Statistische Hefte 25, 261–295 (1984)

Statistische Hefte
Statistical Papers

Aufsätze/Articles

Optimal triangulation of large real world input-output matrices

M. Grötschel, M. Jünger and G. Reinelt

Received: 25.7.83; Revised Version: 27.12.83

In this paper we present optimum triangulations of a large number of input-output matrices. In particular, we report about a series of (44,44)-matrices of the years 1959, 1965, 1970, 1975 of the countries of the European Community, about all (56,56)-matrices compiled by Deutsches Institut für Wirtschaftsforschung for the Federal Republic of Germany, and about the (60,60)-matrices of the Statistisches Bundesamt of the Federal Republic of Germany. These optimum triangulations were obtained with a code developed by the authors which utilizes new polyhedral results for the triangulation problem in a linear programming cutting plane framework. With this code the range of solvability of triangulation problems was more than doubled (in terms of sector numbers) compared to previous work. In particular, for none of the triangulation problems mentioned above optimum solutions were known before. Moreover, we discuss various claims about properties of optimum solutions made in the literature and question some common concepts of analysing triangulated input-output matrices.

1. Introduction

Triangulation of input-output matrices is a problem which has received substantial attention in the literature during the past twentyfive years. The aim of this paper is to present optimum triangulations of a large number of important input-output matrices for which no such triangulations could be found before. In particular, we report about a series of (44,44)-matrices of the Statistical Office of the European Community, about the (56,56)-matrices of the Deutsches Institut für Wirtschaftsforschung, and about three (60,60)-matrices of the Statistisches Bundesamt of the Federal Republic of Germany. Moreover, we discuss some of the claims about optimum triangulations and some prejudices about properties of the latter that can be found in the literature.

A review of the mathematical methods that have been designed for the solution of the triangulation problem and of the developments of the economical analysis and interpretation of this problem is beyond the scope of this paper. The latter area is widely covered in the book WESSELS (1981), while descriptions of recent progress in algorithms for the triangulation problem can be found in GRÖTSCHEL, JÜNGER & REINELT (1982c) or KAAS (1981). The mathematical background for the code used to obtain the results presented here is given in GRÖTSCHEL, JÜNGER & REINELT (1982a) and (1982b).

2. Mathematical Background

Let A be an input-output matrix with n rows and columns, then the *triangulation problem* is to find a simultaneous permutation of the rows and columns of A such that the sum of the entries which are above the main diagonal (of the permuted matrix) is as large as possible.

It has been observed (empirically) by many authors a long time ago that it is quite difficult to find an optimum triangulation. The results of complexity theory imply that this is also true in a precise theoretical sense. Namely, the triangulation problem belongs to the class of NP-hard problems (like the travelling salesman problem or general integer programming) for which efficient (i. e. polynomial time) algorithms very likely do not exist. This follows from the fact that the triangulation problem is (trivially) equivalent to the acyclic subgraph problem and the feedback arc set problem which are well-known to belong to the class of NP-hard problems. The latter equivalence is described in GRÖTSCHEL, JÜNGER & REINELT (1982a); a good source on the theoretical background of the theory of NP-hardness resp. NP-completeness is GAREY & JOHNSON (1979).

It has become apparent in the recent years that those algorithms for NP-hard problems are most successful which combine cutting plane LP-methods with various heuristic features and branch & bound techniques. We have designed and implemented such an algorithm which is mainly a cutting plane method enriched by several heuristics and governed by branch & bound. This algorithm is described in detail in GRÖTSCHEL, JÜNGER & REINELT (1982c) and has been used to obtain the results discussed here. The algorithm is based on results about the facial structure of the polytope related to the triangulation problem. These theoretical investigations can be found in GRÖTSCHEL, JÜNGER & REINELT (1982b).

The triangulation problem for an (n,n)-matrix A can be formulated as an integer linear program as follows. Let x_{ij}, $1 \leq i, j \leq n$, $i \neq j$, be a variable corresponding to the deliveries from sector i to sector j (note that the main diagonal of A does not play a role in the triangulation problem, so it is not necessary to consider the variables x_{ii}, $i = 1,\ldots,n$).

Let $\mathbb{R}^m$, where $m = n(n-1)$, be the vector space of vectors $x = (x_{12}, x_{13}, \ldots, x_{1n}, x_{21}, x_{23}, x_{24}, \ldots, x_{2n}, x_{31}, \ldots, x_{n,n-2}, x_{n,n-1})$, and consider the following system of equations and inequalities

$$x_{ij} \geq 0 \qquad 1 \leq i,\ j \leq n,\ i \neq j \tag{2.1}$$

$$x_{ij} + x_{ji} = 1 \qquad 1 \leq i < j \leq n \tag{2.2}$$

$$x_{ij} + x_{jk} + x_{ki} \leq 2 \qquad 1 \leq i < j < k \leq n \tag{2.3}$$

$$x_{ik} + x_{kj} + x_{ji} \leq 2 \qquad 1 \leq i < j < k \leq n \tag{2.4}$$

To be consistent with our paper GRÖTSCHEL, JÜNGER & REINELT (1982c) we denote the solution set of this system by P_2^n, i. e.

$$P_2^n = \{x \in \mathbb{R}^m \mid x \text{ satisfies } (2.1), \ldots (2.4)\},$$

and we set

$$P_1^n = \{x \in \mathbb{R}^m \mid x \text{ satisfies } (2.1), (2.2)\}.$$

Linear programs over P_1^n are trivial to solve, and linear programs over P_2^n are theoretically solvable in polynomial time with the ellipsoid method. A practical method to do this is outlined in GRÖTSCHEL, JÜNGER & REINELT (1982c)

It is a well-known fact that every optimum solution of the integer linear program

$$\alpha_0 = \max\left\{\sum_{i \neq j}^{n} a_{ij}\, x_{ij} \;\middle|\; x \in P_2^n \quad \text{and} \quad x \in \{0,1\}^m\right\} \tag{2.5}$$

determines an optimum triangulation of the input-output matrix $A = (a_{ij})$, namely, sector i is in the k-th position of the optimum linear ordering if and only if $\sum_{j=1, j \neq i}^{n} x_{ji} = k-1$.

Setting

$$\alpha_1 := \max\left\{\sum_{i \neq j}^{n} a_{ij} x_{ij} \;\middle|\; x \in P_1^n\right\},$$

$$\alpha_2 := \max\left\{\sum_{i \neq j}^{n} a_{ij} x_{ij} \;\middle|\; x \in P_2^n\right\},$$

$$\alpha := \sum_{i \neq j}^{n} a_{ij},$$

then

$$\lambda := 100\,\frac{\alpha_0}{\alpha}$$

is the celebrated *degree of linearity* (of A) which is used to measure the mutual dependence or circularity of the sectors. The parameter

$$\bar{\lambda} := 100\,\frac{\alpha_1}{\alpha}$$

is called *theoretical degree of linearity* (of A). We think that the use of the adjective "theoretical" in this name is a poor choice. According to HELMSTÄDTER (1964) and WESSELS (1981) "theoretical" indicates that, since $\lambda \leq \bar{\lambda}$, this is a number which the degree of linearity λ could theoretically achieve. But, of course, this is true for every other number as well which bounds λ from above. Probably the only significance of $\bar{\lambda}$ is that it is an upper bound on λ which is very easy to calculate.

We now want to introduce another upper bound, denoted by λ', for λ which we think is a much more natural parameter than $\bar{\lambda}$. Note that

$$\left\{x \in P_2^n \mid x \in \{0,1\}^m\right\} \subseteq P_2^n \subseteq P_1^n$$

and therefore

$$\alpha_0 \leq \alpha_2 \leq \alpha_1.$$

Let us set

$$\lambda' := 100\,\frac{\alpha_2}{\alpha}$$

then clearly

$$\lambda \leq \lambda' \leq \bar{\lambda}.$$

We call λ' the *fractional degree of linearity* (of A). The use of the name "fractional" here is standard terminology in combinatorial optimization. Namely, if we have an integer linear programming formulation of a combinatorial optimization problem (like (2.5) is such a formulation of the triangulation problem) then the linear program obtained from this by removing the integrality stipulations is usually called the fractional version of the combinatorial optimization problem. So, max $\left\{\sum_{i\neq j}^{n} a_{ij}x_{ij} \mid x \in P_2^n\right\}$ is the fractional version of the triangulation problem. It is also customary to call the optimum solution value of the fractional version its fractional value. Thus, it is appropiate to call α_2 the fractional value of the triangulation problem, which in turn gives rise to the name fractional degree of linearity for λ'.

The "philosophical" background for introducing "fractional" values as above is to get values which are very good bounds for the corresponding integral values and which are efficiently computable. The theoretical degree of linearity $\bar{\lambda}$ is trivial to calculate, but we do not know of any real-world input-output matrix where $\bar{\lambda} = \lambda$ holds.

In our opinion, the fractional degree of linearity λ' is much more attractive. As shown in GRÖTSCHEL, JÜNGER & REINELT (1982c), α_2 and thus λ' are computable in reasonable time (although with more effort than $\bar{\lambda}$) and - quite surprisingly - in fourty-one of the fourty-four examples we shall present later, λ' was in fact equal to λ. This shows that λ' achieves both goals (fast computation and good bound) simultaneously and much better than $\bar{\lambda}$.

3. Triangulation of the I-O Tables of the European Community

A major problem in comparing the structures of economies of different states via the analysis of their input-output tables is the fact that quite often the systems of dividing the economy into sectors do not agree. And even if in the I-O-tables of two countries some sector names are identical, the statistical definition of the sectors may be different (due to an incomparable statistical data basis) so that corresponding flows of value are in fact incomparable.

The I-O-program of the European Community (EC) tries to overcome this drawback. The statistical offices of the members of the EC compile I-O-tables of size (44,44) which are based on the same definitions. An effort is made to use the statistical data in such a way that in all countries the compilation of the entries of the tables is as consistent as possible compared with the definition. This program is probably the first successful attempt to make international comparisons possible.

The data of these I-O-tables are available from "Office Statistique des Communautés Européennes (EUROSTAT), Boite postale 1907, Kirchberg, Luxembourg" on tape. We present here the triangulations of the (44,44)-matrices of intermediate consumption at current prices for the years 1959, 1965, 1970 and 1975. We want to remark that the word "triangulated matrix", whenever we use it, means that this matrix represents an optimum triangulation. This has to be stressed, since this notion is quite often misused in the literature. In many cases we encountered, a "triangulated matrix" is nothing but the (nonoptimal) result of a heuristic procedure.

In the sequel, the I-O-tables for the members of the EC are denoted by the name of the country and the reference year. EUR6 resp. EUR9 denote the I-O-tables in which the original six resp. first nine EC-members' I-O-tables have been aggregated into one I-O-table. (Table 3.2 shows in addition for each I-O-table the code used by EUROSTAT to identify these matrices uniquely.) Table 3.1 contains the sector names according to their standard numbering.

Sector numbering for EUROSTAT I-O-tables

1. Agricultural, forestry and fishery products
2. Coal, lignite (brown coal) and briquettes
3. Products of coking
4. Crude petroleum, natural gas and petroleum products
5. Electric power, gas, steam and water
6. Production and processing of radio-active materials and ores
7. Ferrous and non-ferrous ores and metals, other than radioactive
8. Non-metallic mineral products
9. Chemical products
10. Metal products except machinery and transport equipment
11. Agricultural and industrial machinery
12. Office and data processing machines, precision and optical instruments
13. Electrical goods
14. Motor vehicles
15. Other transport equipment
16. Meats, meat preparations and preserves, other products from slaughtered animals
17. Milk and dairy products
18. Other food products
19. Beverages
20. Tobacco products
21. Textiles and clothing
22. Leathers, leather and skin goods, footwear
23. Timber, wooden products and furniture
24. Paper and printing products
25. Rubber and plastic products
26. Other manufacturing products
27. Building and construction
28. Recovery and repair services
29. Wholesale and retail trade
30. Lodging and catering services
31. Inland transport services
32. Maritime and air transport services
33. Auxiliary transport services
34. Communication services
35. Services of credit and insurance institutions
36. Business services provided to enterprises
37. Services of renting of immovable goods
38. Market services of education and research
39. Market services of health
40. Recreational and cultural services, personal services, other market services n.e.c.
41. General public services
42. Non-market services of education and research provided by general government and private non-profit institutions
43. Non-market services of health provided by general government and private non-profit institutions
44. Domestic services and other non-market services n.e.c.

Table 3.1

For the year 1959, there are five I-O-tables of the type described above available, whose collection is shortly denoted by EUROSTAT-59. Table 3.2 displays an optimum ranking and the theoretical degree of linearity $\bar{\lambda}$, the fractional degree of linearity λ' and the ("optimum") degree of linearity λ for each of those tables.

Rankings and degrees of linearity for EUROSTAT-59

	Rankings				
Number of sector	BELGIUM-59 T59B11XX.B	GERMANY-59 T59D11XX.B	FRANCE-59 T59F11XX.B	ITALY-59 T59I11XX.B	NETHER-59 T59N11XX.B
1	22	17	19	18	28
2	8	5	5	4	3
3	10	6	6	13	4
4	4	13	3	3	2
5	11	8	7	5	5
6	36	14	36	36	36
7	16	36	15	14	12
8	15	29	13	15	16
9	19	37	17	16	8
10	18	18	16	17	13
11	21	2	26	27	15
12	17	19	34	26	19
13	20	12	25	25	17
14	33	3	33	31	20
15	26	21	35	28	23
16	23	4	21	22	30
17	29	15	28	32	29
18	30	20	29	33	32
19	32	16	32	35	34
20	34	24	31	34	33
21	31	23	23	19	31
22	24	27	22	23	35
23	25	28	24	21	14
24	6	25	11	9	7
25	3	34	20	20	9
26	35	22	30	24	18
27	28	1	27	30	27
28	5	26	8	6	21
29	14	7	14	12	26
30	38	11	38	38	38
31	9	33	10	8	22
32	27	31	12	29	25
33	13	30	9	7	24
34	2	32	4	11	6
35	7	38	2	2	11
36	1	35	1	1	1
37	41	41	41	41	42
38	37	9	37	37	37
39	39	39	39	39	39
40	12	10	18	10	10
41	44	44	44	44	41
42	43	43	43	43	44
43	40	40	40	40	40
44	42	42	42	42	43
	Degrees of linearity				
Theoretical degree of linearity	87.575	91.432	88.962	91.129	85.863
Fractional degree of linearity	83.822	88.148	85.516	88.443	82.991
Degree of linearity	83.818	88.148	85.516	88.443	82.991

Table 3.2

For instance in the I-O-table for BELGIUM-59 sector 1 (Agricultural, forestry and fishery products) appears in position 22 of the displayed optimum linear ordering, sector 2 (Coal, lignite and briquettes) in position 8, etc. We also say that sector 1 has rank 22, sector 2 has rank 8 etc. This means that sector 36 (Business services provided to enterprises) is at the top of the displayed hierarchy (having rank 1) while sector 41 (General public services) is at the bottom (having rank 44). As we shall see later, this kind of ranking will be subject to drastic changes in the course of time.

Our lists contain one optimum solution of the triangulation problems for each country. It has often been claimed in the literature that "due to economical reasons" optimum solutions of real-world triangulation problems should be unique. Reality disproves this conjecture. Each of the problems we solved has several optimum solutions! We shall comment on this in more detail in section 5.

As one can see from Table 3.2 the degree of linearity λ and the fractional degree of linearity λ' are equal in four out of five cases. Only for BELGIUM-59 there is a deviation of less than 0.005 %. Note that the theoretical degree of linearity $\bar{\lambda}$ is more than 3.7 % larger than λ on the average over the five cases in Table 3.2.

A convenient notation for a linear ordering O of the sectors $\{1,2,\ldots,n\}$ of an I-O-table is $\langle O_1, O_2,\ldots,O_n\rangle$ denoting a bijective mapping (or permutation) $R : \{1,\ldots,n\} \rightarrow \{1,\ldots,n\}$ with $R(O_i) = i$ for the sector O_i occuring at the i-th position of the linear ordering O. We also say that for any sector $k \in \{1,\ldots,n\}$, sector k has the ranking R(k) in the linear ordering O. For two linear orderings O_1 and O_2 of the collection of sectors $\{1,2,\ldots,n\}$ and the corresponding ranking functions R_1 and R_2 the *Spearman rank correlation coefficient* $\sigma(R_1,R_2)$ defined by

$$\sigma(R_1,R_2) = 1 - \frac{6 \cdot \sum_{i=1}^{n} (R_1(i)-R_2(i))^2}{n(n^2-1)}$$

is often used to measure the similarity of the linear orderings. Calculated for two optimum triangulations of two different countries this coefficient is interpreted as a measure of similarity of the structure of the economies of the two countries. It ranges from 1 (if O_1 and O_2 are equal) to -1 (if

O_2 is the reverse linear ordering of O_1). For the optimum linear orderings displayed in Table 3.2 the mutual rank correlation coefficients are presented in Table 3.3.

Spearman rank correlation coefficients for EUROSTAT-59

	BELGIUM-59	GERMANY-59	FRANCE-59	ITALY-59	NETHER-59
BELGIUM-59	-	0.146018	0.913742	0.928823	0.866385
GERMANY-59	0.146018	-	0.133897	0.150669	0.232417
FRANCE-59	0.913742	0.133897	-	0.948978	0.813249
ITALY-59	0.928823	0.150669	0.948978	-	0.838337
NETHER-59	0.866385	0.232417	0.813249	0.838337	-

Table 3.3

For sake of completeness we list here and in the following the Spearman rank correlation coefficients. The interpretations of these should, however, be handled with some care, cf. section 5.

In Tables 3.4 and 3.5 we have compiled the corresponding data for the EUROSTAT-matrices of 1965, and in Tables 3.6 and 3.7 for 1970. Note that there is no I-O-table IRELAND-70. Instead, the results for IRELAND-69 are presented since this I-O-table was used in the aggregated I-O-table EUR9-70. Also, EUROSTAT has two I-O-tables GERMANY-70, here denoted by GERM-70a and GERM-70b (EUROSTAT coding for both: T70D11XX.B). GERM-70b is a revised version of GERM-70a, but since GERM-70a was used in the aggregated I-O-tables EUR6-70 and EUR9-70, we have listed both of them. Some interesting observations about these two I-O-tables will be presented in section 5.

Tables 3.8 and 3.9 list optimum rankings, degrees of linearity and Spearman rank correlation coefficients for the EUROSTAT-matrices of 1975.

Rankings and degrees of linearity for EUROSTAT-65

Number of sector	BELGIUM-65 T65B11XX.B	GERMANY-65 T65D11XX.B	FRANCE-65 T65F11XX.B	ITALY-65 T65I11XX.B	LUXEMB-65 T65L11XX.B	NETHER-65 T65N11XX.B	EUR6-65 T65W11XX.B
	Rankings						
1	24	24	24	22	34	27	23
2	6	7	4	4	22	5	4
3	18	15	13	12	21	19	14
4	3	5	2	3	18	4	1
5	7	8	5	5	23	6	5
6	2	4	1	2	17	3	3
7	19	18	15	20	36	20	17
8	22	17	14	19	32	21	16
9	23	19	17	13	31	14	18
10	20	21	16	21	26	22	20
11	21	23	21	25	25	26	22
12	30	35	34	29	16	25	29
13	31	22	20	30	37	24	21
14	15	40	22	39	15	32	37
15	40	41	35	37	14	41	38
16	25	25	27	26	44	28	24
17	34	30	25	32	43	33	32
18	35	31	26	33	33	34	33
19	36	33	32	34	42	35	34
20	32	32	44	36	41	40	28
21	28	28	28	23	24	29	25
22	26	26	42	27	13	39	36
23	29	29	29	24	35	30	26
24	8	13	8	14	19	13	10
25	5	20	18	15	40	15	19
26	27	27	31	28	12	23	27
27	33	38	30	31	38	31	30
28	16	9	23	6	27	7	6
29	12	16	19	18	30	16	15
30	37	34	33	35	39	36	35
31	17	10	7	10	28	12	9
32	41	3	36	38	11	42	39
33	14	2	12	9	10	11	8
34	4	6	3	11	29	8	2
35	11	14	10	17	20	18	13
36	10	1	6	8	9	10	7
37	44	39	11	7	8	44	31
38	13	11	41	44	7	9	12
39	39	36	37	43	6	37	41
40	9	12	9	16	5	17	11
41	43	44	40	40	4	43	43
42	42	43	43	42	3	38	42
43	38	37	38	41	1	1	44
44	1	42	39	1	2	2	40
	Degrees of linearity						
Theoretical degree of linearity	88.647	86.486	87.643	89.945	91.888	85.836	86.080
Fractional degree of linearity	85.030	83.007	84.395	85.812	90.805	82.585	82.921
Degree of linearity	85.030	83.007	84.395	85.812	90.805	82.585	82.921

Table 3.4

Spearman rank correlation coefficients for EUROSTAT-65

	BELGIUM-65	GERMANY-65	FRANCE-65	ITALY-65	LUXEMB-65	NETHER-65	EUR6-65
BELGIUM-65	-	0.672304	0.665539	0.738689	0.020014	0.820296	0.787879
GERMANY-65	0.672304	-	0.704440	0.603946	-0.043834	0.626075	0.869063
FRANCE-65	0.665539	0.704440	-	0.786751	-0.127696	0.571106	0.831431
ITALY-65	0.738689	0.603946	0.786751	-	-0.021424	0.663989	0.745032
LUXEMB-65	0.020014	-0.043834	-0.127696	-0.021424	-	0.129105	-0.166878
NETHER-65	0.820296	0.626075	0.571106	0.663989	0.129105	-	0.713460
EUR6-65	0.787879	0.869063	0.831431	0.745032	-0.166878	0.713460	-

Table 3.5

Rankings and degrees of linearity for EUROSTAT-70

Number of sector	IRELAND-69 T69R11XX.B	BELGIUM-70 T70B11XX.B	GERM-70a T70D11XX.B	GERM-70b T70D11XX.B	FRANCE-70 T70F11XX.B	ITALY-70 T70I11XX.B	DENMARK-70 T70K11XX.B	LUXEMB-70 T70L11XX.B	NETHER-70 T70N11XX.B	UK-70 T70U11XX.B	EUR6-70 T70W11XX.B	EUR9-70 T70X11XX.B
	Rankings											
1	32	28	26	21	25	20	28	30	31	34	24	24
2	9	7	5	10	8	3	9	24	5	18	4	4
3	8	17	14	11	13	15	27	19	23	22	14	14
4	6	6	2	2	7	2	4	18	8	9	2	2
5	20	9	6	12	9	4	10	20	9	19	5	5
6	19	5	1	1	6	1	3	17	4	17	1	1
7	10	19	17	18	16	18	16	32	13	24	17	17
8	15	18	16	17	14	16	21	29	27	23	16	16
9	16	21	19	20	17	17	13	26	12	26	19	19
10	11	20	18	19	19	19	17	25	18	25	18	18
11	12	26	25	35	22	24	20	22	16	31	23	23
12	29	27	23	34	21	28	18	16	15	30	22	22
13	18	24	22	31	20	25	19	15	17	29	21	21
14	21	25	39	36	23	42	23	11	20	42	39	39
15	23	39	38	37	36	29	22	10	30	33	36	36
16	38	29	33	26	31	26	35	37	33	38	32	32
17	37	35	32	25	30	33	34	36	32	36	31	31
18	39	36	34	27	32	34	36	38	35	37	33	33
19	44	37	35	28	33	35	37	39	36	43	34	34
20	43	33	30	24	44	41	39	9	38	41	30	30
21	33	31	27	22	26	21	29	8	34	40	25	25
22	42	30	37	32	35	38	41	7	41	39	38	38
23	34	32	28	30	27	23	30	31	26	28	26	26
24	13	10	8	13	3	8	7	13	10	13	8	8
25	17	22	20	23	18	22	14	35	14	27	20	20
26	14	23	21	33	28	27	25	6	19	14	27	27
27	35	34	29	39	29	31	31	33	28	32	28	28
28	22	13	7	6	24	5	24	12	21	21	6	6
29	30	15	15	16	15	12	15	21	25	20	15	15
30	31	38	36	29	34	36	38	34	37	12	35	35
31	24	16	13	9	12	13	5	23	22	16	13	13
32	28	41	40	38	37	30	33	44	39	35	37	37
33	27	14	12	8	2	11	11	42	24	11	7	7
34	7	8	3	3	1	6	6	14	3	10	3	3
35	26	12	11	14	11	9	12	28	11	15	12	12
36	5	11	4	7	4	7	8	27	6	8	9	9
37	41	43	31	15	10	10	40	41	44	7	29	29
38	4	4	9	5	5	14	44	5	29	6	10	10
39	3	40	24	41	39	44	26	40	40	5	40	40
40	25	3	10	4	38	32	32	4	7	4	11	11
41	2	44	43	43	43	43	42	43	43	44	42	41
42	1	42	44	44	42	40	2	3	42	3	44	44
43	40	1	42	42	40	37	1	1	1	1	41	43
44	36	2	41	40	41	39	43	2	2	2	43	42
	Degrees of linearity											
Theoretical degree of linearity	90.408	87.180	86.378	86.497	88.976	89.455	88.755	90.878	85.444	83.515	86.459	85.643
Fractional degree of linearity	87.862	84.132	83.183	82.199	85.115	85.613	85.893	89.808	82.810	80.636	83.401	82.668
Degree of linearity	87.862	84.132	83.181	82.199	85.115	85.613	85.893	89.808	82.810	80.636	83.401	82.668

Table 3.6

Spearman rank correlation coefficients for EUROSTAT-70

	IRELAND-69	BELGIUM-70	GERMANY-70a	GERMANY-70b	FRANCE-70	ITALY-70	DENMARK-70	LUXEMB-70	NETHER-70	UK-70	EUR6-70	EUR9-70
IRELAND-69	–	0.252854	0.464412	0.228753	0.385342	0.313883	0.448344	0.095278	0.281889	0.358140	0.378858	0.384778
BELGIUM-70	0.252854	–	0.699648	0.633122	0.556448	0.585765	0.493587	0.475123	0.859901	0.557576	0.694151	0.687949
GERMANY-70a	0.464412	0.699648	–	0.881748	0.821424	0.860183	0.527696	0.094151	0.599154	0.404228	0.972093	0.972093
GERMANY-70b	0.228753	0.633122	0.881748	–	0.779422	0.843129	0.336575	0.045384	0.424383	0.334884	0.909655	0.909796
FRANCE-70	0.385342	0.556448	0.821424	0.779422	–	0.896124	0.531501	-0.067512	0.489782	0.309514	0.851868	0.851304
ITALY-70	0.313883	0.585765	0.860183	0.843129	0.896124	–	0.540381	-0.021001	0.502185	0.329387	0.907118	0.905990
DENMARK-70	0.448344	0.493587	0.527696	0.336575	0.531501	0.540381	–	0.191402	0.628189	0.407611	0.526991	0.515292
LUXEMB-70	0.095278	0.475123	0.094151	0.045384	-0.067512	-0.021001	0.191402	–	0.385906	0.293869	0.077942	0.071882
NETHER-70	0.281889	0.859901	0.599154	0.424383	0.489782	0.502185	0.628189	0.385906	–	0.444820	0.594221	0.588161
UK-70	0.358140	0.557576	0.404228	0.334884	0.309514	0.329387	0.407611	0.293869	0.444820	–	0.320507	0.314306
EUR6-70	0.378858	0.694151	0.972093	0.909655	0.851868	0.907118	0.526991	0.077942	0.594221	0.320507	–	0.999577
EUR9-70	0.384778	0.687949	0.972093	0.909796	0.851304	0.905990	0.515292	0.071882	0.588161	0.314306	0.999577	–

Table 3.7

Rankings and degrees of linearity for EUROSTAT-75

Number of sector	Rankings					
	GERMANY-75 T75D11XX.B	SPAIN-75 T75E11XX.B	ITALY-75 T75I11XX.B	DENMARK-75 T75K11XX.B	NETHER-75 T75N11XX.B	UK-75 T75U11XX.B
1	22	22	21	27	32	31
2	7	6	3	5	31	9
3	8	14	15	26	30	18
4	1	4	2	6	6	5
5	9	7	4	8	7	10
6	6	5	1	4	5	4
7	18	17	18	17	14	20
8	19	15	16	22	11	19
9	21	16	17	14	12	22
10	20	18	20	18	17	21
11	35	21	26	21	19	29
12	34	28	29	20	15	39
13	33	20	24	19	16	27
14	38	41	42	3	20	28
15	39	32	37	23	29	40
16	24	23	27	38	34	33
17	27	29	31	35	33	32
18	28	30	32	36	35	34
19	29	31	33	37	36	35
20	23	36	36	42	38	36
21	25	25	22	29	26	23
22	32	24	39	28	41	38
23	31	26	23	30	27	25
24	10	9	7	10	9	7
25	26	19	19	15	13	24
26	36	27	28	31	18	26
27	37	34	30	32	28	30
28	4	8	5	24	21	17
29	16	12	13	16	24	16
30	30	37	35	39	37	37
31	17	13	12	7	23	14
32	3	33	38	34	39	15
33	15	11	11	13	22	13
34	14	1	6	9	8	6
35	12	10	9	12	10	12
36	5	3	8	11	3	3
37	13	40	10	43	44	11
38	11	44	14	41	25	2
39	41	43	34	25	40	1
40	2	35	25	33	4	8
41	43	42	43	44	43	44
42	44	39	41	2	42	43
43	42	38	44	1	1	41
44	40	2	40	40	2	42
	Degrees of linearity					
Theoretical degree of linearity	86.742	89.915	89.338	88.945	85.402	86.190
Fractional degree of linearity	82.773	87.715	85.136	86.054	82.943	82.678
Degree of linearity	82.773	87.715	85.136	86.054	82.943	82.678

Table 3.8

Spearman rank correlation coefficients for EUROSTAT-75

	GERMANY-75	SPAIN-75	ITALY-75	DENMARK-75	NETHER-75	UK-75
GERMANY-75	-	0.538125	0.798450	0.186892	0.299507	0.776885
SPAIN-75	0.538125	-	0.720648	0.488795	0.602114	0.420437
ITALY-75	0.798450	0.720648	-	0.411276	0.431994	0.794926
DENMARK-75	0.186892	0.488795	0.411276	-	0.541367	0.329105
NETHER-75	0.299507	0.602114	0.431994	0.541367	-	0.348555
UK-75	0.776885	0.420437	0.794926	0.329105	0.348555	-

Table 3.9

It is interesting to consider the development of the economic structure re-presented by an I-O-table) of a fixed country in the course of time. Since in section 4 we shall present numerical data for two collections of I-O-tables of the Federal Republic of Germany, we take the EUROSTAT-matrices for West Germany as an example in order to allow comparisons. For ease of reading we summarize in Table 3.10 the rankings and degrees of linearity for the appropriate EUROSTAT-matrices presented in previous tables. For the year 1970 we have selected the revised I-O-table GERMANY-70b.

Rankings and degrees of linearity for EUROSTAT-GERMANY-tables

	Rankings			
Number of sector	GERMANY-59 T59011XX.8	GERMANY-65 T65011XX.8	GERMANY-70 T70011XX.8	GERMANY-75 T75011XX.8
1	17	24	21	22
2	5	7	10	7
3	6	15	11	8
4	13	5	2	1
5	8	8	12	9
6	14	4	1	6
7	36	18	18	18
8	29	17	17	19
9	37	19	20	21
10	18	21	19	20
11	2	23	35	35
12	19	35	34	34
13	12	22	31	33
14	3	40	36	38
15	21	41	37	39
16	4	25	26	24
17	15	30	25	27
18	20	31	27	28
19	16	33	28	29
20	24	32	24	23
21	23	28	22	25
22	27	26	32	32
23	28	29	30	31
24	25	13	13	10
25	34	20	23	26
26	22	27	33	36
27	1	38	39	37
28	26	9	6	4
29	7	16	16	16
30	11	34	29	30
31	33	10	9	17
32	31	3	38	3
33	30	2	8	15
34	32	6	3	14
35	38	14	14	12
36	35	1	7	5
37	41	39	15	13
38	9	11	5	11
39	39	36	41	41
40	10	12	4	2
41	44	44	43	43
42	43	43	44	44
43	40	37	42	42
44	42	42	40	40
	Degrees of linearity			
Theoretical degree of linearity	91.432	86.486	86.497	86.742
Fractional degree of linearity	88.148	83.007	82.199	82.773
Degree of linearity	88.148	83.007	82.199	82.773

Table 3.10

Table 3.11 shows all mutual Spearman rank correlation coefficients for these four I-O-tables. It reflects the already mentioned drastic changes from 1959 to 1965.

Spearman rank correlation coefficients for EUROSTAT-GERMANY-tables

	GERMANY-59	GERMANY-65	GERMANY-70b	GERMANY-75
GERMANY-59	-	0.154334	0.179422	0.169556
GERMANY-65	0.154334	-	0.812967	0.871459
GERMANY-70b	0.179422	0.812967	-	0.884426
GERMANY-75	0.169556	0.871459	0.884426	-

Table 3.11

A visual impression of the development of the rank of a sector $i \in \{1,\dots,44\}$ in the optimum linear orderings over the years can be obtained from the so-called migration diagram presented in Figure 3.12. E.g., sector 27 (Building and construction) fell from position 1 in 1959 to position 38 in 1965 and remained relatively stable since then (positions 39 in 1970 and 37 in 1975). Figure 3.12 also shows the considerable changes from 1959 to 1965. But again we have to refer to the discussion of section 5 concerning the interpretation of these data.

Migration diagram for EUROSTAT-GERMANY-tables

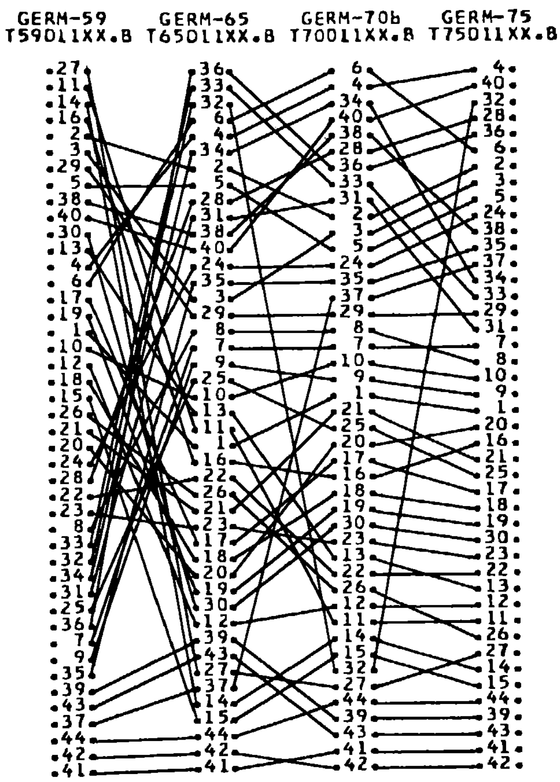


Figure 3.12

4. Triangulation of two collections of I-O-tables for the Federal Republic of Germany

There are two major collections of Input-Output-tables for the West German economy. One is compiled by the Deutsches Institut für Wirtschaftsforschung (DIW) in Berlin and the other by the Statistisches Bundesamt in Wiesbaden.

From the DIW-matrices, we have chosen the 56-sector tables for the years 1954 - 1972. The matrices can be found in "Beiträge zur Strukturforschung", Heft 27/1973, Heft 38/1975 and "Vierteljahreshefte zur Wirtschaftsforschung", Heft 1/1974, but can also be obtained from the DIW on tape. Table 4.1 lists the 56 sector names along with their standard numbering.

Sector numbering for DIW-I-O-tables

1. Land- und Forstwirtschaft, Fischerei
2. Elektrizitätswirtschaft
3. Gas- und Wasserwirtschaft
4. Kohlenbergbau
5. Eisenerzbergbau
6. Kali- und Steinsalzbergbau
7. Erdöl- und Erdgasgewinnung
8. Restlicher Bergbau
9. Industrie der Steine und Erden
10. Eisenschaffende Industrie
11. Eisen-,Stahl- und Tempergießerei
12. Ziehereien und Kaltwalzwerke
13. NE-Metallindustrie
14. Chemische Industrie
15. Mineralölverarbeitung
16. Gummi- und asbestverarb. Industrie
17. Sägewerke und holzbearb. Industrie
18. Zellstoff- und papiererz. Industrie
19. Stahlbau
20. Maschinenbau
21. Straßenfahrzeugbau
22. Luftfahrzeugbau
23. Schiffbau
24. Elektronische Industrie
25. Feinmech. und optische Industrie
26. Stahlverformung
27. EBM-Industrie
28. Feinkeramische Industrie
29. Glasindustrie
30. Holzverarbeitende Industrie
31. Musikinstr. - und Spielw.-Industrie
32. Papier- und pappeverarb.-Industrie
33. Druckerei- und Vervielf.-Industrie
34. Kunststoffverarbeitende Industrie
35. Lederindustrie
36. Textilindustrie
37. Bekleidungsindustrie
38. Mühlenindustrie
39. Ölmühlen- und Margarine-Industrie
40. Zuckerindustrie
41. Brauerei und Malzerei
42. Tabakverarbeitende Industrie
43. Sonst. Nahr- und Genußm.-Industrie
44. Verarb. Handwerk u. sonst. prod. Gew.
45. Baugewerbe
46. Großhandel
47. Einzelhandel
48. Eisenbahnen
49. Schiffahrt, Wasserstraßen und Häfen
50. Übriger Verkehr
51. Nachrichtenübermittlung (Bundespost)
52. Kreditinstitute und Versicherungsgew.
53. Wohnungsvermietung
54. Sonstige Dienstleistungen
55. Staat (einschl. Sozialversicherung)
56. Private Haushalte (Häusliche Dienste)

Table 4.1

Optimum rankings and the values for the theoretical degrees of linearity $\bar{\lambda}$, the fractional degrees of linearity λ' and the ("optimum") degrees of linearity λ can be found in Table 4.2 and Table 4.3. The code name DIW56XYZ is to be interpreted as follows: The first part DIW56 states that we have a 56-sector-I-O-table compiled by DIW, X indicates whether the matrix entries reflect the values of the deliveries among the sectors in current prices (code N for "nominal") or in the prices of the year 1962 (Code R for "real"), XY denotes the year.

Rankings and degrees of linearity for DIW-I-O-tables at current prices

	Rankings					
Number of sector	DIW56N54	DIW56N58	DIW56N62	DIW56N66	DIW56N67	DIW56N72
1	30	31	31	31	32	33
2	13	8	12	15	13	18
3	12	6	11	14	12	17
4	2	2	10	13	11	16
5	17	21	20	16	24	26
6	22	15	16	22	10	20
7	4	5	4	10	7	1
8	1	1	9	12	8	14
9	16	17	15	18	23	25
10	19	26	26	19	26	27
11	34	28	41	41	28	42
12	20	27	27	20	27	28
13	18	18	17	17	15	19
14	24	20	19	24	19	22
15	5	7	8	11	9	2
16	27	24	24	27	31	31
17	35	32	32	32	33	34
18	7	9	13	1	14	4
19	42	46	46	46	41	43
20	43	47	47	47	48	47
21	50	49	49	49	49	50
22	51	50	51	51	51	51
23	53	53	52	52	52	52
24	41	42	45	45	46	45
25	48	48	48	48	47	46
26	21	29	28	21	29	29
27	38	30	29	28	30	30
28	28	25	25	29	25	35
29	29	33	30	30	36	36
30	34	34	33	33	37	41
31	47	43	43	42	53	53
32	8	10	14	2	16	5
33	9	11	22	3	20	7
34	26	23	21	25	22	24
35	36	40	39	39	35	40
36	25	22	23	26	21	23
37	45	44	40	40	45	48
38	32	36	34	34	39	38
39	23	19	18	23	18	21
40	31	35	35	35	38	37
41	44	37	36	37	43	8
42	40	38	37	36	34	6
43	33	39	38	38	40	39
44	37	41	42	43	42	44
45	46	51	53	53	50	49
46	15	16	7	9	6	19
47	52	52	50	50	17	32
48	3	3	1	4	3	11
49	11	13	5	7	4	12
50	14	14	6	8	5	13
51	6	4	2	5	1	3
52	10	12	3	6	2	10
53	55	55	55	55	55	55
54	49	45	44	44	44	9
55	54	54	54	54	54	54
56	56	56	56	56	56	56
	Degrees of linearity					
Theoretical degree of linearity	84.706	84.563	84.730	84.960	84.856	83.133
Fractional degree of linearity	81.299	81.605	81.435	81.791	81.063	79.812
Degree of linearity	81.299	81.605	81.435	81.791	81.063	79.812

Table 4.2

Rankings and degrees of linearity for DIW-I-O-tables at prices of 1962

Number of sector	Rankings					
	DIW56R54	DIW56R58	DIW56R62	DIW56R66	DIW56R67	DIW56R72
1	32	31	31	31	31	28
2	15	13	12	15	13	12
3	14	12	11	14	12	11
4	2	2	10	13	11	10
5	17	21	20	17	24	19
6	22	15	16	22	10	16
7	4	4	4	1	7	1
8	1	1	9	12	8	8
9	16	17	15	16	23	47
10	19	26	26	18	25	23
11	28	28	41	41	27	38
12	20	27	27	20	26	24
13	18	18	17	19	15	14
14	24	20	19	24	19	18
15	5	5	8	2	9	2
16	27	24	24	27	30	27
17	33	32	32	32	32	29
18	7	6	13	3	14	13
19	44	46	46	46	41	39
20	45	47	47	47	48	45
21	50	49	49	49	49	49
22	51	50	51	51	51	50
23	53	53	52	52	52	52
24	43	42	45	45	46	43
25	48	48	48	48	47	44
26	21	29	28	21	28	25
27	29	30	29	28	29	26
28	30	25	25	29	34	30
29	31	33	30	30	36	31
30	34	34	33	33	37	36
31	47	43	43	42	53	53
32	8	7	14	4	16	15
33	9	8	22	5	20	21
34	26	23	21	25	22	22
35	40	40	39	39	35	35
36	25	22	23	26	21	20
37	41	44	40	40	45	46
38	36	36	34	34	39	33
39	23	19	18	23	18	17
40	35	35	35	35	38	32
41	37	37	36	36	43	41
42	38	38	37	37	33	37
43	39	39	38	38	40	34
44	42	41	42	43	42	40
45	46	51	53	53	50	48
46	13	16	7	11	6	9
47	52	52	50	50	17	51
48	3	3	1	6	2	5
49	11	11	5	9	4	6
50	12	14	6	10	5	7
51	6	9	2	7	3	3
52	10	10	3	8	1	4
53	55	55	55	55	54	55
54	49	45	44	44	44	42
55	54	54	54	54	56	54
56	56	56	56	56	55	56
	Degrees of linearity					
Theorotical degree of linearity	84.500	84.265	84.730	85.028	85.579	83.390
Fractional degree of linearity	80.785	81.061	81.435	81.941	81.801	79.966
Degree of linearity	80.785	81.061	81.435	81.941	81.801	79.966

Table 4.3

Table 4.4 shows the mutual Spearman rank correlation coefficients for all 56-sector tables of the DIW. Thus, it also shows ranking comparisons between the "nominal" and "real" versions of the two tables compiled for one year.

Spearman rank correlation coefficients for DIW-I-O-tables

	DIW56N54	DIW56N58	DIW56N62	DIW56N66	DIW56N67	DIW56N72	DIW56R54	DIW56R58	DIW56R62	DIW56R66	DIW56R67	DIW56R72
DIW56N54	–	0.973206	0.952085	0.963295	0.899658	0.799111	0.987560	0.975871	0.952085	0.968968	0.898565	0.929118
DIW56N58	0.973206	–	0.970540	0.955707	0.920232	0.828230	0.981135	0.995694	0.970540	0.960766	0.916815	0.932468
DIW56N62	0.952085	0.970540	–	0.963021	0.938688	0.834723	0.960014	0.967327	1.000000	0.965550	0.935065	0.947505
DIW56N66	0.963295	0.955707	0.963021	–	0.902119	0.848189	0.972659	0.963090	0.963021	0.992823	0.902324	0.921941
DIW56N67	0.899658	0.920232	0.938688	0.902119	–	0.832468	0.909501	0.917157	0.938688	0.900137	0.996514	0.920301
DIW56N72	0.799111	0.828230	0.834723	0.848189	0.832468	–	0.823103	0.835748	0.834723	0.856254	0.834997	0.822556
DIW56R54	0.987560	0.981135	0.960014	0.972659	0.909501	0.823103	–	0.985304	0.960014	0.978127	0.910800	0.929460
DIW56R58	0.975871	0.995694	0.967327	0.963090	0.917157	0.835748	0.985304	–	0.967327	0.968900	0.914559	0.930349
DIW56R62	0.952085	0.970540	1.000000	0.963021	0.938688	0.834723	0.960014	0.967327	–	0.965550	0.935065	0.947505
DIW56R66	0.968968	0.960766	0.965550	0.992823	0.900137	0.856254	0.978127	0.968900	0.965550	–	0.900342	0.926384
DIW56R67	0.898565	0.916815	0.935065	0.902324	0.996514	0.834997	0.910800	0.914559	0.935065	0.900342	–	0.920779
DIW56R72	0.929118	0.932468	0.947505	0.921941	0.920301	0.822556	0.929460	0.930349	0.947505	0.926384	0.920779	–

Table 4.4

Finally, as for the EUROSTAT-matrices of the Federal Republic of Germany, we have displayed migration diagrams for the "nominal" matrices in Figure 4.5 and the "real" matrices in Figure 4.6.

Migration diagram for DIW-I-O-tables at current prices

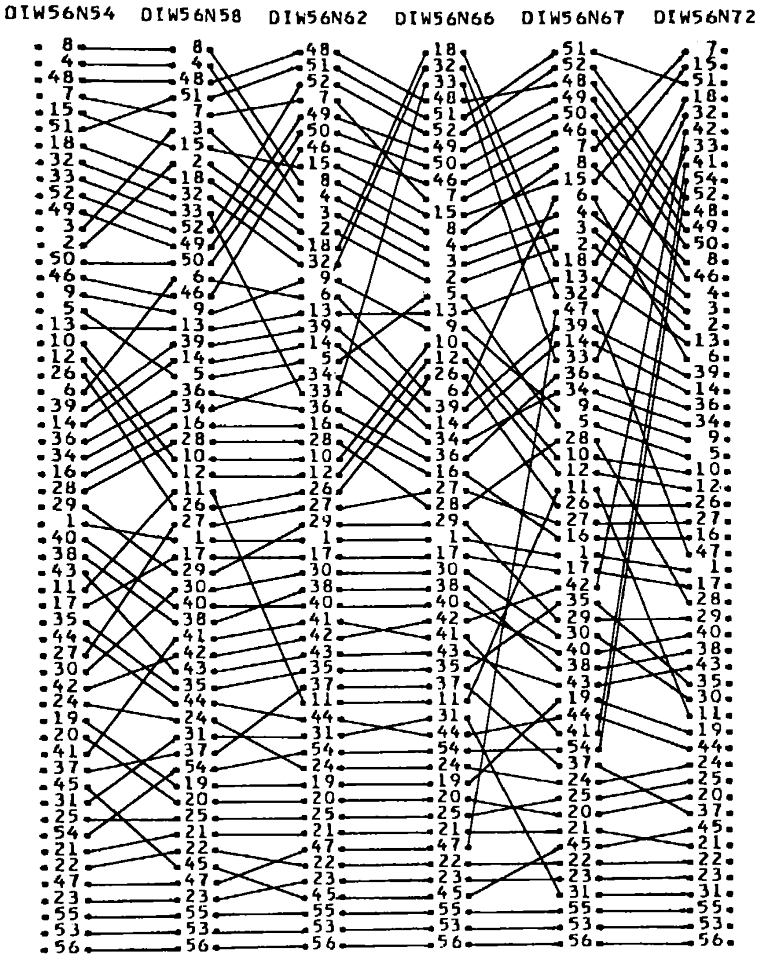


Figure 4.5

Migration diagram for DIW-I-O-tables at prices of 1962

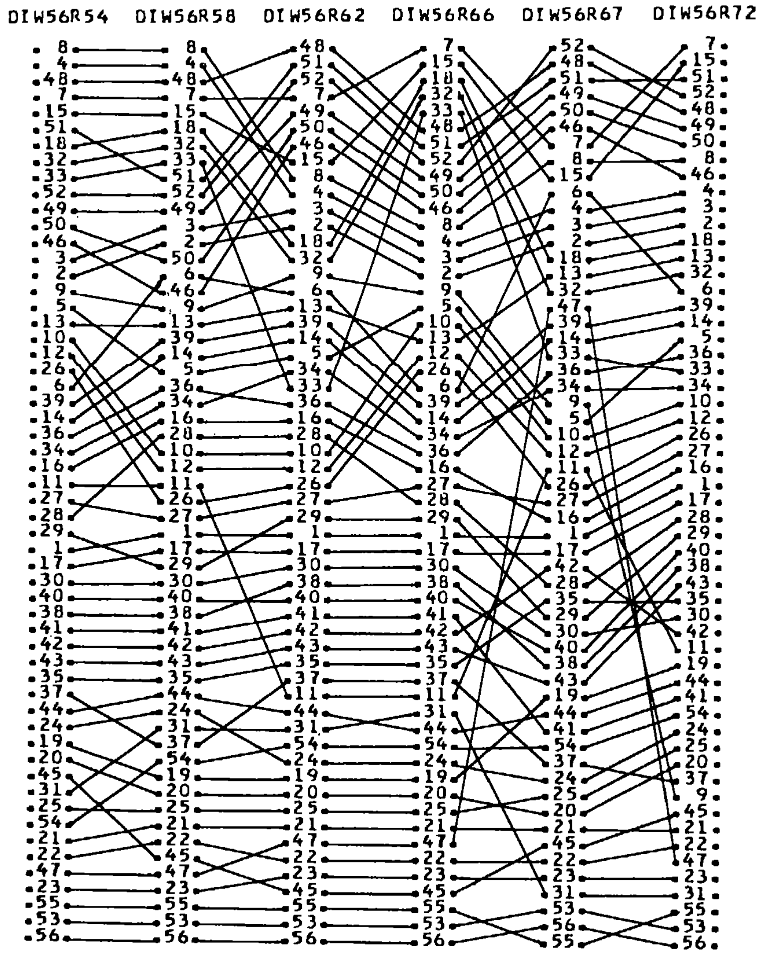


Figure 4.6

From the collection of Input-Output-Tables compiled by the German Statistisches Bundesamt in Wiesbaden we have chosen the three existing 60-sector tables of intermediate consumption for the years 1970, 1974 and 1975 called "Input-Output-Tabelle zu Ab-Werk-Preisen - Inländische Produktion". These matrices can be obtained from the "Statistisches Bundesamt, Postfach 55 28, D-6200 Wiesbaden, West Germany" on tape. As for the previous two cases, Table 4.7 lists the sector definitions in the canonical order used by the Statistisches Bundesamt.

Sector numbering for the I-O-tables of the Statistisches Bundesamt der Bundesrepublik Deutschland (STABU)

1. Gew. v. Erzeugnissen d. Landwirtschaft
2. Gew. v. Erzeugnissen d. Forstwirtschaft u. d. gewerblichen Jagd
3. Gew. v. Erzeugnissen d. Fischerei u. d. Fischzucht

4. Gew. u. Vertlg. v. Elektrischem Strom
5. Vertlg. v. Gas, Gew. u. Vertlg. v. Dampf
6. Gew. u. Vertlg. v. Wasser a. Öffentlicher Versorgung
7. Gew. v. Kohle, h. v. Erzeugnissen d. Kohlenbergbaues
8. Gew. v. Bergbauerzeugnissen (oh. Kohle, Erdöl, Erdgas)
9. Gew. v. Erdöl, Erdgas, Bituminösen Gesteinen

10. H. v. Chemischen Erzeugnissen
11. H. v. Mineralölerzeugnissen
12. H. v. Kunststofferzeugnissen
13. H. v. Gummi- u. Asbestwaren
14. Gew. v. Baumaterial u. feuerf. Erden, h. v. Grobker. Erzeugn. usw.
15. H. v. Zement, Kalk, Gips u. Baustoffen daraus
16. H. v. Feinkeramischen Erzeugnissen
17. H. v. Glas, Glaswaren

18. H. v. Eisen, Stahl usw., Erzeugnissen d. Ziehereien, Kalt-Walzwerke u. Stahlverformung, d. Schlossereien u. S., a. n. g.
19. H. v. Ne-Metallen u. Ne-Metallhalbzeug
20. H. v. Gießereierzeugnissen

21. H. v. Stahl- u. Leichtmetallbauerzeugnissen (oh. Waggons)
22. H. v. Fahrzeugen (oh. Kraftwagen u. -Zubehör)
23. H. v. Maschinenbauerzeugn. (oh. Lokomotiv., Büromasch., ADV-Anl.)
24. H. v. Kraftwagen u. -Zubehör, Rep. v. Strassenfahrzeugen
25. H. v. Büromaschinen, ADV-Geräten u. -Einrichtungen

26. H. v. Elektrotechnischen Erzeugnissen (oh. ADV-Anlagen)
27. H. v. Feinmechanischen u. optischen Erzeugnissen sowie Uhren
28. H. v. Eisen-, Blech- u. Metallwaren (oh. Kraftwagenzubehör)
29. H. v. Musikinstrumenten, Sportgeräten, Spiel- u. Schmuckwaren

30. H. v. Schnittholz, Holzhalbfabrikaten, Holzwaren
31. H. v. Holzschliff, Zellstoff, Papier und Pappe
32. H. v. Papier- u. Pappewaren, Druckerei- u. V. Erzeugnissen
33. H. v. Leder u. Lederwaren
34. H. v. Textilien
35. H. v. Bekleidung u. Bettwaren, Polsterei- u. Dekorateurarbeiten

36. H. v. Nahrungs- u. Genußmitteln, a. n. g.
37. Bearb. v. Milch, h. v. Milchpräparaten, Butter, Käse
38. H. v. Fleisch u. Fleischerzeugn., Gew. v. roh. Häuten u. Fellen
39. H. v. Getränken
40. H. v. Tabakwaren

41. Erstellung v. Bauten

42. Leistg. d. Großhandels u. d. Einfuhr- u. Vorratsstellen
43. Leistg. d. Handelsvermittlung
44. Leistg. d. Einzelhandels
45. Leistg. d. Eisenbahnen (Schienengebundener Verkehr)
46. Leistg. d. Schiffahrt, Wasserstraßen, Häfen
47. Sonst. Verkehrsleistungen
48. Leistg. d. Nachrichtenübermittlung

49. Bankdienstleistungen gegen tatsächliche Entgelte
50. Bankdienstleistungen gegen unterstellte Entgelte
51. Dienstleistg. d. Versicherungen (oh. Vermittlung, Sozialvers.)
52. Marktbestimmte Gaststätten- u. Beherbergungsleistungen
53. Verlags-, Literatur- u. Presseleistungen
54. Marktbestimmte Gesundheits- u. Veterinärleistungen
55. Vermietung von Grundstücken u. Räumen
56. Marktbest. Forschungs- u. Unterrichtsleistungen, Leistg. d. f. Unternehmen tätigen Organisationen oh. Erwerbscharakter
57. Übr. marktbest. Dienstleistungen, Rep. v. Gebrauchsgütern

58. Leistg. d. priv. Organisationen oh. Erwerbscharakter
59. Häusliche Dienste
60. Leistg. d. Gebietskörperschaften u. d. Sozialversicherung

Table 4.7

As before, Tables 4.8, 4.9 and Figure 4.10 display optimum rankings and degrees of linearity, Spearman rank correlation coefficients and a migration diagram, respectively.

Rankings and degrees of linearity for STABU-I-O-tables

	Rankings		
Number of sector	STABU70	STABU74	STABU75
1	44	45	45
2	10	9	11
3	36	37	30
4	13	12	14
5	12	11	13
6	2	13	2
7	11	10	12
8	20	21	20
9	1	1	1
10	21	22	22
11	3	2	3
12	24	24	26
13	31	29	32
14	27	27	27
15	56	41	40
16	26	25	21
17	32	32	33
18	28	28	24
19	25	26	23
20	29	30	28
21	53	42	39
22	54	55	55
23	41	39	36
24	42	40	38
25	34	35	37
26	33	34	35
27	40	33	34
28	30	31	29
29	39	43	41
30	37	38	31
31	14	14	15
32	15	15	16
33	38	54	54
34	22	23	25
35	47	49	49
36	49	50	50
37	48	47	47
38	46	48	48
39	50	51	51
40	45	46	46
41	57	44	42
42	19	19	19
43	18	20	18
44	35	36	44
45	8	7	9
46	55	59	59
47	9	8	10
48	4	3	7
49	17	17	6
50	51	52	52
51	7	6	8
52	52	53	53
53	16	16	17
54	43	57	57
55	23	18	43
56	5	4	4
57	6	5	5
58	58	56	56
59	60	60	60
60	59	58	58
	Degrees of linearity		
Theoretical degree of linearity	88.259	88.429	88.417
Fractional degree of linearity	83.187	83.965	83.799
Degree of linearity	83.186	83.965	83.799

Table 4.8

Spearman rank correlation coefficients for STABU-I-O-tables

	STABU70	STABU74	STABU75
STABU70	-	0.965157	0.943929
STABU74	0.965157	-	0.967046
STABU75	0.943929	0.967046	-

Table 4.9

Migration diagram for STABU-I-O-tables

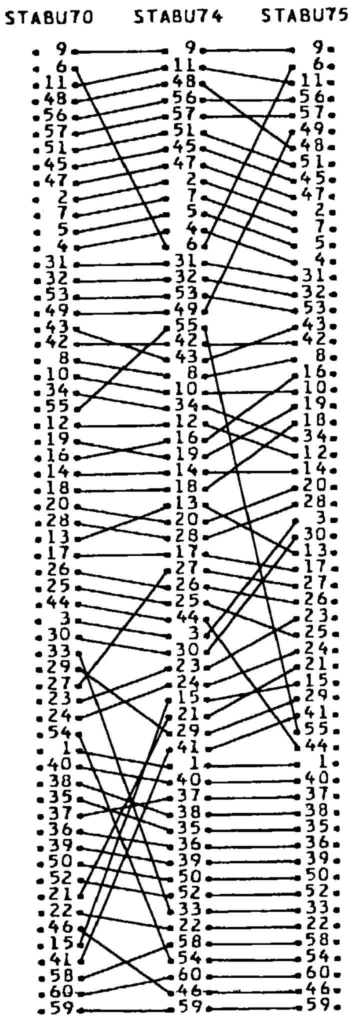


Figure 4.10

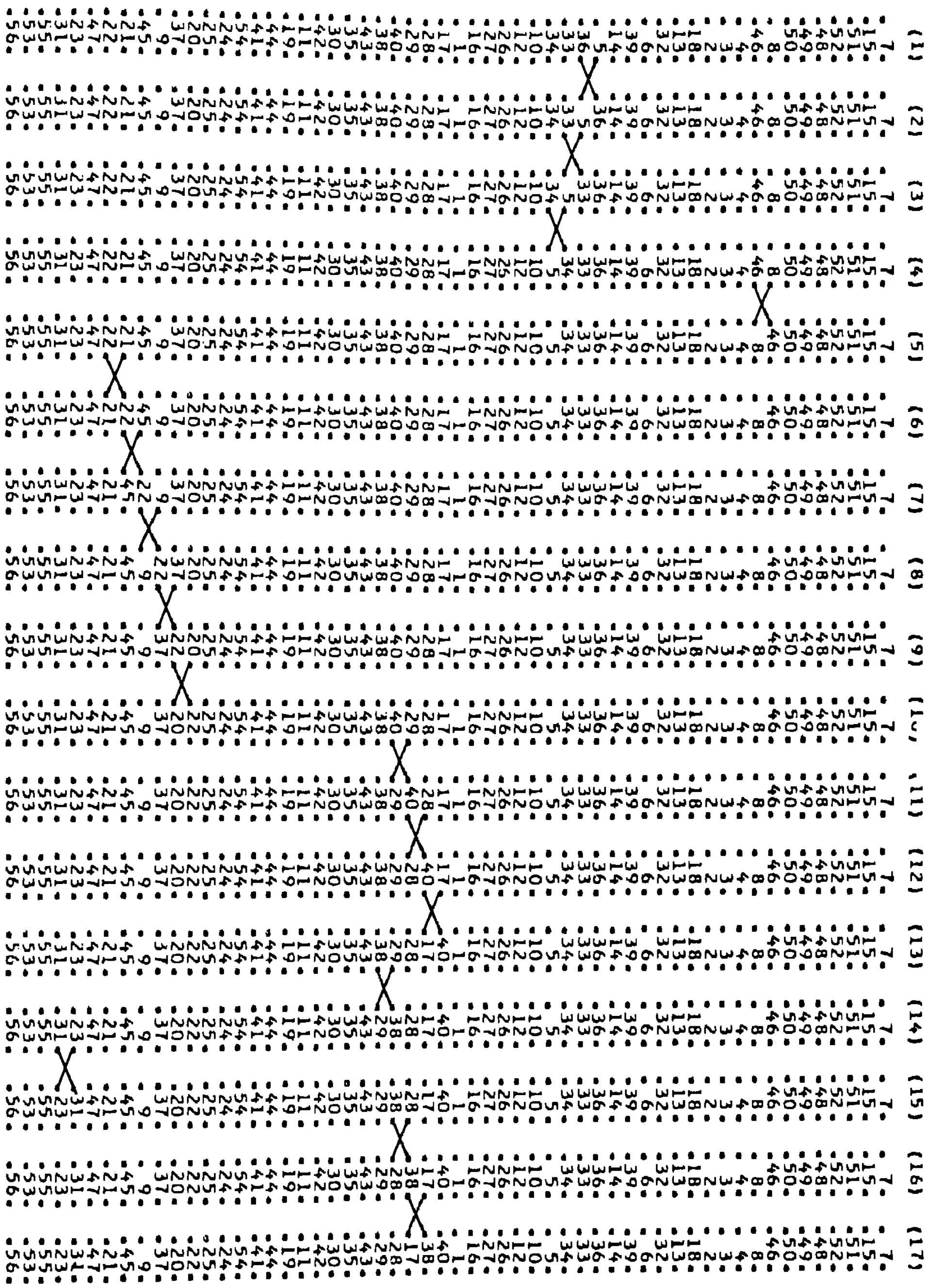
As in section 3 we have resisted the attempt of trying to give economic interpretations of the optimum solutions presented above. We shall discuss the reason for this in the following section.

5. Discussion

In sections 3 and 4 we have presented optimum solutions of the triangulation problem for three major collections of real world Input-Output-tables. To our knowledge, all previous analyses for matrices of the sizes considered here were based on suboptimal solutions. This is true even for very recent publications as e.g. WESSELS (1981) in the case of the 56-sector DIW-matrices whose optimum solutions are presented in section 4. We can conclude from the optimum solutions we found, that the usual methods of analysis of optimum linear orderings have to be regarded with much more care than it is usually done.

A trivial observation is that, due to the fact that real-world input-output-tables often contain sectors whose rows and columns consist of zero-entries only, the Spearman rank correlation coefficient can be an arbitrarily poor measure of "similarity" between two (optimum) linear orderings of the sectors of an input-output-table. One example is the I-O-table GERMANY-59 of EUROSTAT whose last 12 rows and last 7 columns contain zero entries only. Of course, each of the sectors numbered from 38 to 44 can be moved to any arbitrary position to obtain an alternative optimum linear ordering from a given one. We have utilized this fact to compute an optimum linear ordering for GERMANY-59 such that "free" sectors are positioned in "reasonable" locations.

In the case of the I-O-table DIW56R72 sector 56 appears in the last position of the optimum linear ordering shown in Table 4.3. However, the linear ordering obtained from this one by moving sector 56 into position one and all other sectors down by one is also an optimum linear ordering. The Spearman rank correlation coefficient of these two optimum linear orderings is 0.369993 which would usually be interpreted as reflecting very little correlation. But even apart from such extreme cases, there is usually quite a number of alternative optimum linear orderings making the usual way of analysing such orderings questionable. Figure 5.1 shows a series of 17 alternative optimum linear orderings for the I-O-table DIW56R72 where the ordering (i+1) was obtained form the ordering i (i=1,2,...,16) by the exchange of two "adjacent" sectors.



In Figure 5.2, we have compared our optimum linear ordering for the I-O-table DIW56N72 shown in Figure 4.2 to the (nonoptimal) one presented in WESSELS (1981) by a migration diagram. The danger of misinterpretations of the "hierarchical structure" is obvious. The Spearman rank correlation coefficient for these two linear orderings is 0.812303.

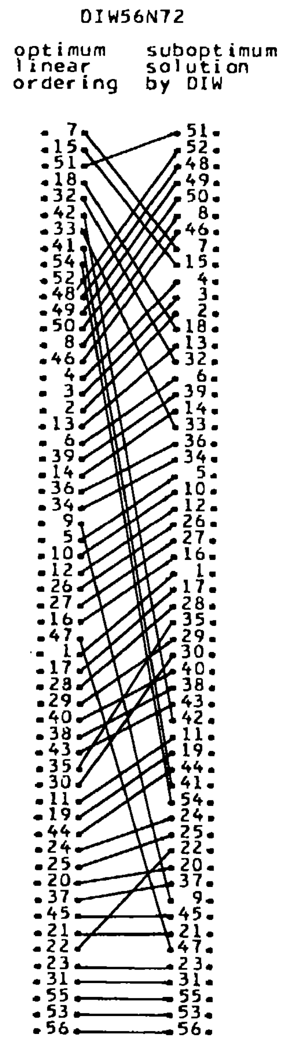


Figure 5.2

The questionability of migration diagrams is illustrated even more impressively in Figure 5.3. Here we have displayed a migration diagram for the two different EUROSTAT-I-O-tables T70D11XX.B (GERMANY-70a and GERMANY-70b, see section 3). Recall that the latter is a revision of the former, but the changes are considered so small that the tables EUR6-70 and EUR9-70 based on the old version were not recomputed. Applying pairwise exchange (as in Figure 5.1) we have tried to obtain optimum linear orderings of both tables "as similar as possible" in terms of the value of the Spearman rank correlation coefficient. The maximum Spearman rank correlation coefficient we could obtain using this method was 0.881748. These two optimum linear orderings are displayed in the migration diagram shown in Figure 5.3. The result seems to contradict the "insignificance of the changes".

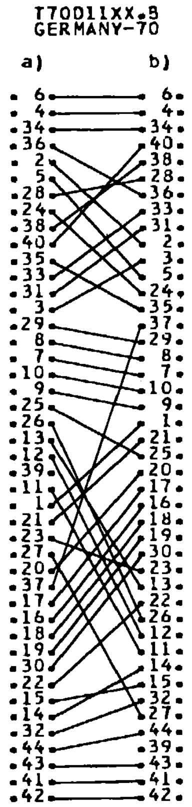


Figure 5.3

It seems that it is generally accepted that the degree of linearity can be used for international comparisons, for measuring the technological state of a country, and for evaluating the technological development of one country over time, see e. g. HELMSTÄDTER (1964) or WESSELS (1981) for a broad discussion of this issue. In particular the following statements are often made.

1) Highly developed countries have a lower degree of linearity than less developed ones.

2) For highly developed countries the degree of linearity is almost constant or slightly decreasing over time.

3) Large countries have a lower degree of linearity than small ones.

4) The degree of linearity is a structural constant that can be used to compare the technological standards of different countries.

5) For highly developed countries the ranking of the sectors in different countries is similar.

We think that the results of this paper clearly disprove 5), as can be seen from the various tables presented. But moreover, in our opinion it does not seem that the degree of linearity can carry the interpretational load put on it in statements 1),...,4). To compare these statements with reality we have compiled Table 5.4 which shows the development of the degrees of linearity for Germany over time using the three different collections of I-O-tables for Germany. Table 5.5 gives the degrees of linearity in increasing order for the members of the EC computed from the EUROSTAT-matrices of 1970.

STABU	λ
1970	83.186
1974	83.965
1975	83.799

EUROSTAT	λ
GERM-59	88.148*)
GERM-65	83.007
GERM-70b	82.199
GERM-75	82.773

DIWN (nominal)	λ
1954	81.299
1958	81.605
1962	81.435
1966	81.791
1967	81.063
1972	79.812

DIWR (real)	λ
1954	80.785
1958	81.061
1962	81.435
1966	81.941
1967	81.801
1972	79.766

Table 5.4

EUROSTAT 1970

1.	UK-70	80.636
2.	GERM-70b	82.199
3.	EUR9-70	82.668
4.	NETHER-70	82.810
5.	GERM-70a	83.181
6.	EUR6-70	83.401
7.	BELGIUM-70	84.132
8.	FRANCE-70	85.115
9.	ITALY-70	85.613
10.	DENMARK-70	85.893
11.	IRELAND-69	87.862
12.	LUXEMB-70	89.808

Table 5.5

*) We have the impression that something is wrong with the I-O-table GERM-59. This table somehow does not fit into the series of EUROSTAT-matrices of Germany and the other countries. This can be seen from Table 5.4, as well as from Tables 3.3, 3.11 and Figure 3.12. Our only explanation is that it was probably computed from an insufficient data basis.

6. Conclusions

There is a vast amount of papers in which various real-world input-output matrices are nonoptimally triangulated by heuristic methods. These linear orderings are often used to make comparisons between different countries, e.g. by means of the Spearman rank correlation coefficient, they are used to determine a hierarchy of the sectors of an economy or to describe the structural changes of an economy over time.

We found that most of these interpretations are rather questionable, due to the fact that triangulation of input-output matrices is a highly sensitive instrument of economic analysis. We have shown in this paper that small revisions of I-O-tables may result in drastic changes of the hierarchy of sectors, even worse, that there are often different optimum rankings for one I-O-table which seem to be quite nonsimilar. Furthermore, using various types of tables compiled for one and the same year for one and the same country the optimum triangulations are rather uncomparable and show different hierarchical structures of the sectors; moreover, even the "real" and "nominal" versions of one I-O-table lead to quite different rankings. Since all our observations are based on optimum triangulations, they apply even more drastical to the (usual) case where nonoptimal rankings are compared.

However, we do not believe that our computational results imply that triangulation of input-output-matrices is a useless tool of economic analysis. They only show that the interpretations have to be done with much more care making intensive use of various kinds of sensitivity analysis and taking into account that quite a number of entries of I-O-tables are nothing but expert guesses.

It would probably be a good idea to give up to aim at a total linear ordering. We think analysts should confine themselves with partial orderings of the sectors which are - in some sense to be made precise - stable, e. g. where a reversion of adjacent sectors would result in a considerable change of the degree of linearity.

7. Acknowledgement

Thanks are due to M. PECCI-BORIANI of EUROSTAT for making important informations about the EUROSTAT-matrices for the year 1970 available to us.

8. References

[1] Garey, M.R. ; Johnson, D.S. [1979] : Computers and intractibility: a guide to the theory of NP-completeness. Freeman, San Francisco, 1979.

[2] Grötschel, M. ; Jünger, M. ; Reinelt, G. [1982a]: On the acyclic subgraph polytope. WP 82215-OR, Institut für Operations Research, Universität Bonn, 1982.

[3] Grötschel, M. ; Jünger, M. ; Reinelt, G. [1982b]: Facets of the linear ordering polytope. WP 82217-OR, Institut für Operations Research, Universität Bonn, 1982.

[4] Grötschel, M. ; Jünger, M. ; Reinelt, G. [1982c]: A cutting plane algorithm for the linear ordering problem. WP 82219-OR, Institut für Operations Research, Universität Bonn, 1982.

[5] Helmstädter, E. [1964]: Die Dreiecksform der Input-Output-Matrix und ihre möglichen Wandlungen im Wachstumsprozeß. In F. Neumark (Hrsg.): Strukturwandlungen einer wachsenden Wirtschaft. Schriften des Vereins für Socialpolitik, NF Band 30/II (1964).

[6] Kaas, R. [1981]: A branch and bound algorithm for the acyclic subgraph problem. European Journal of Operational Research 8 (1981), 355-362.

[7] Wessels, H. [1981]: Triangulation und Blocktriangulation von Input-Output-Tabellen. Deutsches Institut für Wirtschaftsforschung: Beiträge zur Strukturforschung, Heft 63, Berlin 1981.

Prof. Dr. Martin Grötschel
Dr. Michael Jünger
Dipl.-Informatiker Gerhard Reinelt
Institut für Mathematik
Lehrstuhl für Angewandte Mathematik II
Universität Augsburg
Memminger Str. 6
8900 Augsburg
West Germany