two types of variables in our model, the **decision** and the **routing** variables.

The decision variables are zero-one variables indicating which capacity is chosen for the edges of the supply graph. For each $e \in E$ the decision variable x_e^t equals 1 for all breakpoints $t \le \tau$, where $\tau \le T_e$ is the chosen breakpoint, and equals 0 otherwise.

The routing variables are needed in order to obtain the routings in each operating state. The routing variable $f(uv, s, P)$, for an operating state *s* and a demand edge $uv \in D_s$ denotes the part of the demand $r_{uv} d_{uv}$ that is routed on the **short** path $P \in P(uv, s)$, where $P(uv, s)$ is the set of all short paths in G_s. By short path here, we mean any path that satisfies the path length restriction ℓ_{uv}.

The objective is to minimize the total capacity installation cost. With this information we can write the mixed-integer linear programming model as follows:

$$\min \sum_{e \in E} \sum_{t=1}^{T_e} k_e^t x_e^t$$

subject to

$$0 \le x_e^{T_e} \le \cdots \le x_e^1 \le x_e^0 = 1 \quad \forall e \in E \qquad (1)$$

$$x_e^t \in \{0,1\} \qquad \forall e \in E, \forall t \qquad (2)$$

$$y_e = \sum_{t=0}^{T_e} m_e^t x_e^t \qquad \forall e \in E \qquad (3)$$

$$\sum_{uv \in D_s} \sum_{P \in P(uv,s): e \in P} f(uv,s,P) \le y_e \quad \forall s \quad \forall e \in E_s \qquad (4)$$

$$\sum_{P \in P(uv,0)} f(uv,0,P) = d_{uv} \qquad \forall uv \in D \qquad (5)$$

$$\sum_{P \in P(uv,s)} f(uv,s,P) = r_{uv} d_{uv} \quad \forall s \quad \forall uv \in D_s \qquad (6)$$

$$\sum_{P \in P(uv,0): w \in P} f(uv,0,P) \le \delta_{uv} d_{uv} \quad \forall uv \in D \quad \forall w \in V \quad w \ne u, v \qquad (7)$$

$$f(uv,0,P) \le \delta_{uv} d_{uv} \quad \forall uv \in D, P = \{uv\} \qquad (8)$$

$$f(uv,s,P) \ge 0 \qquad \forall s, \forall uv \in D_s, \quad \forall P \in P(uv,s) \qquad (9)$$

A **feasible solution** to our problem is an x-vector that has integer (zero-one) components and that corresponds to a **feasible capacity vector** *y*. The capacity vector *y*, which is calculated from the x-vector from (3), is **feasible** if it permits feasible

routings for all operating states, i.e., if the system of linear equalities and inequalities (4),…,(9) has a feasible solution for the given *y*.

3. The algorithmic approach

In this section we give a high-level description of a cutting-plane algorithm that we developed to solve the problem described in the previous section; see e.g. Nemhauser and Wolsey [NW88], Padberg [Pad95] for a general description of a cutting plane algorithm. This type of algorithm has been applied successfully to other problems in telecommunications; see e.g. Grötschel, Monma and Stoer [GMS92a, GMS92b], and Stoer [Sto92].

In Figure 3 we show the flow chart of the algorithm.

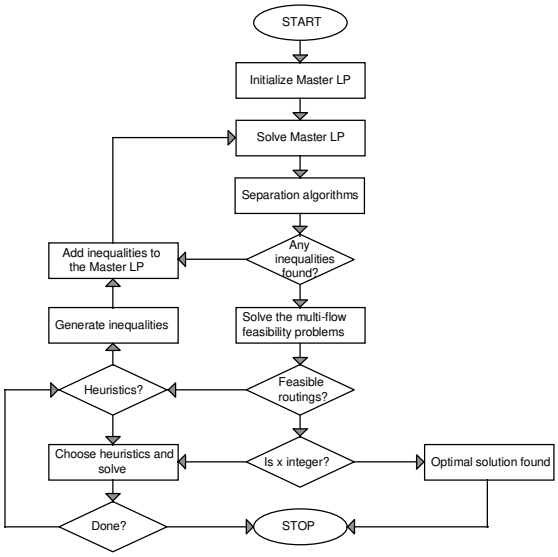

Figure 3: Flow chart of the algorithm

The algorithm consists of three main parts:

1. the cutting plane part which determines a lower bound for the objective function value and a starting point for the LP-based heuristics,
2. the multicommodity flow problems, which are solved to determine whether a given set of capacities permits feasible routings, or not, and,
3. the heuristic algorithms which are used to obtain feasible solutions.

In the cutting plane part we solve the **master LP**, which contains constraints in x variables, only. In particular, these are the ordering constraints (1), and a subset of the valid inequalities for the 0-1 polytope of feasible x-vectors. The valid inequalities (cutting planes) we use are

(i) strengthened band inequalities, introduced by Dahl and Stoer [DS92],

(ii) strengthened metric inequalities, see Alevras, Grötschel and Wessäly [AGW96] and

(iii) diversification cuts, see Alevras, Grötschel and Wessäly [AGW96].

These inequalities are identified automatically during the execution of the algorithm via so-called separation routines, which are algorithms that, given an x-vector, identify a violated inequality if one exists. There are two reasons why a separation algorithm might fail to identify a violated inequality: either such an inequality does not exist or, the separation algorithm is heuristic, i.e., it does not guarantee that a violated inequality will be found even if it exists. If we fail to separate a violated inequality we switch to the multicommodity flow problems to determine whether the (possibly fractional) solution obtained for the master LP, which corresponds to a feasible capacity vector *y*. If so, and if the x solution from the master LP is integer, then we have found an optimal solution and we are done. If the feasibility test fails for at least one operating state, then a violated metric inequality (see Iri [Iri71], and Onaga and Kakusho [OK71]) has been identified. From the derived metric inequalities we try to separate violated strengthened band inequalities. The underlying problem for the separation is the multiple-choice knapsack problem (see Martello and Toth [MT90]). We apply a dynamic-programming based exact procedure and a heuristic procedure suggested by Stoer and Dahl [SD92] to solve the separation problem for strengthened band inequalities. If we fail to generate violated strengthened-band inequalities, we derive strengthened metric inequalities, applying a divide-and-round procedure to the metric inequalities. The identified violated inequalities are then added to the master LP and the whole procedure is repeated. If feasible routings are found for all operating states and we cannot find violated inequalities, but the x variables are not integer, then we resort to various LP-based heuristics to obtain "good" integer solutions. The heuristics are of two types. In the first type one branch of the branch-

and-cut tree (see e.g. Padberg [Pad95]) is examined. Different selection criteria for the branching variable x_e^t give rise to different heuristics of this type. In the other type, we first derive a feasible capacity vector from the fractional master LP solution and then, we try to improve the solution using various criteria to reduce the capacities of the supply edges.

The cutting plane phase provides a lower bound z_{LP} and the best heuristic solution provides an upper bound z_{IP}. Thus we get a quality guarantee for the best solution found by the algorithm, i.e., an upper bound to the gap between the best solution found and an optimal solution, given by the quantity $\frac{z_{IP} - z_{LP}}{z_{LP}} 100$.

4. Planning process and results

In this section we describe how the supply and demand graphs are obtained, and we present a typical output of the algorithm.

In principle, the supply graph can be a complete graph with parallel edges. However, it is evident from the model of Section 2 that careful selection of the possible links and breakpoint capacities is desired, since this will reduce the number of integer variables in the model. Moreover, we have observed in practice that working with complete graphs increases significantly the running time of the algorithm, and, in most cases, gives solutions the quality of which is not better than that of solutions derived with supply graphs with fewer edges. We remark that the supply graph must satisfy certain connectivity requirements that are imposed by the diversification and reservation parameters. In Figure 5 we show an example of a supply graph.

The demands between nodes, i.e., the edges of the demand graph, are obtained in the switching planning process by applying a *call* based routing algorithm to map the forecast traffic between nodes (in Erlangs) to logical demands (in channels). The other input parameters of the demand graph are provided as follows. The length restriction can be any integer number bigger than 1. The reservation parameter r_{uv} is set to a value between 0 and 1, i.e., $0 \le r_{uv} \le 1$, while the diversification parameter δ_{uv} is set to a positive value between 0 and 1, i.e., $0 < \delta_{uv} \le 1$. Figure 6 shows an example of a demand graph.

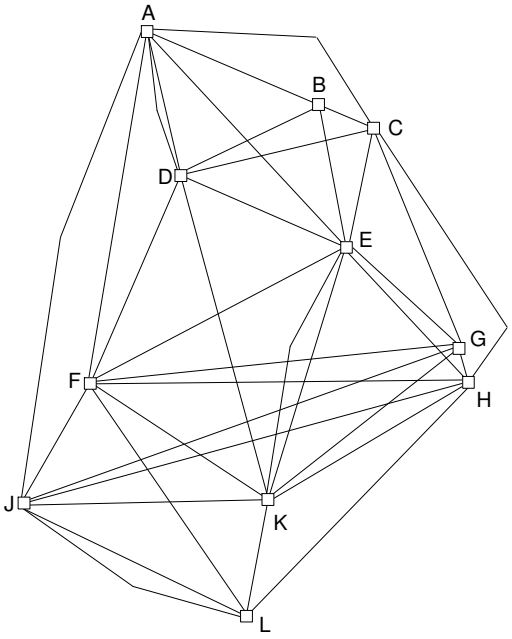

Figure 5: Supply graph showing all possible transmission systems.

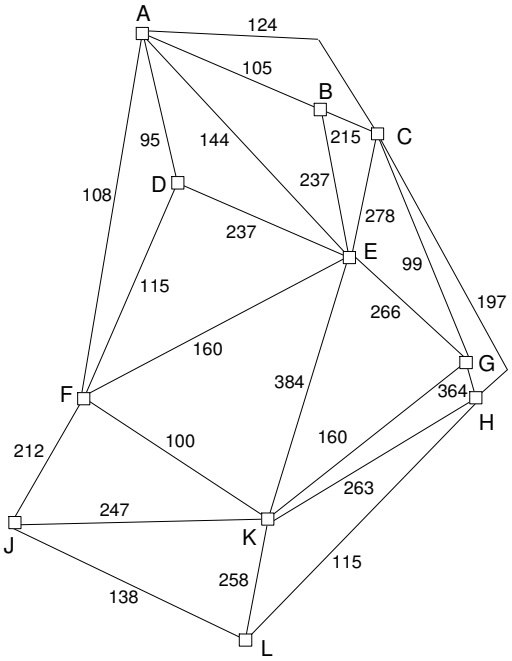

Figure 6: Logical switching network (demand graph). The demands are given in number of 64kbit/s-channels.

The primary result is the dimensioning of the physical network, i.e., the assignment of capacities to the links which permit the satisfaction of the demands in the normal state and of a given percentage of the demands in each single failure state (see Figures 7 and 8 for two examples of the

output for the graphs of Figures 5 and 6). Depending on the applied heuristics we get different network topologies with different total costs. Thus the planner can choose one of these topologies according to possible special criteria regarding the cost and the network topology.

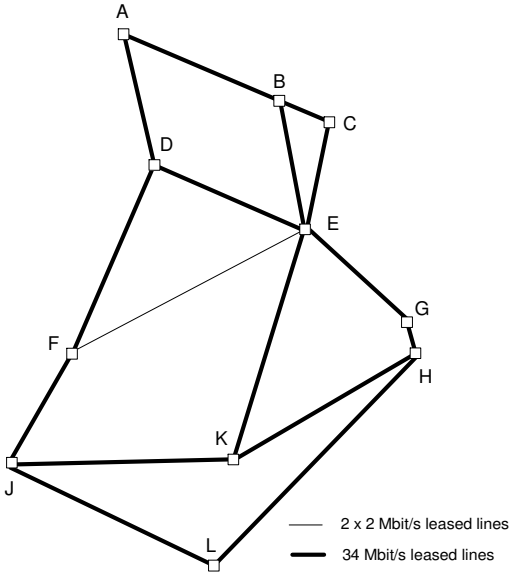

Figure 7: Resulting physical network with a planned survivability of 50 % for a single link or node failure.

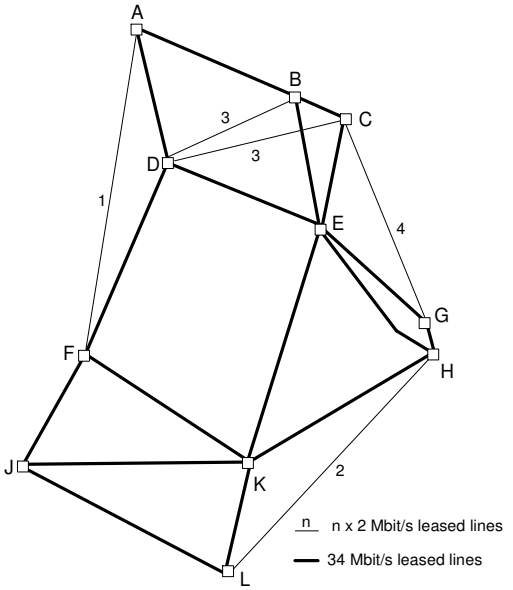

Figure 8: Calculated topology applying a diversification of 50%.

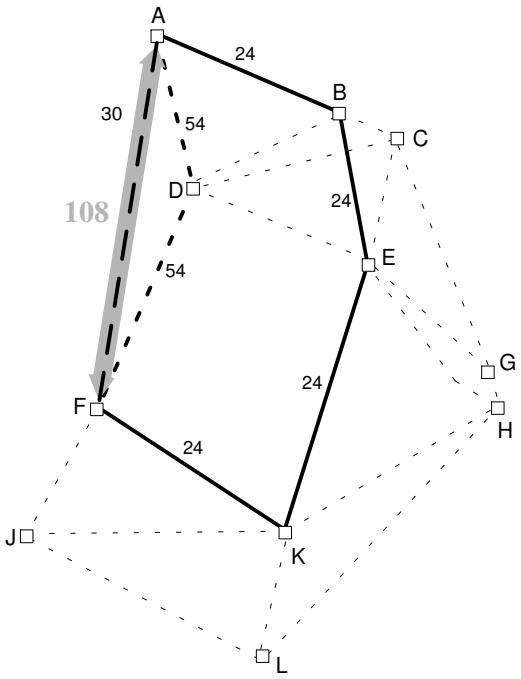

Figure 9: In this example the demand (A - F) is routed via three different paths under the condition that no more than 50% of the total demand (108 channels) is carried by one path.

Another important part of the output is the information about the routing of the channels in the normal and the failure states. This output is given in tabular form; Figure 10 shows a part of the routings table for the example of Figures 5 and 6 with the capacities shown in Figure 7. Figure 9 shows the routing of a demand in the case of diversification for the example of Figures 5 and 6 and the capacities of Figure 8.

For every operating state (normal, node failure and link failure) the routings table provides information of routings and reroutings and may be used as a data base for network management systems which are involved in network recovery.

Normal operating state:

Demand (K - L)	
Flow	Path nodes
131	K - H - L
127	K - J - L
Demand (H - L)	
Flow	Path nodes
115	H - L
	•••
Demand (F - J)	
Flow	Path nodes
212	F - J

Failing edge (H - L):

Demand (K - L)	
Flow	Path nodes
129	K - E - D - F - J - L
Demand (H - L)	
Flow	Path nodes
58	H - G - E - D - F - J - L
	•••
Demand (F - J)	
Flow	Path nodes
93	F - J
13	F - D - A - B - E - K - J

Figure 10: Example of the routing information provided by DISCNET.

5. Implementing Survivability

The model presented in this paper, gives the network designer various ways to introduce survivability at the Transport Level. In this section we discuss and compare these ways.

The physical network is said to have survivability of α%, if at least α% of each demand can be satisfied in case of a single node or single edge failure. In our model we have two input parameters which are used to introduce survivability: the diversification parameter and the reservation parameter. These two parameters can be set one at a time or in any combination.

Setting the diversification parameter δ_{uv} for the demand d_{uv} of the logical switching network, we require that at most $100\delta_{uv}$% of d_{uv} is routed through any node (other than *u* and *v*) or any link of the physical network. This implies that we get routings which provide node disjoint paths, each of them carrying at most $\delta_{uv} d_{uv}$ channels, and therefore, only that many channels of the demand can be lost in a single node or single link failure. That is, $(1-\delta_{uv})d_{uv}$ channels survive without any rerouting effort. There are two drawbacks, however. First, setting the diversification parameter to δ_{uv}, implies that the demand d_{uv} will be routed through at least $\lceil 1/\delta_{uv} \rceil$ node disjoint paths. For example, setting δ_{uv} to 0.49, we "request" at least 3 node disjoint paths through which we route d_{uv}. Evidently, we cannot achieve 100% survivability with this parameter. Moreover, diversification values below 0.34 are undesirable by the network operator, because this would force at least 4 paths each of them carrying only a small fraction of the demand. The second drawback is the high cost of the resulting network; see Figure 11.

Using the reservation parameter to introduce survivability we take advantage of possible redundancy in the network by allowing rerouting in failure situations. Depending on the particular failure all demands might be rerouted. For a specific demand of d_{uv} channels, the reservation parameter r_{uv} **guarantees** that at least $r_{uv} d_{uv}$ channels will be still satisfied in a failure state. In our tests we have observed, that much more can actually survive. For instance, by maximizing - in a post processing step - the total satisfied demand, we found that all but few demands are indeed fully satisfied. However, it should be noted that this is an empirical observation, and in theory one can guarantee only that $r_{uv} d_{uv}$ channels will survive a failure. The advantage of this method is the low cost network compared to that of the previous method; see Figure 11. The obvious disadvantage of this method is the need for rerouting in case of a failure. Indeed, as we have observed in practice, this rerouting may be extensive, making

the management of the network rather difficult.

Therefore, one has two methods of introducing survivability to the network, namely, by setting the diversification parameter and by setting the reservation parameter. To compare the costs of the two methods, we make several runs for the example shown in Figures 5 and 6. We choose as survivability values 0%, 25%, 50%, 66%, 75% and 100%, where the last value can be achieved only by setting the reservation parameter equal to 1.0. Although 75% survivability can be achieved by setting the diversification parameter equal to 0.25, we do not consider this option because, as we mentioned above, this forces too many paths for each demand. For the other values of survivability, the correspond diversification/reservation values are 1.0/0.0, 0.75/0.25, 0.5/0.5 and 0.34/0.66. The best solution values we get with our network dimensioning tool DISCNET are shown in Figure 11.

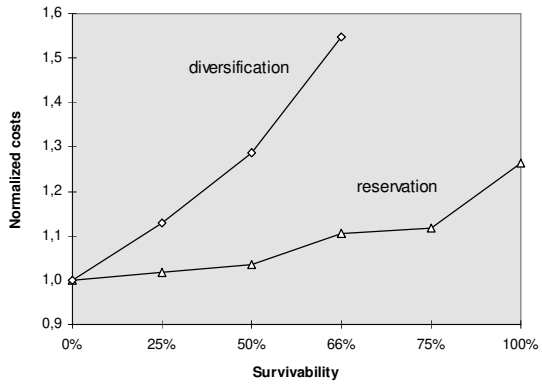

Figure 11: Comparison of costs of introducing survivability by setting the diversification or the reservation parameters.

In general, there is a trade-off between the easiness of the network management provided by the first method and the low total installation cost provided by the second. Since the network management costs are not included in the installation costs, it is up to the network operator to decide whether these costs counterbalance the difference in the installation costs.

A third way we consider to introduce survivability in the network, is a combination of the first two methods. A minimum survivability is achieved by the diversification parameter setting, with the advantage of easy network management. Additional survivability is introduced by the reservation parameter setting. In case of a failure situation the operator has to decide whether to reconfigure the network, or not. This decision depends on various aspects, e.g., on the affected traffic, the expected recovery time, and the required effort to reconfigure the network.

To compare the cost of implementing the third method, to those of the previous ones, we run two additional series of tests, combining diversification and reservation parameters. In the first series we keep a minimum survivability of 25% (achieved by setting the diversification parameter to 0.75) and we increase survivability by setting the reservation parameter to 0.50, 0.66, 0.75 and 1.0. In the second series we change the minimum survivability value to 50% and increase survivability by setting the reservation parameter to 0.66, 0.75 and 1.0. We only consider reservation parameter settings bigger than the minimum survivability, since the diversification parameter setting dominates the other cases.

The best solution values we get with DISCNET are shown in Figure 12. The third curve in Figure 12 is the reservation curve of Figure 11 (minimum survivability of 0%).

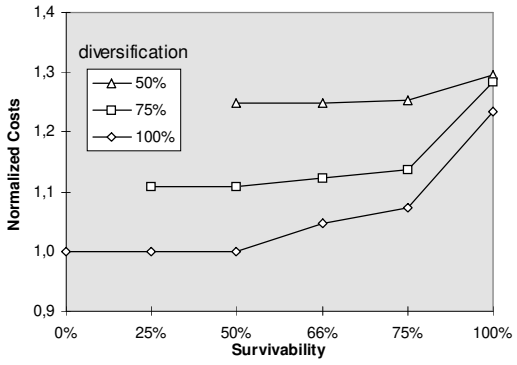

Figure 12: Comparison of costs for the different methods of introducing survivability.

6. Conclusions

Costs, quality, and operational aspects are among the most important issues that have to be considered in the process of

designing a telecommunications network efficiently. The presented network planning toolkit, DISCNET, provides different low cost network topologies that guarantee a specified survivability.

We showed that the two applicable protection schemes, *diversification* and *reservation*, lead to network topologies that differ in the transmission costs, and in the effort to manage the network. In general, the reservation method leads to networks with lower transmission costs, while the diversification method leads to networks that are easier to manage. The combination of the two protection schemes gives the network operator the opportunity to evaluate the importance of the failure of a network component, and to react accordingly.

Taking into account the ongoing liberalization process of the communication environment, efficiently designed telecommunications networks are very important both for the present and for the future. For this reason, planning toolkits like DISCNET are needed, not only to calculate the actual networks, but also to provide insight when used in studies of different network scenarios.

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