Implementation of Cutting Plane Separators for Mixed Integer Programs

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Chapter 1

Introduction

A *mixed integer program* (MIP) is an optimization problem where the objective is to minimize or maximize a function of many variables subject to inequality and equality constraints and integrality restrictions on some or all of the variables. The integrality restrictions are the crucial point. On the one hand, they give us the power to formulate many real-world problems as MIPs, but on the other hand, they make MIPs hard to solve; MIP-solving is NP-hard.

One technique to solve MIPs is the general cutting plane method. The idea is to drop the integrality restrictions and to solve the resulting relaxation of the MIP by efficient methods from linear programming. The dropped integrality restrictions are then reintroduced by cutting planes. Algorithms that generate cutting planes are called *cutting plane separators*. The actual power of this technique arises when it is combined with a linear programming based branch-and-bound algorithm, which is another technique to solve MIPs. Most state-of-the-art MIP solvers are based on this combination, also known as linear programming based branch-and-cut algorithm.

SCIP [1], developed at the Zuse Institute Berlin by T. Achterberg, is such a framework. It integrates constraint and mixed integer programming. The aim of this thesis is to find an *efficient implementation* of different cutting plane separators which embedded in SCIP help to improve the overall performance of the solver.

In [15], it has been shown that cutting plane separators are one of the most important features of MIP solvers. But of course, their effect depends on the way they are integrated into the MIP solver, i.e., on the way in which they interact with other features of the solver. In this thesis, we concentrate on the performance of the cutting plane separators when they are used basically isolated. We suppose that this is a good starting point for developing cutting plane separators which are effective within MIP solvers. At the end of this thesis, we remove the isolated application of our developed cutting plane separators and present computational results concerning their impact on the overall performance of SCIP.

We deal with six different cutting plane separators. For three of them, namely the cutting plane separators for the class of GMI inequalities, the node packing problem, and the class of implied bound inequalities, we give only a brief introduction to the classes of valid inequalities which are separated and mention some of the algorithmic aspects.

The main part of this thesis is dedicated to the other three, namely the cutting plane separators for

- the class of c-MIR inequalities,
- the 0-1 knapsack problem, and
- the 0-1 single node flow problem.

Different classes of strong valid inequalities for the 0-1 knapsack polytope have been studied extensively in the literature and the corresponding cutting plane separators have been used successively in various MIP solvers. For our implementation, the following question arises: Which of these classes of valid inequalities do we want to separate in our cutting plane separator, i.e., which of them lead to the best performance in practice? To answer that question, we perform a computational study which includes three classes of valid inequalities. Two of them involve sequential up- and down-lifting and the other one involves superadditive up-lifting. To our knowledge, no paper has been published presenting computational results for separating the later class. Thus, our computational study also covers this subject.

For the 0-1 single node flow set, a variety of classes of valid inequalities have been derived in the literature, too. Thus, the same question arises here. It is well known that different classes of valid inequalities for the 0-1 single node flow problem can also be obtained as particular c-MIR inequalities for specific mixed knapsack relaxations of the 0-1 single node flow set (c-MIR flow cover inequalities and c-MIR flow pack inequalities). The separation heuristic for the class of c-MIR inequalities of Marchand and Wolsey [42], which we use in our cutting plane separator for the class of c-MIR inequalities, is designed in such a way that it is able to generate these inequalities. We decided to separate the class of c-MIR flow cover inequalities and c-MIR flow pack inequalities directly in our cutting plane separator for the 0-1 single node flow problem. In this thesis, we will investigate whether using this cutting plane separator in addition to the one for the class of c-MIR inequalities helps to improve the overall performance of SCIP.

For each of the three cutting plane separators, we perform extensive computational studies where we investigate the effect of different algorithmic and implementation choices on the performance of the separation algorithms. The study covers different heuristical choices used by other researchers as well as some modifications of these heuristics. We will use the results to develop our final cutting plane separators. One crucial aspect of implementing efficient cutting plane separators is to reduce the time spent in the separation routines to an acceptable level. In our computational study we will also go into this subject.

Outline of the Thesis

In Chapter 2, we give a brief introduction to the general cutting plane method and provide the most important concepts and definitions from mixed integer programming used in this thesis. In addition, we show how valid inequalities can be strengthened by a procedure called lifting. This technique will be used in our cutting plane separator for the 0-1 knapsack problem. Furthermore, we state general information concerning the computational studies which we performed for developing the individual cutting plane separators.

Chapter 3, 4, and 5 form the main part of this thesis. Here, we address the development of the cutting plane separators for the class of c-MIR inequalities, for the 0-1 knapsack problem, and for the 0-1 single node flow problem. They are all organized in a similar way. We always start with giving a literature review, then we introduce the classes of valid inequalities which we want to separate. This is followed by a discussion of different algorithmic aspects of the corresponding separation algorithms. Here, we will also state the heuristics used by other researchers. Finally, we present the results of our computational study concerning the effect of the discussed algorithmic and implementation choices. The results will then be used to develop our final cutting plane separators. At the end of each chapter, we state some concluding remarks.

In Chapter 6, we briefly introduce three further cutting plane separators, namely the cutting plane separators for the class of GMI inequalities, for the node packing problem, and for the class of implied bound inequalities.

In Chapter 7, we present computational results concerning the comparison of our cutting plane separators to the ones provided by other MIP solvers. Here, the cutting plane separators are still applied basically isolated. Finally, we remove the isolated application and evaluate the impact of the individual cutting plane separators on the overall performance of SCIP.

Chapter 2

Preliminaries

In this chapter, we provide the most important concepts and definitions from mixed integer programming used in this thesis. Furthermore, we state general information concerning the computational studies which we carried out for developing the individual cutting plane separators.

2.1 General Cutting Plane Method

In this section, we introduce the general cutting plane method and state basic concepts and definitions needed in this connection. The description of the cutting plane method is taken from [27] and the presentation of the basic concepts and definitions follows [46, 47].

We start with the formal definition of a MIP and some of its special cases.

Definition 2.1. Let $n \in \mathbb{Z}_+ \backslash \{0\}$, $p \in \mathbb{Z}_+$ with $p \leq n$, $m \in \mathbb{Z}_+ \backslash \{0\}$, $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, and $b \in \mathbb{R}^m$. Optimization problems of the form

$$z_{\text{MIP}} = \min\{c^T x : Ax \leq b, x \in \mathbb{Z}^p \times \mathbb{R}^{n-p}\},$$

$$\min\{c^T x : Ax \leq b, x \in \{0,1\}^p \times \mathbb{R}^{n-p}\},$$

$$\min\{c^T x : Ax \leq b, x \in \mathbb{Z}^n\},$$

and

$$\min\{c^T x : Ax \leq b, x \in \{0,1\}^n\}$$

are called *mixed integer program* (MIP), *mixed 0-1 integer program* (BMIP), *integer program* (IP), and *0-1 integer program* (BIP), respectively.

Note that in later chapters, we will denote the vector of the integer variables of a MIP by x and the vector of the real variables of a MIP by y. Here, in order to shorten the presentation we do not use this distinction for the notation.

To solve a MIP by the general cutting plane method we first drop the integrality restriction and obtain the so-called *linear programming relaxation* (LP relaxation) of the MIP

$$z_{\text{LP}} = \min\{c^T x : Ax \leq b, x \in \mathbb{R}^n\}.$$

The set $P = \{x \in \mathbb{R}^n : Ax \leq b\}$ is called the *feasible region* of the LP. In terms of polyhedral theory, P is a polyhedron.

Definition 2.2 ([46]).

1. A *polyhedron* $P \subseteq \mathbb{R}^n$ is the set of vectors in $\mathbb{R}^n$ that satisfy a finite number of linear inequalities, i.e., $P = \{x \in \mathbb{R}^n : Ax \leq b\}$, where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$.

2. A *polyhedron* $P \subseteq \mathbb{R}^n$ is said to be *rational* if there exits $A^{'} \in \mathbb{Q}^{m \times n}$ and $b^{'} \in \mathbb{Q}^m$ such that $P = \{x \in \mathbb{R}^n : A^{'}x \leq b^{'}\}$.

3. A polyhedron $P \subseteq \mathbb{R}^n$ is *bounded* if there exists an $R \in \mathbb{R}_+$ such that $P \subseteq \{x \in \mathbb{R}^n : \|x\| \leq R\}$, where $\| \cdot \| := \sqrt{x^T x}$ is the Euclidean norm. A bounded polyhedron is called a *polytope*.

In analogy to P, we call the set $X = \{x \in \mathbb{Z}^p \times \mathbb{R}^{n-p} : Ax \leq b\}$ the *feasible region* of a MIP. It is a subset of the polyhedron P as $X = P \cap (\mathbb{Z}^p \times \mathbb{R}^{n-p})$. Because of the integrality restrictions in a MIP we cannot directly use the concept of a polyhedron to describe X; we use the concept of a *convex hull*.

Definition 2.3 ([46]). Let $X \subseteq \mathbb{R}^n$.

1. A vector $x \in \mathbb{R}^n$ is a *convex combination* of vectors in X if there exists a finite set of vectors $\{x^1, \ldots, x^k\}$ in X and a $\lambda \in \mathbb{R}^k_+$ with $\sum_{i=1}^k \lambda_i = 1$ and $x = \sum_{i=1}^k \lambda_i x^i$.

2. The *convex hull* of X, denoted by $\text{conv}(X)$, is the set of all vectors in $\mathbb{R}^n$ that are convex combinations of vectors in X. By convention, $\text{conv}(\emptyset) = \emptyset$.

3. X is called a *convex set* if $X = \text{conv}(X)$.

The convex hull of X is not always a polyhedron or even a polytope, but in special cases it is.

Theorem 2.4 ([47]). *If P is a rational polyhedron and $X = P \cap (\mathbb{Z}^p \times \mathbb{R}^{n-p}) \neq \emptyset$, then conv$(X)$ is a rational polyhedron whose extreme points are a subset of X and whose extreme rays are the extreme rays of P.*

Therefore, throughout this thesis we consider only rational polyhedra and assume that if P is stated as $P = \{x \in \mathbb{R}^n : Ax \leq b\}$, then $A \in \mathbb{Q}^{m \times n}$ and $b \in \mathbb{Q}^m$. Now, by Theorem 2.4, we can formulate a MIP with $X \neq \emptyset$ as the linear program

$$\min\{c^T x : x \in \text{conv}(X)\} \tag{2.1}$$

(see [47]). However, this observation, by itself, is not computationally helpful, since to use it, we would need to know a linear inequality description of $\text{conv}(X)$, and generally the number of inequalities needed to describe $\text{conv}(X)$ is extremely large. The idea of the general cutting plane method is to construct a polyhedron Q with $\text{conv}(X) \subseteq Q \subseteq P$ such that $\min\{c^T x : x \in Q\}$ gives an optimal solution to the MIP (see [47]). One part of this construction is to add valid inequalities for X to P.

Definition 2.5 ([46]). Let $X \subseteq \mathbb{R}^n$. The inequality $\alpha^T x \leq \alpha_0$, where $\alpha \in \mathbb{R}^n$ and $\alpha_0 \in \mathbb{R}$, is called a *valid inequality* for X if it is satisfied by all vectors in X.

Definition 2.6 ([46]). Let $X \subseteq \mathbb{R}^n_+$. Furthermore, let $\alpha^T x \leq \alpha_0$ and $\gamma^T x \leq \gamma_0$, where $\alpha, \gamma \in \mathbb{R}^n$ and $\alpha_0, \gamma_0 \in \mathbb{R}$, be valid inequalities for X.

1. If there exists $\lambda > 0$ such that $(\alpha, \alpha_0) = \lambda(\gamma, \gamma_0)$, we call $\alpha^T x \leq \alpha_0$ and $\gamma^T x \leq \gamma_0$ *equivalent*.

2. If there exists $\mu > 0$ such that $\gamma \geq \mu\alpha$ and $\gamma_0 \leq \mu\alpha_0$, then $\{x \in \mathbb{R}^n_+ : \gamma^T x \leq \gamma_0\} \subseteq \{x \in \mathbb{R}^n_+ : \alpha^T x \leq \alpha_0\}$. In this case, we say that $\gamma^T x \leq \gamma_0$ *is at least as strong as* $\alpha^T x \leq \alpha_0$.

We assume that the MIP has an optimal solution, i.e., that the MIP is neither infeasible nor unbounded. The general cutting plane method starts by solving the LP relaxation of the MIP with methods from linear programming (see e.g. [52]). Obviously, $z_{\mathrm{LP}} \leq z_{\mathrm{MIP}}$ holds. Let x^*_{LP} be an optimal solution of the LP relaxation of the MIP. If x^*_{LP} is in $\mathbb{Z}^p \times \mathbb{R}^{n-p}$, we are done; x^*_{LP} is an optimal solution of the MIP. Otherwise, it is a well-known result that there exists a valid inequality for X that is not satisfied by x^*_{LP}, or 'cuts off' x^*_{LP} from $\mathrm{conv}(X)$ (see [46]). Such an inequality is called *cutting plane* and the problem of determining whether x^*_{LP} is in $\mathrm{conv}(X)$ and if not of finding a cutting plane is called *separation problem*. If we found a cutting plane, we add it to P and obtain a polyhedron Q with $\mathrm{conv}(X) \subseteq Q \subset P$. This process is iterated until the solution is in $\mathbb{Z}^p \times \mathbb{R}^{n-p}$, i.e., an optimal solution of the MIP is found.

The separation problem can be formulated for different class of valid inequalities for X. Gomory has shown that a cutting plane method based on iteratively adding Gomory mixed integer cuts (see Chapter 6) solves a MIP under certain conditions in a finite number of steps (see [40, 46]).

MIPs can also be solved by the *linear programming based branch-and-bound algorithm*. The algorithm uses a divide-and-conquer strategy to explore the feasible region of the MIP and therefore guarantees to find an optimal solution, if one exists. But, instead of exploring the whole feasible region, it makes use of lower and upper bounds and therefore avoids touching certain (large) parts of the feasible region (see [27]). This algorithm can be used in combination with the general cutting plane method described above. For a MIP in minimization form the cutting plane method can help to improve the lower bound used in the branch-and-bound algorithm (also called *dual bound*). The resulting algorithm is called *linear programming based branch-and-cut algorithm*. We do not give a detailed description here, but refer the interested reader to [27, 46].

2.2 Lifting Theory

For implementing efficient cutting plane separators, it seems important to use cutting planes that are *strong* in the sense that they define facets of $\mathrm{conv}(X)$, where X is the feasible region of the MIP, or at least faces of $\mathrm{conv}(X)$ of reasonably high dimension (see [46]).

Definition 2.7 ([46]). Let $P \subseteq \mathbb{R}^n$ be a polyhedron, and $\alpha^T x \leq \alpha_0$, where $\alpha \in \mathbb{R}^n$ and $\alpha_0 \in \mathbb{R}$, be a valid inequality for P.

1. A set of vectors $x^1, \ldots, x^k \in \mathbb{R}^n$ is *affinely independent* if the unique solution of $\sum_{i=1}^k \lambda_i x^i = 0$, $\sum_{i=1}^k \lambda_i = 0$ is $\lambda_i = 0$ for $i = 1, \ldots, k$.

2. P is of *dimension* k, denoted by $\dim(P) = k$, if the maximum number of affinely independent vectors in P is $k+1$.

3. P is *full-dimensional* if $\dim(P) = n$.

4. If $F = \{x \in P : \alpha^T x = \alpha_0\}$, F is called a *face* of P, and we say that $\alpha^T x \leq \alpha_0$ *defines* F.

5. A face F of P is a *facet* of P if $\dim(F) = \dim(P) - 1$.

To derive strong valid inequalities for $\text{conv}(X)$ we use the principle of *lifting*, where a valid inequality for the restriction of X to some lower-dimensional space is extended to a strong valid inequality for $\text{conv}(X)$. The following description of the lifting theory for BMIPs is taken from [31]; we only modify the notation. The theory is given in a very general way and can be used for special cases, such as the 0-1 knapsack problem and the 0-1 single node flow problem.

Definition 2.8 ([46]). Let $N = \{1, \ldots, n\}$ be a finite set. We say that $\{C_k : k = 0, \ldots, t\}$ is a *partition* of N, if $\cup_{k=0}^{t} C_k = N$ and $C_k \cap C_l = \emptyset$ for $k, l = 0, \ldots, t$ with $k \neq l$.

Note that if a partition of N consists of only two sets C_0 and C_1, we also use the notation (C_0, C_1).

Consider the feasible region of a BMIP, given in the form

$$
\begin{aligned}
X = \{x \in \mathbb{R}^{|N|} \quad : \quad & \sum_{k=0}^{t} \sum_{j \in C_k} a_j x_j \leq a_0^{'}, \\
& \sum_{j \in C_k} w_j x_j \leq r_k \text{ for } k = 0, \ldots, t, \\
& x_j \in \{0,1\} \text{ for all } j \in I\},
\end{aligned}
$$

where $\{C_k : k = 0, \ldots, t\}$ is a partition of $N = \{1, \ldots, n\}$, $I \subseteq N$, $m \in \mathbb{Z}_+ \backslash \{0\}$, $a_j \in \mathbb{Q}^m$ for all $j \in N$, $a_0^{'} \in \mathbb{Q}^m$, $m_k \in \mathbb{Z}_+ \backslash \{0\}$ for $k = 0, \ldots, t$, $w_j \in \mathbb{Q}^{m_k}$ for all $j \in C_k$ and for $k = 0, \ldots, t$, and $r_k \in \mathbb{Q}^{m_k}$ for $k = 0, \ldots, t$. We assume that each variable x_j has at least one finite bound given by b_j, i.e., $x_j \geq b_j$ or $x_j \leq b_j$.

Initially, we consider the subset of X with $x_j = b_j$ for all $j \in N \backslash C_0$ given by

$$
\begin{aligned}
X^0 = \{x \in \mathbb{R}^{|C_0|} \quad : \quad & \sum_{j \in C_0} a_j x_j \leq a_0, \\
& \sum_{j \in C_0} w_j x_j \leq r_0, \\
& x_j \in \{0,1\} \text{ for all } j \in I \cap C_0\},
\end{aligned}
$$

where $a_0 = a_0^{'} - \sum_{j \in N \backslash C_0} a_j b_j$. Let

$$
0 \leq \alpha_0 - \sum_{j \in C_0} \alpha_j x_j \tag{2.2}
$$

be an arbitrary valid inequality for X^0. To construct a valid inequality for X of the form

$$0 \le \alpha_0 - \sum_{j \in C_0} \alpha_j x_j - \sum_{k=1}^{t} \sum_{j \in C_k} \alpha_j (x_j - b_j), \tag{2.3}$$

we start with inequality (2.2) and lift the variables in $N \backslash C_0$. Without loss of generality, we assume that the variables with indices in $C_1, \ldots, C_t$ are lifted sequentially in that order and that in a given set C_k they are lifted simultaneously. Note that this contains as special cases *simultaneous lifting* of all variables ($t = 1$) and *sequential lifting* of all variables ($|C_k| = 1$ for $k = 1, \ldots, t$).

The intermediate feasible regions X^i for $i = 1, \ldots, t$ are defined by

$$\begin{aligned} X^i = \{x \in \mathbb{R}^{\sum_{k=0}^{i} |C_k|} \quad : \quad & \sum_{j \in C_0} a_j x_j + \sum_{k=1}^{i} \sum_{j \in C_k} a_j (x_j - b_j) \le a_0, \\ & \sum_{j \in C_k} w_j x_j \le r_k \text{ for } k = 0, \ldots, i, \\ & x_j \in \{0,1\} \text{ for all } j \in I \cap (\cup_{k=0}^{i} C_k)\}. \end{aligned}$$

For $i = 1, \ldots, t$, the *lifting problem* L_i associated with C_i, given a valid inequality

$$0 \le \alpha_0 - \sum_{j \in C_0} \alpha_j x_j - \sum_{k=1}^{i-1} \sum_{j \in C_k} \alpha_j (x_j - b_j), \tag{2.4}$$

for X^{i-1}, is to find α_j for all $j \in C_i$ such that the inequality

$$\sum_{j \in C_i} \alpha_j (x_j - b_j) \le \alpha_0 - \sum_{j \in C_0} \alpha_j x_j - \sum_{k=1}^{i-1} \sum_{j \in C_k} \alpha_j (x_j - b_j) \tag{2.5}$$

is valid for X^i.

For $i = 1, \ldots, t$, let

$$Z^i = \{z \in \mathbb{R}^m : \exists x \in X^i : \sum_{j \in C_i} a_j (x_j - b_j) = z, \sum_{j \in C_0} a_j x_j + \sum_{k=1}^{i-1} \sum_{j \in C_k} a_j (x_j - b_j) \le a_0 - z\}.$$

Furthermore, for $z \in Z^i$, let

$$\begin{aligned} h_i(z; \alpha_j, j \in C_i) = \max\{ \sum_{j \in C_i} \alpha_j (x_j - b_j) \quad : \quad & \sum_{j \in C_i} a_j (x_j - b_j) = z, \\ & \sum_{j \in C_i} w_j x_j \le r_i, \\ & x_j \in \{0,1\} \text{ for all } j \in I \cap C_i\}, \end{aligned}$$

and

$$
\begin{aligned}
f_i(z) = \min\{\ & \alpha_0 - \sum_{j\in C_0}\alpha_j x_j - \sum_{k=1}^{i-1}\sum_{j\in C_k}\alpha_j(x_j - b_j)\ : \\
& \sum_{j\in C_0} a_j x_j + \sum_{k=1}^{i-1}\sum_{j\in C_k} a_j(x_j - b_j) \le a_0 - z, \\
& \sum_{j\in C_k} w_j x_j \le r_k \text{ for } k = 0,\dots,i-1, \\
& x_j \in \{0,1\} \text{ for all } j \in I \cap [\cup_{k=0}^{i-1} C_k]\ \}.
\end{aligned}
$$

Theorem 2.9 ([31]). *For $i = 1,\dots,t$, inequality (2.5) is valid for X^i for any choice of α_j for all $j \in C_i$ such that $h_i(z;\alpha_j, j\in C_i) \le f_i(z)$ for all $z \in Z^i$.*

Let b be the vector of the bounds b_j for all $j \in \cup_{k=0}^{i} C_k$. When α_j for all $j \in C_i$ are such that

$$h_i(z;\alpha_j, j\in C_i) = f_i(z)$$

has $|C_i|$ solutions $x^1,\dots,x^{|C_i|}$ such that the components in C_i of $x^1 - b,\dots,x^{|C_i|} - b$ are linearly independent, we say that the lifting is *maximal*.

Theorem 2.10 ([31]). *For $i = 1,\dots,t$, if conv(X^{i-1}) and conv(X^i) are full-dimensional, inequality (2.4) defines a facet of conv(X^{i-1}), and $\alpha_0 \ne 0$, then inequality (2.5) defines a facet of conv(X^i) if and only if the lifting is maximal.*

Corollary 2.11 ([31]). *Given an arbitrary valid inequality (2.2) for X^0, we can construct a valid inequality (2.3) for X by sequentially lifting sets C_i for $i = 1,\dots,t$. At each step i, the lifting coefficients α_j for all $j \in C_i$ have to be such that $h_i(z;\alpha_j, j \in C_i) \le f_i(z)$ for all $z \in Z^i$. If inequality (2.2) defines a facet of conv(X^0), conv(X^i) is full-dimensional for $i = 0,\dots,t-1$, and at each step i the lifting is maximal, then inequality (2.3) defines a facet of conv(X).*

Lifting coefficients α_j for all $j \in \cup_{k=1}^{t} C_k$ are, in general, dependent on the lifting sequence $C_1,\dots,C_t$. Superadditive functions are important for the development of sequence independent lifting techniques.

Definition 2.12. A function f is *superadditive* on Z if f is bounded for all $z \in Z$ and $f(z_1) + f(z_2) \le f(z_1 + z_2)$ for all z_1, z_2 and $z_1 + z_2 \in Z$.

Let Z be a bounded convex set such that $Z^i \subseteq Z$ for $i = 1,\dots,t$.

Definition 2.13. The *lifting function* f with respect to the valid inequality (2.2) for X^0 is defined to be $f(z) = f_1(z)$ for all $z \in Z$.

Definition 2.14. If $f(z) = f_i(z)$ for all $z \in Z$, for $i = 2,\dots,t$ and all lifting sequences, then the lifting is said to be *sequence independent.*

The following result is fundamental to the development of sequence independent lifting techniques.

Theorem 2.15 ([32]). *If f is superadditive on Z, then lifting is sequence independent.*

Obviously, a superadditive lifting function significantly reduces the computational burden of the lifting process. Instead of having to compute functions f_i for each $i = 1, \ldots, t$, we only have to compute the lifting function f. Unfortunately, most lifting functions are not superadditive. To be able to profit from the computational advantages of sequence independent lifting, we need a superadditive function that yields a valid inequality for X.

Corollary 2.16. *If g is superadditive, $g(z) \leq f(z)$ for all $z \in Z$, and if α_j for all $j \in C_i$ are such that $h_i(z; \alpha_j, j \in C_i) \leq g(z)$ for all $z \in Z^i$ and for $i = 1, \ldots, t$, then the lifted inequality (2.3) is valid for X.*

A function g that satisfies the conditions of Corollary 2.16 is called a *superadditive valid lifting function* for f. Next, we adress the problem of choosing a 'good' superadditive valid lifting function. A desirable property is that g should not be *dominated* by another superadditive valid lifting function $g^{'}$, i.e., there is no superadditive $g^{'}$ with $g(z) \leq g^{'}(z)$ for all $z \in Z$ and $g(z^{'}) < g^{'}(z^{'})$ for some $z^{'} \in Z$. A more interesting property is maximality. Let $E = \{z \in Z : f_i(z) = f(z)$ for $i = 1, \ldots, t$ independent of the coefficients a_j for all $j \in C_k$ and for $k = 1, \ldots, t\}$. Note that if f is superadditive, then $E = Z$. We say that g is a *maximal superadditive valid lifting function* if $g(z) = f(z)$ for all $z \in E$.

2.3 Computational Study

In this section, we state general information about the computational experiments which we conducted to develop efficient cutting plane separators for the class of c-MIR inequalities (Chapter 3), the 0-1 knapsack problem (Chapter 4), and the 0-1 single node flow problem (Chapter 5).

All computational tests described in Chapter 3, 4, and 5 were carried out on a Sun V40z with a 2.20 GHz AMD Opteron CPU (1024 KB cache) and 32 GB RAM. In each test, we used a time limit 3,600 seconds of CPU time and a memory limit of 4 GB for each instance contained in the considered test set.

Our implementations were embedded in SCIP 0.81 [1] which is a framework that integrates constraint and mixed integer programming. As underlying LP solver in SCIP 0.81 we used CPLEX 10.01 [21].

The efficiency of the individual cutting plane separators depends on the way they are integrated into SCIP 0.81, i.e., on the way they interact with other features of the solver including other cutting plane separators. For this thesis, we decided to concentrate on the performance of the individual cutting plane separators when they are used basically isolated. A next step would be to improve the way they are integrated into SCIP 0.81. Therefore, in all test runs for developing the cutting plane separators, we considered only the root node of the branch-and-cut tree and called up SCIP 0.81 with its default settings except the following changes.

Primal heuristics If a primal heuristic is successful, this might cause further dual propagations, which could lead to the generation of further cuts. Therefore, we disabled all primal heuristics.

Strong branching Every branching strategy employing strong branching can detect infeasibility of subproblems of a MIP (see [2]) and may therefore cause fixing of variables. As this influences the dual bound, we did not use strong branching, but *most infeasible branching* (see [2]).

Cutting plane separators We disabled all cutting plane separators except the tested one.

Restarts This feature may lead to the generation of further cuts and therefore, we disabled it.

We used the same test set, which we call *initial test set*, for all three cutting plane separators, except for the one for the 0-1 knapsack problem. For this separator, we extended the initial test set in order to get a reasonable number of instances for which cuts were generated. The initial test set consists of 134 instances, which were taken from Miplib 2003 [3], Miplib 3.0 [14], and from the MIP collection of Mittelmann [45]. For each cutting plane separator, we divided the test set into two sets. One was used to evaluate the effect of different versions of the cutting plane separators and the other one was used to ensure that the CPU time spent in the final separator is on an acceptable level for all instances in the test set.

For each cutting plane separator, we present two tables (Table B.1, B.2, B.20, B.21, B.46, and B.47), where the main characteristics of the instances in these two sets are summarized. In each table, the column headed Type contains the problem type and the columns headed Conss and Vars contain the number of constraints and the number of variables. z_{LP} denotes the optimal objective function value of the LP relaxation at the root node before cutting planes are added, and z_{MIP} represents the optimal objective function value of the MIP. The names of some instances are given in italic face. For these instances, we do not know the optimal objective function value. For these problems, we set z_{MIP} as the objective function value of the best known feasible solution, which was generated by Cplex 10.01 running for one hour of CPU time with default settings.

To evaluate the performance of our cutting plane separators we have chosen four measures. Gap closed % (β) denotes the percentage of the initial gap (gap between z_{LP} and z_{MIP}) that is closed by using the separator. It is defined as

$$\beta = 100 \cdot \frac{db - z_{\mathrm{LP}}}{z_{\mathrm{MIP}} - z_{\mathrm{LP}}},$$

where db is the dual bound at the root node after adding cutting planes. Note that $z_{\mathrm{MIP}} > z_{\mathrm{LP}}$ holds for all instances in our test sets since we only use instances which are not solved to optimality at the root node when all cutting plane separators are disabled. Cuts is the number of cutting planes generated by the cutting plane separator at the root node. Sepa Time is the elapsed CPU time in seconds for the separation routine at the root node. Sepa Time Average (γ) is the average elapsed CPU time in seconds for the separation routine per separation round at the root node. It is defined as

$$\gamma = \frac{\text{Sepa Time}}{\text{number of separation rounds performed at the root node}}.$$

As we will see in Chapter 3, 4, and 5, there are many algorithmic and implementation choices which have to be made when implementing the cutting plane separators. Thus, evaluating the performance of each separator for all possible combinations of these choices is practically impossible. Inspired by the work of Gu, Nemhauser and Savelsbergh [30], we use for each separator the following approach in our computational study. We give a default algorithm, which represents a set of choices which are basic, i.e., the set does not contain choices which are modifications of algorithmic aspects that may lead to an improved performance. Then, we present computational results which compare the performance of the default algorithm to the performance of an algorithm in which a single choice has been altered. We only modify this approach if not using a basic choice for an algorithmic aspect leads to such a small number of cuts found by the default algorithm that the results for altering other single choices would not be very meaningful.

The results will be presented in the following way. In the tables containing the results for the default algorithms (Table B.3, B.22, B.28, B.36, and B.48), we state the performance measures Gap closed %, Cuts, Sepa Time and Sepa Time Average (there will be three default algorithms for the cutting plane separator for the 0-1 knapsack problem, because we consider three classes of valid inequalities). At the bottom of the tables, in the row labelled Total, we give the sum of the values of each performance measure over all instances, and in the row labelled Geom. Mean, we give the geometric mean of the values of each performance measure over all instances where individual values smaller than one were replaced by one. In the tables containing the results for altering a single choice for each cutting plane separator (see e.g. Table B.4), we report in addition for each performance measure the difference to the corresponding default algorithm. Here numbers in blue color indicate that the value of the performance measure obtained by the altered algorithm is better than the one obtained by the default algorithm, and numbers in red color indicate that the value of the performance measure obtained by the altered algorithm is worse than the one obtained by the default algorithm. Note that for Gap closed % the $\triangle$ value for each instance, for Total and for Geom. Mean is given in percentage points, not in percentage.

Chapter 3

Cutting Plane Separator for the Class of C-MIR Inequalities

In this chapter, we investigate the implementation of an efficient cutting plane separator which generates *general cutting planes* for MIPs. More precisely, we separate the class of *complemented mixed integer rounding inequalities* (c-MIR inequalities).

3.1 Introduction

Cutting planes which do not require or exploit any knowledge about the underlying problem are called *general cutting planes*. Several families of general cutting planes for MIPs have been proposed in the literature over the last fifty years, including *Gomory mixed integer cuts* (GMI cuts) [28], *intersection cuts* [7], *disjunction cuts* [9], *split cuts* [16], and *mixed integer rounding cuts* (MIR cuts) [48]. It is well-known that many of these inequalities, such as GMI inequalities and MIR inequalities, are equivalent (see [19, 20]).

In contrast to cutting plane separators that are based on the polyhedral analysis of the problem formulation, for decades, general cutting planes were considered to be impractical for solving large size instances of MIPs. However, in the 1990s, motivated by their results for implementing *lift-and-project cuts* [10] for BMIPs, Balas, Ceria, Cornuéjols, and Natraj [11] revisited GMI cuts and demonstrated their practical usefulness within a linear programming based branch-and-cut algorithm (see also [18]).

Marchand and Wolsey [39, 42] also pursued the idea of using general cutting planes for MIPs. They investigated the practical usefulness of MIR cuts. However, their work is based on the idea that even general cuts should be based, if possible, on problem structure, and that this problem structure is to be found in the original problem matrix (see [39]). They considered *mixed knapsack sets* which are obtained by taking linear combinations of constraints (transformed into equality form) of the original MIP and by substituting bounds imposed on the real variables. For these sets, they introduced the class of c-MIR inequalities, which are MIR inequalities derived for the mixed knapsack sets after scaling the mixed knapsack constraint and complementing some of the integer variables. This idea was motivated by the observation that several strong valid inequalities based on specific problem structure, such as *flow cover inequalities* (see Chapter 5) can be derived as MIR inequalities when taking linear combinations of specific types of constraints and using the bounds imposed on the variables.

Our implementation of the cutting plane separator for the class of c-MIR inequalities follows the separation heuristic given in [39, 42]. See also [29], for experiences with implementing this separation heuristic. In [42], it was suggested to further test some of the heuristic choices made in the c-MIR separation heuristic and to develop more elaborate strategies for creating mixed knapsack sets. To Marchand and Wolsey it appeared to be especially important to study and understand how to reduce the number of cuts generated. In this chapter, we partly go into these subjects.

The structure of the remaining chapter is as follows. In Section 3.2, we give a brief introduction to the classes of MIR and c-MIR inequalities both defined for mixed knapsack sets and state a procedure to construct such sets from a MIP. In Section 3.3, we give an outline of the c-MIR separation heuristic introduced by Marchand and Wolsey and discuss different algorithmic aspects of this separation routine. Our computational results for implementing the cutting plane separator for the class of c-MIR inequalities are reported in Section 3.4. Finally, conclusions are given in Section 3.5.

3.2 The Class of C-MIR Inequalities

We consider the *mixed knapsack inequality*

$$\sum_{j\in N} a_j x_j \leq a_0 + s, \tag{3.1}$$

where a_0 and a_j are rational numbers for all $j \in N = \{1,\dots,n\}$, x_j are nonnegative integer variables for all $j \in N$, and s is a nonnegative real variable. The *mixed knapsack set* X^{MK} associated with inequality (3.1) is the set of all vectors $(x,s) \in \mathbb{Z}^n_+ \times \mathbb{R}_+$ satisfying inequality (3.1). In addition, we assume that all integer variables are bounded, i.e.,

$$X^{MK} = \{(x,s) \in \mathbb{Z}^n_+ \times \mathbb{R}_+ : \sum_{j\in N} a_j x_j \leq a_0 + s, x_j \leq b_j \text{ for all } j \in N\},$$

where b_j are nonnegative rational numbers for all $j \in N$.

Theorem 3.1 ([39, 42]). *For $d \in \mathbb{R}$, let $f_d = d - \lfloor d \rfloor$ and $d^+ = max\{d, 0\}$. The inequality*

$$\sum_{j\in N} (\lfloor a_j \rfloor + \frac{(f_{a_j} - f_{a_0})^+}{1 - f_{a_0}}) x_j \leq \lfloor a_0 \rfloor + \frac{s}{1 - f_{a_0}} \tag{3.2}$$

is valid for X^{MK}.

Inequality (3.2) is called *mixed integer rounding inequality* (MIR inequality).

Remark 3.2. The MIR inequality (3.2) can also be written in the form

$$\sum_{j\in N} F_{f_{a_0}}(a_j) x_j + \bar{F}_{f_{a_0}}(-1) s \leq F_{f_{a_0}}(a_0),$$

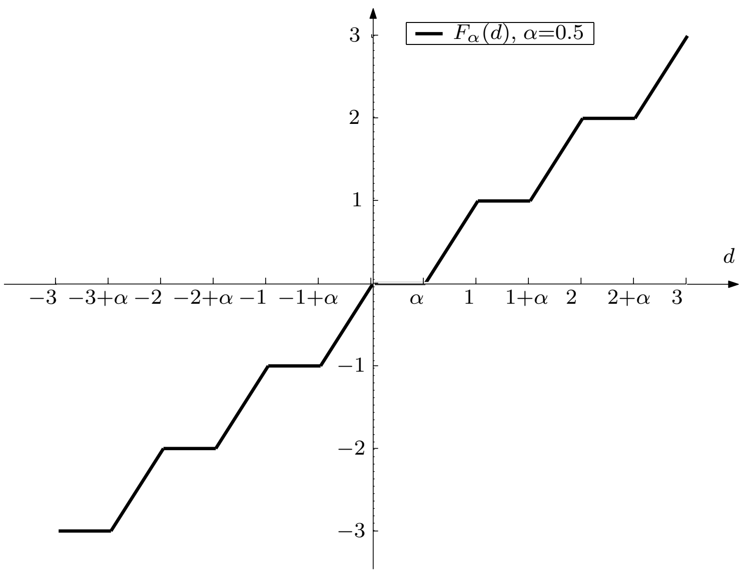

Figure 3.1: MIR function F_α for $\alpha = 0.5$.

where $F_\alpha : \mathbb{R} \to \mathbb{R}$ for $0 \le \alpha < 1$ is given by

$$F_\alpha(d) = \lfloor d \rfloor + \frac{(f_d - \alpha)^+}{1 - \alpha}$$

and $\bar{F}_\alpha(d) = \min\{0, \frac{d}{1-\alpha}\}$. Function F_α is called *MIR function* and is given in Figure 3.1 for α equal to 0.5. It is well-known that the MIR function is nondecreasing and superadditive on $\mathbb{R}$ (see [46, 48]).

The MIR inequality (3.2) derived for X^{MK} depends on the formulation of X^{MK}, i.e., scaling the mixed knapsack inequality (3.1) and complementing some of the integer variables may lead to different MIR inequalities valid for X^{MK}. This observation was used by Marchand and Wolsey [39, 42].

Theorem 3.3 ([39, 42]). *If (T, U) is any partition of N and $\delta \in \mathbb{Q}_+ \backslash \{0\}$, then inequality*

$$\sum_{j \in T} F_{f_\beta}(\frac{a_j}{\delta}) x_j + \sum_{j \in U} F_{f_\beta}(-\frac{a_j}{\delta})(b_j - x_j) \le \lfloor \beta \rfloor + \frac{s}{\delta(1 - f_\beta)}, \tag{3.3}$$

is valid for X^{MK}, where $\beta = \frac{a_0 - \sum_{j \in U} a_j b_j}{\delta}$.

Proof. Complementing x_j for all $j \in U$, i.e., substituting $x_j = b_j - \bar{x}_j$ with $\bar{x}_j \in \mathbb{Z}_+$ for all $j \in U$, and dividing the mixed knapsack inequality (3.1) by δ, we obtain the mixed knapsack set

$$\begin{aligned} \bar{X}^{MK} = \{(x, \bar{x}, s') \in \mathbb{Z}_+^{|T|} \times \mathbb{Z}_+^{|U|} \times \mathbb{R}_+ :\ & \sum_{j \in T} \frac{a_j}{\delta} x_j + \sum_{j \in U} -\frac{a_j}{\delta} \bar{x}_j \le \beta + s', \\ & x_j \le b_j \text{ for all } j \in T, \\ & \bar{x}_j \le b_j \text{ for all } j \in U\}, \end{aligned}$$

where $s' = \frac{s}{\delta}$ and $\beta = \frac{a_0 - \sum_{j\in U} a_j b_j}{\delta}$. By Theorem 3.1, the MIR inequality

$$\sum_{j\in T} F_{f_\beta}(\frac{a_j}{\delta})x_j + \sum_{j\in U} F_{f_\beta}(-\frac{a_j}{\delta})\bar{x}_j \leq \lfloor \beta \rfloor + \frac{s'}{1-f_\beta}$$

is valid for $\bar{X}^{MK}$. Substitution $\bar{x}_j = b_j - x_j$ for all $j \in U$ and $s' = \frac{s}{\delta}$ leads to inequality (3.3) valid for X^{MK}. □

Inequality (3.3) is called *complemented mixed integer rounding inequality* (c-MIR inequality) associated with (T, U) and δ. Furthermore, the family of all c-MIR inequalities associated with some (T, U) and some δ is called *class of c-MIR inequalities.*

The aim of this chapter is to implement a cutting plane separator which generates general cutting planes for MIPs. Therefore, we consider the *mixed integer set*

$$\begin{aligned} X = \{(x,y) \in \mathbb{Z}^n_+ \times \mathbb{R}^m_+ :\ & \sum_{j\in N} a^i_j x_j + \sum_{j\in M} c^i_j y_j = a^i_0 \text{ for all } i \in P, \\ & x_j \leq b_j \text{ for all } j \in N\}, \end{aligned} \tag{3.4}$$

where a^i_0 and a^i_j are rational numbers for all $j \in N$ and $i \in P = \{1,\dots,p\}$, c^i_j are rational numbers for all $j \in M = \{1,\dots,m\}$ and $i \in P$, and b_j are nonnegative rational numbers for all $j \in N$. Constraints of the form $\sum_{j\in N} a^i_j x_j + \sum_{j\in M} c^i_j y_j = a^i_0$ are called *single mixed integer constraints*. Note that each row of a MIP can be transformed into an equality constraint by adding a nonnegative slack variable if necessary. In addition, let y_j be bounded by a simple and variable *lower* bound and by a simple and variable *upper* bound for all $j \in M$ defined as follows.

Definition 3.4. Let $l_j \in \mathbb{Q}_+$, $\tilde{l}_j \in \mathbb{Q}_+$, and x_j be a nonnegative integer variable. Further, let y_j be a nonnegative real variable with $l_j \leq y_j$ and $\tilde{l}_j x_j \leq y_j$. Then l_j is called *simple lower bound* imposed on y_j and $\tilde{l}_j x_j$ is called *variable lower bound* imposed on y_j.

Definition 3.5. Let $u_j \in \mathbb{Q}_+ \cup \{\infty\}$, $\tilde{u}_j \in \mathbb{Q}_+ \cup \{\infty\}$, and x_j be a nonnegative integer variable. Furthermore, let y_j be a nonnegative real variable with $y_j \leq u_j$ and $y_j \leq \tilde{u}_j x_j$. Then u_j is called *simple upper bound* imposed on y_j and $\tilde{u}_j x_j$ is called *variable upper bound* imposed on y_j.

Remark 3.6. Note that in Definition 3.5, $\tilde{u}_j = \infty$ is only allowed to simplify the notation for the variable upper bounds. If $\tilde{u}_j = \infty$, then $\tilde{u}_j x^*_j = \infty$ for all $x^*_j \in [0, 1]$.

Furthermore, without loss generality, we assume throughout this chapter that $n = m$ holds.

To derive valid inequalities for X, we relax X to a mixed knapsack set X^{MK} and generate c-MIR inequalities valid for X^{MK}. The relaxation X^{MK} of X is also called *mixed knapsack relaxation* of X. It can be constructed the following way.

Aggregation: Choose $\omega^i \in \mathbb{Q}$ for all $i \in P$ and relax X to obtain the set

$$\begin{aligned} X' = \{(x,y) \in \mathbb{Z}^n_+ \times \mathbb{R}^m_+ :\ & \sum_{j\in N} \alpha_j x_j + \sum_{j\in M} \gamma_j y_j = \alpha_0, \\ & x_j \leq b_j \text{ for all } j \in N\}, \end{aligned} \tag{3.5}$$

where $\alpha_j = \sum_{i \in P} \omega^i a^i_j$ for all $j \in N$, $\gamma_j = \sum_{i \in P} \omega^i c^i_j$ for all $j \in M$, and $\alpha_0 = \sum_{i \in P} \omega^i a^i_0$.

Bound substitution: Now, choose for each real variable y_j, $j \in M$ one of the following substitutions

$$y_j = u_j - \bar{y}_j, \quad y_j = \tilde{u}_j x_j - \bar{y}_j, \quad y_j = l_j + \bar{y}_j, \text{ or } \quad y_j = \tilde{l}_j x_j + \bar{y}_j$$

where $\bar{y}_j$ is a nonnegative real variable. Note that the first and the second substitution are only allowed to be chosen if $u_j < \infty$ and $\tilde{u}_j < \infty$, respectively. Then, X' is equivalent to the set

$$\begin{aligned} X'' = \{(x, \bar{y}) \in \mathbb{Z}^n_+ \times \mathbb{R}^m_+ : & \sum_{j \in N} \alpha'_j x_j + \sum_{j \in M, \gamma'_j \geq 0} \gamma'_j \bar{y}_j = \alpha'_0 - \sum_{j \in M, \gamma'_j < 0} \gamma'_j \bar{y}_j, \\ & x_j \leq b_j \text{ for all } j \in N\}, \end{aligned} \tag{3.6}$$

for appropriate rational numbers α'_j for all $j \in N$, γ'_j for all $j \in M$ and α'_0. Finally, relax X'' to obtain the mixed knapsack set

$$\begin{aligned} X^{MK} = \{(x, s) \in \mathbb{Z}^n_+ \times \mathbb{R}_+ : & \sum_{j \in N} \alpha'_j x_j \leq \alpha'_0 + s, \\ & x_j \leq b_j \text{ for all } j \in N\}, \end{aligned} \tag{3.7}$$

where $s = -\sum_{j \in M, \gamma'_j < 0} \gamma'_j \bar{y}_j$.

3.3 Algorithmic Aspects

In the last section, we introduced the class of c-MIR inequalities valid for a mixed knapsack set. Furthermore, we stated a procedure which constructs mixed knapsack relaxations of a mixed integer set. In this section, we investigate algorithmic aspects of a separation algorithm for the class of c-MIR inequalities.

Let $(x^*, y^*) \in (\mathbb{R}^n_+ \backslash \mathbb{Z}^n_+) \times \mathbb{R}^m_+$ be a fractional vector and let X be a mixed integer set given in the form (3.4). We want to solve the following separation problem.

Separation problem for the class of c-MIR inequalities

Find a mixed knapsack relaxation

$$X^{MK} = \{(x, s) \in \mathbb{Z}^n_+ \times \mathbb{R}_+ : \sum_{j \in N} \alpha'_j x_j \leq \alpha'_0 + s, x_j \leq b_j \text{ for all } j \in N\},$$

of X and let $(x^*, s^*) \in (\mathbb{R}^n_+ \backslash \mathbb{Z}^n_+) \times \mathbb{R}_+$ be the corresponding fractional vector. Find a partition (T, U) of N and a constant $\delta \in \mathbb{Q}_+ \backslash \{0\}$ such that

$$\sum_{j \in T} F_{f_\beta}(\frac{\alpha'_j}{\delta}) x^*_j + \sum_{j \in U} F_{f_\beta}(-\frac{\alpha'_j}{\delta})(b_j - x^*_j) > \lfloor \beta \rfloor + \frac{s^*}{\delta(1 - f_\beta)},$$

where $\beta = \frac{\alpha'_0 - \sum_{j \in U} \alpha'_j b_j}{\delta}$, or show that no inequality in the class of c-MIR inequalities for any mixed knapsack relaxation of X is violated by the corresponding fractional vector (x^*, s^*).

We solve the separation problem for the class of c-MIR inequalities heuristically using the separation algorithm introduced by Marchand and Wolsey [39, 42]. The algorithm uses the procedure given in the last section to construct different mixed knapsack relaxations of X and tries to generate c-MIR inequalities for these relaxations. It consists of three steps, each of them heuristic.

Step 1 *Aggregation heuristic.* A single mixed integer constraint is generated by taking a linear combination of constraints defining X (see (3.5)).

Step 2 *Bound substitution heuristic.* A mixed knapsack relaxation of X is derived from the single mixed integer constraint generated in Step 1 using bounds imposed on the real variables (see (3.7)).

Step 3 *Cut generation heuristic.* For the mixed knapsack relaxation derived in Step 2, a violated c-MIR inequality is generated if possible.

Determining useful linear combinations of constraints defining X in Step 1 plays a crucial role in the separation algorithm. Marchand and Wolsey [39, 42] suggested to loop through the set P of constraints defining X. For each constraint $i \in P$, which we call *starting constraint*, the separation algorithm first takes the linear combination with scalar equal to one for this constraint and scalar equal to zero for all other constraints. If no violated c-MIR inequality for a mixed knapsack set based on this linear combination is found, the algorithm selects, in the aggregation heuristic, another constraint $r \in P \setminus \{i\}$ in order to eliminate a real variable appearing in the starting constraint. In the new linear combination, only the scalar of the selected constraint r changes. The process is repeated until a violated c-MIR inequality for a mixed knapsack set based on the current linear combination is found or a maximum number of constraints added to the starting constraint, denoted by the parameter MAXAGGR, is reached. The number of constraints added to each starting constraint is limited in order to reduce the time spent in the separation algorithm.

The complete separation algorithm for the class of c-MIR inequalities using MAXAGGR $= 6$ is given in Algorithm 3.1. Note that the cuts derived by the separation algorithm have to be restated in terms of the original real variables y_j, $j \in M$ before they are added to the MIP.

In [39], Marchand used MAXAGGR $= 6$. Marchand and Wolsey [42] and Gonçalves and Ladanyi [29] tested different values of MAXAGGR. Note that in [42] and [29], MAXAGGR denotes the maximum number of single mixed integer constraints defining X which are allowed to be *combined* to form the aggregated constraint.

We decided to state the complete separation algorithm already here in order to give an idea of the management of the three steps. In the next sections, we give a detailed description of the three steps and discuss different algorithmic aspects of each heuristic.

3.3.1 Aggregation Heuristic

The aggregation heuristic is the first part of the procedure to construct a mixed knapsack relaxation of X.

This heuristic adds in each iteration of the while-loop in Algorithm 3.1 (Line 10) another single mixed integer constraint from the formulation of X to the current

Input : Mixed integer set X defined as in (3.4), simple and variable lower bounds and simple and variable upper bounds imposed on y_j for all $j \in M$ defined as in Definition 3.4 and 3.5, and $(x^*, y^*) \in (\mathbb{R}^n_+ \backslash \mathbb{Z}^n_+) \times \mathbb{R}^m_+$ fractional vector.

Output: Set of violated (with respect to the fractional vectors $(x^*, s^*) \in (\mathbb{R}^n_+ \backslash \mathbb{Z}^n_+) \times \mathbb{R}_+$ which correspond to the constructed mixed knapsack relaxations of X) inequalities from the class of c-MIR inequalities. (This set can be empty.)

MAXAGGR $\leftarrow$ 6
$C \leftarrow \emptyset$
for $i \leftarrow 1$ **to** p **do**
    /* Use starting constraint. */
    Step 1. $Aggrcons \leftarrow \sum_{j \in N} a^i_j x_j + \sum_{j \in M} c^i_j y_j = a^i_0$ and $Q \leftarrow \{i\}$
    Step 2. Call Algorithm 3.3 for X, simple and variable lower and upper bounds imposed on y_j for all $j \in M$, (x^*, y^*) and $Aggrcons$. (Let X^{MK} be the constructed mixed knapsack set and (x^*, s^*) be the corresponding fractional vector.)
    Step 3. Call Algorithm 3.4 for X^{MK} and (x^*, s^*). (If a c-MIR inequality valid for X^{MK} is found, let $\sum_{j \in N} \omega_j x_j \leq \omega_0 + \omega s$ be the found inequality.)
    if *Inequality was found* ***and*** $\sum_{j \in N} \omega_j x^*_j > \omega_0 + \omega s^*$ **then**
        $C \leftarrow C \cup \{\sum_{j \in N} \omega_j x_j \leq \omega_0 + \omega s\}$
        continue
    /* Use aggregated constraints. */
    while $|Q| \leq$ MAXAGGR **do**
        Step 1. Call Algorithm 3.2 for X, simple and variable lower and upper bounds imposed on y_j for all $j \in M$, (x^*, y^*), $Aggrcons$ and Q.
        if *No aggregation took place* **then**
            break
        Step 2. Call Algorithm 3.3 for X, simple and variable lower and upper bounds imposed on y_j for all $j \in M$, (x^*, y^*) and $Aggrcons$. (Let X^{MK} be the constructed mixed knapsack set and (x^*, s^*) be the corresponding fractional vector.)
        Step 3. Call Algorithm 3.4 for X^{MK} and (x^*, s^*). (If a c-MIR inequality valid for X^{MK} is found, let $\sum_{j \in N} \omega_j x_j \leq \omega_0 + \omega s$ be the found inequality.)
        if *Inequality was found* ***and*** $\sum_{j \in N} \omega_j x^*_j > \omega_0 + \omega s^*$ **then**
            $C \leftarrow C \cup \{\sum_{j \in N} \omega_j x_j \leq \omega_0 + \omega s\}$
            break
return C

Algorithm 3.1: Separation algorithm for the class of c-MIR inequalities. Use MAXAGGR $= 6$.

aggregated constraint. Let $Q \subseteq P$ be the set of single mixed integer constraints in the formulation of X which have been aggregated to form the current aggregated constraint

$$\sum_{j \in N} \alpha_j x_j + \sum_{j \in M} \gamma_j y_j = \alpha_0. \tag{3.8}$$

The aggregation heuristic selects a single mixed integer constraint $r \in P \backslash Q$ in order to eliminate a real variable appearing in the aggregated constraint (3.8). This idea of refining progressively the structure on which to generate valid inequalities by adding to an initial constraint other ones was first used by Van Roy and Wolsey [55], where they created paths in fixed charge networks.

For $j \in M$, let $d_j = \min\{y_j^* - lb_j^*, ub_j^* - y_j^*\}$ be the *bound distance* of the real variable y_j, where $lb_j^* = \max\{l_j, \tilde{l}_j x_j^*\}$ and $ub_j^* = \min\{u_j, \tilde{u}_j x_j^*\}$. Let

$$M^* = \{j \in M \,:\, \gamma_j \neq 0, d_j > 0 \text{ and } \exists i \in P \backslash Q : c_j^i \neq 0\}$$

be the set of real variables which are candidates to be eliminated. If M^* is empty, we stop the aggregation heuristic; Algorithm 3.1 now continues with a new starting constraint. Otherwise, we choose a real variable y_k with $k \in M^*$ and a constraint $r \in P \backslash Q$ where this variable appears. For this selection we take into account the bound distances of the real variables in M^* and values $\mathsf{AGGRSCORE}_j^i \in \mathbb{R}$ for all $j \in M^*$ and $i \in P \backslash Q$. Note that $\mathsf{AGGRSCORE}_j^i \in \mathbb{R}$, $j \in M^*$ and $i \in P \backslash Q$ may depend on the iteration of the for-loop and while-loop in Algorithm 3.1 (Line 3 and 10). We select the real variable y_k with $k \in M^*$ which has the greatest bound distance and then the constraint $r \in P \backslash Q$ with $c_k^r \neq 0$ which has the greatest value of $\mathsf{AGGRSCORE}_k^r$. The current aggregated constraint (3.8) and the selected constraint r are now aggregated in such a way that the coefficient of y_k becomes zero. The corresponding scalar $\omega^r = -\frac{\gamma_k}{c_k^r}$ of constraint r in the new linear combination is called *aggregation factor*. The following types of scores can be used to select the constraint.

Score Type 1 Use $\mathsf{AGGRSCORE}_j^i = -i$ for all $j \in M^*$ and $i \in P \backslash Q$, i.e., for $k \in M^*$ select $r \in P \backslash Q$ with $c_r^k \neq 0$ such that r is the index of the constraint found first.

Score Type 2 Use $\mathsf{AGGRSCORE}_j^i = \textit{random number}$ for all $j \in M^*$ and $i \in P \backslash Q$, i.e., for $k \in M^*$ select $r \in P \backslash Q$ with $c_r^k \neq 0$ randomly.

Score Type 3 For $i \in P$, let db_i be the LP solution value of the *dual variable* corresponding to the original MIP row, i.e., of the row of the MIP before introducing the nonnegative slack variables, which corresponds to constraint i. See [52], for the definition of the dual LP. Let

$$dens_i = \frac{|\{j \in N : a_j^i \neq 0\}| + |\{j \in \tilde{M} : c_j^i \neq 0\}|}{|N| + |\tilde{M}|},$$

where $\tilde{M}$ is the index set of the real variables in the original MIP, be the *density* of the original MIP row corresponding to constraint i. Furthermore, let $s^i \in M$ be the index of the nonnegative slack variable introduced in the original MIP row corresponding to constraint i. Use

$$\begin{aligned}\mathsf{AGGRSCORE}^{\mathsf{i}}_{\mathsf{j}} = 0.9^l \; &(\max\{\tfrac{db_i}{\max\{\|(c,d)\|,1.0\}}, 0.0001\} + \\ &0.0001(1 - dens_i) + \\ &0.001(1 - \tfrac{y^*_{si}}{\max\{\|(a^i,c^i)\|,0.1\}}))\end{aligned}$$

for all $j \in M^*$ and $i \in P\backslash Q$, where l is the number of times constraint i has already been involved in an aggregation in Algorithm 3.1, $(c, d) \in \mathbb{Q}^n \times \mathbb{Q}^{|\tilde{M}|}$ is the vector of the coefficients of all variables in the objective function of the original MIP, $(a^i, c^i) \in \mathbb{Q}^n \times \mathbb{Q}^{|\tilde{M}|}$ is the vector of the coefficients of all variables in the original MIP row corresponding to constraint i and $\|\cdot\|$ is the Euclidean norm. That means, for $k \in M^*$, prefer constraints $r \in P\backslash Q$ (with $c^k_r \neq 0$) for which the dual variable corresponding to the original MIP row has a great LP solution value, for which the corresponding original MIP row has a small density, which are tight and which have not been involved in an aggregation in the separation algorithm yet.

Score Type 4 Use $\mathsf{AGGRSCORE}^{\mathsf{i}}_{\mathsf{j}} = -$ (*number of real variables with nonzero coefficient in the new aggregated constraint when the real variable* y_j *and the constraint* i *are selected*) for all $j \in M^*$ and $i \in P\backslash Q$.

The complete aggregation heuristic for using Score Type 1 is given in Algorithm 3.2. This algorithm can be extended straightforward for using one of the other types of scores.

Marchand and Wolsey [39, 42] did not go into details about how they choose the constraint $r \in P\backslash Q$ with $c^r_k \neq 0$.

Gonçalves and Ladanyi [29] tested Score Type 1 and Score Type 2. In addition, they suggested to use all possible constraints $r \in P\backslash Q$ with $c^r_k \neq 0$ for the selected real variable y_k with $k \in M^*$ for the aggregation. The version of their code using this strategy turned out to be too slow since the number of aggregated constraints can become very large. However, they tested this version for some instances for $\mathsf{MAXAGGR} = 2$ and the performance with respect to the dual bound did not seem to be much better than using Score Type 1 or Score Type 2.

3.3.2 Bound Substitution Heuristic

The bound substitution heuristic is the second part of the procedure to construct a mixed knapsack relaxation of X.

Let

$$\sum_{j\in N} \alpha_j x_j + \sum_{j\in M} \gamma_j y_j = \alpha_0 \tag{3.9}$$

be the single mixed integer constraint generated in the aggregation heuristic. The bound substitution heuristic performs for each real variable y_j, $j \in M$ one of the following substitutions

$$y_j = u_j - \bar{y}_j, \quad y_j = \tilde{u}_j x_j - \bar{y}_j, \quad y_j = l_j + \bar{y}_j, \text{ or } \quad y_j = \tilde{l}_j x_j + \bar{y}_j,$$

where $\bar{y}_j$ is a nonnegative real variable. Note that the first and the second substitution are only performed if $u_j < \infty$ and $\tilde{u}_j < \infty$, respectively. The set defined by the

Input : Mixed integer set X defined as in (3.4), simple and variable lower bounds and simple and variable upper bounds imposed on y_j for all $j \in M$ defined as in Definition 3.4 and 3.5, $(x^*, y^*) \in (\mathbb{R}^n_+ \setminus \mathbb{Z}^n_+) \times \mathbb{R}^m_+$ fractional vector, aggregated constraint $\sum_{j \in N} \alpha_j x_j + \sum_{j \in M} \gamma_j y_j = \alpha_0$, and $Q \subseteq P$ set of single mixed integer constraints in the formulation of X which have been combined to form the aggregated constraint.

Output: Updates aggregated constraint and updates the set $Q \subseteq P$, or returns notification that no aggregation took place.

for $j \leftarrow 1$ **to** m **do**
    $lb^*_j \leftarrow \max\{l_j, \tilde{l}_j x^*_j\}$
    $ub^*_j \leftarrow \min\{u_j, \tilde{u}_j x^*_j\}$
    $d_j \leftarrow \min\{y^*_j - lb^*_j, ub^*_j - y^*_j\}$
$M^* \leftarrow \{j \in M : \gamma_j \neq 0, d_j > 0 \text{ and } \exists i \in P \setminus Q : c^i_j \neq 0\}$
if $M^* = \emptyset$ **then**
    return *No aggregation took place*
$bestaggrscore \leftarrow -\infty$
$bestbounddist \leftarrow 0$
foreach $j \in M^*$ **do**
    if $d_j < bestbounddist$ **then continue**
    foreach $i \in P \setminus Q$ *with* $c^i_j \neq 0$ **do**
        $\mathsf{AGGRSCORE}^i_j \leftarrow -i$
        if $d_j > bestbounddist$ ***or*** $\mathsf{AGGRSCORE}^i_j > bestaggrscore$ **then**
            $bestaggrscore \leftarrow \mathsf{AGGRSCORE}^i_j$
            $bestbounddist \leftarrow d_j$
            $k \leftarrow j$
            $r \leftarrow i$
$\omega^r \leftarrow -\frac{\gamma_k}{c^r_k}$
for $j \leftarrow 1$ **to** n **do** $\alpha_j \leftarrow \alpha_j + \omega^r a^r_j$
for $j \leftarrow 1$ **to** m **do** $\gamma_j \leftarrow \gamma_j + \omega^r c^r_j$
$\alpha_0 \leftarrow \alpha_0 + \omega^r a^r_0$
$Q \leftarrow Q \cup \{r\}$

Algorithm 3.2: Aggregation heuristic. Use Score Type 1.

resulting single mixed integer constraint and by the bounds imposed on the integer variables is than relaxed to obtain the mixed knapsack relaxation X^{MK} of X, as explained in Section 3.2.

We want to comprehend the strategies suggested by Marchand and Wolsey [39, 42] for the selection of the substitutions, and we also want to develop new strategies. Furthermore, in [39, 42], a real variable is either bounded by a simple upper bound *or* by a variable upper bound. The same hold for the lower bounds: a real variable is either bounded by a simple lower bound *or* by a variable lower bound. Thus, we have to extend the bound substitution heuristic given in [39, 42] to the more general situation considered here.

Therefore, let the MIR inequality for X^{MK} be given in the form

$$\sum_{j \in N} F_{f_{\alpha_0'}}(\alpha_j')x_j + \sum_{j \in M} \bar{F}_{f_{\alpha_0'}}(\gamma_j')\bar{y}_j \leq \lfloor \alpha_0' \rfloor. \tag{3.10}$$

Note that for $d \geq 0$ and $0 \leq \alpha < 1$, $\bar{F}_\alpha(d) = 0$. Furthermore, let $k \in M$ be the index of a real variable with γ_k not equal to zero in (3.9), $u_k < \infty$, and $\tilde{u}_k < \infty$.

To understand the effect of the different bound substitutions performable for y_k, we analyze the effect on the MIR inequality (3.10) when we use one of the different bounds imposed on y_k for the substitution of y_k in comparison to the case where no bound substitution is performed.

Using a simple bound for the substitution of y_k has a different effect than using a variable bound. On the one hand, among other changes, using a simple bound may change the value of α_0' and therefore, may change the value of $f_{\alpha_0'}$, which would lead to different values of the coefficients of all variables in the MIR inequality (3.10). On the other hand, among other changes, using a variable bound does not change the value of α_0', whereas the value of α_k' for the integer variable x_k involved in the variable bound used may change, which would lead to a different value of the coefficient of x_k in the MIR inequality (3.10).

In addition, using a lower bound for the substitution of y_k has a different effect than using an upper bound. On the one hand, using a lower bound does not change the value of γ_k'. On the other hand, using an upper bound changes the value of γ_k', i.e., if $\gamma_k \geq 0$ in (3.9), γ_k' becomes negative when using an upper bound for the substitution of y_k, and the other way around. Thus, the decision of using a lower bound or an upper bound influences the value of γ_k' and therefore, also the coefficient of $\bar{y}_k$ in the MIR inequality (3.10). If γ_k is nonnegative in (3.9), we assume that in practice using a lower bound performs better than using an upper bound. If γ_k is negative in (3.9), we can not decide in advance whether using a lower bound or using an upper bound leads to a better performance.

We suggest to use a two step procedure for deciding which bound is used for the substitution for each real variable. In the first step, we decide whether a simple or variable bound is used, i.e., we select a lower bound lb_j (simple or variable bound) and an upper bound ub_j (simple or variable). And, in the second step we decide whether a lower or upper bound is used, i.e., we decide which of the two bounds selected in the first step we will actually use for the substitution. From our analysis the question arises, which of the following three criteria for the first step does lead to the best performance of the separation algorithm in practice.

Criterium F1 *Use only simple bounds if possible.* For each $j \in M$, we define the lower bound $lb_j = l_j$ with value $lb_j^* = l_j$.

Furthermore, for each $j \in M$, we define the upper bound $ub_j = \tilde{u}_j x_j$ if $u_j = \infty$ and $ub_j = u_j$ otherwise and ub_j^* to be the corresponding bound value.

Criterium F2 *Use only variable bounds if possible.* For each $j \in M$, we define the lower bound $lb_j = \tilde{l}_j x_j$ with value $lb_j^* = \tilde{l}_j x_j^*$.

Furthermore, for each $j \in M$, we define the upper bound $ub_j = u_j$ if $\tilde{u}_j = \infty$ and $ub_j = \tilde{u}_j x_j$ otherwise and ub_j^* to be the corresponding bound value.

Criterium F3 *Use for each real variable y_j, $j \in M$ the simple bound if it is closer to y_j^* than the variable bound and the variable bound otherwise.* For each $j \in M$, we define the lower bound $lb_j = l_j$ if $l_j > \tilde{l}_j x_j^*$ and $lb_j = \tilde{l}_j x_j$ otherwise and lb_j^* to be the corresponding bound value.

Furthermore, for each $j \in M$, we define the upper bound $ub_j = u_j$ if $u_j < \tilde{u}_j x_j^*$ and $ub_j = \tilde{u}_j x_j$ otherwise and ub_j^* to be the corresponding bound value.

For some special cases, we select the following bounds in the second step. If $ub_j^* = \infty$ or $y_j^* = lb_j^*$, we substitute $y_j = lb_j + \bar{y}_j$. If $y_j^* = ub_j^*$, we substitute $y_j = ub_j - \bar{y}_j$. For the remaining cases, from our analysis the question arises, which of the following four criteria for the second step does lead to the best performance of the separation algorithm in practice.

Criterium S1 *Minimize the value of $s^* = -\sum_{j \in M, \gamma_j' < 0} \gamma_j' \bar{y}_j^*$, i.e., try to obtain a single mixed integer constraint where all real variables $\bar{y}_j$ have a nonnegative coefficient.* Substitute

$$y_j = \begin{cases} lb_j + \bar{y}_j & : \quad \gamma_j > 0, \\ ub_j - \bar{y}_j & : \quad \gamma_j < 0. \end{cases}$$

Criterium S2 *Opposite of Criterium S1: Minimize the value of $\sum_{j \in M, \gamma_j' > 0} \gamma_j' \bar{y}_j^*$, i.e., try to obtain a single mixed integer constraint where all real variables $\bar{y}_j$ have a negative coefficient.* Substitute

$$y_j = \begin{cases} lb_j + \bar{y}_j & : \quad \gamma_j < 0, \\ ub_j - \bar{y}_j & : \quad \gamma_j > 0. \end{cases}$$

Criterium S3 *Use for each real variable y_j, $j \in M$ the lower bound if it is closer to y_j^* than the upper bound and the upper bound otherwise.* Substitute

$$y_j = \begin{cases} lb_j + \bar{y}_j & : \quad y_j^* - lb_j^* \leq ub_j^* - y_j^*, \\ ub_j - \bar{y}_j & : \quad y_j^* - lb_j^* > ub_j^* - y_j^*. \end{cases}$$

Criterium S4 *Mixture of Criterium S1 and Criterium S3: Try to obtain a single mixed integer constraint where all real variables $\bar{y}_j$, $j \in M$ have a nonnegative coefficient if γ_j is nonnegative in (3.9). For the remaining variables y_j, $j \in M$*

use the lower bound if it is closer to y_j^* than the upper bound and the upper bound otherwise. Substitute

$$y_j = \begin{cases} lb_j + \bar{y}_j & : \quad \gamma_j > 0 \quad \text{or} \quad (\quad \gamma_j < 0 \text{ and } y_j^* - lb_j^* \leq ub_j^* - y_j^* \quad), \\ ub_j - \bar{y}_j & : \quad \gamma_j < 0 \text{ and } y_j^* - lb_j^* > ub_j^* - y_j^*. \end{cases}$$

Gonçalves and Ladanyi [29] suggested to perform the bound substitution heuristic in addition to the current aggregated constraint (3.9) multiplied by minus one. This modification was used because their bound substitution heuristic frequently failed to return mixed knapsack constraints. They concluded that that happened when the real variable $s = -\sum_{j \in M, \gamma_j^{'} < 0} \gamma_j^{'} \bar{y}_j$ was zero due to the in-existence of coefficients $\gamma_j^{'} < 0$ after bound substitution.

The complete bound substitution heuristic for using Criterium F3 in the first step and Criterium S3 in the second step and for not multiplying the current aggregated constraint (3.9) by minus one in addition is given in Algorithm 3.3. This algorithm can be extended straightforward for using one of the other criteria in the first and second step and for multiplying the current aggregated constraint (3.9) by minus one in addition.

Marchand and Wolsey [39, 42] suggested to use Criterium S1, Criterium S2 or Criterium S3 in the second step of the bound substitution heuristic. As already mentioned, they did not have to perform the first step, since they considered only one lower bound (simple or variable) and one upper bound (simple or variable) imposed on each real variable. Neither in [39], nor in [42] the modification of multiplying the current aggregated constraint (3.9) by minus one in addition has been mentioned.

Gonçalves and Ladanyi [29] tested the three criteria suggested by Marchand and Wolsey for the second step. The first step was also not performed because the considered the same bounds as in [39, 42].

Note that the bound substitution heuristic presented here can easily be extended to the more general case where the real variables are not restricted to be nonnegative. Furthermore, it can be extended to the case where the variable bounds are given in the form $\tilde{l}_j x_j + d_j^l$ and $\tilde{u}_j x_j + d_j^u$, where $d_j^l, d_j^u \in \mathbb{Q}$. These extensions are used in SCIP 0.81.

3.3.3 Cut Generation Heuristic

The cut generation heuristic tries to generate a c-MIR inequality which is valid for the mixed knapsack set constructed in the aggregation heuristic and the bound substitution heuristic and which is violated by the fractional vector corresponding to the constructed mixed knapsack set.

Let

$$X^{MK} = \{(x, s) \in \mathbb{Z}_+^n \times \mathbb{R}_+ : \sum_{j \in N} \alpha_j^{'} x_j \leq \alpha_0^{'} + s, x_j \leq b_j \text{ for all } j \in N\}$$

be the constructed mixed knapsack relaxation of X and $(x^*, s^*) \in (\mathbb{R}_+^n \setminus \mathbb{Z}_+^n) \times \mathbb{R}_+$ be the corresponding fractional vector. The main aspect of the cut generation heuristic is to choose a useful partition (T, U) of N and a useful constant $\delta \in \mathbb{Q}_+ \setminus \{0\}$.

Input : Mixed integer set X defined as in (3.4), simple and variable lower bounds and simple and variable upper bounds imposed on y_j for all $j \in M$ defined as in Definition 3.4 and 3.5, $(x^*, y^*) \in (\mathbb{R}^n_+ \setminus \mathbb{Z}^n_+) \times \mathbb{R}^m_+$ fractional vector, and aggregated constraint $\sum_{j \in N} \alpha_j x_j + \sum_{j \in M} \gamma_j y_j = \alpha_0$.

Output: Mixed knapsack relaxation X^{MK} of X and (x^*, s^*) corresponding fractional vector.

for $j \leftarrow 1$ **to** n **do** $\alpha_j^{'} \leftarrow \alpha_j$

$\alpha_0^{'} \leftarrow \alpha_0$

for $j \leftarrow 1$ **to** m **do**

  `/* First step: Select simple or variable bound. */`

  if $l_j > \tilde{l}_j x_j^*$ **then** $lb_j \leftarrow l_j$ and $lb_j^* \leftarrow l_j$

  else $lb_j \leftarrow \tilde{l}_j x_j$ and $lb_j^* \leftarrow \tilde{l}_j x_j^*$

  if $u_j < \tilde{u}_j x_j^*$ **then** $ub_j \leftarrow u_j$ and $ub_j^* \leftarrow u_j$

  else $ub_j \leftarrow \tilde{u}_j x_j$ and $ub_j^* \leftarrow \tilde{u}_j x_j^*$

  `/* Second step: Select lower or upper bound. */`

  if $y_j^* - lb_j^* \leq ub_j^* - y_j^*$ **then** `/* Substitute` $y_j = lb_j + \bar{y}_j$. `*/`

    if $lb_j = \tilde{l}_j x_j$ **then**

      $\gamma_j^{'} \leftarrow \gamma_j$ and $\alpha_j^{'} \leftarrow \alpha_j^{'} + \gamma_j \tilde{l}_j$

    else

      $\gamma_j^{'} \leftarrow \gamma_j$ and $\alpha_0^{'} \leftarrow \alpha_0^{'} - \gamma_j l_j$

    $\bar{y}_j^* \leftarrow y_j^* - lb_j^*$

  else `/* Substitute` $y_j = ub_j - \bar{y}_j$. `*/`

    if $ub_j = \tilde{u}_j x_j$ **then**

      $\gamma_j^{'} \leftarrow -\gamma_j$ and $\alpha_j^{'} \leftarrow \alpha_j^{'} + \gamma_j \tilde{u}_j$

    else

      $\gamma_j^{'} \leftarrow -\gamma_j$ and $\alpha_0^{'} \leftarrow \alpha_0^{'} - \gamma_j u_j$

    $\bar{y}_j^* \leftarrow ub_j^* - y_j^*$

$s \leftarrow -\sum_{j \in M, \gamma_j^{'} < 0} \gamma_j^{'} \bar{y}_j$ and $s^* \leftarrow -\sum_{j \in M, \gamma_j^{'} < 0} \gamma_j^{'} \bar{y}_j^*$

$X^{MK} \leftarrow \{(x, s) \in \mathbb{Z}^n_+ \times \mathbb{R}_+ : \sum_{j \in N} \alpha_j^{'} x_j \leq \alpha_0^{'} + s, x_j \leq b_j \text{ for all } j \in N\}$

return X^{MK} ***and*** (x^*, s^*)

Algorithm 3.3: Bound substitution heuristic. Use Criterium F3 in the first step and Criterium S3 in the second step. Do not multiply the given single mixed integer constraint by minus one in addition.

As for the bound substitution heuristic, we want to comprehend the strategy suggested by Marchand and Wolsey [39, 42] for choosing the partition, and we also want to develop new strategies. Therefore, for (T, U) partition of N and $\delta = 1$, let the c-MIR inequality for X^{MK} be given in the form

$$\sum_{j \in T} F_{f_\beta}(\alpha_j^{'}) x_j + \sum_{j \in U} F_{f_\beta}(-\alpha_j^{'})(b_j - x_j) + \sum_{j \in M} \bar{F}_{f_\beta}(\gamma_j^{'}) \bar{y}_j \leq \lfloor \beta \rfloor. \tag{3.11}$$

Note that for $d \geq 0$ and $0 \leq \alpha < 1$, $\bar{F}_\alpha(d) = 0$. Furthermore, let $k \in N$ be the index of an integer variable with $\alpha_k^{'}$ not equal to zero in X^{MK}. To understand the effect of complementing x_k, we analyze the effect on the violation of (3.11) when we complement only x_k, in comparison to the case where none of the integer variables is complemented.

The effect of complementing x_k depends on the value of $\alpha_k^{'}$ in X^{MK}. If $\alpha_k^{'}$ is negative in X^{MK}, complementing x_k leads, among other changes, to a greater or equal value of β in (3.11). In addition, the value of $F_{f_\beta}(-\alpha_k^{'})(b_k - x_k^*)$ (≥ 0) in (3.11) when complementing x_k is greater than or equal to the value of $F_{f_\beta}(\alpha_k^{'}) x_k^*$ (≤ 0) in (3.11) when not complementing x_k.

If $\alpha_k^{'}$ is nonnegative in X^{MK}, complementing x_k leads, among other changes, to a smaller or equal value of β in (3.11). In addition, the value of $F_{f_\beta}(-\alpha_k^{'})(b_k - x_k^*)$ (≤ 0) in (3.11) when complementing x_k is smaller than or equal to the value of $F_{f_\beta}(\alpha_k^{'}) x_k^*$ (≥ 0) in (3.11) when not complementing x_k.

This raises the question, whether complementing all integer variables x_j, $j \in N$

- with $\alpha_j^{'} < 0$ in X^{MK},
- with $\alpha_j^{'} > 0$ in X^{MK}, or
- for which x_j^* is closer to b_j than to the lower bound zero

leads to a better performance of the separation algorithm in practice. The following cut generation procedure suggested by Marchand [39] uses the last strategy for choosing the partition (T, U) of N.

Procedure 1 Take the initial partition (T, U) of N with

$$U = \{j \in N : x_j^* \geq \frac{b_j}{2}\}.$$

Select

$$\delta \in N^* = \{|\alpha_j^{'}| : j \in N, \alpha_j^{'} \neq 0 \text{ and } 0 < x_j^* < b_j\}$$

such that the resulting c-MIR inequality for X^{MK} has the greatest violation, i.e., such that

$$\sum_{j \in T} F_{f_\beta}(\frac{\alpha_j^{'}}{\delta}) x_j^* + \sum_{j \in U} F_{f_\beta}(-\frac{\alpha_j^{'}}{\delta})(b_j - x_j^*) - \lfloor \beta \rfloor - \frac{s^*}{\delta(1 - f_\beta)}$$

is maximized, where $\beta = \frac{\alpha_0^{'} - \sum_{j \in U} \alpha_j^{'} b_j}{\delta}$. If $f_\beta = 0$ for all $\delta \in N^*$, some additional integer variables x_j, $j \in T$ lying strictly between their bounds are complemented, ordered by nonincreasing value of $x_j^* - \frac{b_j}{2}$.

Try to improve the violation of the c-MIR inequality by

a) modifying δ dividing it by 2, 4 and 8 and
b) successively complementing each integer variable x_j, $j \in T$ lying strictly between its bounds, ordered by nonincreasing value of $x_j^* - \frac{b_j}{2}$.

We suggest to test in addition the following two procedures, which differ from Procedure 1 by the chosen initial partition (T, U) of N.

Procedure 2 Like Procedure 1 but take the initial partition (T, U) of N with

$$U = \{j \in N : \alpha_j^{'} < 0 \text{ and } x_j^* > 0\} \cup \{j \in N : x_j^* = b_j\}.$$

Procedure 3 Like Procedure 1 but take the initial partition (T, U) of N with

$$U = \{j \in N : \alpha_j^{'} > 0, \text{ and } x_j^* > 0\} \cup \{j \in N : x_j^* = b_j\}.$$

To understand the strategy used by Marchand and Wolsey [39, 42] in Procedure 1 for choosing the constant δ, note that for $d \in \mathbb{Q}$ and $0 < \alpha < 1$,

$$\left\{ \begin{array}{r} \lfloor d \rfloor \leq F_\alpha(d) < d \\ F_\alpha(d) = d \end{array} \right\} \text{ if } \left\{ \begin{array}{l} d - \lfloor d \rfloor > 0 \\ d - \lfloor d \rfloor = 0 \end{array} \right\} .$$

This is, the maximum value of $F_\alpha(d)$ is d, and $F_\alpha(d)$ is equal to d if d is integral.

Furthermore, for $0 < d - \lfloor d \rfloor \leq \alpha$, the value of $d - F_\alpha(d)$ is the smaller the smaller the value of $d - \lfloor d \rfloor$ is, since $F_\alpha(d) = \lfloor d \rfloor$. For $\alpha < d - \lfloor d \rfloor < 1$, the value of $d - F_\alpha(d)$ is the smaller the larger the value of $d - \lfloor d \rfloor$ is.

Therefore, choosing $\delta \in \mathbb{Q}_+ \backslash \{0\}$ such that $\frac{\alpha_j^{'}}{\delta} - \lfloor \frac{\alpha_j^{'}}{\delta} \rfloor$ for $j \in T$ and $-\frac{\alpha_j^{'}}{\delta} - \lfloor -\frac{\alpha_j^{'}}{\delta} \rfloor$ for $j \in U$ are very small (at best equal to zero) or very large, may improve the chance of finding a violated c-MIR inequality valid for X^{MK}.

In Procedure 1, the candidate set for the value of δ is defined in such a way that for at least one integer variable x_j, $j \in N$, $\frac{\alpha_j^{'}}{\delta}$ and $-\frac{\alpha_j^{'}}{\delta}$, respectively is integral. We suggest to extend the candidate set for the value of δ to the set

$$N^* \cup \{1 + \max\{|\alpha_j^{'}| : j \in N\}\}.$$

The additional candidate for the value of δ is chosen such that for some of the integer variables x_j, $j \in N$, $\frac{\alpha_j^{'}}{\delta} - \lfloor \frac{\alpha_j^{'}}{\delta} \rfloor$ and $-\frac{\alpha_j^{'}}{\delta} - \lfloor -\frac{\alpha_j^{'}}{\delta} \rfloor$, respectively may be very small or very large.

The complete cut generation heuristic for using Procedure 1 and not using the extended candidate set for the value of δ is given in Algorithm 3.4. This algorithm can be extended straightforward for using one of the other procedures and the extended candidate set for the value of δ. Note that the violation of a c-MIR inequality with small value of f_β is probably very small. Therefore, in SCIP 0.81, we do not further use c-MIR inequalities generated within the cut generation heuristic with $f_\beta <$ MINFRAC for MINFRAC $= 0.05$.

Marchand and Wolsey [39, 42] and Gonçalves and Ladanyi [29] use Procedure 1, but with $N^* = \{\alpha_j^{'} : j \in N \text{ and } 0 < x_j^* < b_j\}$ as candidate set for the value of δ. We changed the definition of N^* in Procedure 1 because in Theorem 3.3, δ is restricted

Input : Mixed knapsack set X^{MK} given in the form (3.7) and $(x^*, s^*) \in (\mathbb{R}^n_+ \backslash \mathbb{Z}^n_+) \times \mathbb{R}_+$ fractional vector.

Output: Valid c-MIR inequality for X^{MK} or notification that no such inequality was found.

`/* Use initial partition and initial value of δ. */`

$U \leftarrow \{j \in N : x^*_j \geq \frac{b_j}{2}\}$, $T \leftarrow N \backslash U$ and $t \leftarrow |T|$

Sort T by nonincreasing value of $x^*_j - \frac{b_j}{2}$. (Let $\{j_1, \ldots, j_t\}$ be the ordered set.)

$N^* \leftarrow \{|\alpha'_j| : j \in N, \alpha'_j \neq 0$ and $0 < x^*_j < b_j\}$

$\delta_{found} \leftarrow FALSE$, $v_{best} \leftarrow -\infty$, and $l \leftarrow 0$

repeat

  `/* Complement an additional variable. */`

  if $l > 0$ ***and*** $x^*_{j_l} > 0$ **then** $U \leftarrow U \cup \{j_l\}$ and $T \leftarrow T \backslash \{j_l\}$

  else if $l > 0$ **then** $l \leftarrow l + 1$ and **continue**

  foreach $\delta \in N^*$ **do**

    $\beta \leftarrow \frac{\alpha'_0 - \sum_{j \in U} \alpha'_j b_j}{\delta}$

    if $\beta \in \mathbb{Z}$ **then continue**

    else

      $\delta_{found} \leftarrow TRUE$

      $v \leftarrow \sum_{j \in T} F_{f_\beta}(\frac{\alpha'_j}{\delta}) x^*_j + \sum_{j \in U} F_{f_\beta}(-\frac{\alpha'_j}{\delta})(b_j - x^*_j) - \lfloor \beta \rfloor - \frac{s^*}{\delta(1 - f_\beta)}$

      if $v > v_{best}$ **then** $\delta_{best} \leftarrow \delta$ and $v_{best} \leftarrow v$

  $l \leftarrow l + 1$

until $\delta_{found} = TRUE$ ***or*** $l = t$

if $\delta_{found} = FALSE$ **then return** *No inequality found*

`/* Improve violation by modifying δ. */`

$\bar{\delta} \leftarrow \delta_{best}$

foreach $\delta \in \{\frac{\bar{\delta}}{2}, \frac{\bar{\delta}}{4}, \frac{\bar{\delta}}{8}\}$ **do**

  $\beta \leftarrow \frac{\alpha'_0 - \sum_{j \in U} \alpha'_j b_j}{\delta}$

  $v \leftarrow \sum_{j \in T} F_{f_\beta}(\frac{\alpha'_j}{\delta}) x^*_j + \sum_{j \in U} F_{f_\beta}(-\frac{\alpha'_j}{\delta})(b_j - x^*_j) - \lfloor \beta \rfloor - \frac{s^*}{\delta(1 - f_\beta)}$

  if $v > v_{best}$ **then** $\delta_{best} \leftarrow \delta$ and $v_{best} \leftarrow v$

$\delta \leftarrow \delta_{best}$

`/* Improve violation by complementing additional variables. */`

$U_{best} \leftarrow U$, $T_{best} \leftarrow T$ and $t \leftarrow |T|$

Sort T by nonincreasing value of $x^*_j - \frac{b_j}{2}$. (Let $\{j_1, \ldots, j_t\}$ be the ordered set.)

for $l \leftarrow 1$ **to** t **do**

  if $x^*_{j_l} > 0$ **then**

    $U \leftarrow U \cup \{j_l\}$ and $T \leftarrow T \backslash \{j_l\}$

    $\beta \leftarrow \frac{\alpha'_0 - \sum_{j \in U} \alpha'_j b_j}{\delta}$

    $v \leftarrow \sum_{j \in T} F_{f_\beta}(\frac{\alpha'_j}{\delta}) x^*_j + \sum_{j \in U} F_{f_\beta}(-\frac{\alpha'_j}{\delta})(b_j - x^*_j) - \lfloor \beta \rfloor - \frac{s^*}{\delta(1 - f_\beta)}$

    if $v > v_{best}$ **then** $U_{best} \leftarrow U$, $T_{best} \leftarrow T$ and $v_{best} \leftarrow v$

$U \leftarrow U_{best}$ and $T \leftarrow T_{best}$

return $\sum_{j \in T} F_{f_\beta}(\frac{\alpha'_j}{\delta}) x_j + \sum_{j \in U} F_{f_\beta}(-\frac{\alpha'_j}{\delta})(b_j - x_j) \leq \lfloor \beta \rfloor + \frac{s}{\delta(1 - f_\beta)}$

Algorithm 3.4: Cut generation heuristic. Use Procedure 1 and do not use the extended candidate set for the value of δ.

to be in $\mathbb{Q}_+\backslash\{0\}$. Note that in [42], as well as in [29] complementing of additional integer variables if $f_\beta = 0$ for all $\delta \in N^*$ in Procedure 1 has not been mentioned.

Note that if we complement an nonnegative integer variable, we actually perform a bound substitution using the upper bound imposed on the variable. And analog, if we do not complement a nonnegative integer variable, we perform a bound substitution using the lower bound imposed on the variable. If we use this interpretation, the cut generation heuristic can easily be extended to the more general case where the integer variables are not restricted to be nonnegative. This extension is used in SCIP 0.81.

3.3.4 Numerical Issues

Numerical difficulties may occur in floating point computing. To avoid numerical troubles for our separation algorithm, we take the following measures.

The first measure concerns the aggregation heuristic. Let

$$\sum_{j\in N} \alpha_j x_j + \sum_{j\in M} \gamma_j y_j = \alpha_0$$

be the current aggregated constraint. Let $k \in M$ be the index of a real variable which is candidate to be eliminated and $r \in P\backslash Q$ be a constraint where this variable appears. Let ω^i for all $i \in Q$ be the aggregation factors of the constraints which have been aggregated to form the current aggregated constraint. We only allow constraint r to be added to the current aggregated constraint if

$$\tfrac{\max\{|\omega^i|:i\in Q\}}{|\omega^r|} \leq 10,000 \qquad \text{and} \qquad \tfrac{|\omega^r|}{\min\{|\omega^i|:i\in Q\}} \leq 10,000,$$

where ω^r is the aggregation factor of constraint r if this constraint is used in the aggregation heuristic. Thus, we guarantee

$$\frac{\max\{|\omega^i| : i \in Q\}}{\min\{|\omega^i| : i \in Q\}} \leq 10,000$$

for all aggregated constraints generated by the aggregation heuristic. This is done to avoid the summation of numbers with extremely different values, i.e., the summation of very small and very large numbers.

The second measure concerns the cut generation heuristic. Let

$$\sum_{j\in T} F_{f_\beta}(\frac{\alpha_j^{'}}{\delta})x_j + \sum_{j\in U} F_{f_\beta}(-\frac{\alpha_j^{'}}{\delta})(b_j - x_j) \leq \lfloor\beta\rfloor + \frac{s}{\delta(1-f_\beta)}$$

be a c-MIR inequality found within the cut generation heuristic. We do not further use this c-MIR inequality if $f_\beta >$ MAXFRAC for MAXFRAC $= 0.95$. This is done to avoid large coefficients of the integer and real variables in the generated c-MIR cuts.

3.4 Computational Study

In Section 3.3, we gave an outline of the separation algorithm for the class of c-MIR inequalities and discussed different algorithmic and implementation choices which have to be made when implementing this separation algorithm. In this section, we describe our computational experience with these choices.

We divided the initial test set (see Section 2.3) into two sets; the *main test set* and the *remaining test set*.

Main test set Contains all instances of the initial test set for which the default algorithm or at least one of the different versions of the default algorithm where a single aspect is altered leads to an initial gap closed of more than zero percent.

Remaining test set Contains the remaining instances of the initial test set.

We use the main test set to evaluate the effect of using the different versions of our default separation algorithm and to develop the final efficient separation algorithm. This set consists of 78 MIPs, 31 are various instances from Miplib 2003 [3], 23 are instances from Miplib 3.0 [14] and 24 are members of the MIP collection of Mittelmann [45]. Table B.1 summarizes the main characteristics of the instances in the main test set. The remaining test set will only be used to ensure that the CPU time spent in our final separation algorithm is on an acceptable level for all instances in the initial test set. Table B.2 summarizes the main characteristics of the instances in the remaining test set.

See Section 2.3, for information about the workstation on which we performed our computational experiments, about the implementation environment of the cutting plane separator and about the representation of our test sets and our computational results.

Default Algorithm

Our default algorithm for separating the class of c-MIR inequalities is given in Algorithm 3.1, which calls Algorithm 3.2 (aggregation heuristic), Algorithm 3.3 (bound substitution heuristic) and Algorithm 3.4 (cut generation heuristic) as subroutines, i.e., in Section 3.3 we have already stated the four algorithms using the procedures, criteria and parameters of the default algorithm.

For our main test set, the results for applying our default algorithm are given in Table B.3 and a summary of the results is contained in Table 3.1. By using our default algorithm for the main test set, we are able to close 16.29 percent of the initial gap in geometric mean. The CPU time spent in the separation routine is 8259.5 seconds in total. For 11 instances in the main test set, the separation time is greater than 60 seconds of CPU time, for 4 instances, namely atlanta-ip, momentum2, msc98-ip and net12, the separation time is even greater than 600 seconds of CPU time. Thus, the separation time is unacceptable high.

As we will see, the large amount of time spent in the separation routine for some of the instances in our test set is caused by the large number of starting constraints used in each separation round. We have decided, to test first the different algorithmic and implementation choices discussed in Section 3.3 in order to obtain a separation algorithm which uses the best choices with respect to the effect of the separation algorithm, i.e., with respect to the initial gap closed ignoring the separation time. We will refer to this algorithm as *resulting algorithm (slow version)*. Afterwards we

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)		Sepa Time > 60 sec (Number)		Sepa Time > 600 sec (Number)	
	Value	△	Value	△	Value	△	Value	△
Default algorithm	16.29	0.00	8259.5	0.0	11	0	4	0
Aggr. heur. - 1. modification[1]	21.46	5.17	12210.5	3951.0	15	4	7	3
Aggr. heur. - 2. modification[2]	14.69	-1.60	8960.3	700.8	10	-1	5	1
B. subst. heur. - 1. modification[3]	11.52	-4.77	5191.8	-3067.7	6	-5	2	-2
B. subst. heur. - 2. modification[4]	16.27	-0.02	8327.8	68.3	11	0	4	0
B. subst. heur. - 3. modification[5]	11.40	-4.89	6669.4	-1590.1	8	-3	2	-2
B. subst. heur. - 4. modification[6]	13.27	-3.02	6490.2	-1769.3	8	-3	4	0
B. subst. heur. - 5. modification[7]	15.23	-1.06	6985.2	-1274.3	8	-3	3	-1
B. subst. heur. - 6. modification[8]	15.10	-1.19	13902.3	5642.8	14	3	7	3
Cut gen. heur. - 1. modification[9]	13.01	-3.28	5677.0	-2582.5	11	0	4	0
Cut gen. heur. - 2. modification[10]	15.65	-0.64	6290.1	-1969.4	10	-1	3	-1
Cut gen. heur. - 3. modification[11]	17.22	0.93	9255.2	995.7	12	1	4	0
Cut gen. heur. - 4. modification[12]	15.56	-0.73	5886.7	-2372.8	11	0	2	-2
Cut gen. heur. - 5. modification[13]	15.49	-0.80	8279.5	20.0	11	0	4	0
Resulting algo. (slow v.)[1] [11] [14]	22.78	6.49	11995.6	3736.1	14	3	5	1

Table 3.1: Summary of the computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Default algorithm*, *default algorithm where a single algorithmic aspect is altered* and *resulting algorithm (slow version)*. (△ with respect to the default algorithm)

describe methods for reducing the separation time of the obtained algorithm and state the resulting fast and effective separation algorithm.

Parameter MAXAGGR

In the default algorithm, we use MAXAGGR $= 6$, i.e., we allow not more than six constraints to be added to a starting constraint when generating mixed knapsack relaxations of X. Marchand [39] used MAXAGGR $= 6$ and Marchand and Wolsey [42] set MAXAGGR to 5, because they had not observed a significant increase in the efficacy of their separation heuristic with MAXAGGR greater than 5.

In our computational study, we tested the effect of using differ values of the parameter MAXAGGR for our main test set. In Figure 3.2, the initial gap closed in geometric mean achieved and the separation time in CPU seconds in geometric mean required when setting MAXAGGR $= 0$, i.e., not performing any aggregation, to MAXAGGR $= 10$ are given. As one can see, using aggregation leads to a significantly

[1]Use Score Type 3.
[2]Use Score Type 4.
[3]Use Criterium F1 in the first step.
[4]Use Criterium F2 in the first step.
[5]Use Criterium S1 in the second step.
[6]Use Criterium S2 in the second step.
[7]Use Criterium S4 in the second step.
[8]Multiply the given single mixed integer constraint by minus one in addition.
[9]Use Procedure 2.
[10]Use Procedure 3.
[11]Use the extended candidate set for the value of δ
[12]If $f_\beta = 0$ for all $\delta \in N^*$, do not complement additional integer variables x_j, $j \in T$ lying strictly between their bounds.
[13]Do not try to improve the violation of the c-MIR inequality by successively complementing each integer variable x_j, $j \in T$ lying strictly between its bounds.
[14]Use MAXAGGR $= 5$.

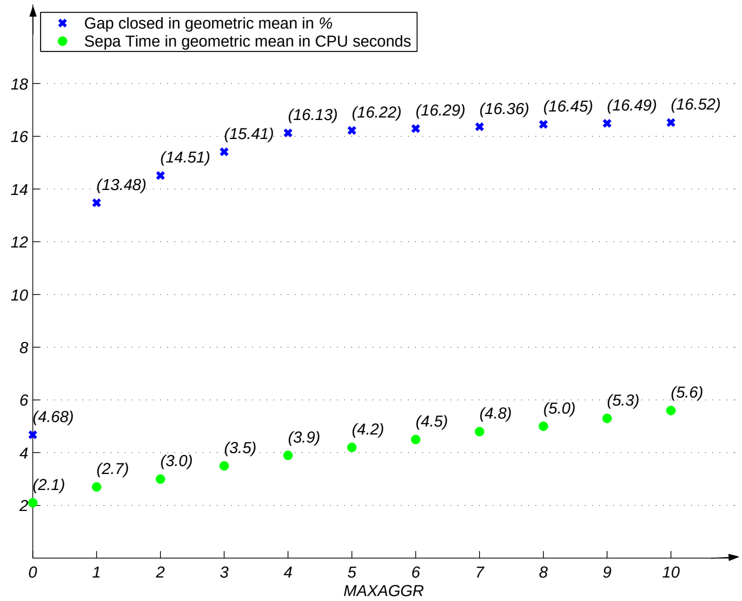

Figure 3.2: Computational results for the cutting plane separator for the class of c-MIR inequalities. *Parameter* MAXAGGR. Use MAXAGGR $= 0, \ldots, 10$.

improved performance of the separation algorithm. The initial gap closed in geometric mean increases by 8.80 percentage points from 4.68 percent to 13.48 percent when allowing one constraint to be added to a starting constraint. The results also confirm Marchand's and Wolsey's observation that there is no significant increase in the efficacy with MAXAGGR greater than 5. Thus, in our resulting algorithm (slow version) we use MAXAGGR $= 5$.

Aggregation Heuristic

In Section 3.3.1, we stated an algorithm which adds a single mixed integer constraint from the formulation of X to the current aggregated constraint in order to eliminate a real variable appearing in the current aggregated constraint. This algorithm is the first part of the construction of a mixed knapsack relaxation of X. The crucial aspect of the aggregation heuristic is the selection of the real variable to be eliminated and the constraint added. In Section 3.3.1, we stated four types of scores which can be used to select the constraint used to eliminate a selected real variable. In the default algorithm we use Score Type 1, i.e., we select the constraint found first. This criterium is very simple, since it does not take into account any information about the constraints which are candidates to be added.

In our computational study, we tested Score Type 3 and Score Type 4, but not Score Type 2, since we want to obtain a deterministic separation algorithm. The results for using Score Type 3 obtained on our main test set are given in Table B.4 and a summary of the results is contained in Table 3.1. The initial gap closed in geometric mean increases by 5.17 percentage points. Thus, by taking into account the LP solution of the dual variable corresponding to the original MIP row, the density of the original MIP row, the slack value of the constraint and the number of times the constraint has already been involved in an aggregation for the selection of the constraint to be added, we significantly improve the efficacy of the separation algorithm.

The results for using Score Type 4 obtained on the main test set are given in Table B.5 and a summary of the results is contained in Table 3.1. Using this type of score, i.e., minimizing the number of real variables with a nonzero coefficient in the new aggregated constraint when eliminating a selected real variable, does not lead to an improved performance of the separation algorithm. The initial gap closed reduces by 1.60 percentage points. Thus, in our resulting algorithm (slow version) we use Score Type 3.

Bound Substitution Heuristic

In Section 3.3.2, we have explained why it is reasonable to use a two step procedure for deciding which bound is used for the substitution of each real variable appearing in the single mixed integer constraint constructed in the aggregation heuristic. In our default algorithm, we use Criterium F3 in the first step, i.e., we use for each real variable y_j, $j \in M$ the simple bound if it is closer to y_j^* than the variable bound and the variable bound otherwise, and Criterium S3 in the second step, i.e., we use for each real variable y_j, $j \in M$ the lower bound if it is closer to y_j^* than the upper bound and the upper bound otherwise.

For the first step of the bound substitution heuristic, we have tested to use Criterium F1 and Criterium F2. The results obtained on the main test set for using Criterium F1 are given in Table B.6 (see also Table 3.1, for a summary of the results). Using Criterium F1, i.e., using only simple bounds if possible, reduces the initial gap closed in geometric mean by 4.77 percentage points. Using Criterium F2 (see Table 3.1, for a summary of the results obtained on the main test set), i.e., using only variable bounds if possible, leads for all instances in our main test set except vpm2 to the same value of the initial gap closed and the same number of cuts. This suggests that for our main test set by applying Criterium F3 and Criterium S3 (as it is done in the default algorithm), variable bounds are mostly selected for the bound substitution of each real variable y_j, $j \in M$ if possible.

For the second step of the bound substitution heuristic, we have tested to use Criterium S1, Criterium S2 and Criterium S4. The results obtained on the main test set for using the first two criteria are given in Table B.7 and Table B.8 and a summary of the results is contained in Table 3.1. As one can see, neither using Criterium S1, i.e., trying to obtain a single mixed integer constraint where all real variables have a nonnegative coefficient, nor using Criterium S2, i.e., trying to obtain a single mixed integer constraint where all real variables have a negative coefficient, leads to an improved performance of the separation algorithm. For the main test set, using Criterium S4 (see Table B.9 and the summary contained in Table 3.1), which is a mixture of Criterium S1 and Criterium S3, leads to a greater value of the initial gap closed in geometric mean than using Criterium S1 or Criterium S2, but the initial gap closed in geometric mean is smaller than the one achieved by using Criterium S3.

We conclude that using the closest bound for the substitution for each real variable leads to the best performance of the separation algorithm. As we will see, this is also the case for substituting the integer variables in the cut generation heuristic. Thus, in our resulting algorithm (slow version) we use the same criteria for selecting the bounds for the substitution as in the default algorithm, i.e., Criterium F3 and Criterium S3.

We also tested the modification to perform the bound substitution heuristic in addition to the current aggregated constraint multiplied by minus one, which Gonçalves and Ladanyi suggested in [29]. The results for our main test set are given in Table B.10 and a summary of the results is contained in Table 3.1. For some of the instances in our main test set, the modification leads to a greater value of the initial gap closed while for others it leads to a smaller value. The initial gap closed in geometric mean reduces by 1.19 percentage points. In addition, the time spent in the separation algorithm in total increases by 5642.8 seconds of CPU time. Thus, we have decided not to use this modification in our resulting algorithm (slow version).

Cut Generation Heuristic

In Section 3.3.3, we introduced the cut generation heuristic suggested by Marchand and Wolsey [39, 42], which we denoted by Procedure 1. This procedure is used in our default algorithm.

We tested it against Procedure 2 and Procedure 3, which differ from Procedure 1 by the chosen initial partition (T, U) of N. The results for our main test set are given in Table B.11 and Table B.12 (see also Table 3.1, for a summary of the results). Neither of them leads to an improved performance of the separation algorithm. The initial gap closed in geometric mean reduces by 3.28 percentage points when using Procedure 2, i.e., when complementing initially all integer variables x_j, $j \in N$ with a *negative* coefficient in the constructed mixed knapsack set, and by 0.64 percentage points when using Procedure 3, i.e., when complementing initially all integer variables x_j, $j \in N$ with a *nonnegative* coefficient in the constructed mixed knapsack set. We conclude that, analogue to the bound substitution heuristic, in practice complementing all integer variables x_j, $j \in N$ for which x_j^* is closer to the upper bound than to the lower bound leads to the best performance of the separation algorithm. Thus, in our resulting algorithm (slow version) we use the same procedure for the cut generation heuristic as in the default algorithm, i.e., we use Procedure 1.

In addition, we tested to use the extended candidate set for the value of δ. The results for our main test set given in Table B.13 show that this modification improves the performance of the separation algorithm (see also Table 3.1, for a summary of the results). The initial gap closed in geometric mean increases by 0.93 percentage points. Thus, we will use the extended candidate set for the value of δ in our resulting algorithm (slow version).

In Procedure 1, we complement additional integer variables if $f_\beta = 0$ for all $\delta \in N^*$. Since this step is not mentioned in [42] and [29], we have investigated the effect of this step on the performance of the separation algorithm, by testing Procedure 1 without this step. The results for our main test set are given in Table B.14 and a summary of the results is contained in Table 3.1. The initial gap closed in geometric mean reduces by 0.73 percentage points. We conclude that the step is useful for an efficient cutting plane separator and will therefore not leave it in our resulting

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)		Sepa Time > 60 sec (Number)		Sepa Time > 600 sec (Number)	
	Value	△	Value	△	Value	△	Value	△
Resulting algo. (slow v.)	22.78	0.00	11995.6	0.0	14	0	5	0
Resulting algo. (fast v.)[15]	20.90	-1.88	675.4	-11320.2	2	-12	0	-5

Table 3.2: Summary of the computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Resulting algorithm (slow version)* and *resulting algorithm (fast version).* (△ with respect to the resulting algorithm (slow version))

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)		Sepa Time > 60 sec (Number)		Sepa Time > 600 sec (Number)	
	Value	△	Value	△	Value	△	Value	△
Resulting algo. (slow v.)	1.09	0.00	3778.1	0.0	8	0	1	0
Resulting algo. (fast v.)[15]	1.09	0.00	310.8	-3467.3	2	-6	0	-1

Table 3.3: Summary of the computational results for the cutting plane separator for the class of c-MIR inequalities on the remaining test set. *Resulting algorithm (slow version)* and *resulting algorithm (fast version).* (△ with respect to the resulting algorithm (slow version))

algorithm (slow version).

In Procedure 1, we try to improve the violation of the found c-MIR inequality by successively complementing each integer variable x_j, $j \in T$ lying strictly between its bounds. As this might by very time consuming, we have tested the effect of this step on the performance of the separation algorithm, by using Procedure 1 without this step. The results for our main test set are given in Table B.15 and a summary of the results is contained in Table 3.1. The initial gap closed in geometric mean reduces by 0.80 percentage points. Since furthermore the time spent in the separation routine in total is nearly the same as for the default algorithm, we conclude that the step is also useful for an efficient cutting plane separator and we will therefore not leave it in the resulting algorithm (slow version).

In summary, in our resulting algorithm (slow version), we perform Procedure 1 with the extended candidate set for the value of δ.

Resulting Algorithm

From the results of our computational study, we obtain the following best algorithmic and implementation choices for the separation algorithm for the class of c-MIR inequalities.

Parameter MAXAGGR Use MAXAGGR = 5.

Aggregation heuristic Use Score Type 3 for selecting the constraint to be added to the current aggregated constraint.

Bound substitution heuristic As is the default algorithm, use Criterium F3 in the first step and Criterium S3 in the second step for selecting the bound used for substituting each real variable and do not perform the bound substitution heuristic in addition for the current aggregated constraint multiplied by minus one.

[15] Use MAXTESTDELTA = 10. Select starting constraints $i \in P$ by nonincreasing value of CONSSCORE^i. Use MAXFAILS = 150, MAXCONTS = 20, MAXCUTS = 100 and MAXROUNDS = 15.

Cut generation heuristic Perform Procedure 1, but use the extended candidate set for the value of δ.

We call the corresponding separation algorithm *resulting algorithm (slow version)*. For our main test set, the results for our resulting algorithm (slow version) are given in Table B.16 and a summary of the results is contained in Table 3.1 and Table 3.2. By using this version of the separation algorithm we close 22.78 percent of the initial gap in geometric mean in contrast to 16.29 percent for using our default algorithm. Thus, we have significantly improved the performance of our default separation algorithm with respect to the initial gap closed. But, as for the default algorithm, the CPU time spent the separation routine is on an unacceptable level. It even increases to 11,995.6 seconds of CPU time in total. For 14 instances in the main test set the separation time is greater than 60 seconds of CPU time and for 5 instances, namely a1c1s1, atlanta-ip, momentum2, msc98-ip and net12, it is even greater than 600 seconds of CPU time. For 8 of the 14 instances, the initial gap closed is smaller than 5 percent, including atlanta-ip, momentum2, msc98-ip and net12, i.e., the separation algorithm is time consuming but not very effective for them.

For our remaining test set, the results for using our resulting algorithm (slow version) are given in Table B.17 and a summary of the results is contained in Table 3.3. The initial gap closed is zero percent for all instances in the remaining test set except for swath1 (5.91 percent), swath2 (5.20 percent) and swath3 (4.16 percent). That means, for these instances, where neither using the default algorithm nor using the default algorithm with a single aspect altered leads to an initial gap closed of more than zero percent, the combination of the best aspects improves the performance of the cutting plane separator. The separation time is 3,778.1 seconds of CPU time in total. For 8 instances in the remaining test set, the separation time is greater than 60 seconds of CPU time and for one instance, namely neos19, it is even greater than 600 seconds of CPU time. Thus, as for the main test set, the separation time is on an unacceptable level. Since we want to implement an efficient cutting plane separator for the class of c-MIR inequalities, we have to find methods for reducing the separation time without loosing too much of the initial gap closed.

The resulting algorithm (slow version), as well as the default algorithm, uses each constraint $i \in P$ as a starting constraint, i.e., it tries for each constraint $i \in P$ to generate violated c-MIR inequalities for mixed knapsack relaxations of X based on linear combinations of constraints defining X including constraint i. We suppose that the large amount of time spent in the separation routine is cause by a large number of starting constraints. Therefore, we do not want to use all possible starting constraints, but only those which may lead to cuts with a great violation. From our computational study, we know that using Score Type 3 for selecting the constraint to be added to the current aggregated constraint in the aggregation heuristic is very useful. Therefore, we use the same type of score for each possible starting constraint. For $i \in P$, let

$$\begin{aligned}\mathsf{CONSSCORE}^{\mathsf{i}} = {} & 0.9^l(\max\{\tfrac{db_i}{\max\{\|(c,d)\|,1.0\}}, 0.0001\} + 0.0001(1 - dens_i) + \\ & 0.001(1 - \tfrac{y^*_{si}}{\max\{\|(a^i,c^i)\|,0.1\}})),\end{aligned}$$

where l, db_i, (c,d), $dens_i$, s^i and (a^i, c^i) are defined as in Section 3.3.1. We select starting constraints $i \in P$ by nonincreasing value of CONSSCORE^i and limit the number of starting constraints by the following parameters.

MAXFAILS The parameter denotes the maximum number of starting constraints per separation round for which we consecutively did not obtain a violated c-MIR inequality (including the aggregation). Note that in early separation rounds we increase this value up to the double value, i.e., we allow up to MAXFAILS $+$ (MAXFAILS $- 2k)^+$ consecutive fails, where k is the number of separation rounds which have already been performed at the current branch-and-bound node.

MAXCONTS The parameter denotes the maximum number of real variables in a starting constraint which have a coefficient not equal to zero and lie strictly between their bounds. Constraints $i \in P$ for which the value is exceeded are not used as starting constraints.

We will also use the parameter MAXCONTS in addition to the parameter MAXAGGR to further limit the number of constraints added to a starting constraint. We stop adding constraints to a starting constraint if the number of real variables in the new aggregated constraint which have a coefficient not equal to zero and lie strictly between their bounds is greater than MAXCONTS.

If the separation algorithm generates violated c-MIR inequalities for nearly every starting constraint, the parameter MAXFAILS does not help to reduce the separation time. Therefore, we suggest to use in addition the following parameters.

MAXCUTS The parameter denotes the maximum number of violated c-MIR inequalities generated per separation round.

MAXROUNDS The parameter denotes the maximum number of separation rounds performed at the current branch-and-bound node.

Another point which may cause a large separation time for our separation algorithm is the fact that we test all candidates for the value of δ contained in the set N^*. If a MIP has a large number of variables, the cardinality of N^* can be very large. We suggest to limit the time spent in the cut generation heuristic by the following parameter.

MAXTESTDELTA The parameter denotes the maximum number of different values of δ from the candidate set N^* for which we generate c-MIR inequalities in the cut generation heuristic.

Note that the additional candidate for the value of δ in the extended candidate set is not counted, i.e., the c-MIR inequality for this value of δ is generated in any case.

In order to find useful values of the five parameters introduced above, we selected all instances of our main test set and of our remaining test set for which the separation time in the last test was greater than 60 seconds of CPU time and some instances of the main test set with small separation time. We applied our resulting algorithm (slow version) with the modification to select the starting constraints by nonincreasing value of CONSSCORE^i, $i \in P$ to theses instances and analyzed the behavior of the separation algorithm with respect to the five parameters.

For our main test set, the results for using MAXFAILS = 150, MAXCONTS = 20, MAXCUTS = 100, MAXROUNDS = 15 and MAXTESTDELTA = 10 are given in Table B.18 and a summary of the results is contained in Table 3.2. Here the △ values are given with respect to the resulting algorithm (slow version). The initial gap closed in geometric mean reduces only by 1.88 percentage points and the time spent in the separation routine is now 675.4 seconds of CPU time. For none of the instances in our main test set, the separation time is greater than 600 seconds of CPU time, but for two instances it is greater than 60 seconds of CPU time (neos3 (72.5 seconds) and net12 (92.2 seconds)). For our remaining test set, the results for the same test are given in Table B.19 and a summary is contained in Table 3.3. Here the △ values are also given with respect to the resulting algorithm (slow version). The initial gap closed does not change for any instance in the remaining test set and the separation time in total reduces to 310.8 seconds of CPU time, but for two instances in the remaining test set, the separation time is still greater than 60 seconds of CPU time (ds (74.4 seconds) and neos12 (85.7 seconds)). Thus, for both test sets, we were able to reduce the separation time to an acceptable level without loosing too much of the initial gap closed. However, the results also show that the cutting plane separator for the class of c-MIR inequalities is one of the most time consuming ones implemented in SCIP 0.81 (see also Chapter 7).

To give a little insight into the effect of the parameters MAXROUNDS and MAXTESTDELTA, we present some further results. Figure 3.3 summarizes the results obtained on our main test set for using MAXTESTDELTA = ∞, MAXFAILS = 150, MAXCONTS = 20, MAXCUTS = 100 and MAXROUNDS = $5, 10, \ldots, 50$. Note that the separation time in total is 317.9 seconds of CPU time for using MAXROUNDS = 5 (here only one instance in the main test set has separation time greater than 60 seconds of CPU time) and 1,670.3 seconds of CPU time for using MAXROUNDS = 50 (here seven instances in the main test set have separation time greater than 60 seconds of CPU time). From these results we conclude that for our main test, the most efficient cuts are generated in early separation rounds. Furthermore, one can see that limiting the number of separation rounds does help to reduce the time spent in the separation routine. As stated above, we decided to use MAXROUNDS = 15.

The following table summarizes the results obtained on our main test set for using MAXFAILS = 150, MAXCONTS = 20, MAXCUTS = 100 and MAXROUNDS = 15, MAXTESTDELTA = $5, 10, 50, 100$ and MAXTESTDELTA = ∞.

MAXTESTDELTA	Gap Closed % (Geom. Mean)	Sepa Time sec (Total)
∞	20.90	717.2
100	20.90	713.2
50	20.90	712.6
10	20.90	675.4
5	20.80	611.8

As one can see on, on our main test set, allowing the generation of c-MIR inequalities in the cut generation heuristic only for a small number of different values of δ from the set N^* does not causes a significant change in the performance of the separation algorithm with respect to the initial gap closed, but a small reduction of the separation time. Note that the results may also indicate that for most of the

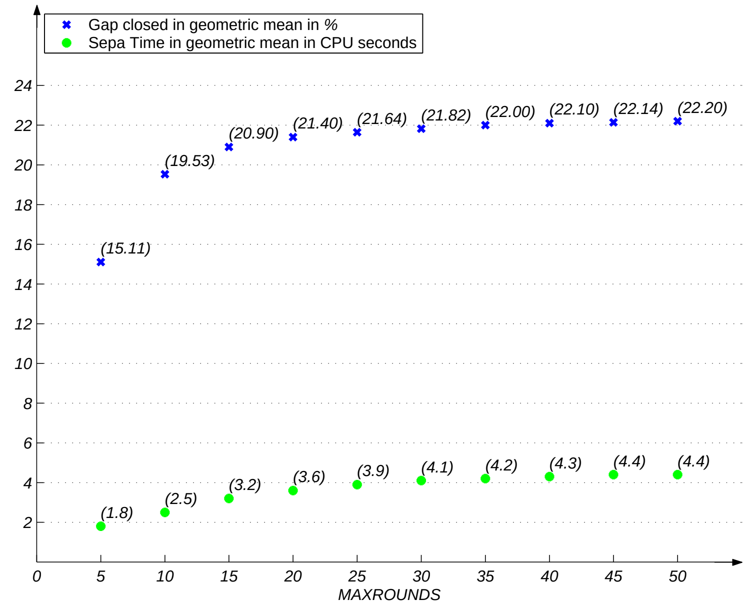

Figure 3.3: Computational results for the separation algorithm for the class of c-MIR inequalities. *Resulting algorithm (different fast versions).* Use MAXTESTDELTA $= \infty$. Select starting constraints $i \in P$ by nonincreasing value of CONSSCORE$^{\text{i}}$. Use MAXFAILS $= 150$, MAXCONTS $= 20$, MAXCUTS $= 100$ and MAXROUNDS $= 5, 10, \ldots, 50$.

instances in the main test set, the number of different values in the candidate set for the value of δ is small. Nevertheless, we decided to use MAXTESTDELTA $= 10$, since we suppose that if this cutting plane separator is combined with others, using a small value of MAXTESTDELTA may be important to keep the time spent in the cutting plane separator for the class of c-MIR inequalities on an acceptable level.

3.5 Conclusion

The results of our computational study suggest that the strategy used for constructing mixed knapsack sets strongly affects the performance of the cutting plane separator. This includes the selection of the starting constraints, the number of constraints added to the starting constraints, the selection these constraints and the selection of the bounds used for the substitution of the real variables in the aggregated constraints.

The first aspect strongly influences the time spent in the separation algorithm, which was unacceptable high for some of the instances in our test set when using all possible starting constraints. The other aspect influence the effect of the separation algorithm.

Marchand and Wolsey [39, 42] take all possible constraints and did not go into details about the selection of the constraints to be added to a starting constraint. We suppose that in our resulting cutting plane separator we use a more elaborated strategy for creating mixed knapsack sets.

Chapter 4

Cutting Plane Separator for the 0-1 Knapsack Problem

In this chapter, we investigate the implementation of an efficient cutting plane separator for the 0-1 knapsack problem. The algorithm generates strong valid inequalities for the polytope associated with the 0-1 knapsack problem, the so-called 0-1 knapsack polytope. This polytope is the convex hull of all 0-1 vectors satisfying a given linear inequality whose coefficients and right-hand side are integers. The cutting plane separator can be applied to each MIP row which represents such a linear inequality. The cuts generated for the corresponding 0-1 knapsack polytope are valid for the feasible region of the MIP, and can therefore be used in a branch-and-cut algorithm.

4.1 Introduction

Strong valid inequalities for the 0-1 knapsack polytope have been studied extensively in the literature, and many researches have investigated their usefulness to solve IPs and MIPs.

The study of the polyhedral structure of the 0-1 knapsack problem dates back to the 1970s, where Balas [8], Hammer, Johnson and Peled [33] and Wolsey [59] gave a complete characterization of the class of facets of the 0-1 knapsack polytope defined by so-called canonical inequalities. These inequalities are based on a structure called strong cover. Building on these results, Balas and Zemel [12] have investigated facets of the 0-1 knapsack polytope associated with minimal covers. The corresponding inequalities are called minimal cover inequalities. In the 1980s and 1990s, more general classes of valid inequalities were explored. Padberg [51] has introduced the class of (1,k)-configuration inequalities, which includes the class of minimal cover inequalities, and Weismantel [58] has introduced the class of extended weight inequalities, which includes the class of (1,k)-configuration inequalities.

In all these studies, the concept of lifting was used to extend inequalities which are valid for the restriction of the 0-1 knapsack polytope to some lower-dimensional space to inequalities which are valid for the original polytope. One type of lifting is called up-lifting, where variables fixed at their lower bounds are lifted. Padberg's sequential up-lifting procedure, which has been introduced in [50], was used in [8, 33, 12, 51, 58]. This procedure involves the solution of a knapsack problem in every lifting step. Zemel [61] has introduced a polynomial algorithm which uses dynamic programming to solve these knapsack problems. Another type of lifting is called down-lifting, where variables fixed at their upper bounds are lifted. The idea of sequential down-lifting has been introduced in [59], where it was used to strengthen canonical inequalities.

The inequalities derived by sequential lifting, in general, depend on the sequence in which the variables are lifted. A simultaneous up-lifting procedure to strengthen minimal cover inequalities was studied by Balas and Zemel [12], but its computational burden prevents it from being applied in practice. In [60], the property of superadditivity of the lifting function has been explored which leads to sequence independent lifting. Building on the results of [60], Gu, Nemhauser and Savelsbergh [32] and Atamturk [5] have investigated the concept of superadditive up-lifting to strengthen minimal cover inequalities. Here, the lifting function which, in general, is not superadditive is approximated by a so-called superadditive valid lifting function to obtain sequence independent lifting (see Section 2.2).

The results of the theoretical study of the 0-1 knapsack polytope have been used in linear programming based branch-and-cut algorithms to solve IPs and MIPs. Crowder, Johnson and Padberg [22] pioneered the use of lifted inequalities and successfully solved several instances of IPs which were, at the time, considered to be unsolvable. They separated the class of lifted minimal cover inequalities using sequential up-lifting and the class of lifted (1,k)-configuration inequalities using sequential up-lifting. Since then, there have been several other successful applications of lifted valid inequalities for the 0-1 knapsack polytope. Van Roy and Wolsey [55] separated the class of lifted cover inequalities using sequential up- and down-lifting. Hoffman and Padberg [35] and Gu, Nemhauser and Savelsbergh [30] implemented a cutting plane separator which separates the class of lifted minimal cover inequalities using sequential up- and down-lifting (LMCI1). In [30], a computational study was presented in which many of the algorithmic and implementation choices were evaluated which have to be made when implementing this cutting plane separator. Especially, it turned out that using both up-lifting and down-lifting instead of using only up-lifting leads to a better performance of the cutting plane separator. Martin [44] separated the class of lifted extended weight inequalities using sequential up- and down-lifting (LEWI).

For our cutting plane separator, we have to decide which of the above mentioned classes of valid inequalities we want to separate. To our knowledge, no paper has been published presenting computational results for separating the class of lifted minimal cover inequalities using superadditive up-lifting (LMCI2). Besides, we know of no paper in which the performance of a separation algorithm for the class of LMCI1 has been compared to that of a separation algorithm for the class of LEWI. Thus, it is not clear, which of these three classes of valid inequalities leads to a best performing cutting plane separator. Therefore, we investigate separation algorithms for all three classes of valid inequalities in this chapter.

In Section 4.2, we give a brief introduction to these classes of valid inequalities, and in Section 4.3, we discuss different algorithmic aspects of the corresponding separation algorithms. In Section 4.4, we present a computational study. It evaluates the effect of using the different algorithmic and implementation choices described in Section 4.3 on the performance of the three separation algorithms. We use the computational results to design three resulting separation algorithms. Finally, we investigate the effectiveness of using different combinations of our resulting separation algorithms and conclude with an efficient cutting plane separator for the 0-1 knapsack problem. In Section 4.5, we give some concluding remarks.

4.2 Strong Valid Inequalities for the 0-1 Knapsack Polytope

We consider the inequality

$$\sum_{j \in N} a_j x_j \leq a_0, \tag{4.1}$$

where a_0 and a_j are integers for all $j \in N = \{1, \ldots, n\}$ and $x_j \in \{0, 1\}$ for all $j \in N$. We assume, without loss of generality, that $a_j > 0$ for all $j \in N$ (since 0-1 variables can be complemented) and $a_j \leq a_0$ for all $j \in N$ (since $a_j > a_0$ implies $x_j = 0$). The *0-1 knapsack polytope* P associated with (4.1) is the convex hull of all 0-1 vectors satisfying (4.1), i.e., $P = \text{conv}(X^{BK})$, where

$$X^{BK} = \{x \in \{0, 1\}^n : \sum_{j \in N} a_j x_j \leq a_0\}. \tag{4.2}$$

By the above assumptions, P is full-dimensional (see [8]).

The main idea for generating strong valid inequalities for P is to start with an inequality which is valid for the restriction of X^{BK} to some lower-dimensional space and to use the concept of lifting to obtain a valid inequality for X^{BK}. In Section 2.2, we introduced the lifting theory for BMIPs. It can be used for the special case of the 0-1 knapsack problem. Let $M \subseteq N$ and $(j_1, \ldots, j_t)$ be the lifting sequence of the variables in $N \backslash M$. Let

$$0 \leq \alpha_0 - \sum_{j \in M} \alpha_j x_j \tag{4.3}$$

be a valid inequality for the restriction of X^{BK} obtained by fixing $x_j = b_j$ with $b_j \in \{0, 1\}$ for all $j \in N \backslash M$, which is given by $X^0 = \{x \in \{0, 1\}^{|M|} : \sum_{j \in M} a_j x_j \leq a_0 - \sum_{k=1}^{t} a_{j_k} b_{j_k}\}$. To construct a valid inequality

$$0 \leq \alpha_0 - \sum_{j \in M} \alpha_j x_j - \sum_{k=1}^{t} \alpha_{j_k}(x_{j_k} - b_{j_k}) \tag{4.4}$$

for X^{BK}, we start with (4.3) and lift the variables in $N \backslash M$ in the given lifting sequence. The intermediate restrictions of X^{BK} are defined by $X^i = \{x \in \{0, 1\}^{|M|+i} : \sum_{j \in M} a_j x_j + \sum_{k=1}^{i} a_{j_k}(x_{j_k} - b_{j_k}) \leq a_0 - \sum_{k=1}^{t} a_{j_k} b_{j_k}\}$ for $i = 1, \ldots, t$. Note that $X^t = X^{BK}$.

For $i = 1, \ldots, t$, the *lifting problem* L_i associated with j_i, given a valid inequality

$$0 \leq \alpha_0 - \sum_{j \in M} \alpha_j x_j - \sum_{k=1}^{i-1} \alpha_{j_k}(x_{j_k} - b_{j_k}) \tag{4.5}$$

for X^{i-1}, is to find α_{j_i} such that the inequality

$$\alpha_{j_i}(x_{j_i} - b_{j_i}) \le \alpha_0 - \sum_{j\in M} \alpha_j x_j - \sum_{k=1}^{i-1} \alpha_{j_k}(x_{j_k} - b_{j_k}) \tag{4.6}$$

is valid for X^i.

For $i = 1, \ldots, t$, let

$$Z^i = \{z \in \mathbb{R} : \exists x \in X^i \,:\, a_{j_i}(x_{j_i} - b_{j_i}) = z \text{ and } \sum_{j\in M} a_j x_j + \sum_{k=1}^{i-1} a_{j_k}(x_{j_k} - b_{j_k}) \le a_0 - \sum_{k=1}^{t} a_{j_k} b_{j_k} - z\}.$$

Furthermore, for $z \in Z^i$, let

$$\begin{aligned} h_i(z, \alpha_{j_i}) \;=\; \max\{\;& \alpha_{j_i}(x_{j_i} - b_{j_i}) \,: \\ & a_{j_i}(x_{j_i} - b_{j_i}) \;=\; z, \\ & x_{j_i} \in \{0,1\} \;\}, \end{aligned}$$

and

$$\begin{aligned} f_i(z) \;=\; \min\{\;& \alpha_0 - \sum_{j\in M} \alpha_j x_j - \sum_{k=1}^{i-1} \alpha_{j_k}(x_{j_k} - b_{j_k}) \,: \\ & \sum_{j\in M} a_j x_j + \sum_{k=1}^{i-1} a_{j_k}(x_{j_k} - b_{j_k}) \;\le\; a_0 - \sum_{k=1}^{t} a_{j_k} b_{j_k} - z, \\ & x_j \in \{0,1\} \text{ for all } j \in M \cup \{j_1, \ldots, j_{i-1}\} \;\}. \end{aligned}$$

If $b_{j_i} = 0$, observe that $Z^i = \{0, a_{j_i}\}$ or $Z^i = \{0\}$ (the last case may arise because we consider also down-lifting). Furthermore, observe that $h_i(0, \alpha_{j_i}) = 0$ and $h_i(a_{j_i}, \alpha_{j_i}) = \alpha_{j_i}$.

Consider $Z^i = \{0, a_{j_i}\}$. Then the lifting is maximal (see the definition in Section 2.2) if and only if the lifting coefficient α_{j_i} is equal to $f_i(a_{j_i})$, since this is the unique value of α_{j_i} such that

$$h_i(z, \alpha_{j_i}) = f_i(z)$$

has one solution $x \in \{0,1\}^{|M|+i}$ such that the component $x_{j_i} - b_{j_i}$ of $x - b$ is linear independent (for $z = a_{j_i}$, $h_i(z, \alpha_{j_i}) = f_i(z)$ has solution x with $x_{j_i} = 1$). Therefore, if conv(X^{i-1}) and conv(X^i) are full-dimensional and (4.5) defines a facet of conv(X^{i-1}), to obtain a facet defining inequality (4.6) for conv(X^i), the lifting coefficient α_{j_i} has to be equal to $f_i(a_{j_i})$ (see Theorem 2.10).

Now consider $Z^i = \{0\}$. Since for $z = 0$, $h_i(z, \alpha_{j_i}) \le f_i(z)$ holds for any value of α_{j_i}, inequality (4.6) is valid for X^i for any value of the lifting coefficient α_{j_i} (see Theorem 2.9).

See [32], for the special case of sequential up-lifting for minimal cover inequalities and [30], for the special case of sequential up- and down-lifting for minimal cover inequalities.

If $b_{j_i} = 1$, observe that $Z^i = \{-a_{j_i}, 0\}$, and that $h_i(-a_{j_i}, \alpha_{j_i}) = -\alpha_{j_i}$ and $h_i(0, \alpha_{j_i}) = 0$. Thus, the lifting is maximal if and only if the lifting coefficient α_{j_i} is equal to $-f_i(-a_{j_i})$, since this is the unique value of α_{j_i} such that

$$h_i(z, \alpha_{j_i}) = f_i(z)$$

has one solution $x \in \{0,1\}^{|M|+i}$ such that the component $x_{j_i} - b_{j_i}$ of $x - b$ is linear independent (for $z = -a_{j_i}$, $h_i(z, \alpha_{j_i}) = f_i(z)$ has solution x with $x_{j_i} = 0$).

Therefore, if $\text{conv}(X^{i-1})$ and $\text{conv}(X^i)$ are full-dimensional and (4.5) defines a facet of $\text{conv}(X^{i-1})$, to obtain a facet defining inequality (4.6) for $\text{conv}(X^i)$, the lifting coefficient α_{j_i} has to be equal to $-f_i(-a_{j_i})$ (see Theorem 2.10).

See [30], for the special case of sequential up- and down-lifting for minimal cover inequalities.

These observations lead to the following Theorem.

Theorem 4.1. *For each $i = 1, \ldots, t$, consider the knapsack problem $K_{\alpha_{j_i}}$ defined recursively as follows*

$$\begin{aligned} z_{j_i} = \max\{ & \sum_{j \in M} \alpha_j x_j + \sum_{k=1}^{i-1} \alpha_{j_k} x_{j_k} : \\ & \sum_{j \in M} a_j x_j + \sum_{k=1}^{i-1} a_{j_k} x_{j_k} \le a_0 - \sum_{k=i}^{t} a_{j_k} b_{j_k} - z, \\ & x_j \in \{0,1\} \text{ for all } j \in M \cup \{j_1, \ldots, j_{i-1}\} \}, \end{aligned}$$

with

$$z = \begin{cases} a_{j_i} & : \ b_{j_i} = 0, \\ -a_{j_i} & : \ b_{j_i} = 1. \end{cases}$$

For each $i = 1, \ldots, t$, let

$$\alpha_{j_i} = \begin{cases} (\alpha_0 + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}) - z_{j_i} & : \ b_{j_i} = 0 \text{ and } K_{\alpha_{j_i}} \text{ is feasible}, \\ (\alpha_0 + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}) & : \ b_{j_i} = 0 \text{ and } K_{\alpha_{j_i}} \text{ is infeasible}, \\ z_{j_i} - (\alpha_0 + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}) & : \ b_{j_i} = 1. \end{cases}$$

Then, for $i = 1, \ldots, t$, each inequality (4.6) is valid for X^i, in particular, (4.4) is valid for X^{BK}. For $i = 1, \ldots, t$, if conv(X^{i-1}) and conv(X^i) are full-dimensional, (4.5) defines a facet of conv(X^{i-1}), and $K_{\alpha_{j_i}}$ is feasible, then inequality (4.6) defines a facet of conv(X^i). In particular, if (4.3) defines a facet of conv(X^0), conv(X^i) is full-dimensional for $i = 0, \ldots, t-1$, and $K_{\alpha_{j_1}}, \ldots, K_{\alpha_{j_t}}$ are feasible, then (4.4) defines a facet of P.

In [50], Theorem 4.1 was given for the special case of using only up-lifting. In [46], a theorem similar to Theorem 4.1 can be found, but there the case of $K_{\alpha_{j_i}}$ infeasible is not handled.

In the following, we introduce two classes of strong valid inequalities for the 0-1 knapsack polytope which use the above described idea; the class of LMCI1 and the class of LEWI. In addition, we introduce the class of LMCI2, where the lifting function, which, in general, is not superadditive, is approximated by a superadditive valid lifting function to obtain sequence independent lifting.

Class of LMCI1

A set $C \subseteq N$ is called a *cover* for X^{BK} if $\sum_{j \in C} a_j > a_0$, and is called a *minimal cover* if, in addition, $\sum_{j \in C \setminus \{i\}} a_j \leq a_0$ holds for all $i \in C$. For any cover $C \subseteq N$ for X^{BK}, inequality

$$\sum_{j \in C} x_j \leq |C| - 1 \tag{4.7}$$

is called *cover inequality*, and it is valid for X^{BK}. If C is a minimal cover, inequality (4.7) is called *minimal cover inequality*, and it defines a facet of the convex hull of the restriction of X^{BK} to some lower-dimensional space.

Theorem 4.2 ([46]). *If C is a minimal cover for X^{BK} and (C_1, C_2) is any partition of C, with $C_1 \neq \emptyset$, then*

$$\sum_{j \in C_1} x_j \leq |C_1| - 1 \tag{4.8}$$

defines a facet of

$$conv(X^{BK} \cap \{x \in \{0,1\}^n : x_j = 0 \text{ for all } j \in N \setminus C, x_j = 1 \text{ for all } j \in C_2\}).$$

Theorem 4.2 generalizes the result of [8, 33, 59], where it was given for $C_2 = \emptyset$.

To obtain a strong valid inequality for P, we sequentially lift back the fixed variables. Let $(j_1, \ldots, j_t)$ be the lifting sequence of the variables in $N \setminus C_1$. For $i = 1, \ldots, t$, let $b_{j_i} \in \{0,1\}$ be the bound variable x_{j_i} is fixed to, i.e., $b_{j_i} = 0$ for all $j_i \in N \setminus C$ and $b_{j_i} = 1$ for all $j_i \in C_2$. Using Theorem 4.1 for inequality (4.8), we get the following corollary.

Corollary 4.3. *For each $i = 1, \ldots, t$, consider the knapsack problem $K_{\alpha_{j_i}}$ defined recursively as follows*

$$\begin{aligned} z_{j_i} = \max\{ & \sum_{j \in C_1} x_j + \sum_{k=1}^{i-1} \alpha_{j_k} x_{j_k} : \\ & \sum_{j \in C_1} a_j x_j + \sum_{k=1}^{i-1} a_{j_k} x_{j_k} \leq a_0 - \sum_{k=i}^{t} a_{j_k} b_{j_k} - z, \\ & x_j \in \{0,1\} \text{ for all } j \in C_1 \cup \{j_1, \ldots, j_{i-1}\} \ \}, \end{aligned}$$

with

$$z = \begin{cases} a_{j_i} & : \ b_{j_i} = 0, \\ -a_{j_i} & : \ b_{j_i} = 1. \end{cases}$$

For each $i = 1, \ldots, t$, let

$$\alpha_{j_i} = \begin{cases} (|C_1| - 1 + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}) - z_{j_i} & : \ b_{j_i} = 0 \text{ and } K_{\alpha_{j_i}} \text{ is feasible}, \\ (|C_1| - 1 + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}) & : \ b_{j_i} = 0 \text{ and } K_{\alpha_{j_i}} \text{ is infeasible}, \\ z_{j_i} - (|C_1| - 1 + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}) & : \ b_{j_i} = 1. \end{cases}$$

Then

$$\sum_{j \in C_1} x_j + \sum_{j \in N \setminus C} \alpha_j x_j + \sum_{j \in C_2} \alpha_j x_j \leq |C_1| - 1 + \sum_{j \in C_2} \alpha_j \tag{4.9}$$

is valid for X^{BK}*. If* $conv(X^i)$*, where* $X^i = \{x \in \{0,1\}^{|C_1|+i} : \sum_{j \in C_1} a_j x_j + \sum_{k=1}^{i} a_{j_k} x_{j_k} \leq a_0 - \sum_{k=i+1}^{t} a_{j_k} b_{j_k}\}$ *is full-dimensional for* $i = 0, \dots, t-1$ *and* $K_{\alpha_{j_1}}, \dots, K_{\alpha_{j_t}}$ *are feasible, then (4.9) defines a facet of* P*.*

We call inequality (4.9) *lifted minimal cover inequality using sequential up- and down-lifting* (LMCI).

Example 4.4. Consider the 0-1 knapsack polytope $P = \text{conv}(X^{BK})$ with

$$X^{BK} = \{x \in \{0,1\}^5 : x_1 + x_2 + 6x_3 + 2x_4 + 2x_5 \leq 8\}.$$

Then $C = \{3, 4, 5\}$ is a minimal cover for X^{BK}. For the partition (C_1, C_2) of C with $C_2 = \{5\}$, by Theorem 4.2, the inequality $x_3 + x_4 \leq 1$ defines a facet of

$$\text{conv}(X^{BK} \cap \{x \in \{0,1\}^5 : x_1 = x_2 = 0, x_5 = 1\}).$$

Choosing the lifting sequence $(1, 2, 5)$, by Corollary 4.3, we obtain the lifting coefficients $\alpha_1 = 0$, $\alpha_2 = 0$ and $\alpha_3 = 1$. The resulting inequality

$$x_3 + x_4 + x_5 \leq 2 \tag{4.10}$$

defines a facet of P. For the partition (C_1, C_2) of C with $C_2 = \{3\}$, by Theorem 4.2 the inequality $x_4 + x_5 \leq 1$ defines a facet of

$$\text{conv}(X^{BK} \cap \{x \in \{0,1\}^5 : x_1 = x_2 = 0, x_3 = 1\}).$$

Choosing the lifting sequence $(1, 2, 3)$, by Corollary 4.3 we obtain the lifting coefficients $\alpha_1 = 1$, $\alpha_2 = 0$ and $\alpha_3 = 2$. The resulting inequality

$$x_1 + 2x_3 + x_4 + x_5 \leq 3 \tag{4.11}$$

defines a facet of P.

Class of LEWI

In [58], the class of extended weight inequalities has been introduced. The definition of an extended weight inequality given in [58] is based on mutually disjoint subsets T, I and $\{z\}$ of N. Here, we give the definition for $I = \emptyset$.

Definition 4.5. Let $T \subseteq N$ and $z \in N \backslash T$ satisfying $\sum_{j \in T} a_j \leq a_0$ and $\sum_{j \in T} a_j + a_z > a_0$. Setting $r = a_0 - \sum_{j \in T} a_j$, the *extended weight inequality* defined for $T \cup \{z\}$ is of the form

$$\sum_{j \in T} x_j + c_z x_z \leq |T|, \tag{4.12}$$

with $c_z = \min\{\sum_{j \in T} x_j : \sum_{j \in T} a_j x_j \geq a_z - r, x_j \in \{0,1\} \text{ for all } j \in T\}$.

We call the set T in Definition 4.5 *feasible set* for X^{BK}.

Proposition 4.6 ([58]). *The extended weight inequality defined for* $T \cup \{z\}$ *introduced in Definition 4.5 is valid for*

$$X^{BK} \cap \{x \in \{0,1\}^n : x_j = 0 \textit{ for all } j \in N \backslash (T \cup \{z\})\}.$$

Weismantel [58] used sequential up-lifting for the inequality (4.12) to obtain a valid inequality for X^{BK}. In [44], it has been pointed out that the coefficient c_z of x_z in inequality (4.12) can be obtained by starting with inequality $\sum_{j\in T} x_j \leq |T|$, which is valid for $X^{BK} \cap \{x \in \{0,1\}^n : x_j = 0 \text{ for all } j \in N\backslash T\}$, and lifting first x_z. Using Theorem 4.1, the corresponding lifting coefficient α_z calculates as

$$\begin{aligned}\alpha_z &= |T| - \max\{\sum_{j\in T} x_j : \sum_{j\in T} a_j x_j \leq a_0 - a_z, x_j \in \{0,1\} \text{ for all } j \in T\} \\ &= \min\{\sum_{j\in T}(1-x_j) : \sum_{j\in T} a_j(x_j - 1) \leq r - a_z, x_j \in \{0,1\} \text{ for all } j \in T\}.\end{aligned}$$

By complementing x_j, i.e., substituting $\bar{x}_j = 1 - x_j$, for all $j \in T$, we get

$$\alpha_z = \min\{\sum_{j\in T} \bar{x}_j : \sum_{j\in T} a_j \bar{x}_j \geq a_z - r, \bar{x}_j \in \{0,1\} \text{ for all } j \in T\},$$

which is exactly the definition of c_z in Definition 4.5. Thus, the extended weight inequality defined for $T \cup \{z\}$ coincides with the inequality associated with T where x_z is lifted first. Therefore, instead of speaking of an extended weight inequality defined for $T \cup \{z\}$, we speak in the sequel of an extended weight inequality defined for T and view z as the index of the variable lifted first.

Martin [44] fixed some variables to their upper bound in addition and used sequential up- and down-lifting to obtain a valid inequality for X^{BK}. Let $T \subseteq N$ be a feasible set for X^{BK} and (T_1, T_2) be any partition of T, then trivially, inequality

$$\sum_{j\in T_1} x_j \leq |T_1| \tag{4.13}$$

is valid for $X^{BK} \cap \{x \in \{0,1\}^n : x_j = 0 \text{ for all } j \in N\backslash T, x_j = 1 \text{ for all } j \in T_2\}$. Furthermore, let $(j_1, \ldots, j_t)$ with $j_1 = z$ be the lifting sequence of the variables in $N\backslash T_1$, and for $i = 1, \ldots, t$, let $b_{j_i} \in \{0,1\}$ be the bound variable x_{j_i} is fixed to, i.e., $b_{j_i} = 0$ for all $j_i \in N\backslash T$ and $b_{j_i} = 1$ for all $j_i \in T_2$. Using Theorem 4.1 for inequality (4.13), we get the following corollary.

Corollary 4.7. *For each $i = 1, \ldots, t$, consider the knapsack problem $K_{\alpha_{j_i}}$ defined recursively as follows*

$$\begin{aligned}z_{j_i} = \max\{ & \sum_{j\in T_1} x_j + \sum_{k=1}^{i-1} \alpha_{j_k} x_{j_k} : \\ & \sum_{j\in T_1} a_j x_j + \sum_{k=1}^{i-1} a_{j_k} x_{j_k} \leq a_0 - \sum_{k=i}^{t} a_{j_k} b_{j_k} - z, \\ & x_j \in \{0,1\} \text{ for all } j \in T_1 \cup \{j_1, \ldots, j_{i-1}\} \},\end{aligned}$$

with

$$z = \begin{cases} a_{j_i} & : \quad b_{j_i} = 0, \\ -a_{j_i} & : \quad b_{j_i} = 1. \end{cases}$$

For each $i = 1, \ldots, t$, let

$$\alpha_{j_i} = \begin{cases} (|T_1| + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}) - z_{j_i} & : \quad b_{j_i} = 0 \text{ and } K_{\alpha_{j_i}} \text{ is feasible,} \\ (|T_1| + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}) & : \quad b_{j_i} = 0 \text{ and } K_{\alpha_{j_i}} \text{ is infeasible,} \\ z_{j_i} - (|T_1| + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}) & : \quad b_{j_i} = 1. \end{cases}$$

Then

$$\sum_{j\in T_1} x_j + \sum_{j\in N\setminus T} \alpha_j x_j + \sum_{j\in T_2} \alpha_j x_j \le |T_1| + \sum_{j\in T_2} \alpha_j \tag{4.14}$$

is valid for X^{BK}.

We call inequality (4.14) *lifted extended weight inequality using sequential up- and down-lifting* (LEWI). This inequality does not necessarily define a facet of P, but it may do so.

Example 4.8. Consider the 0-1 knapsack polytope $P = \mathrm{conv}(X^{BK})$ with

$$X^{BK} = \{x \in \{0,1\}^6 : 3x_1 + 4x_2 + 6x_3 + 7x_4 + 9x_5 + 18x_6 \le 21\}.$$

Let $T = \{1,2,3,4\}$ and $z = \{5\}$. Then T is a feasible set for X^{BK}. Trivially, for the partition (T_1, T_2) of T with $T_2 = \{1\}$, inequality $x_2 + x_3 + x_4 \le 3$ is valid for

$$X^{BK} \cap \{x \in \{0,1\}^6 : x_5 = x_6 = 0, x_1 = 1\}.$$

Choosing the lifting sequence $(5, 6, 1)$ (here, the variable with index z is lifted first), by Corollary 4.7, we obtain the lifting coefficients $\alpha_5 = 2$, $\alpha_6 = 3$ and $\alpha_1 = 1$. The resulting inequality

$$x_1 + x_2 + x_3 + x_4 + 2x_5 + 3x_6 \le 4 \tag{4.15}$$

is valid for X^{BK}. In fact, inequality (4.15) defines a facet of P.

Class of LMCI2

Let C be a minimal cover for X^{BK}. We use the partition (C_1, C_2) of C with $C_1 = C$ and $C_2 = \emptyset$. By Theorem 4.2, inequality

$$\sum_{j\in C} x_j \le |C| - 1 \tag{4.16}$$

defines a facet of $\mathrm{conv}(X^0)$, where $X^0 = \{x \in \{0,1\}^{|C|} : \sum_{j\in C} a_j x_j \le a_0\}$. Let $Z = [0, a_0]$, then the lifting function f with respect to the valid inequality (4.16) for X^0, is given by

$$\begin{aligned} f(z) \;=\; \min\{\; & |C| - 1 - \sum_{j\in C} x_j \;: \\ & \sum_{j\in C} a_j x_j \;\le\; a_0 - z, \\ & x_j \in \{0,1\} \text{ for all } j \in C \;\}, \end{aligned}$$

for all $z \in Z$.

Suppose, without loss of generality, that $C = \{1, \ldots, |C|\}$ with $a_1 \ge \ldots \ge a_{|C|}$. Let $\lambda = \sum_{j\in C} a_j - a_0$, $A_0 = 0$ and $A_h = \sum_{j=1}^{h} a_j$ for $h = 1, \ldots, |C|$. Then, f can be expressed in a closed form as

$$f(z) = \begin{cases} 0 & : \quad 0 \le z \le A_1 - \lambda, \\ h & : \quad A_h - \lambda < z \le A_{h+1} - \lambda \text{ and } h = 1, \ldots, |C| - 1, \end{cases}$$

for all $z \in Z$ (see [32]).

The function f is not superadditve on Z in general. Therefore, we define a superadditive valid lifting function g for f, given by

$$g(z) = \begin{cases} 0 & : \quad z = 0, \\ h & : \quad A_h - \lambda + \rho_h < z \leq A_{h+1} - \lambda \text{ and} \\ & \quad\;\; h = 0, \ldots, |C| - 1, \\ h - \frac{A_h - \lambda + \rho_h - z}{\rho_1} & : \quad A_h - \lambda < z \leq A_h - \lambda + \rho_h \text{ and} \\ & \quad\;\; h = 1, \ldots, |C| - 1, \end{cases} \tag{4.17}$$

for all $z \in Z$, where $\rho_h = \max\{0, a_{h+1} - (a_1 - \lambda)\}$ for $h = 0, \ldots, |C| - 1$.

Theorem 4.9 ([32]). *The function g is a superadditve valid lifting function for f that is nondominated and maximal on Z.*

Therefore, by Corollary 2.16, the inequality

$$\sum_{j \in C} x_j + \sum_{j \in N \setminus C} g(a_j) x_j \leq |C| - 1 \tag{4.18}$$

is valid for X^{BK}. We call inequality (4.18) *lifted minimal cover inequality using superadditive up-lifting* (LMCI2). Although, this inequality does not necessarily define a facet of P, it may do so. In fact, it may define a facet of P which cannot be obtained by sequential up-lifting.

Example 4.10 ([32]). Consider the 0-1 knapsack polytope $P = \text{conv}(X^{BK})$ with

$$X^{BK} = \{x \in \{0,1\}^7 : 8x_1 + 7x_2 + 6x_3 + 4x_4 + 6x_5 + 6x_6 + 6x_7 \leq 22\}.$$

Then $C = \{1, 2, 3, 4\}$ is a minimal cover for X^{BK}. For the partition (C_1, C_2) of C with $C_2 = \emptyset$, by Theorem 4.2, the inequality $x_1 + x_2 + x_3 + x_4 \leq 3$ defines a facet of

$$\text{conv}(X^{BK} \cap \{x \in \{0,1\}^7 : x_5 = x_6 = x_7 = 0\}).$$

Let $Z = [0, 22]$. The lifting function $f : Z \to \mathbb{R}$ and the superadditive valid lifting function $g : Z \to \mathbb{R}$ are given in Figure 4.1. Note that f is not superadditive on Z, since $f(6) + f(6) = 2 > f(12) = 1$. If we use the superadditive valid lifting function g, we obtain $g(a_5) = g(a_6) = g(a_7) = 0.5$. By Corollary 2.16, the resulting inequality

$$x_1 + x_2 + x_3 + x_4 + 0.5x_5 + 0.5x_6 + 0.5x_7 \leq 3 \tag{4.19}$$

is valid for X^{BK}. In fact, inequality (4.19) defines a facet of P.

Classification of Facet Defining Inequalities

In our computational study, we will investigate which combination of our separation algorithms for the classes of LMCI1, LEWI and LMCI2 leads to the best performance of our cutting plane separator for the 0-1 knapsack problem in practice. In order to explain, why we assume that using a combination leads to better results than using only one class, we state a classification of facet defining inequalities for the 0-1 knapsack polytope which is taken from [34].

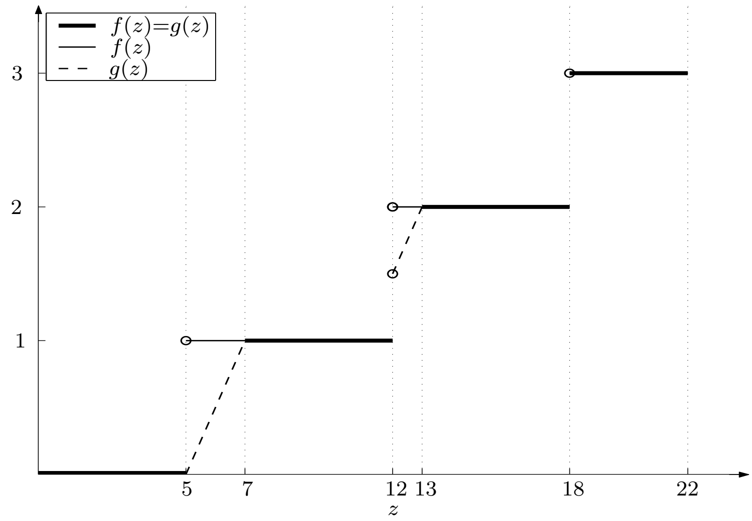

Figure 4.1: Functions f and g for Example 4.10.

Let MC be the set of all minimal covers for X^{BK}. For $C \in MC$, a facet defining inequality for P is called *lifted from* C if it can be scaled to the form

$$\sum_{j \in C} x_j + \sum_{j \in N \setminus C} \alpha_j x_j \leq |C| - 1. \tag{4.20}$$

A facet defining inequality for P lifted from $C \in MC$ is called *integral* if all its coefficients are integers when scaled to the form (4.20), otherwise it is called *nonintegral*. For $C \in MC$, let FLI(C) be the class of all integral facet defining inequalities lifted from C and FLNI(C) be the class of all nonintegral facet defining inequalities lifted from C. Furthermore, let FLI $= \cup_{C \in MC}$FLI(C), FLNI $= \cup_{C \in MC}$FLNI(C) and FR be the set of all facet defining inequalities which are not lifted from any $C \in MC$.

Let $C \in MC$. It is known that every integral facet defining inequality for P lifted from C can, in fact, be obtained from C by applying Padberg's sequential up-lifting procedure to some sequence of $N \setminus C$ (see [12]). This means, that by enumerating all sequences of $N \setminus C$ and applying Padberg's sequential up-lifting procedure to each sequence of $N \setminus C$, one generates exactly the class FLI(C). It is also known that there exist facet defining inequalities lifted from C which are not integral (see [12, 61]). Such nonintegral facet defining inequalities lifted from C cannot be obtained from C by Padberg's sequential up-lifting procedure, but by the simultaneous up-lifting procedure given in [12, 61]. In general, there may exist facet defining inequalities which are not lifted from any $C \in MC$.

We assume that a cutting plane separator which is able to generate inequalities from all three classes of facet defining inequalities (FLI, FLNI and FR) leads to good results in practice. Separating the class of LMCI1 may lead to inequalities in FLI $\cup$ FR. The same holds for separating the class of LEWI. The computational burden of the simultaneous up-lifting procedure given in [12, 61] prevents it from being applied in practice. Therefore, we separate the class LMCI2, which may lead to inequalities in FLI $\cup$ FLNI.

In our computational study, we will also evaluate whether the separation algorithms for the classes of LMCI1 and LEWI lead to different inequalities in practice.

Example 4.11. Consider Example 4.4. We obtained two facet defining inequalities from the class of LMCI1. The first inequality $x_3 + x_4 + x_5 \leq 2$ is in FLI, since $C = \{3, 4, 5\}$ is a minimal cover for X^{BK} with cardinality 3, and all coefficients of the inequality are integral. The second inequality $x_1 + 2x_3 + x_4 + x_5 \leq 3$ is in FR, since none of the subsets of $\{1, 4, 5\}$ is a minimal cover for X^{BK} with cardinality 4 and $\{3\}$ is not a minimal cover for X^{BK} with cardinality $\frac{5}{2}$.

Consider Example 4.8. We obtained the facet defining inequality $x_1 + x_2 + x_3 + 2x_5 + 3x_6 \leq 4$ from the class of LEWI. This inequality is in FR, since none of the subsets of $\{1, 2, 3\}$ is a minimal cover for X^{BK} with cardinality 5, $\{5\}$ is not a minimal cover for X^{BK} with cardinality 3 and $\{6\}$ is not a minimal cover for X^{BK} with cardinality $\frac{7}{3}$.

Consider Example 4.10. We obtained the facet defining inequality $x_1 + x_2 + x_3 + x_4 + 0.5x_5 + 0.5x_6 + 0.5x_7 \leq 3$ from the class of LMCI2. This inequality is in FLNI, since $C = \{1, 2, 3, 4\}$ is a minimal cover for X^{BK} with cardinality 4 and three coefficients of the inequality are nonintegral and since none of the subsets of $\{5, 6, 7\}$ is a minimal cover for X^{BK} with cardinality 7.

4.3 Algorithmic Aspects

In the last section, we have introduced three classes of strong valid inequalities for the 0-1 knapsack polytope, the classes of LMCI1, LEWI and LMCI2. Separation algorithms for these classes of inequalities can be part of a cutting plane separator for the 0-1 knapsack problem. In this section, we investigate algorithmic aspects of these separation algorithms.

Let $x^* \in [0, 1]^n \backslash \{0, 1\}^n$ be a fractional vector with $\sum_{j \in N} a_j x_j^* \leq a_0$. We want to solve the following separation problems.

Separation problem for the class of LMCI1

Find $C \subseteq N$ with $\sum_{j \in C} a_j > a_0$ and $\sum_{j \in C \backslash \{i\}} a_j \leq a_0$ for all $i \in C$, a partition (C_1, C_2) of C with $C_1 \neq \emptyset$ and a lifting sequence $(j_1, \ldots, j_t)$ of the variables in $N \backslash C_1$ such that

$$\sum_{j \in C_1} x_j^* + \sum_{j \in N \backslash C} \alpha_j x_j^* + \sum_{j \in C_2} \alpha_j x_j^* > |C_1| - 1 + \sum_{j \in C_2} \alpha_j,$$

where α_j for all $j \in N \backslash C_1$ are defined as in Corollary 4.3, or show that no inequality in the class of LMCI1 is violated by x^*.

Separation problem for the class of LEWI

Find $T \subseteq N$ and $z \in N \backslash T$ with $\sum_{j \in T} a_j \leq a_0$ and $\sum_{j \in T} a_j + a_z > a_0$, a partition (T_1, T_2) of T and a lifting sequence $(j_1, \ldots, j_t)$ of the variables in $N \backslash T$ with $j_1 = z$ such that

$$\sum_{j \in T_1} x_j^* + \sum_{j \in N \backslash T} \alpha_j x_j^* + \sum_{j \in T_2} \alpha_j x_j^* > |T_1| + \sum_{j \in T_2} \alpha_j,$$

where α_j for all $j \in N \backslash T_1$ are defined as in Corollary 4.7, or show that no inequality in the class of LEWI is violated by x^*.

Separation problem for the class of LMCI2

Find $C \subseteq N$ with $\sum_{j \in C} a_j > a_0$ and $\sum_{j \in C \backslash \{i\}} a_j \leq a_0$ for all $i \in C$ such that

$$\sum_{j \in C} x_j^* + \sum_{j \in N \backslash C} g(a_j) x_j^* > |C| - 1,$$

where the function g is defined by (4.17), or show that no inequality in the class of LMCI2 is violated by x^*.

In the following, we assume that a cover exists, i.e., $\sum_{j \in N} a_j > a_0$. We solve the separation problem for the class of LMCI1 heuristically in a three stage process. In the first stage, we solve the *separation problem for the class of cover inequalities*, which is: Find $C \subseteq N$ with $\sum_{j \in C} a_j > a_0$ and $\sum_{j \in C} x_j^* > |C| - 1$, or show that no inequality in the class of cover inequalities is violated by x^*. We call C an *initial cover*. In the second stage, we make the initial cover minimal by removing variables from C if necessary and partition it into (C_1, C_2) with $C_1 \neq \emptyset$. In the third stage, we determine a lifting sequence of the variables in $N \backslash C_1$ and lift the inequality $\sum_{j \in C_1} x_j \leq |C_1| - 1$ using sequential up- and down-lifting.

The separation problem for the class of LEWI is solved heuristically in an analog three stage process. The first stage is the same as for the separation algorithm for the class of LMCI1. In the second stage, we construct a feasible set $T \subseteq N$ by removing variables from the initial cover until the remaining variables satisfy $\sum_{j \in T} a_j \leq a_0$. We set z as the index of the variable removed last from the initial cover and partition T into (T_1, T_2). In the third stage, we determine a lifting sequence of the variables in $N \backslash T_1$ such that the variable with index z is lifted first, and lift the inequality $\sum_{j \in T_1} x_j \leq |T_1|$ using sequential up- and down-lifting.

The three stage process for heuristically solving the separation problem for the class of LMCI2 also starts with solving the separation problem for the class of cover inequalities. In the second stage, we make the initial cover C minimal by removing variables from C if necessary. In the third stage, we lift the inequality $\sum_{j \in C} x_j \leq |C| - 1$ using superadditive up-lifting.

Note that in all these separation algorithms it is not necessary for the cover inequality corresponding to the initial cover, to be the most violated one. Rather it is more important, with respect to the overall procedure, to obtain an initial cover which admits a ‘good’ lifting (see [30]). Because of the lifting step, we even continue the separation procedure if the initial cover does not correspond to a violated cover inequality, as lifting may still lead to a violated inequality. Because of the heuristic nature of the separation algorithms, we can not guarantee to find a violated inequality of the associated class if one exists. But, it will turn out that the cuts determined by these separation algorithms significantly improve the performance of the linear programming based branch-and-cut algorithm.

As one can see, the algorithmic aspects which we have to address when implementing these three separation algorithms are similar. We have to decide

(1) Which algorithm do we use to find the initial cover?,

(2) In which order do we remove the variables from the initial cover to construct a minimal cover and feasible set, respectively?

and for the separation algorithms for the classes of LMCI1 and LEWI

(3) Which partition of the minimal cover and feasible set, respectively do we use?,

(4) Which lifting sequence do we use?, and

(5) Which algorithm do we use to solve the knapsack problems which occur in the sequential lifting procedure?.

These questions will be addressed in the next sections. In addition, we will state how the researches mentioned in the introduction of this chapter have answered these questions.

4.3.1 Initial Cover

In the first stage of all three separation algorithms, we determine an initial cover C for X^{BK}. We represent C by the characteristic vector $z \in \{0,1\}^n$, i.e., $C = \{j \in N : z_j = 1\}$. It has to satisfy $\sum_{j \in N} a_j z_j > a_0$. The separation problem for the class of cover inequalities can be formulated as, solve the knapsack problem

$$\begin{aligned} \min\{ \ & \sum_{j \in N} (1 - x_j^*) z_j \ : \\ & \sum_{j \in N} a_j z_j \ \geq \ a_0 + 1, \\ & z_j \in \{0,1\} \text{ for all } j \in N \ \} . \end{aligned} \tag{KP1^{BK}}$$

Let ξ be the objective function value of an optimal solution of KP1^{BK}. If $\xi \geq 1$, there exists no cover inequality which is violated by x^*. If $\xi < 1$, the cover inequality corresponding to the initial cover C represented by an optimal solution of KP1^{BK} is violated by x^* (see [46]). By complementing all variables, i.e., substituting $z_j = 1 - \bar{z}_j$ for all $j \in N$, we transform KP1^{BK} into a knapsack problem in maximization form

$$\begin{aligned} \max\{ \ & \sum_{j \in N} (1 - x_j^*) \bar{z}_j \ : \\ & \sum_{j \in N} a_j \bar{z}_j \ \leq \ \sum_{j \in N} a_j - (a_0 + 1), \\ & \bar{z}_j \in \{0,1\} \text{ for all } j \in N \ \} . \end{aligned} \tag{KP1$^{BK}_{max}$}$$

Input : (KP) $\max\{\sum_{j\in N} p_j\tilde{z}_j : \sum_{j\in N} w_j\tilde{z}_j \leq c, \tilde{z}_j \in \{0,1\}$ for all $j \in N\}$, where $p_j \geq 0$ and $w_j \in \mathbb{Z}_+\backslash\{0\}$ for all $j \in N$, $c \in \mathbb{Z}_+$ and $n = |N|$.
Output: $\tilde{z}^* \in \{0,1\}^n$ an optimal solution of KP.

for $d \leftarrow 0$ **to** c **do**
    $A_0(d) \leftarrow 0$
for $j \leftarrow 1$ **to** n **do**
    for $d \leftarrow 0$ **to** $\min\{w_j - 1, c\}$ **do**
        $A_j(d) \leftarrow A_{j-1}(d)$
    for $d \leftarrow w_j$ **to** c **do**
        if $A_{j-1}(d - w_j) + p_j > A_{j-1}(d)$ **then**
            $A_j(d) \leftarrow A_{j-1}(d - w_j) + p_j$
        else
            $A_j(d) \leftarrow A_{j-1}(d)$
$d \leftarrow c$
for $j \leftarrow n$ **down to** 1 **do**
    if $A_j(d) > A_{j-1}(d)$ **then**
        $\tilde{z}_j^* \leftarrow 1$
        $d \leftarrow d - w_j$
    else
        $\tilde{z}_j^* \leftarrow 0$
return $\tilde{z}^*$

Algorithm 4.1: Exact algorithm to solve a knapsack problem in maximization form.

KP1^{BK}_{max} can be solved exactly using Algorithm 4.1, which uses dynamic programming (see [37]). Algorithm 4.1 has time and space complexity of $O(nc)$, where c is the right-hand side of the knapsack problem and $n = |N|$ (see [37]). We also call the right-hand side of a knapsack problem the *capacity* of a knapsack problem. To reduce the time and space complexity, KP1^{BK}_{max} can be solved approximately by applying Algorithm 4.2. This algorithm solves a knapsack problem in maximization form approximately by solving its LP relaxation using Dantzig's method and rounding down the solution (see [23] and [37]). The time complexity of Algorithm 4.2 is $O(n)$, and besides storing the input data and the solution no additional space is required (see [37]). Note that the set N has to be ordered when applying Algorithm 4.2. Sorting can be done in $O(n \log n)$ time using, e.g., the sorting algorithm *merge sort* (see [17]).

As mentioned before, the initial cover C merely assists us in identifying a lower-dimensional space and a valid inequality for the restriction of X^{BK} to this lower-dimensional space which will be lifted afterwards to a valid inequality for X^{BK}. That is, why our main focus is not to find an initial cover which corresponds to a most violated cover inequality, but to find an initial cover that admits a lifted inequality which is valid for X^{BK}, but violated by x^*. In the following, we will state two modifications of KP1^{BK} which will hopefully contribute to this aim.

The first modification of KP1^{BK} is to fix variables in advance. If a variable with

Input : (KP) $\max\{\sum_{j\in N} p_j \tilde{z}_j : \sum_{j\in N} w_j \tilde{z}_j \leq c, \tilde{z}_j \in \{0,1\}$ for all $j \in N\}$, where $p_j \geq 0$ and $w_j \in \mathbb{Q}_+ \backslash \{0\}$ for all $j \in N$, $c \in \mathbb{Q}_+$, $n = |N|$ and N is ordered such that $\frac{p_1}{w_1} \geq \frac{p_2}{w_2} \geq \ldots \geq \frac{p_n}{w_n}$.
Output: $\tilde{z}^* \in \{0,1\}^n$ a feasible solution of KP.

$\bar{w} \leftarrow 0$
for $j \leftarrow 1$ **to** n **do**
  if $\bar{w} + w_j \leq c$ **then**
    $\tilde{z}^*_j \leftarrow 1$
    $\bar{w} \leftarrow \bar{w} + w_j$
  else
    while $j \leq n$ **do**
      $\tilde{z}^*_j \leftarrow 0$
      $j \leftarrow j + 1$
return $\tilde{z}^*$

Algorithm 4.2: Approximate algorithm to solve a knapsack problem in maximization form.

x^*_j equal to zero ends up in the initial cover, the right-hand side of the lifted inequality increases, whereas the activity of the left-hand side remains unchanged. Thus, in this case the chance of finding a violated lifted inequality reduces. Therefore, we fix these variables to zero in advance, i.e., we set $z_j = 0$ for all $j \in N_0$, where $N_0 = \{j \in N : x^*_j = 0\}$. Note that as a consequence of this modification, the existence of a cover is no longer guaranteed. In addition, we fix all variables with x^*_j equal to one to one in advance, i.e., we set $z_j = 1$ for all $j \in N_1$, where $N_1 = \{j \in N : x^*_j = 1\}$. This is done to improve the chance of finding a violated lifted inequality, since out of the set of variables in the cover, the variables with x^*_j equal to one have the greatest contribution to the violation of the lifted inequality. Proposition 4.12 and Proposition 4.13 show that Algorithm 4.1 and Algorithm 4.2 choose the variables with x^*_j equal to one to be in the initial cover anyway. Thus, applying the last fixing is actually done to reduce the time and space complexity.

Proposition 4.12. *Let $\tilde{z}^* \in \{0,1\}^n$ be the solution vector obtained by applying Algorithm 4.1 to $KP1^{BK}_{max}$. Then $\tilde{z}^*_j = 0$ for all $j \in N_1$.*

Proof. Let $c = \sum_{j\in N} a_j - (a_0 + 1)$. Furthermore, for $d = 0, \ldots, c$ and $j = 0, \ldots, n$, let $A_j(d)$ be defined as in Algorithm 4.1, i.e., for $d = 0, \ldots, c$, $A_0(d) = 0$ and for $d = 0, \ldots, c$ and $j = 1, \ldots, n$,

$$A_j(d) = \begin{cases} A_{j-1}(d) & : \quad d < a_j, \\ \max\{A_{j-1}(d), A_{j-1}(d - a_j) + (1 - x^*_j)\} & : \quad d \geq a_j. \end{cases}$$

Note that for $d = 0, \ldots, c$ and $j = 0, \ldots, n$,

$$A_j(d) = \max\{\sum_{k=1}^{j} (1 - x^*_k)\tilde{z}_k : \sum_{k=1}^{j} a_k \tilde{z}_k \leq d, \\ \tilde{z}_k \in \{0,1\} \text{ for } k = 1, \ldots, j\}$$

(see [37]).

Let $l \in N_1$. We show that $A_l(d) \le A_{l-1}(d)$ holds for $d = 0, \ldots, c$. Then, by the definition of Algorithm 4.1, it follows that $\tilde{z}_l^* = 0$.

We have

$$\begin{aligned} A_{l-1}(d) &= \max\{\sum_{k=1}^{l-1}(1-x_k^*)\tilde{z}_k \;:\; \sum_{k=1}^{l-1} a_k\tilde{z}_k \le d, \\ &\qquad\qquad \tilde{z}_k \in \{0,1\} \text{ for } k = 1,\ldots,l-1\} \\ &= \max\{\sum_{k=1}^{l}(1-x_k^*)\tilde{z}_k \;:\; \sum_{k=1}^{l} a_k\tilde{z}_k \le d, \\ &\qquad\qquad \tilde{z}_k \in \{0,1\} \text{ for } k = 1,\ldots,l-1,\; \tilde{z}_l = 0\}. \end{aligned}$$

And, since $(1 - x_l^*) = 0$ and $\{\tilde{z} \in \{0,1\}^l : \sum_{k=1}^{l} a_k\tilde{z}_k \le d \text{ and } \tilde{z}_l = 0\} \subseteq \{\tilde{z} \in \{0,1\}^l : \sum_{k=1}^{l} a_k\tilde{z}_k \le d\}$, we obtain

$$\begin{aligned} A_{l-1}(d) &\le \max\{\sum_{k=1}^{l}(1-x_k^*)\tilde{z}_k \;:\; \sum_{k=1}^{l} a_k\tilde{z}_k \le d, \\ &\qquad\qquad \tilde{z}_k \in \{0,1\} \text{ for } k = 1,\ldots,l\} \\ &= A_l(d). \qquad \square \end{aligned}$$

Proposition 4.13. *Let $KP1^{BK}_{max}$ be given with N ordered such that $\frac{1-x_1^*}{a_1} \ge \ldots \ge \frac{1-x_n^*}{a_n}$. Let $\tilde{z}^* \in \{0,1\}^n$ be the solution vector obtained by applying Algorithm 4.2 to $KP1^{BK}_{max}$. Then $\tilde{z}_j^* = 0$ for all $j \in N_1$.*

Proof. By the definition of Algorithm 4.2, $\tilde{z}^*$ is of the form $\tilde{z}_j^* = 1$ for $j = 1, \ldots, k$ and $\tilde{z}_j^* = 0$ for $j = k+1, \ldots, n$, where $k \in \{0, \ldots, n\}$ is uniquely determined by

$$\sum_{j=1}^{k} a_j \le \sum_{j\in N} a_j - (a_0 - 1) \text{ and } \sum_{j=1}^{k+1} a_j > \sum_{j\in N} a_j - (a_0 + 1).$$

Let $l \in N_1$. We show $l \ge k+1$, which proofs $z_l^* = 0$. For that we show $\sum_{j=1}^{l} a_j > \sum_{j\in N} a_j - (a_0+1)$ or equivalently $\sum_{j=l+1}^{n} a_j \le a_0$. By the assumptions, we have

$$0 = \frac{1-x_l^*}{a_l} \ge \ldots \ge \frac{1-x_n^*}{a_n}.$$

In addition, since $0 \le x_j^* \le 1$ and $a_j > 0$ for all $j \in N$, $\frac{1-x_j^*}{a_j} \ge 0$ holds for all $j \in N$. Therefore, $\frac{1-x_j^*}{a_j}$ has to be equal to zero for $j = l+1, \ldots, n$ and since $a_j > 0$ for all $j \in N$, x_j^* has to be equal to one for $j = l+1, \ldots, n$. Thus, $\{l+1, \ldots, n\} \subseteq N_1$. Since, in addition, x^* satisfies $\sum_{j\in N} a_j x_j^* \le a_0$, $0 \le x_j^* \le 1$ and $a_j > 0$ for all $j \in N$, we obtain

$$a_0 \ge \sum_{j\in N} a_j x_j^* \ge \sum_{j\in N_1} a_j x_j^* = \sum_{j\in N_1} a_j \ge \sum_{j=l+1}^{n} a_j. \quad \square$$

The second modification of $\mathrm{KP1}^{BK}$ was suggested in [30]. When $\mathrm{KP1}^{BK}_{max}$ is solved by applying Algorithm 4.1 or 4.2, the following problems may occur. One problem is that a variable with large a_j but small x^*_j may be selected for the initial cover. This can be interpreted as fixing this variable to one. However, since x^*_j is small, i.e., closer to zero than to one, it probably should be fixed to zero. Another problem is that there is a tendency to pick variables with large a_j for the initial cover, most of which will get a cut coefficient equal to one. However, if these variables are not in the initial cover, they may get a larger lifting coefficient, which would lead to a larger contribution to the violation. The idea of [30] is to take into account the weight of the variables in the objective function. Their modified version of $\mathrm{KP1}^{BK}$ is

$$\begin{aligned} \min\{ \; & \sum_{j \in N} (1 - x^*_j) a_j z_j \; : \\ & \sum_{j \in N} a_j z_j \; \geq \; a_0 + 1, \\ & z_j \in \{0, 1\} \text{ for all } j \in N \; \}. \end{aligned} \tag{$\mathrm{KP2}^{BK}$}$$

Note that this changes the ordering of the variables required for Algorithm 4.2. We denote $\mathrm{KP2}^{BK}$ in maximization form by $\mathrm{KP2}^{BK}_{max}$.

The researchers mentioned in Section 4.1 have used the following methods to find the initial cover in the first stage of their separation routines.

Crowder, Johnson and Padberg [22] *Class of lifted minimal cover inequalities using sequential up-lifting.* Fix all variables in N_0 to zero in advance and solve $\mathrm{KP1}^{BK}_{max}$ approximately using Algorithm 4.2.

Van Roy and Wolsey [55] *Class of lifted cover inequalities using sequential up- and down-lifting.* Solve $\mathrm{KP1}^{BK}$ approximately. Note that the algorithm used has not been mentioned.

Hoffman and Padberg [35] *Class of LMCI1.* Fix all variables in N_0 to zero and all variables in N_1 to one in advance and solve $\mathrm{KP1}^{BK}_{max}$ approximately using Algorithm 4.2.

Gu, Nemhauser and Savelsbergh [30] *Class of LMCI1.* (1) Fix all variables in N_0 to zero in advance and solve $\mathrm{KP1}^{BK}$ exactly. Note that the algorithm used has not been mentioned. (2) Fix all variables in N_0 to zero in advance and solve $\mathrm{KP1}^{BK}_{max}$ approximately using Algorithm 4.2. (3) Fix all variables in N_0 to zero in advance and solve $\mathrm{KP2}^{BK}_{max}$ approximately using Algorithm 4.2.

Martin [44] *Class of LEWI.* Fix all variables in N_0 to zero and all variables in N_1 to one in advance and solve $\mathrm{KP2}^{BK}_{max}$ approximately using Algorithm 4.2.

4.3.2 Minimal Cover, Feasible Set and Partition

In the second stage of the separation algorithms, we use the initial cover obtained in the first stage to identify a lower-dimensional space and a valid inequality for the restriction of X^{BK} to this lower-dimensional space.

For the class of LMCI1, we use Theorem 4.2. First, we make the initial cover minimal by removing variables if necessary. Note that a variable will only be removed if the remaining variables still form a cover. A natural way is to remove the variables in the reverse order in which Algorithm 4.2 would have chosen them to be in an initial cover. That means, removing variables in nonincreasing order of $\frac{1-x_j^*}{a_j}$ if $KP1_{max}^{BK}$ was solved in the first stage of the separation algorithm, and in nondecreasing order of x_j^* if $KP2_{max}^{BK}$ was solved in the first stage. In both cases, variables with x_j^* equal to one will be removed last. Especially for these variables, the first order criterium for removing variables is not unique. Thus, we work with a second order criterium for removing variables which uses a_j. On the one hand, in behalf of nonincreasing a_j as a second order criterium speaks that a larger value of a_j leads to a larger lifting coefficient for this variable. On the other hand, in behalf of nondecreasing a_j as a second order criterium speaks that the lifting coefficient of the variables not in the minimal cover depends on the sum of a_j of the variables in C_2. The larger this sum the larger the lifting coefficient of the variables which are not in the minimal cover (see Corollary 4.3). As we will choose C_2 as the set of all variables with x_j^* equal to one, removing these variables will reduce the sum. Therefore, it will possibly be better to remove variables with smallest a_j. Next, we have to partition the constructed minimal cover C into $(C_1,\ C_2)$ with $C_1 \neq \emptyset$ in order to obtain the lower-dimensional space $\{x \in \{0,1\}^n : x_j = 0 \text{ for all } j \in N \backslash C, x_j = 1 \text{ for all } j \in C_2\}$ and the facet defining inequality $\sum_{j \in C_1} x_j \leq |C_1| - 1$ for $\text{conv}(X^{BK} \cap \{x \in \{0,1\}^n : x_j = 0 \text{ for all } j \in N \backslash C, x_j = 1 \text{ for all } j \in C_2\})$. A natural way to partition C is to choose $C_2 = \{j \in C : x_j^* = 1\}$. Note that since x^* satisfies $\sum_{j \in N} a_j x_j^* \leq a_0$ there exists no cover for X^{BK} which contains only variables with x_j^* equal to one, and therefore $C_1 \neq \emptyset$ holds. A second scheme for partitioning C was suggested by Gu, Nemhauser and Savelsbergh [30]. They fix all but two variables with smallest possible x_j^* to one.

For the class of LEWI, we use the trivial observation concerning inequality (4.13) stated in Section 4.2. We construct a feasible set T from the initial cover again by removing variables. As for the class of LMCI1, a natural way is to remove variables in the reverse order in which Algorithm 4.2 would have chosen them to be in the initial cover. If we use this ordering, no variable with x_j^* equal to one will be removed from the initial cover, since x^* satisfies $\sum_{j \in N} a_j x_j^* \leq a_0$ and the remaining variables will therefore form a feasible set before the variables with x_j^* equal to one will be reached in the ordering. Thus, we do not investigate on a second order criterium for removing variables. Next, we have to partition the feasible set T into $(T_1,\ T_2)$ in order to obtain a lower-dimensional space $\{x \in \{0,1\}^n : x_j = 0 \text{ for all } j \in N \backslash T, x_j = 1 \text{ for all } j \in T_2\}$ and the valid inequality $\sum_{j \in T_1} x_j \leq |T_1|$ for $X^{BK} \cap \{x \in \{0,1\}^n : x_j = 0 \text{ for all } j \in N \backslash T, x_j = 1 \text{ for all } j \in T_2\}$. Analog to the class of LMCI1, a natural way to partition T is to set $T_2 = \{j \in T : x_j^* = 1\}$.

One problem may occur in the separation algorithms for the classes of LMCI1 and LEWI when we perform the second stage as stated above. The sequential lifting procedure of the third stage starts with up-lifting some variables of $N \backslash C$ and $N \backslash T$, respectively (see Section 4.3.4). By Corollary 4.3 and Corollary 4.7, these variables will get a lifting coefficient equal to zero if $|C_1| = 1$ and $|T_1| = 0$, respectively. This disadvantage can be fixed by the following modification of the partition. If $|C_1| = 1$, we choose the variable with smallest a_j from C_2 and put it to C_1, and analog, if $|T_1| = 0$ we choose the variable with smallest a_j from T_2 and put it to T_1. We choose the variable with smallest a_j, since we want the sum of a_j of the variables in C_2 and T_2, respectively, to remain as large as possible, as this sum influences the lifting coefficient of the variables lifted up in the first part of the lifting procedure.

For the class of LMCI2, we use the special case of Theorem 4.2 where $C_2 = \emptyset$. Thus, as for the class of LMCI1, we have to make the initial cover minimal, but we do not have to partition the minimal cover. The construction of the minimal cover can be performed as in the separation algorithm for the class of LMCI1. Only the argument for using nondecreasing a_j as a second order criterium for removing variables changes. Here, in behalf of nondecreasing a_j speaks that the number of variables removed may be larger which would lead to a reduced right-hand side of the lifted inequality and thus, may improve the chance of finding a violated lifted inequality.

The second stages of the separation algorithms of the researchers mentioned in Section 4.1 are the following. Note that except Martin [44] none of the researchers did go into details about the order in which the variables are removed from the initial cover.

Crowder, Johnson and Padberg [22] *Class of lifted minimal cover inequalities using sequential up-lifting.* Make the initial cover C minimal by removing variables from C if necessary and set $C_2 = \emptyset$.

Van Roy and Wolsey [55] *Class of lifted cover inequalities using sequential up- and down-lifting.* Do not make the initial cover C minimal. Start with $C_2 = \emptyset$, if no violated inequality was found, set $C_2 = \{j^*\}$, where $a_{j^*} = \max_{j \in C}\{a_j : x^*_j > 0\}$.

Hoffman and Padberg [35] *Class of LMCI1.* Make the initial cover C minimal by removing variables from C if necessary and set $C_2 = \{j \in C : x^*_j = 1\}$.

Gu, Nemhauser and Savelsbergh [30] *Class of LMCI1.* (1) Make the initial cover C minimal by removing variables from C if necessary and set $C_2 = \{j \in C : x^*_j = 1\}$. (2) Make the initial cover C minimal by removing variables from C if necessary and choose C_1 as the set of the two variables with smallest x^*_j.

Martin [44] *Class of LEWI.* Construct a feasible set T from the initial cover C by removing variables from C if necessary in the reverse order in which Algorithm 4.2 has chosen them to be in the initial cover and set $T_2 = \{j \in T : x^*_j = 1\}$.

4.3.3 Lifting Sequence

In the third stage of the separation algorithms, we lift the valid inequality for the restriction of X^{BK} to the lower-dimensional space identified in the second stage of the separation algorithms to a valid inequality for X^{BK}.

For the class of LMCI2, we perform superadditive up-lifting, which is sequence independent. For the classes of LMCI1 and LEWI, we use sequential up- and down-lifting. Here, different lifting sequences usually lead to different lifted inequalities. We will use a two-level lifting sequence. At the first level, we specify sets of variables that are lifted in a certain order, and at the second level, we specify the lifting order within these sets.

For the class of LMCI1, we partition $N\backslash C$ into the sets F and R with $F = \{j \in N\backslash C : x_j^* > 0\}$ and $R = \{j \in N\backslash C : x_j^* = 0\}$, and we use the following first level lifting order: first up-lift all variables in F, then down-lift all variables in C_2 and finally up-lift all variables in R. This is done because the lifting coefficients of the variables in C_2 with x_j^* equal to one and of the variables in R have no effect on whether the lifted inequality is violated or not, which suggests that the variables in F should be lifted first. In addition, first down-lifting all variables in C_2 would lead to a lifting coefficient of one for all variables in C_2 (since C is a minimal cover) and would, therefore, undo the partition of C. We still have to choose the lifting order within the sets specified at the first level. Note that in case of up-lifting, the *maximum* lifting coefficient is obtained when a variable is lifted first. And, in case of down-lifting, the *minimum* lifting coefficient is obtained when a variable is lifted first (see [12, 30]). Several sequences have been suggested in the literature (see [30]).

Sequence 1 Lift the variables in order of nonincreasing absolute difference between x_j^* and the value the variables are fixed to.

Sequence 2 Lift the variables in order of nondecreasing magnitude of reduced costs.

Sequence 3 At each lifting step, lift a variable with maximum $\alpha_j x_j^*$.

Sequence 4 Lift the variables in order of nonincreasing a_j.

In [30], the following explanations for these sequences are given. Sequence 1 is the most natural one, since the larger the absolute difference between x_j^* and the value the variable is fixed to, the greater is the effect of the variable on the violation of the lifted inequality. This sequence only applies to fractional variables. The rational behind Sequence 2 is that variables with a reduced cost of small magnitude are more important (at least locally) than variables with a reduced cost far away from zero. This sequence only applies to nonbasic variables. Sequence 3 chooses at each lifting step the variable with the highest contribution to the left-hand side of the lifted inequality. This sequence only applies to fractional variables which have to be up-lifted. Sequence 4 takes into account that the right-hand side of the knapsack problem which has to be solved at each lifting step depends on a_j.

For the class of LEWI, we can use an analog two-level lifting sequence as described for the class of LMCI1, except that the variable x_z removed last from the initial cover has to be lifted first. We suggest to test a modification of this two-level lifting sequence where the restriction to lift the variable removed last from the initial cover first is released, since the lifting sequence within the set F (containing z) may choose a different z which may lead to an improvement in the violation of the lifted inequality.

The researchers mentioned in Section 4.1 have used the following lifting sequences in their separation algorithms.

Crowder, Johnson and Padberg [22] *Class of lifted minimal cover inequalities using sequential up-lifting.* Let C be the minimal cover, set $F = \{j \in N\backslash C : x_j^* > 0\}$ and $R = \{j \in N\backslash C : x_j^* = 0\}$ and use the following lifting order:

first up-lift all variables in F and then up-lift all variables in R. Note that the lifting sequence used within the sets F and R has not been mentioned.

Van Roy and Wolsey [55] *Class of lifted cover inequalities using sequential up- and down-lifting.* Let C be the cover and use the following lifting order: first up-lift all variables in $N \backslash C$ using Sequence 3 and then down-lift the variable in C_2 if C_2 is not empty.

Hoffman and Padberg [35] *Class of LMCI1.* Let C be the minimal cover, set $F = \{j \in N\backslash C : x_j^* > 0\}$ and $R = \{j \in N\backslash C : x_j^* = 0\}$ and use the following lifting order: first up-lift all variables in F, then down-lift all variables in C_2 and finally up-lift all variables in R. Within the sets F, C_2 and R use a lifting sequence that is based on both the first-order lifting coefficient and the reduced cost of nonbasic variables.

Gu, Nemhauser and Savelsbergh [30] *Class of LMCI1.* Let C be the minimal cover and set $F = \{j \in N\backslash C : x_j^* > 0\}$ and $R = \{j \in N\backslash C : x_j^* = 0\}$. (1) Use the following lifting order: first up-lift all variables in F using Sequence 3, then down-lift all variables in C_2 using Sequence 2 and finally up-lift all variables in R using Sequence 2. (2) Use the following lifting order: first up-lift all variables in F using Sequence 4, then down-lift all variables in C_2 using Sequence 4 and finally up-lift all variables in R using Sequence 4. (3) Use the following lifting order: first up-lift all variables in F using a random lifting order, then down-lift all variables in C_2 using a random lifting order and finally up-lift all variables in R using a random lifting order.

Martin [44] *Class of LEWI.* Let T be the feasible set and set $F = \{j \in N\backslash T : x_j^* > 0\}$ and $R = \{j \in N\backslash T : x_j^* = 0\}$ and use the following lifting order: first up-lift all variables in F using Sequence 1 (note that Sequence 1 chooses the variable removed last from the initial cover as the variable lifted first automatically), then down-lift all variables in T_2 using Sequence 4 and finally up-lift all variables in R using Sequence 4.

4.3.4 Computing the Lifting Coefficients

In this section, we deal with methods of calculating the lifting coefficients in the third stage of the separation algorithms.

For the class of LMCI1, we start with inequality $\sum_{j \in C_1} x_j \leq |C_1| - 1$ and perform sequential up- and down-lifting. Let $(j_1, \ldots, j_t)$ be the lifting sequence of the variables in $N\backslash C_1$. For each $i = 1, \ldots, t$, we have to solve the knapsack problem $K_{\alpha_{j_i}}$ defined in Corollary 4.3. Each of these knapsack problems can be solved exactly using Algorithm 4.1 or approximately using Algorithm 4.2. For the class of lifted minimal cover inequalities using sequential up-lifting, Zemel [62] developed an exact procedure to calculate all lifting coefficients in time complexity $O(n^2)$. This algorithm uses dynamic programming to solve a reformulation of the knapsack problems $K_{\alpha_{j_i}}$ in which the role of the objective function and the constraint is reversed. In [30], Gu, Nemhauser and Savelsbergh used an extension of Zemel's procedure for the class of LMCI1, which has time complexity of $O(n^4)$ to calculate all lifting coefficients. As the algorithm is not explicitly given in [30], we do that here.

Consider for each $i = 1, \dots, t$, the set of dual knapsack problems $D_{\alpha_{j_i}}(\omega)$ for $\omega = 0, \dots, |C_1| + \sum_{k=1}^{i-1} \alpha_{j_k}$

$$\begin{aligned} A_{j_i}(\omega) \;=\; \min\{ \;& \sum_{j \in C_1} a_j x_j + \sum_{k=1}^{i-1} a_{j_k} x_{j_k} \;: \\ & \sum_{j \in C_1} x_j + \sum_{k=1}^{i-1} \alpha_{j_k} x_{j_k} \;\geq\; \omega \\ & x_j \in \{0,1\} \text{ for all } j \in C_1 \cup \{j_1, \dots, j_{i-1}\} \;\}. \end{aligned}$$

Clearly, if the problem $K_{\alpha_{j_i}}$ is feasible, i.e., $a_0 - \sum_{k=i}^{t} a_{j_k} b_{j_k} - z \geq 0$, where $z = a_{j_i}$ if $b_{j_i} = 0$ and $z = -a_{j_i}$ if $b_{j_i} = 1$, it is related to the set of problems $D_{\alpha_{j_i}}(\omega)$, $\omega = 0, \dots, |C_1| + \sum_{k=1}^{i-1} \alpha_{j_k}$ via the relation

$$z_{j_i} = \max\{\omega : A_{j_i}(\omega) \leq a_0 - \sum_{k=i}^{t} a_{j_k} b_{j_k} - z\}. \tag{4.21}$$

Proposition 4.14. *Let* $i \in \{1, \dots, t\}$ *with* $b_{j_i} = 0$. *Then* $z_{j_i} \leq |C_1| - 1 + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}$.

Proof. By Theorem 4.2 and Theorem 4.1, inequality

$$\sum_{j \in C_1} x_j + \sum_{k=1}^{i-1} \alpha_{j_k} x_{j_k} \leq |C_1| - 1 + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k} \tag{4.22}$$

is valid for $X^{i-1} = \{x \in \{0,1\}^{|C_1|+i-1} : \sum_{j \in C_1} a_j x_j + \sum_{k=1}^{i-1} a_{j_k} x_{j_k} \leq a_0 - \sum_{k=i}^{t} a_{j_k} b_{j_k}\}$.

Since $a_{j_i} > 0$, inequality (4.22) is also valid for $\{x \in \{0,1\}^{|C_1|+i-1} : \sum_{j \in C_1} a_j x_j + \sum_{k=1}^{i-1} a_{j_k} x_{j_k} \leq a_0 - \sum_{k=i}^{t} a_{j_k} b_{j_k} - a_{j_i}\}$. Therefore,

$$\begin{aligned} z_{j_i} \;=\; \max\{ \;& \sum_{j \in C_1} x_j + \sum_{k=1}^{i-1} \alpha_{j_k} x_{j_k} \;: \\ & \sum_{j \in C_1} a_j x_j + \sum_{k=1}^{i-1} a_{j_k} x_{j_k} \;\leq\; a_0 - \sum_{k=i}^{t} a_{j_k} b_{j_k} - a_{j_i}, \\ & x_j \in \{0,1\} \text{ for all } j \in C_1 \cup \{j_1, \dots, j_{i-1}\} \;\} \\ \leq\; & |C_1| - 1 + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}. \end{aligned}$$

□

By Proposition 4.14, if $b_{j_i} = 0$, we only have to consider ω with $0 \leq \omega \leq |C_1| - 1 + \sum_{k=1}^{i-1} \alpha_{j_k} b_{j_k}$ when calculating z_{j_i} via the equation (4.21).

Consider for each $i = 1, \dots, t-1$, the set of dual knapsack problems $D_{\alpha_{j_{i+1}}}(\omega)$ for $\omega = 0, \dots, |C_1| + \sum_{k=1}^{i} \alpha_{j_k}$ of the next lifting step

$$\begin{aligned} A_{j_{i+1}}(\omega) \;=\; \min\{ \;& \sum_{j \in C_1} a_j x_j + \sum_{k=1}^{i-1} a_{j_k} x_{j_k} + a_{j_i} x_{j_i} \;: \\ & \sum_{j \in C_1} x_j + \sum_{k=1}^{i-1} \alpha_{j_k} x_{j_k} + \alpha_{j_i} x_{j_i} \;\geq\; \omega \\ & x_j \in \{0,1\} \text{ for all } j \in C_1 \cup \{j_1, \dots, j_i\} \;\}. \end{aligned}$$

If we set $A_{j_i}(\omega) = \infty$ for $\omega > |C_1| + \sum_{k=1}^{i-1} \alpha_{j_k}$, we get

$$A_{j_{i+1}}(\omega) = \begin{cases} \min\{A_{j_i}(\omega), a_{j_i}\} & : \quad \omega < \alpha_{j_i}, \\ \min\{A_{j_i}(\omega), A_{j_i}(\omega - \alpha_{j_i}) + a_{j_i}\} & : \quad \omega \geq \alpha_{j_i}. \end{cases}$$

This leads to Algorithm 4.3. Note that the lifting algorithm is given for first up-lifting all variables in F, then down-lifting all variables in C_2 and finally up-lifting all variables in R.

For the class of LEWI, we start with inequality $\sum_{j \in T_1} x_j \leq |T_1|$ and perform sequential up- and down-lifting. Let $(j_1, \ldots, j_t)$ be the lifting sequence of the variables in $N \backslash T_1$. We have to solve for each $i = 1, \ldots, t$ the knapsack problem $K_{\alpha_{j_i}}$ defined in Corollary 4.7. Each of this knapsack problems can be solved exactly using Algorithm 4.1 or approximately using Algorithm 4.2. As for the class of LMCI1, Zemel's procedure can be extended to compute the lifting coefficients for the class of LEWI in an analogue way.

For the class of LMCI2, we start with inequality $\sum_{j \in C} x_j \leq |C| - 1$ and perform superadditive lifting using the superadditive valid lifting function g defined in Section 4.2. As explained in Section 4.2, we only have to compute $g(a_j)$ for all $j \in N \backslash C$ to obtain the valid inequality $\sum_{j \in C} x_j + \sum_{j \in N \backslash C} g(a_j) x_j \leq |C| - 1$ for X^{BK}. The complete procedure is given in Algorithm 4.4.

The researchers mentioned in Section 4.1 performed lifting in the following way.

Crowder, Johnson and Padberg [22] *Class of lifted minimal cover inequalities using sequential up-lifting.* Calculate the lifting coefficients approximately using Algorithm 4.2 to solve each knapsack problem.

Van Roy and Wolsey [55] *Class of lifted cover inequalities using sequential up- and down-lifting.* Calculate the lifting coefficients exactly using Algorithm 4.1 to solve each knapsack problem.

Hoffman and Padberg [35] *Class of LMCI1.* Calculate the lifting coefficients approximately using Algorithm 4.2 to solve each knapsack problem.

Gu, Nemhauser and Savelsbergh [30] *Class of LMCI1.* (1) Calculate the lifting coefficients exactly using the extension of Zemel's procedure for the class of LMCI1. (2) Calculate the lifting coefficients approximately using the algorithm suggested in [43] to solve each knapsack problem, which provides a slightly better bound than the value of the LP relaxation, and has time complexity $O(n \log n)$ for the computation of a single lifting coefficient.

Martin [44] *Class of LEWI.* Calculate the lifting coefficient of the variables in F and T_2 exactly using the extension of Zemel's procedure for the class of LEWI and calculate the lifting coefficient of the variables in R approximately using Algorithm 4.2 to solve each knapsack problem.

Input : X^{BK} defined as is (4.2), (C_1, C_2) partition of C minimal cover for X^{BK}, where $C_1 = \{l_1, \ldots, l_{|C_1|}\} \neq \emptyset$ with $a_{l_1} \leq \ldots \leq a_{l_{|C_1|}}$ and $C_2 = \{j_{f+1}, \ldots, j_g\}$, (F, R) partition of $N \backslash C$ with $F = \{j_1, \ldots, j_f\}$ and $R = \{j_{g+1}, \ldots, j_t\}$, and $\sum_{j \in C_1} x_j \leq |C_1| - 1$ facet defining inequality for $\text{conv}(X^{BK} \cap \{x \in \{0,1\}^n : x_j = 0 \text{ for all } j \in N \backslash C, x_j = 1 \text{ for all } j \in C_2\})$.

Output: $\sum_{j \in C_1} x_j + \sum_{j \in N \backslash C} \alpha_j x_j + \sum_{j \in C_2} \alpha_j x_j \leq |C_1| - 1 + \sum_{j \in C_2} \alpha_j$ valid inequality for X^{BK}.

$A(0) \leftarrow 0$
for $\omega \leftarrow 1$ **to** $|C_1|$ **do**
  $A(\omega) \leftarrow A(\omega - 1) + a_{l_\omega}$
$\alpha_0 \leftarrow |C_1| - 1$
for $i \leftarrow 1$ **to** f **do** /* up-lifting of the variables in F */
  if $a_0 - \sum_{k=f+1}^{g} a_{j_k} - a_{j_i} < 0$ **then** $z_{j_i} \leftarrow 0$
  else $z_{j_i} \leftarrow \max\{\omega : 0 \leq \omega \leq \alpha_0, A(\omega) \leq a_0 - \sum_{k=f+1}^{g} a_{j_k} - a_{j_i}\}$
  $\alpha_{j_i} \leftarrow \alpha_0 - z_{j_i}$
  for $\omega \leftarrow |C_1| + \sum_{k=1}^{i-1} \alpha_{j_k} + 1$ **to** $|C_1| + \sum_{k=1}^{i} \alpha_{j_k}$ **do**
    $A(\omega) \leftarrow \infty$
  for $\omega \leftarrow |C_1| + \sum_{k=1}^{i} \alpha_{j_k}$ **down to** 0 **do**
    if $\omega < \alpha_{j_i}$ **then** $A(\omega) \leftarrow \min\{A(\omega), a_{j_i}\}$
    else $A(\omega) \leftarrow \min\{A(\omega), A(\omega - \alpha_{j_i}) + a_{j_i}\}$
for $i \leftarrow f + 1$ **to** g **do** /* down-lifting of the variables in C_2 */
  $z_{j_i} \leftarrow \max\{\omega : 0 \leq \omega \leq |C_1| + \sum_{k=1}^{i-1} \alpha_{j_k}, A(\omega) \leq a_0 - \sum_{k=i}^{g} a_{j_k} + a_{j_i}\}$
  $\alpha_{j_i} \leftarrow z_{j_i} - \alpha_0$
  $\alpha_0 \leftarrow \alpha_0 + \alpha_{j_i}$
  for $\omega \leftarrow |C_1| + \sum_{k=1}^{i-1} \alpha_{j_k} + 1$ **to** $|C_1| + \sum_{k=1}^{i} \alpha_{j_k}$ **do**
    $A(\omega) \leftarrow \infty$
  for $\omega \leftarrow |C_1| + \sum_{k=1}^{i} \alpha_{j_k}$ **down to** 0 **do**
    if $\omega < \alpha_{j_i}$ **then** $A(\omega) \leftarrow \min\{A(\omega), a_{j_i}\}$
    else $A(\omega) \leftarrow \min\{A(\omega), A(\omega - \alpha_{j_i}) + a_{j_i}\}$
for $i \leftarrow g + 1$ **to** t **do** /* up-lifting of the variables in R */
  $z_{j_i} \leftarrow \max\{\omega : 0 \leq \omega \leq \alpha_0, A(\omega) \leq a_0 - a_{j_i}\}$
  $\alpha_{j_i} \leftarrow \alpha_0 - z_{j_i}$
  for $\omega \leftarrow \alpha_0$ **down to** 0 **do**
    if $\omega < \alpha_{j_i}$ **then** $A(\omega) \leftarrow \min\{A(\omega), a_{j_i}\}$
    else $A(\omega) \leftarrow \min\{A(\omega), A(\omega - \alpha_{j_i}) + a_{j_i}\}$
return $\sum_{j \in C_1} x_j + \sum_{k=1}^{t} \alpha_{j_k} x_{j_k} \leq \alpha_0$

Algorithm 4.3: Exact lifting algorithm for the class of LMCI1 (extension of Zemel's procedure).

Input : X^{BK} defined as is (4.2), C minimal cover for X^{BK}, and $\sum_{j \in C} x_j \leq |C| - 1$ facet defining inequality for $\text{conv}(X^{BK} \cap \{x \in \{0,1\}^n : x_j = 0 \text{ for all } j \in N \setminus C\})$.

Output: $\sum_{j \in C} x_j + \sum_{j \in N \setminus C} \alpha_j x_j \leq |C| - 1$ valid inequality for X^{BK}.

Sort C by nonincreasing a_j.
$\lambda \leftarrow \sum_{j \in C} a_j - a_0$
$A_0 \leftarrow 0$
for $h \leftarrow 1$ **to** $|C|$ **do**
    $A_h \leftarrow A_{h-1} + a_h$
    $I_{h-1} \leftarrow A_h - \lambda$
    $\rho_{h-1} \leftarrow \max\{0, a_h - (a_1 - \lambda)\}$
Sort $N \setminus C$ by nondecreasing a_j. (Let $\{j_j, \dots, j_t\}$ be the ordered set.)
$h \leftarrow 0$
for $i \leftarrow 1$ **to** t **do**
    while $I_h < a_{j_i}$ **do** /* Search h : $a_{j_i} \in (A_h - \lambda, A_{h+1} - \lambda]$ */
        $h \leftarrow h + 1$
    if $h = 0$ **then**
        $\alpha_{j_i} \leftarrow h$
    else
        if $a_{j_i} \leq I_{h-1} + \rho_h$ **then** /* $a_{j_i} \in (A_h - \lambda, A_h - \lambda + \rho_h]$ */
            $\alpha_{j_i} \leftarrow h - \frac{I_{h-1} + \rho_h - a_{j_i}}{\rho_1}$
        else /* $a_{j_i} \in (A_h - \lambda + \rho_h, A_{h+1} - \lambda]$ */
            $\alpha_{j_i} \leftarrow h$
return $\sum_{j \in C} x_j + \sum_{k=1}^{t} \alpha_{j_k} x_{j_k} \leq |C| - 1$

Algorithm 4.4: Lifting algorithm for the class of LMCI2.

4.4 Computational Study

In Section 4.3, we gave an outline of the separation algorithms for the classes of LMCI1, LEWI and LMCI2 and discussed different algorithmic and implementation choices which have to be made when implementing these separation algorithms. In this section, we describe our computational experience with these choices. We extended the initial test set (see Section 2.3) by 14 instances, which are real-world problems obtained from various resources, and divided the extended initial test set into two sets; the *main test set* and the *remaining test set*. The extension of the initial test set was done in order to obtain a reasonable large main test set.

Main test set Contains all instances of the extended initial test set for which at least one of the three default algorithms or at least one of the different versions of the default algorithms where a single aspect is altered leads to an initial gap closed of more than zero percent.

Remaining test set Contains the remaining instances of the extended initial test set.

We use the main test set to develop the resulting separation algorithms for the classes of LMCI1, LEWI and LMCI2, and to obtain our final cutting plane separator for the 0-1 knapsack problem which uses the most efficient combination of the three resulting separation algorithms. This set consists of 29 MIPs, 9 are various instances from Miplib 2003 [3], 8 are instances from Miplib 3.0 [14] and 12 are real-world problems that we obtained from various resources. Table B.20 summarizes the main characteristics of the instances in the main test set. The remaining test set is only used to show that the CPU time spent in our final cutting plane separator is on an acceptable level for all instances in the extended initial test set. Table B.21 summarizes the main characteristics of the instances in the remaining test set.

See Section 2.3, for information about the workstation on which we performed our computational experiments, about the implementation environment of the separation algorithms and about the representation of our test sets and our computational results.

Finally, note that we always used the algorithm merge sort (see [17]) for sorting sets.

4.4.1 Separation Algorithm for the Class of LMCI1

Our default algorithm for separating the class of LMCI1 is given in Algorithm 4.5. The results for applying this algorithm to the instances in our main test set are given in Table B.22 and a summary of the results is contained in Table 4.1. We close 16.31 percent of the initial gap in geometric mean and the CPU time spent in the separation routine is 1355.9 seconds in total. For four instances in our main test set, the separation time is greater than 10 seconds of CPU time. As we will see, the large separation time for these instances is caused by the fact that we use the exact algorithm to find the initial cover in the first stage of the default algorithm.

Input : X^{BK} defined as is (4.2) and $x^* \in [0,1]^n \setminus \{0,1\}^n$ fractional vector with $\sum_{j \in N} a_j x_j^* \leq a_0$.
Output: Violated (with respect to x^*) inequality from the class of LMCI1 or notification that no such inequality was found.

`/* First stage: Initial cover */`

$N_0 \leftarrow \{j \in N : x_j^* = 0\}$ and $N_1 \leftarrow \{j \in N : x_j^* = 1\}$
if $\sum_{j \in N \setminus N_0} a_j - (a_0 + 1) < 0$ **then return** *No inequality found*
Call Algorithm 4.1 for

$$\max\{ \sum_{j \in N \setminus (N_0 \cup N_1)} (1 - x_j^*) \bar{z}_j : \sum_{j \in N \setminus (N_0 \cup N_1)} a_j \bar{z}_j \leq \sum_{j \in N \setminus N_0} a_j - (a_0 + 1), \quad \bar{z}_j \in \{0,1\} \text{ for all } j \in N \setminus (N_0 \cup N_1) \}$$

(Let $\bar{z}^*$ be the solution.)
$C \leftarrow N_1 \cup \{j \in N \setminus (N_0 \cup N_1) : \bar{z}_j^* = 0\}$

`/* Second stage: Minimal cover and partition */`

Sort C by nonincreasing $\frac{1 - x_j^*}{a_j}$ and use nondecreasing a_j as a second order criterium.
$c \leftarrow |C|$
for $j \leftarrow 1$ **to** c **and** C *is not minimal* **do**
  if $\sum_{i \in C} a_i - a_j > a_0$ **then** $C \leftarrow C \setminus \{j\}$
$C_2 \leftarrow \{j \in C : x_j^* = 1\}$ and $C_1 \leftarrow C \setminus C_2$

`/* Third stage: Lifting sequence and computing the lifting coefficients */`

$F \leftarrow \{j \in N \setminus C : x_j^* > 0\}$ and $R \leftarrow \{j \in N \setminus C : x_j^* = 0\}$
Sort F, C_2, and R according to Sequence 4 and C_1 by nondecreasing a_j.
Call Algorithm 4.3 for X^{BK}, (C_1, C_2), (F, R) and $\sum_{j \in C_1} x_j \leq |C_1| - 1$. (Let $\sum_{j \in C_1} x_j + \sum_{j \in N \setminus C} \alpha_j x_j + \sum_{j \in C_2} \alpha_j x_j \leq |C_1| - 1 + \sum_{j \in C_2} \alpha_j$ be the lifted valid inequality for X^{BK}.)

`/* Result */`

if $\sum_{j \in C_1} x_j^* + \sum_{j \in N \setminus C} \alpha_j x_j^* + \sum_{j \in C_2} \alpha_j x_j^* > |C_1| - 1 + \sum_{j \in C_2} \alpha_j$ **then**
  return $\sum_{j \in C_1} x_j + \sum_{j \in N \setminus C} \alpha_j x_j + \sum_{j \in C_2} \alpha_j x_j \leq |C_1| - 1 + \sum_{j \in C_2} \alpha_j$
else return *No inequality found*

Algorithm 4.5: Separation algorithm for the class of LMCI1. *Default algorithm.*

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)		Sepa Time > 10 sec (Number)	
	Value	△	Value	△	Value	△
Default algorithm	16.31	0.00	1355.9	0.0	4	0
Initial cover - 1. modification[1]	15.61	-0.70	7.6	-1348.3	0	-4
Initial cover - 2. modification[2]	15.81	-0.50	25.3	-1330.6	0	-4
Initial cover - 3. modification[3]	14.50	-1.81	256.9	-1099.0	4	0
Initial cover - 4. modification[4]	16.42	0.11	7.1	-1348.8	0	-4
Minimal cover - modification[5]	17.02	0.71	1738.2	382.3	4	0
Partition - modification[6]	16.40	0.09	1822.4	466.5	4	0
Lifting sequence - modification[7]	16.31	0.00	1468.9	113.0	5	1
Resulting algorithm[4 6]	16.36	0.05	7.4	-1348.5	0	-4

Table 4.1: Summary of the computational results for the separation algorithm for the class of LMCI1 on the main test set. *Default algorithm, default algorithm where a single algorithmic aspect is altered* and *resulting algorithm*. (△ with respect to the default algorithm)

Initial Cover

In Section 4.3.1, we described two modifications of $KP1^{BK}$. We have decided to use the first modification of $KP1^{BK}$ without comparing it to the unmodified version, since most of the researchers mentioned in Section 4.1 have fixed some variables in advance. This is, we fix all variables in N_0 to zero and all variables in N_1 to one in advance. Thus, if we speak in the sequel of solving $KP1^{BK}_{max}$ ($KP2^{BK}_{max}$) we mean solving $KP1^{BK}$ ($KP2^{BK}$) in maximization form after applying this fixing to $KP1^{BK}$ ($KP2^{BK}$).

In the default algorithm, we solve $KP1^{BK}_{max}$ exactly using Algorithm 4.1. We have tested to solve $KP1^{BK}_{max}$ approximately using Algorithm 4.2. See Table 4.1, for a summary of the results obtained for the main test set. It turned out that this reduces the separation time. For all instances in the main test set, the separation time is less than or equal to 10 seconds of CPU time. But this also reduces the initial gap closed for most of the instances in the main test set. We conclude that solving $KP1^{BK}_{max}$ using Algorithm 4.1 performs better than using Algorithm 4.2 with respect to the initial gap closed, but can be very time consuming. Algorithm 4.1 has time and space complexity of $O(nc)$, where n and c are defined as in Algorithm 4.1. We tested another version of the default algorithm where we use Algorithm 4.1 if nc is not greater than 1,000,000 and Algorithm 4.2 otherwise. (In order to find a good bound for switching between both algorithms, we ran our default algorithm on the main test set and examined the value of nc of the knapsack problems that had to be solved to find initial covers.)

The results for the main test set are given in Table B.23 and a summary of the results is contained in Table 4.1. As one can see, this version performs better than the default algorithm, since all instances in the main test set have separation time

[1] Solve $KP1^{BK}_{max}$ approximately using Algorithm 4.2.

[2] Solve $KP1^{BK}_{max}$ exactly using Algorithm 4.1 if nc is not greater than 1,000,000 and approximately using Algorithm 4.2 otherwise.

[3] Solve $KP2^{BK}_{max}$ exactly using Algorithm 4.1.

[4] Solve $KP2^{BK}_{max}$ approximately using Algorithm 4.2.

[5] Use nonincreasing a_j as the second order criterium for removing variables.

[6] Set $C_2 = \{j \in C : x^*_j = 1\}$ and $C_1 = C \backslash C_2$. Change the partition if $|C_1| = 0$.

[7] Use Sequence 1 for the set F and Sequence 4 for the sets C_2 and R.

less than or equal to 10 seconds of CPU time and the gap closed reduces only for those instances for which using the default algorithm led to a large separation time.

The second modification of KP1^{BK} discussed in Section 4.3.1 was assumed to improve the performance of the separation algorithm. It will turn out that this assumption is true. We have solved KP2_{max}^{BK} exactly using Algorithm 4.1. The results for the main test set are given in Table B.24 and a summary of the results is contained in Table 4.1. As one can see, for some of the instances in the main test set, the initial gap closed increases, but for others, it decreases. The initial gap closed in geometric mean reduces by 1.81 percentage points. Better results were obtained for solving KP2_{max}^{BK} approximately using Algorithm 4.2. They are given in Table B.25 and a summary of them is contained in Table 4.1. The initial gap closed increases for most of the instances in our main test set and the initial gap closed in geometric mean increases by 0.11 percentage points. Furthermore, the separation time is 7.1 seconds of CPU time in total. Thus, solving KP2_{max}^{BK} approximately performs better than solving KP2_{max}^{BK} exactly with respect to both, the separation time and the initial gap closed. And it also outperforms the version of the default algorithm where we solve KP1_{max}^{BK} using Algorithm 4.1 if nc is not greater than 1,000,000 and using Algorithm 4.2 otherwise.

Minimal Cover and Partition

In the default algorithm, we make the initial cover minimal by removing variables if necessary, in the reverse order in which Algorithm 4.2 would have chosen them to be in the initial cover. In Section 4.3.2, we have discussed two schemes for the second order criterium for removing variables. In the default algorithm, we use the criterium of nondecreasing a_j.

We have tested it against the criterium of nonincreasing a_j. The results for our main test set are given in Table B.26 and a summary of the results is contained in Table 4.1. As one can see, the second scheme performs better than the first one, as the initial gap closed increases for most of the instances and the initial gap closed in geometric mean increases by 0.71 percentage points. Nevertheless, in our resulting separation algorithm, we use the first scheme, since using the second scheme in our resulting algorithm reduces the initial gap closed. In geometric mean, it is 14.77 percent in contrast to 16.36 percent (see Table B.27) for using the first scheme in the resulting algorithm. A possible explanation is that in the default algorithm we solve KP1_{max}^{BK} to find an initial cover. In Section 4.3.1, we have stated the problem that there is a tendency to pick variables with large a_j for the initial cover and using the second scheme removes some of these variables from the initial cover. However, in the resulting algorithm, we solve KP2_{max}^{BK} which already works against the tendency to pick variables with large a_j for the initial cover.

We have decided to use the natural partition of the minimal cover, i.e., we set $C_2 = \{j \in C : x^* = 1\}$ and $C_1 = C \setminus C_2$, without comparing it to the second scheme suggested by Gu, Nemhauser and Savelsbergh [30], since in their computational study it turned out that there is no significant difference in the performance between this partition and their suggested second scheme.

In Section 4.3.2, we have pointed out a disadvantage of using the natural partition; the fact that it may occur that $|C_1| = 1$. We have tested the modification of the natural partition where we put a variable with smallest a_j from C_2 to C_1 if $|C_1| = 1$. A summary of the results for our main test set is contained in Table 4.1. It turned out that this modification has only a small effect on the performance of the separation algorithm with respect to the initial gap closed for our main test set. The initial gap closed in geometric mean only increases by 0.09 percentage points. But for the resulting algorithm we will use this modification, as there may be other instances where this modification could help to improve the performance of the separation algorithm. Note that this causes the slight difference in the results given in Table B.25 and the results for our resulting algorithm given in Table B.27.

Lifting Sequence

In Section 4.3.3, we have explained why it is reasonable to use a two-level lifting sequence. In the default algorithm, we use Sequence 4 within the sets F, C_2 and R at the second level. We have decided to test this version only against the lifting sequence used by Martin [44] for the class of LEWI, i.e., Sequence 1 for the set F and Sequence 4 for the sets C_2 and R at the second level, as the computational results of Gu, Nemhauser and Savelsbergh [30] indicate that the lifting order within the sets is not very important. Our results for using Sequence 1 instead of Sequence 4 for the set F confirm that, as there is no significant difference with respect to the initial gap closed (see Table 4.1, for a summary of the results for our main test set).

Computing the Lifting Coefficients

We have decided to compute the lifting coefficients exactly using Algorithm 4.3, as Gu, Nemhauser and Savelsbergh [30] concluded from the results of their computational study that this algorithm is fast in practice. Our results for the resulting algorithm (see Table B.27) confirm this conclusion.

Resulting Algorithm

From the results of our computational study we get the following three stages of the separation algorithm for the class of LMCI1.

First stage Fix all variables in N_0 to zero and all variables in N_1 to one in advance and solve KP2^{BK}_{max} approximately using Algorithm 4.2.

Second stage Make the initial cover minimal by removing variables, if necessary, in the reverse order in which Algorithm 4.2 has chosen them to be in the initial cover and use as the second order criterium for removing variables nondecreasing a_j. Set $C_2 = \{j \in C : x^*_j = 1\}$ and put a variable with smallest a_j from C_2 to C_1 if $|C_1| = 1$.

Third stage Use the following lifting order: first up-lift all variables in F using Sequence 4, then down-lift all variables in C_2 using Sequence 4 and finally up-lift all variables in R using Sequence 4. Compute the lifting coefficients exactly using Algorithm 4.3.

The results for our resulting algorithm are given in Table B.27 and a summary of the results is contained in Table 4.1. By using our resulting algorithm for the main

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)		Sepa Time > 10 sec (Number)	
	Value	△	Value	△	Value	△
Default algorithm	16.45	0.00	1510.3	0.0	6	0
Initial cover - 1. modification[8]	16.13	-0.32	7.3	-1503.0	0	-6
Initial cover - 2. modification[9]	15.51	-0.94	20.3	-1490.0	0	-6
Initial cover - 3. modification[10]	13.78	-2.67	420.8	-1089.5	6	0
Initial cover - 4. modification[11]	14.67	-1.78	6.9	-1503.4	0	-6
Partition - modification[12]	3.79	-12.66	379.9	-1130.4	2	-4
Lifting sequence - 1. modification[13]	16.47	0.02	1573.0	62.7	6	0
Lifting sequence - 2. modification[14]	16.42	-0.03	1572.2	61.9	6	0
Resulting algorithm[8 14]	16.79	0.34	8.5	-1501.8	0	-6

Table 4.2: Summary of the computational results for the separation algorithm for the class of LEWI on the main test set. *Default algorithm*, *default algorithm where a single algorithmic aspect is altered* and *resulting algorithm*. (△ with respect to the default algorithm)

test set we close 16.36 percent of the initial gap in geometric mean and the CPU time spent in the separation routine is 7.4 seconds of CPU time in total. Thus, we have significantly improved the performance of our default separation algorithm for the class of LMCI1.

4.4.2 Separation Algorithm for the Class of LEWI

Our default algorithm for separating the class of LEWI is given in Algorithm 4.6. The results for applying this algorithm to the instances in our main test set are given in Table B.28 and a summary of the results is contained in Table 4.2. The initial gap closed in geometric mean is 16.45 percent and the CPU time spent in the separation routine is 1510.3 seconds in total. For six instances in the main test set the separation time is greater than 10 seconds of CPU time. As for the class of LMCI1, we will see that the large separation time for these instances is caused by using the exact algorithm to find the initial cover in the first stage of the default algorithm.

Initial Cover

For the class of LEWI, we use the same method to find an initial cover in the default algorithm as for the class of LMCI1 (see Section 4.4.1). This is, we use the first modification of $KP1^{BK}$ discussed in Section 4.3.1 without comparing it to the unmodified version and we solve $KP1^{BK}_{max}$ exactly using Algorithm 4.1.

As for the class of LMCI1, the large separation time needed for some of the instances in the main test set when using the default algorithm reduces when $KP1^{BK}_{max}$ is solved approximately using Algorithm 4.2 (see Table B.29 and Table 4.2). The

[8] Solve $KP1^{BK}_{max}$ approximately using Algorithm 4.2.

[9] Solve $KP1^{BK}_{max}$ exactly using Algorithm 4.1 if nc is not greater than 1,000,000 and approximately using Algorithm 4.2 otherwise.

[10] Solve $KP2^{BK}_{max}$ exactly using Algorithm 4.1.

[11] Solve $KP2^{BK}_{max}$ approximately using Algorithm 4.2.

[12] Set $T_2 = \{j \in T : x^*_j = 1\}$ and $T_1 = T \setminus T_2$. Do not change the partition if $|T_1| = 0$.

[13] Use Sequence 1 for the set F and Sequence 4 for the sets T_2 and R.

[14] Do not use the restriction to lift first the variable which has been removed last from the initial cover.

Input : X^{BK} defined as is (4.2) and $x^* \in [0,1]^n \backslash \{0,1\}^n$ fractional vector with $\sum_{j \in N} a_j x_j^* \leq a_0$.

Output: Violated (with respect to x^*) inequality from the class of LEWI or notification that no such inequality was found.

`/* First stage: Initial cover */`

$N_0 \leftarrow \{j \in N : x_j^* = 0\}$ and $N_1 \leftarrow \{j \in N : x_j^* = 1\}$

if $\sum_{j \in N \backslash N_0} a_j - (a_0 + 1) < 0$ **then return** *No inequality found*

Call Algorithm 4.1 for

$$\max\{ \sum_{j \in N \backslash (N_0 \cup N_1)} (1 - x_j^*)\bar{z}_j : \sum_{j \in N \backslash (N_0 \cup N_1)} a_j \bar{z}_j \leq \sum_{j \in N \backslash N_0} a_j - (a_0 + 1), \; \bar{z}_j \in \{0,1\} \text{ for all } j \in N \backslash (N_0 \cup N_1) \}$$

(Let $\bar{z}^*$ be the solution.)

$C \leftarrow N_1 \cup \{j \in N \backslash (N_0 \cup N_1) : \bar{z}_j^* = 0\}$

`/* Second stage: Feasible set and partition */`

Sort C by nonincreasing $\frac{1 - x_j^*}{a_j}$.

$T \leftarrow C$ and $t \leftarrow |T|$

for $j \leftarrow 1$ **to** t **and** $\sum_{j \in T} a_j > a_0$ **do**

  $T \leftarrow T \backslash \{j\}$ and $z \leftarrow j$

$T_2 \leftarrow \{j \in T : x_j^* = 1\}$ and $T_1 \leftarrow T \backslash T_2$

if $|T_1| = 0$ **then**

  $T_1 \leftarrow \{j^*\}$ with $a_{j^*} = \min_{j \in T_2}\{a_j\}$, and $T_2 \leftarrow T_2 \backslash \{j^*\}$

`/* Third stage: Lifting sequence and computing the lifting coefficients */`

$F \leftarrow \{j \in N \backslash T : x_j^* > 0\}$ and $R \leftarrow \{j \in N \backslash T : x_j^* = 0\}$

Sort $F \backslash \{z\}$ according to Sequence 4 and set z to be the first variable in the ordered set F, sort T_2 and R according to Sequence 4, and sort T_1 by nondecreasing a_j

Call a lifting algorithm that is the extension of Zemel's procedure to the class of LEWI (analog to Algorithm 4.3) for X^{BK}, (T_1, T_2), (F, R) and $\sum_{j \in T_1} x_j \leq |T_1|$. (Let $\sum_{j \in T_1} x_j + \sum_{j \in N \backslash T} \alpha_j x_j + \sum_{j \in T_2} \alpha_j x_j \leq |T_1| + \sum_{j \in T_2} \alpha_j$ be the lifted valid inequality for X^{BK}.)

`/* Result */`

if $\sum_{j \in T_1} x_j^* + \sum_{j \in N \backslash T} \alpha_j x_j^* + \sum_{j \in T_2} \alpha_j x_j^* > |T_1| + \sum_{j \in T_2} \alpha_j$ **then**

  return $\sum_{j \in T_1} x_j + \sum_{j \in N \backslash T} \alpha_j x_j + \sum_{j \in T_2} \alpha_j x_j \leq |T_1| + \sum_{j \in T_2} \alpha_j$

else return *No inequality found*

Algorithm 4.6: Separation algorithm for the class of LEWI. *Default algorithm.*

separation time is 7.3 seconds of CPU time in total. But similar to the class of LMCI1, in addition, the initial gap closed in geometric mean reduces by 0.32 percentage points. However, note that for some of the instances in the main test set the initial gap closed increases. For the instances in the main test set which had a small separation time in the default algorithm and for which the initial gap closed reduced when we solved $KP1^{BK}_{max}$ approximately, this reduction can be eliminated when we use Algorithm 4.1 if nc is not greater than 1,000,000 (where n and c are defined as in Algorithm 4.1) and Algorithm 4.2 otherwise. The results for this version of the default algorithm for the main test set are given in Table B.30 and a summary of the results is contained in Table 4.2. Unfortunately, we also lose the increase of the initial gap closed achieved for some of the instances when we solved $KP1^{BK}_{max}$ approximately. Altogether, the initial gap closed in geometric mean reduces by 0.93 percentage points. Thus, in contrast to the class of LMCI1, here always solving $KP1^{BK}_{max}$ approximately performs better than the version where we solve $KP1^{BK}_{max}$ approximately only if nc is greater than or equal to 1,000,000.

The second modification of $KP1^{BK}$ discussed in Section 4.3.1 was assumed to improve the performance of the separation algorithm. For the class of LMCI1, this assumption fulfilled, but here it does not. The results for solving $KP2^{BK}_{max}$ exactly using Algorithm 4.1 are given in Table B.31 and a summary of the results is contained in Table 4.2. For the class of LMCI1, the initial gap closed increased for some of the instances in the main test set and for others it decreased when we solved $KP2^{BK}_{max}$ exactly using Algorithm 4.1, and the initial gap closed in geometric mean reduced. Here a similar effect can be observed, except that the gap closed does not increase for as many instances as for the class of LMCI1. The initial gap closed in geometric mean reduces by 2.67 percentage points. For the class of LMCI1, the initial gap closed increased for most of the instances in the main test set when we solved $KP2^{BK}_{max}$ approximately using Algorithm 4.2 and the initial gap closed in geometric mean increased. Here solving $KP2^{BK}_{max}$ approximately does also perform better than solving $KP2^{BK}_{max}$ exactly (see Table B.32 and Table 4.2). But, as one can see, the second modification of $KP1^{BK}$ discussed in Section 4.3.1 does not improve the performance of the separation algorithm; the initial gap closed in geometric mean reduces by 1.78 percentage points.

Feasible Set and Partition

In the default algorithm, we construct a feasible set T from the initial cover by removing variables in the reverse order in which Algorithm 4.2 would have chosen them to be in the initial cover.

As for the class of LMCI1, we have decided to use the natural partition of the feasible set, i.e., we set $T_2 = \{j \in T : x^*_j = 1\}$ and $T_1 = T \backslash T_2$, without comparing it to the second scheme suggested by Gu, Nemhauser and Savelsbergh [30].

In Section 4.3.2, we have pointed out a disadvantage of using the natural partition, the fact that it may occur that $|T_1| = 0$. We also suggested a modification of the natural partition to fix this problem, i.e., to put a variable with smallest a_j from T_2 to T_1. In contrast to the class of LMCI1, we use this modification already in the default algorithm. This is done, because, when we did not use this modification, the number of cuts found was so small that the other tests, especially for finding the initial cover, were not meaningful anymore. The results for using the natural partition without this modification for the main test set are given in Table B.33 and a summary of the results is contained in Table 4.2. As one can see, the modification clearly improves the performance of the separation algorithm.

Lifting Sequence

In the default algorithm, we use at the second level of the two-level lifting sequence for the set F the Sequence 4 with the restriction to lift the variable removed last from the initial cover first, and for the sets T_2 and R we use Sequence 4.

We have tested this scheme against the lifting sequence used by Martin [44]: use Sequence 1 for the set F (here the variable removed last from the initial cover is lifted first automatically) and Sequence 4 for the sets T_2 and R. The results for using Sequence 1 instead of Sequence 4 for the set F show that there is no significant difference with respect to the initial gap closed (see Table 4.2 for a summary of the results). Therefore, our results confirm the conclusion of Gu, Nemhauser and Savelsbergh [30] for the class of LMCI1 that the lifting order within the sets is not very important.

We have tested the modification concerning the handling of the variable removed last from the initial cover, i.e., we used Sequence 4 for the set F without the restriction to lift this variable first. The results for the main test set are given in Table B.34 and a summary of the results is contained in Table 4.2. As one can see, there is no significant difference in the performance with respect to the initial gap closed (the initial gap closed in geometric mean reduces by 0.03 percentage points). Nevertheless, we will use this modification in our resulting algorithm since this version is less restrictive and it improves the performance of the default algorithm when it is used in connection with solving KP1^{BK}_{max} approximately (see Table B.29 and Table B.35).

Computing the Lifting Coefficients

As for the class of LMCI1, we have decided to compute the lifting coefficients exactly using the extension of Zemel's procedure (analog to Algorithm 4.3). The results for our resulting algorithm (see Table B.35) show that this lifting procedure is fast in practice.

Resulting Algorithm

From the results of our computational study we get the following three stages of the separation algorithm for the class of LEWI.

First stage Fix all variables in N_0 to zero and all variables in N_1 to one in advance and solve KP1^{BK}_{max} approximately using Algorithm 4.2.

Second stage Construct a feasible set T from the initial cover by removing variables, in the reverse order in which Algorithm 4.2 has chosen them to be in the initial cover. Set $T_2 = \{j \in T : x^*_j = 1\}$ and put a variable with smallest a_j from T_2 to T_1 if $|T_1| = 0$.

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)		Sepa Time > 10 sec (Number)	
	Value	△	Value	△	Value	△
Default algorithm	14.46	0.00	1220.2	0.0	4	0
Initial cover - 1. modification[15]	13.94	-0.52	5.3	-1214.9	0	-4
Initial cover - 2. modification[16]	14.03	-0.43	15.1	-1205.1	0	-4
Initial cover - 3. modification[17]	13.49	-0.97	181.8	-1038.4	4	0
Initial cover - 4. modification[18]	15.01	0.55	4.1	-1216.1	0	-4
Minimal cover - modification[19]	15.83	1.37	1501.8	281.6	4	0
Resulting algorithm[18]	15.01	0.55	4.1	-1216.1	0	-4

Table 4.3: Summary of the computational results for the separation algorithm for the class of LMCI2 on the main test set. *Default algorithm*, *default algorithm where a single algorithmic aspect is altered* and *resulting algorithm*. (△ with respect to the default algorithm)

Third stage Use the following lifting order: first up-lift all variables in F using Sequence 4, then down-lift all variables in C_2 using Sequence 4 and finally up-lift all variables in R using Sequence 4. Compute the lifting coefficients exactly using the extension of Zemel's procedure (analog to Algorithm 4.3).

The results for using our resulting algorithm for the main test set are given in Table B.35 and a summary of the results is contained in Table 4.2. By using our resulting algorithm for the main test set we close 16.79 percent of the initial gap in geometric mean and the CPU time spent in the separation routine is 8.5 seconds in total. Thus, we have reduced the separation time without reducing the performance of our separation algorithm with respect to the initial gap closed in geometric mean.

4.4.3 Separation Algorithm for the Class of LMCI2

Our default algorithm for separating the class of LMCI2 is given in Algorithm 4.7. The results for the main test set are given in Table B.36 and a summary of the results is contained in Table 4.3. By using our default algorithm for the main test set, we close 14.46 percent of the initial gap in geometric mean and the CPU time spent in the separation routine is 1220.2 seconds in total. For four instances in the main test set, the separation time is greater than 10 seconds of CPU time. As for the classes of LMCI1 and LEWI, we will see that the large separation time for these instances is caused by the fact that we use the exact algorithm to find an initial cover in the first stage of the default algorithm.

Initial Cover

In the default algorithm for the class of LMCI2, we use the same method to find an initial cover as for the classes of LMCI1 and LEWI (see Section 4.4.1 and Section 4.4.2), i.e., we use the first modification of KP1^{BK} discussed in Section 4.3.1

[15] Solve KP1^{BK}_{max} approximately using Algorithm 4.2.
[16] Solve KP1^{BK}_{max} exactly using Algorithm 4.1 if nc is not greater than 1,000,000 and approximately using Algorithm 4.2 otherwise.
[17] Solve KP2^{BK}_{max} exactly using Algorithm 4.1.
[18] Solve KP2^{BK}_{max} approximately using Algorithm 4.2.
[19] Use nonincreasing a_j as the second order criterium for removing variables.

Input : X^{BK} defined as is (4.2) and $x^* \in [0,1]^n \backslash \{0,1\}^n$ fractional vector with $\sum_{j \in N} a_j x_j^* \leq a_0$.
Output: Violated (with respect to x^*) inequality from the class of LMCI2 or notification that no such inequality was found.

`/* First stage: Initial cover */`
$N_0 \leftarrow \{j \in N : x_j^* = 0\}$ and $N_1 \leftarrow \{j \in N : x_j^* = 1\}$
if $\sum_{j \in N \backslash N_0} a_j - (a_0 + 1) < 0$ **then**
  return *No inequality found*
Call Algorithm 4.1 for

$$\max\{ \sum_{j \in N \backslash (N_0 \cup N_1)} (1 - x_j^*) \bar{z}_j : \sum_{j \in N \backslash (N_0 \cup N_1)} a_j \bar{z}_j \leq \sum_{j \in N \backslash N_0} a_j - (a_0 + 1), \quad \bar{z}_j \in \{0,1\} \text{ for all } j \in N \backslash (N_0 \cup N_1) \}$$

(Let $\bar{z}^*$ be the solution.)
$C \leftarrow N_1 \cup \{j \in N \backslash (N_0 \cup N_1) : \bar{z}_j^* = 0\}$

`/* Second stage: Minimal cover and partition */`
Sort C by nonincreasing $\frac{1 - x_j^*}{a_j}$ and use nondecreasing a_j as a second order criterium.
$c \leftarrow |C|$
for $j \leftarrow 1$ **to** c **and** C *is not minimal* **do**
  if $\sum_{i \in C} a_i - a_j > a_0$ **then**
    $C \leftarrow C \backslash \{j\}$

`/* Third stage: Computing the lifting coefficients */`
Call Algorithm 4.4 for X^{BK} and $\sum_{j \in C} x_j \leq |C| - 1$. (Let $\sum_{j \in C} x_j + \sum_{j \in N \backslash C} \alpha_j x_j \leq |C| - 1$ be the lifted valid inequality for X^{BK}.)

`/* Result */`
if $\sum_{j \in C} x_j^* + \sum_{j \in N \backslash C} \alpha_j x_j^* > |C| - 1$ **then**
  return $\sum_{j \in C} x_j + \sum_{j \in N \backslash C} \alpha_j x_j \leq |C| - 1$
else
  return *No inequality found*

Algorithm 4.7: Separation algorithm for the class of LMCI2. *Default algorithm.*

without comparing it to the unmodified version and solve KP1^{BK}_{max} exactly using Algorithm 4.1.

As for the classes of LMCI1 and LEWI, the large separation time needed for some of the instances in the main test set when using the default algorithm reduces when KP1^{BK}_{max} is solved approximately using Algorithm 4.2 (see Table 4.3 for a summary of the results for the main test set). But as for the class of LMCI1, in addition, the initial gap closed reduces for most of the instances in the main test set. This reduction of the initial gap closed can be eliminated for the instances which already had a small separation time in the default algorithm if we use Algorithm 4.1 if nc is not greater than 1,000,000 (where n and c are defined as in Algorithm 4.1) and Algorithm 4.2 otherwise. The results for the last scheme are given in Table B.37 and a summary of the results in contained in Table 4.3. As for the class of LMCI1, one can see that this version performs better than the default algorithm, since all instances in the main test set have separation time less than or equal to 10 seconds of CPU time and the gap closed reduces only for those instances for which using the default algorithm led to a large separation time.

The second modification of KP1^{BK} discussed in Section 4.3.1 was assumed to improve the performance of the separation algorithm, which turned out to be true for the class of LMCI1 and false for the class of LEWI. For the class of LMCI2, the assumption is again fulfilled. The results for solving KP2^{BK}_{max} exactly using Algorithm 4.1 are given in Table B.38 and a summary of the results is contained in Table 4.3. As for the class of LMCI1, one can see that for some of the instances in the main test set the initial gap closed increases, but for others it decreases. The initial gap closed in geometric reduces by 0.97 percentage points. Here we also obtained better results when we solved KP2^{BK}_{max} approximately using Algorithm 4.2. The results are given in Table B.39 and a summary is contained in Table 4.3. The initial gap closed in geometric mean increases by 0.55 percentage points and the separation time is 4.1 seconds of CPU time in total. Thus, as for the class of LMCI1, the last scheme outperforms the version of the default algorithm where we solve KP2^{BK}_{max} exactly. And it also performs better than the version where we solve KP1^{BK}_{max} using Algorithm 4.1 if nc is not greater than 1,000,000 and using Algorithm 4.2 otherwise.

Minimal Cover

As for the class of LMCI1, in the default algorithm, we make the initial cover minimal by removing variables if necessary, in the reverse order in which Algorithm 4.2 would have chosen them to be in the initial cover. In Section 4.3.2, we discussed two schemes for the second order criterium for removing variables. In the default algorithm, we use the criterium of nondecreasing a_j.

We have tested it against the criterium of nonincreasing a_j. The results for our main test set are given in Table B.40 and a summary of the results is contained in Table 4.3. As for the class of LMCI1, the second scheme performs better than the first one. The initial gap closed in geometric mean increases by 1.37 percentage points. Nevertheless as for the class of LMCI1, in our resulting algorithm, we use the first scheme since using the second scheme in our resulting algorithm reduces the initial gap closed. In geometric mean, it is 12.95 percent in contrast to 15.01 percent for using the first scheme in the resulting algorithm (see Table B.39). The possible explanation for the reduction of the performance given in Section 4.4.1 for the class of LMCI1 can also be applied here.

Resulting Algorithm

From the results of our computational study we get the following three stages of the separation algorithm for the class of LMCI2.

First stage Fix all variables in N_0 to zero and all variables in N_1 to one in advance and solve KP2^{BK}_{max} approximately using Algorithm 4.2.

Second stage Make the initial cover minimal by removing variables if necessary, in the reverse order in which Algorithm 4.2 has chosen them to be in the initial cover and use as the second order criterium for removing variables nondecreasing a_j.

Third stage Compute the lifting coefficients exactly using Algorithm 4.4.

Thus, the version of the default algorithm where we solve KP2^{BK}_{max} approximately using Algorithm 4.2 to find the initial cover becomes our resulting algorithm. The results for the main test set were given in Table B.39 and a summary is contained in Table 4.3. By using our resulting algorithm for the main test set we close 15.01 percent of the initial gap in geometric mean and the CPU time spent in the separation routine is 4.1 seconds in total. Thus, we have significantly improved the performance of our default separation algorithm for the class of LMCI2.

4.4.4 Comparison

In the last three sections, we have investigated the effect of using different algorithmic choices on the performance of the separation algorithms for the classes of LMCI1, LEWI and LMCI2. Building on these results we have developed efficient separation algorithms for the three classes. We decided to include all three classes in the computational study as the three separation algorithms may lead to different cuts (see Section 4.2). In this section, we will investigate whether in practice this is true and develop the final cutting plane separator for the 0-1 knapsack problem.

Out of the three developed separation algorithms the one for the class of LEWI performs best (see Table 4.4). We have tested an algorithm which separates both the class of LEWI and the class of LMCI1. The results for the main test set are given in Table B.41 and a summary of the results is contained in Table 4.4. They show that separating both classes of valid inequalities instead of separating only the class of LEWI improves the performance of the separation algorithm. We conclude that in practice our separation algorithms for the classes of LEWI and LMCI1 lead to different cuts.

Furthermore, the separation algorithm for the class of LMCI1 performs better than the separation algorithm for the class of LMCI2 (see Table 4.4). In particular, the separation algorithm for the class of LMCI1 finds more cuts than the one for the class of LMCI2 for most of the instances in our main test set (see Table B.27 and Table B.39). The results for the main test set for separating both the class of LMCI1 and the class of LMCI2 are given in Table B.42 and a summary of the

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)		Sepa Time > 10 sec (Number)	
	Value	△	Value	△	Value	△
LEWI	16.79	0.00	8.5	0.0	0	0
LMCI1	16.36	-0.43	7.4	-1.1	0	0
LMCI2	15.01	-1.78	4.1	-4.4	0	0
LEWI and LMCI1	17.52	0.73	11.6	3.1	0	0
LMCI1 and LMCI2	16.59	-0.20	7.2	-1.3	0	0
LEWI and LMCI2	17.09	0.30	8.9	0.4	0	0
LEWI, LMCI1 and LMCI2	17.27	0.48	12.5	4.0	0	0

Table 4.4: Summary of the computational results for different combinations of the separation algorithms for the classes of LMCI1, LEWI and LMCI2 on the main test set. (△ with respect to the resulting separation algorithm for the class of LEWI)

results is contained in Table 4.4. Note that in Table B.42 the △ values are given with respect to the separation algorithm for the class of LMCI1. As one can see, there is no significant difference between the performance of this algorithm and the performance of the separation algorithm for the class of LMCI1. We conclude that the cuts found by our separation algorithm for the class of LMCI2 are nearly the same as the cuts found by our separation algorithm for the class of LMCI1, i.e., our separation algorithm for the class of LMCI2 does not find many cuts with fractional coefficients. The results for separating both the class of LEWI and the class of LMCI2 (see Table B.43 and Table 4.4) confirm this conclusions, since this algorithm performs better than the separation algorithm for the class of LEWI but not as good as the algorithm separating both the class of LEWI and the class of LMCI1.

Finally, we have tested an algorithm which separates all three classes of valid inequalities for the 0-1 knapsack polytope. The results for the main test set are given in Table B.44 and a summary of the results is contained in Table 4.4. As one can see, this algorithm performs worse than the algorithm which separates only the classes of LEWI and LMCI1.

Thus, in our final cutting plane separator for the 0-1 knapsack problem we separate both the class of LMCI1 and the class of LEWI. By using this cutting plane separator for the main test set, we close 17.52 percent of the initial gap in geometric mean and the CPU time spent in the separation routine is 11.6 seconds in total. The results for applying our final cutting plane separator to the remaining test set are given in Table B.45. They show that the separation time is on an acceptable level for all instances in the extended initial test set.

4.5 Conclusion

In our computational study, we found out that for all three classes of valid inequalities the construction of the initial cover is the crucial point for implementing fast and effective separation algorithms.

This includes both the separation problem which is used to find the initial cover and the algorithm used to solve this separation problem. The best combination differs for the three classes of valid inequalities. The modification of the separation problem for the class of cover inequalities suggested by Gu, Nemhauser, and Savelsbergh [30] helped to improve the performance of the separation algorithms for the classes of LMCI1 and LMCI2 but only if the separation problem was solved approximately. For the class of LEWI, independent of whether we used the approximate or exact algorithm this modification led to a degradation in the performance of the separation algorithm. On our test set, we obtained the best results when we did not apply the construction used in [44] but solved the original separation problem approximately. Here, as in all other test runs carried out in this chapter, the exact algorithm for solving the separation problem led to an unacceptable amount of time spent in the separation algorithm.

Furthermore, we observed that the effect of using different constructions for the minimal cover strongly depends on whether the modification of Gu, Nemhauser, and Savelsbergh [30] is applied.

Finally, our computational experiments showed that our resulting separation algorithm for the class of LEWI performs slightly better than the one for the class of LMCI1 and that the combination of both algorithms leads to the best performance on our test set. Concerning the practical usefulness of separating the class of LMCI2 we found out that the corresponding separation algorithm is inferior to the one for the class of LMCI1. Furthermore, the separation algorithm for the class of LMCI2 does not help to improve the performance of the cutting plane separator when it is used neither in combination with the one for the class of LMCI1 nor with the one separating both the class of LEWI and LMCI1.

Chapter 5

Cutting Plane Separator for the 0-1 Single Node Flow Problem

In this chapter we investigate the implementation of an efficient cutting plane separator for the 0-1 single node flow problem. The algorithm generates valid inequalities for the feasible region of the 0-1 single node flow problem, the so-called 0-1 single node flow set. This set with some additional simple constraints can be obtained as a relaxation of a mixed integer set. Therefore, the cutting plane separator can be applied to each row of a MIP.

5.1 Introduction

In the 1980s, Padberg, Van Roy, and Wolsey [53] introduced a class of valid inequalities for the 0-1 single node flow set with only inflow arcs. The inequalities are based on a structure called flow cover and are denoted by flow cover inequalities (FCIs). These results were generalized to the 0-1 single node flow set with additional outflow arcs by Van Roy and Wolsey [54]. They derived the class of generalized flow cover inequalities (GFC inequalities), which, among other things, depend on a constant $\bar{u}$. In [54], two choices of the value of $\bar{u}$ are emphasized. The corresponding inequalities are denoted by GFC1 and GFC2 inequalities. Note that in later literature, the GFC1 inequality is also called extended generalized flow cover inequality (EGFCI) and the GFC2 inequality is also called simple generalized flow cover inequality (SGFCI). Furthermore, in [54], a procedure is given to transform a mixed 0-1 integer set into a 0-1 single node flow set with some additional simple constraints (see also [46]).

As for the classes of valid inequalities for the 0-1 knapsack polytope, the concept of superadditive lifting was investigated to strengthen FCIs and SGFCIs. Gu, Nemhauser, and Savelsbergh [31] derived the classes of lifted flow cover inequalities (LFCIs) and lifted simple generalized flow cover inequalities (LSGFCIs). In addition, they showed that LFCIs are also at least as strong as EGFCIs.

Marchand [39] showed that the c-MIR approach can also be used to derive valid inequalities for the 0-1 single node flow set which are at least as strong as SGFCIs and EGFCIs respectively, i.e., he showed that particular c-MIR inequalities (see Chapter 3) for particular mixed knapsack relaxations of the 0-1 single node flow set are at least as strong as SGFCIs and EGFCIs respectively (see also [38]). Here, the structure of a flow cover does also play a crucial role. In addition, he showed that the inequality which is at least as strong as the EGFCI is equivalent to the LFCI derived in [31] and that the inequality which is at least as strong as the SGFCI does not necessarily dominate the LSGFCI derived in [31].

In [41], another class of valid inequalities for the 0-1 single node flow set was introduced, namely the class of continuous cover inequalities. This class is also derived from a mixed knapsack relaxation of the 0-1 single node flow set and is based on a structure called k-cover. In [4], it was shown that LSGFCIs are at least as strong as continuous cover inequalities.

Cutting plane separators for the classes of valid inequalities mentioned above have been used successively in linear programming based branch-and-cut algorithms to solve BMIPs. See [55], for the separation of the classes of SGFCIs and EGFCIs, and [41] for the separation of the class of continuous cover inequalities. A computational study comparing the effectiveness of SGFCIs, EGFCIs, LSGFCIs, and LFCIs was presented in [31]. It turned out that separating the class of LSGFCIs yields the best performance of the branch-and-cut algorithm.

The classes of valid inequalities mentioned above are based on the structure of a flow cover and k-cover, respectively. Further classes of valid inequalities for the 0-1 single node flow set have been derived in the literature. They are complementary to the classes mentioned above and are based on the structure of a flow pack and k-reverse cover, respectively. See [57], for the introduction of the class of generalized flow pack inequalities, [4] for the investigation of superadditive lifting of special cases of generalized flow pack inequalities, [38, 39] for the c-MIR approach and [41] for the introduction of the class of continuous reverse cover inequalities. The relationship between the inequalities based on flow packs and k-reverse covers is similar to the relationship between the complementary inequalities based on flow covers and k-covers (see [4, 39]).

Considering the relationship between the classes of valid inequalities for the 0-1 single node flow set and the results of the computational study of Gu, Nemhauser, and Savelsbergh [31], it seems reasonable to us to separate the class of LSGFCI or to use Marchand's c-MIR approach in our cutting plane separator for the 0-1 single node flow problem and to separate in addition the corresponding complementary class of valid inequalities. We decided to use the c-MIR approach, i.e., we implement a cutting plane separator for the 0-1 single node flow problem which generates c-MIR inequalities for particular mixed knapsack relaxations of the 0-1 single node flow set. We call these inequalities c-MIR flow cover inequalities (c-MIRFCIs). Furthermore, we investigate the effectiveness of separating in addition the complementary class of valid inequalities which we call c-MIR flow pack inequalities (c-MIRFPIs).

The remainder of this chapter is organized as follows. In Section 5.2, we introduce the classes of SGFCIs, EGFCIs, and c-MIRFCIs, and review the results concerning the relationship between these inequalities. Furthermore, we show how c-MIRFPIs can be obtained as c-MIRFCIs for a relaxation of the 0-1 single node flow set. In Section 5.3, we develop a separation algorithm for the class of c-MIRFCIs which uses the relationship between this class and the classes of SGFCIs and EGFCIs, and discuss different algorithmic aspects of the separation algorithm. In addition, we state a procedure to construct 0-1 single node flow relaxations of mixed integer sets. Our computational results for using the different algorithmic and implemen-

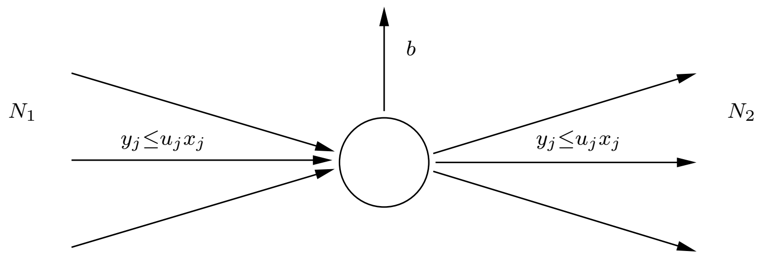

Figure 5.1: 0-1 single node flow set.

tation choices discussed in Section 5.3 are reported in Section 5.4. In Section 5.5, a conclusion and remarks on possible extensions are given.

5.2 Valid Inequalities for the 0-1 Single Node Flow Set

In this section, we introduce three classes of valid inequalities for the 0-1 single node flow set; the classes of simple generalized flow cover inequalities (SGFCIs) and extended generalized flow cover inequalities (EGFCIs) and the class of c-MIR flow cover inequalities (c-MIRFCIs). Furthermore, we show that particular c-MIRFCIs are at least as strong as SGFCIs and EGFCIs respectively. Therefore, in our cutting plane separator for the 0-1 single node flow problem, we separate the class of c-MIRFCIs. The separation algorithm strongly uses the relationship between this class and the other two. This is the reason for introducing not only the class of c-MIRFCIs in this section.

We consider the *0-1 single node flow set*

$$X^{SNF} = \{(x,y) \in \{0,1\}^n \times \mathbb{R}^n_+ : \sum_{j \in N_1} y_j - \sum_{j \in N_2} y_j \leq b, \quad y_j \leq u_j x_j \text{ for all } j \in N\}, \tag{5.1}$$

where b is a rational number, u_j are nonnegative rational numbers for all $j \in N = \{1, \ldots, n\}$ and (N_1, N_2) is a partition of N.

X^{SNF} is the feasible region of a 0-1 single node flow problem with external demand of b and inflow arcs $j \in N_1$ and outflow arcs $j \in N_2$ of the single node (see Figure 5.1). Here, each real variable y_j, $j \in N$ represents the flow on an arc j and each binary variable x_j, $j \in N$ indicates whether arc j is open ($x_j = 1$) or closed ($x_j = 0$). The flow on the inflow and outflow arcs of the single node is constrained by the conservation inequality. In addition, for each arc $j \in N$, the flow y_j on it is bounded by the capacity u_j of arc j if arc j is open and by zero otherwise.

Van Roy and Wolsey [54] derive two classes of valid inequalities for X^{SNF} as special cases of the class of generalized flow cover (GFC) inequalities. Both are based on the structure of a flow cover. A pair (C_1, C_2) is called a *flow cover* for X^{SNF} if $C_1 \subseteq N_1$, $C_2 \subseteq N_2$ and

$$\sum_{j \in C_1} u_j - \sum_{j \in C_2} u_j = b + \lambda$$

with $\lambda > 0$. Note that C_1 and C_2 can be empty.

Theorem 5.1 ([54]). *If (C_1, C_2) is a flow cover for X^{SNF} and $L_2 \subseteq N_2 \backslash C_2$, then the inequality*

$$\sum_{j \in C_1} y_j + (u_j - \lambda)^+(1 - x_j) - \sum_{j \in C_2} u_j - \sum_{j \in L_2} min\{u_j, \lambda\} x_j - \sum_{j \in N_2 \backslash (C_2 \cup L_2)} y_j \leq b \tag{5.2}$$

is valid for X^{SNF}.

We call inequality (5.2) *simple generalized flow cover inequality* (SGFCI), which is the name used in [31]. In [54], it was introduced as GFC2 inequality.

Theorem 5.2 ([54]). *If (C_1, C_2) is a flow cover for X^{SNF} with $C_1 \neq \emptyset$, $L_1 \subseteq N_1 \backslash C_1$, $L_2 \subseteq N_2 \backslash C_2$ and $\bar{u} = max_{j \in C_1} u_j > \lambda$, then the inequality*

$$\begin{aligned}\sum_{j \in C_1} y_j + (u_j - \lambda)^+(1 - x_j) + \sum_{j \in L_1} y_j - (max\{\bar{u}, u_j\} - \lambda) x_j \\ - \sum_{j \in C_2} u_j - min\{\lambda, (u_j - \bar{u} + \lambda)^+\}(1 - x_j) \\ - \sum_{j \in L_2} min\{u_j, max\{u_j - \bar{u} + \lambda, \lambda\}\} x_j - \sum_{j \in N_2 \backslash (C_2 \cup L_2)} y_j \leq b\end{aligned} \tag{5.3}$$

is valid for X^{SNF}.

We call inequality (5.3) *extended generalized flow cover inequality* (EGFCI), which is again the name used in [31]. In [54], it was introduced as GFC1 inequality.

Marchand and Wolsey [39, 42] introduced the class of c-MIR inequalities for the mixed knapsack set (see Chapter 3). Furthermore, in [39], it was shown how a flow cover (C_1, C_2) for X^{SNF} and sets $L_1 \subseteq N_1 \backslash C_1$ and $L_2 \subseteq N_2 \backslash C_2$ can be used to construct a mixed knapsack relaxation of X^{SNF} and how these sets can, in addition, be used to obtain c-MIR inequalities for this relaxation. Thus, the c-MIR approach can be applied to derive another class of valid inequalities for X^{SNF}. This result is given in Theorem 5.3. We state the proof since it involves the construction mentioned above, which we will use in our cutting plane separator for the 0-1 single node flow problem. See also [38], for a review of this result.

Let f_d for $d \in \mathbb{R}$ as well as the MIR function $F_\alpha : \mathbb{R} \to \mathbb{R}$ for $0 \leq \alpha < 1$ be defined as in Section 3.2.

Theorem 5.3 ([38, 39]). *If (C_1, C_2) is a flow cover for X^{SNF}, $L_1 \subseteq N_1 \backslash C_1$, $L_2 \subseteq N_2 \backslash C_2$ and $\bar{u} \in \mathbb{Q}_+ \backslash \{0\}$ with $\bar{u} > \lambda$, then the inequality*

$$\begin{aligned}\sum_{j \in C_1} y_j + (u_j + \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}))(1 - x_j) + \sum_{j \in L_1} y_j - (u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}})) x_j \\ - \sum_{j \in C_2} u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}})(1 - x_j) + \sum_{j \in L_2} \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}) x_j \\ - \sum_{j \in N_2 \backslash (C_2 \cup L_2)} y_j \leq b,\end{aligned} \tag{5.4}$$

where $\beta = -\frac{\lambda}{\bar{u}}$, is valid for X^{SNF}.

Proof. Mixed knapsack relaxation of X^{SNF}. Using the lower and the upper bounds imposed on the real variables, we substitute $y_j = u_j x_j - \bar{y}_j$ for all $j \in N' = C_1 \cup L_1 \cup C_2 \cup L_2$ and use the trivial substitution $y_j = 0 + \bar{y}_j$ for all $j \in N \backslash N'$ in the conservation inequality of X^{SNF} and obtain the set

$$\begin{aligned}\{(x, \bar{y}) \in \{0,1\}^n \times \mathbb{R}^n_+ : \sum_{j \in C_1 \cup L_1} u_j x_j + \sum_{j \in N_1 \backslash (C_1 \cup L_1)} \bar{y}_j - \sum_{j \in C_2 \cup L_2} u_j x_j \\ \leq b + \sum_{j \in C_1 \cup L_1} \bar{y}_j + \sum_{j \in N_2 \backslash (C_2 \cup L_2)} \bar{y}_j - \sum_{j \in C_2 \cup L_2} \bar{y}_j, \\ \bar{y}_j \leq u_j x_j \text{ for all } j \in N\}.\end{aligned}$$

Using the nonnegativity of $\bar{y}_j$ for all $j \in N_1 \backslash (C_1 \cup L_1)$ and for all $j \in C_2 \cup L_2$, we obtain the mixed knapsack relaxation

$$\begin{aligned}X^{MK} = \{(x, s) \in \mathbb{Z}^{n'}_+ \times \mathbb{R}_+ : \sum_{j \in C_1 \cup L_1} u_j x_j - \sum_{j \in C_2 \cup L_2} u_j x_j \leq b + s, \\ x_j \leq 1 \text{ for all } j \in N'\}\end{aligned}$$

of X^{SNF}, where $s = \sum_{j \in C_1 \cup L_1} \bar{y}_j + \sum_{j \in N_2 \backslash (C_2 \cup L_2)} \bar{y}_j$ and $n' = |N'|$.

C-MIR inequality for X^{MK}. By Theorem 3.3 for $(T, U) = (L_1 \cup L_2, C_1 \cup C_2)$ partition of N' and $\delta = \bar{u}$, the c-MIR inequality

$$\begin{aligned}\sum_{j \in C_1} F_{f_\beta}(-\frac{u_j}{\bar{u}})(1 - x_j) + \sum_{j \in L_1} F_{f_\beta}(\frac{u_j}{\bar{u}}) x_j + \sum_{j \in C_2} F_{f_\beta}(\frac{u_j}{\bar{u}})(1 - x_j) \\ + \sum_{j \in L_2} F_{f_\beta}(-\frac{u_j}{\bar{u}}) x_j \leq \lfloor \beta \rfloor + \frac{s}{\bar{u}(1 - f_\beta)},\end{aligned} \tag{5.5}$$

where $\beta = \frac{b - \sum_{j \in C_1} u_j + \sum_{j \in C_2} u_j}{\bar{u}}$, is valid for X^{MK}.

Valid inequality for X^{SNF}. Since $\beta = -\frac{\lambda}{\bar{u}}$ and $\bar{u} > \lambda > 0$, $\lfloor \beta \rfloor = -1$ and therefore $1 - f_\beta = \frac{\lambda}{\bar{u}}$. Thus, inequality (5.5) is equivalent to

$$\begin{aligned}\sum_{j \in C_1} F_{f_\beta}(-\frac{u_j}{\bar{u}})(1 - x_j) + \sum_{j \in L_1} F_{f_\beta}(\frac{u_j}{\bar{u}}) x_j + \sum_{j \in C_2} F_{f_\beta}(\frac{u_j}{\bar{u}})(1 - x_j) \\ + \sum_{j \in L_2} F_{f_\beta}(-\frac{u_j}{\bar{u}}) x_j \leq -1 + \frac{s}{\lambda}.\end{aligned} \tag{5.6}$$

Multiplying inequality (5.6) by $\lambda > 0$, using the definition of λ and restating the inequality in terms of the original variables gives inequality (5.4) valid for X^{SNF}. □

Inequality (5.4) is called *c-MIR flow cover inequality* (c-MIRFCI), and the family of all c-MIRFCIs is called the *class of c-MIRFCIs*.

Marchand [39] showed that particular c-MIRFCIs are at least as strong as EGFCIs. In [38], this result was given for EGFCIs where the coefficient $-\min\{u_j, \max\{u_j - \bar{u} + \lambda, \lambda\}$ of x_j for all $j \in L_2$ is relaxed to $-\max\{u_j - \bar{u} + \lambda, \lambda\}$.

Corollary 5.4 ([39]). *If (C_1, C_2) is a flow cover for X^{SNF}, $L_1 \subseteq N_1 \backslash C_1$, $L_2 \subseteq N_2 \backslash C_2$ and $\bar{u} = max_{j \in C_1} u_j > \lambda$, then the c-MIRFCI (5.4) takes the form*

$$\sum_{j \in C_1} y_j + (u_j - \lambda)^+(1 - x_j) + \sum_{j \in L_1} y_j - (u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}}))x_j$$
$$- \sum_{j \in C_2} u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}})(1 - x_j) + \sum_{j \in L_2} \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}})x_j - \sum_{j \in N_2 \backslash (C_2 \cup L_2)} y_j \leq b,$$

where $\beta = -\frac{\lambda}{\bar{u}}$ and is at least as strong as the EGFCI (5.3).

Marchand [39] also showed that particular c-MIRFCIs are at least as strong as SGFCIs where the coefficient $-\min\{u_j, \lambda\}$ of x_j for all $j \in L_2$ is relaxed to $-\lambda$. In Corollary 5.5, we proof that this also holds when omitting the relaxation.

Corollary 5.5. *If (C_1, C_2) is a flow cover for X^{SNF}, $L_1 \subseteq N_1 \backslash C_1$, $L_2 \subseteq N_2 \backslash C_2$ and $\bar{u} = max_{j \in C_1 \cup L_2} u_j > \lambda$, then the c-MIRFCI (5.4) takes the form*

$$\sum_{j \in C_1} y_j + (u_j - \lambda)^+(1 - x_j) + \sum_{j \in L_1} y_j - (u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}}))x_j$$
$$- \sum_{j \in C_2} u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}})(1 - x_j) - \sum_{j \in L_2} min\{u_j, \lambda\}x_j - \sum_{j \in N_2 \backslash (C_2 \cup L_2)} y_j \leq b, \quad (5.7)$$

where $\beta = -\frac{\lambda}{\bar{u}}$. If in addition $L_1 = \emptyset$, then the c-MIRFCI (5.7) is at least as strong as the SGFCI (5.2).

Proof. In [38], it was observed that $\lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}) = -\min\{u_j, \lambda\}$ for $u_j \leq \bar{u}$. Since $u_j \leq \bar{u}$ for all $j \in C_1 \cup L_2$, $u_j + \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}) = u_j - \min\{u_j, \lambda\} = (u_j - \lambda)^+$ for all $j \in C_1$ and $\lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}) = -\min\{u_j, \lambda\}$ for all $j \in L_2$. Therefore, the c-MIRFCI (5.4) takes the form (5.7).

By the definition of $F_\alpha : \mathbb{R} \to \mathbb{R}$ for $0 \leq \alpha < 1$, $F_{f_\beta}(d) \geq 0$ for $d \geq 0$. Since $u_j \geq 0$ and $x_j \in \{0, 1\}$ for all $j \in C_2$ and $\bar{u} > \lambda > 0$, $\lambda F_{f_\beta}(\frac{u_j}{\bar{u}})(1 - x_j) \geq 0$ for all $j \in C_2$. Therefore, if $L_1 = \emptyset$, the c-MIRFCI (5.7) is at least as strong as the SGFCI (5.2). □

The following example shows that if $L_1 \subseteq N_1 \backslash C_1$ is chosen arbitrarily the c-MIRFCI (5.7) is not necessarily at least as strong as neither the SGFCI nor the relaxed SGFCI. Note that this disproves the statement in [38] (Section 4.2, Corollary 3) where Corollary 5.5 is given for the relaxed SGFCI with $L_1 \subseteq N_1 \backslash C_1$ chosen arbitrarily.

Example 5.6. Consider the 0-1 single node flow set

$$X^{SNF} = \{(x, y) \in \{0, 1\}^6 \times \mathbb{R}^6_+ : y_1 + y_2 - y_3 + y_4 + y_5 - y_6 \leq -8,$$
$$y_1 \leq 10x_1, y_2 \leq 9x_2, y_3 \leq 7x_3,$$
$$y_4 \leq 16x_4, y_5 \leq 5x_5, y_6 \leq 19x_6\}.$$

Taking flow cover $(C_1, C_2) = (\{1, 2\}, \{6\})$ with $\lambda = 8$, sets $L_1 = \{4\}$ and $L_2 = \emptyset$ and $\bar{u} = \max_{j \in C_1 \cup L_2} u_j = 10 > \lambda$ by Corollary 5.5, we obtain the c-MIRFCI

$$[y_1 + 2(1 - x_1)] + [y_2 + 1(1 - x_2)] + [y_4 - 4x_4] - [19 - 15(1 - x_6)] - y_3 \leq -8 \quad (5.8)$$

and by Theorem 5.1, we obtain the SGFCI

$$[y_1 + 2(1 - x_1)] + [y_2 + 1(1 - x_2)] - 19 - y_3 \leq -8. \tag{5.9}$$

We show that the c-MIRFCI (5.8) is not necessarily at least as strong as the SGFCI (5.9) since there exists a point $(x^*, y^*) \in \mathbb{R}^6_+ \times \mathbb{R}^6_+$ with

$$(x^*, y^*) \in \{(x, y) \in \mathbb{R}^6_+ \times \mathbb{R}^6_+ : (x, y) \text{ satisfies } (5.8)\},$$

but

$$(x^*, y^*) \notin \{(x, y) \in \mathbb{R}^6_+ \times \mathbb{R}^6_+ : (x, y) \text{ satisfies } (5.9)\}.$$

Consider $(x^*, y^*) = ((1, 1, 0, 1, 0, 1), (10, 3, 0, 0, 0, 0))$. Then, $(x^*, y^*) \in \{(x, y) \in \mathbb{R}^6_+ \times \mathbb{R}^6_+ : (x, y) \text{ satisfies } (5.8)\}$, since

$$[y^*_1 + 2(1 - x^*_1)] + [y^*_2 + (1 - x^*_2)] + [y^*_4 - 4x^*_4] - [19 - 15(1 - x^*_6)] - y^*_3 = -10 < -8,$$

but $(x^*, y^*) \notin \{(x, y) \in \mathbb{R}^6_+ \times \mathbb{R}^6_+ : (x, y) \text{ satisfies } (5.9)\}$, since

$$[y^*_1 + 2(1 - x^*_1)] + [y^*_2 + (1 - x^*_2)] - 19 - y^*_3 = -6 > -8.$$

Remark 5.7. A pair (C_1, C_2) is called a *flow pack* for X^{SNF} if $C_1 \subseteq N_1$, $C_2 \subseteq N_2$ and

$$\sum_{j \in C_1} u_j - \sum_{j \in C_2} u_j = b - \lambda,$$

with $\lambda > 0$. As mentioned in the introduction of this chapter, classes of valid inequalities for X^{SNF} based on flow packs have been derived in the literature. It is well known that these classes of valid inequalities can also be obtained by applying known results based on flow covers to the relaxation

$$X^{SNF}_{rel} = \{(x, y, s) \in \{0, 1\}^n \times \mathbb{R}^n_+ \times \mathbb{R}_+ : -\sum_{j \in N_1} y_j + \sum_{j \in N_2} y_j - s \leq -b, \\ y_j \leq u_j x_j \text{ for all } j \in N\}$$

of X^{SNF} (see [4, 38, 57]). Note that the flow pack (C_1, C_2) for X^{SNF} is a flow cover for X^{SNF}_{rel}. X^{SNF}_{rel} is constructed by introducing a slack variable $s \in \mathbb{R}_+$ in the conservation constraint defining X^{SNF}, multiplying the obtained equality by minus one and relaxing it to an inequality.

In particular, for X^{SNF}_{rel}, (C_1, C_2) flow cover for X^{SNF}_{rel}, sets $L_1 \subseteq N_1 \backslash C_1$ and $L_2 \subseteq N_2 \backslash C_2$ and a constant $\bar{u} \in \mathbb{Q}_+ \backslash \{0\}$, we can apply the construction used in the proof of Theorem 5.3 (the slack variable is handled like the real variables y_j with $j \in N_1 \backslash (C_1 \cup L_1)$) in order to obtain the inequality

$$\begin{aligned} -\sum_{j \in C_1} u_j - y_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}})(1 - x_j) + \sum_{j \in L_1} y_j + \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}})x_j \\ + \sum_{j \in C_2} (u_j + \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}))(1 - x_j) + \sum_{j \in L_2} -(u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}}))x_j \\ - \textstyle\sum_{j \in N_2 \backslash (C_2 \cup L_2)} y_j \leq 0 \end{aligned} \tag{5.10}$$

valid for X^{SNF}, where $\beta = -\frac{\lambda}{\bar{u}}$ (see [38]). Inequality (5.10) is called *c-MIR flow pack inequality* (c-MIRFPI), and the family of all c-MIRFPIs is called the *class of c-MIRFPIs*.

The theory presented in this chapter is much more general than it appears; it can be used to separate cutting planes for BMIPs. A transformation of a *mixed 0-1 integer set* in which all real variables have simple or variable upper bounds into a 0-1 single node flow set with some additional simple constraints is given in [54] (see also [46]). In Section 5.3.3, we consider *mixed integer sets* in which the real variables have simple lower and upper bounds and SCIP 0.81 specific variable lower and upper bounds. We state a procedure similar to the one given in [54] to construct 0-1 single node flow relaxations with some additional simple constraints of these sets. Note that here integer variables which are not binary variables are simply relaxed to real variables. Using this procedure, we can apply our cutting plane separator for the 0-1 single node flow problem to each row of a MIP.

5.3 Algorithmic Aspects

In the last section, we introduced the class of c-MIRFCIs valid for 0-1 single node flow sets. Here, we first investigate algorithmic aspects of a cutting plane separator for the 0-1 single node flow problem which separates this class of valid inequalities and then state a procedure which constructs 0-1 single node flow relaxations with some additional simple constraints of mixed integer sets.

Let $(x^*, y^*) \in ([0,1]^n \backslash \{0,1\}^n) \times \mathbb{R}^n_+$ be a fractional vector, and let X^{SNF} be a 0-1 single node flow set. We want to solve the following separation problem.

Separation problem for the class of c-MIRFCIs

Find sets $C_1 \subseteq N_1$ and $C_2 \subseteq N_2$ such that $\sum_{j \in C_1} u_j - \sum_{j \in C_2} u_j = b + \lambda$ with $\lambda > 0$, sets $L_1 \subseteq N_1 \backslash C_1$ and $L_2 \subseteq N_2 \backslash C_2$ and a constant $\bar{u} \in \mathbb{Q}_+ \backslash \{0\}$ with $\bar{u} > \lambda$ such that

$$\begin{aligned}\sum_{j \in C_1} y_j^* + (u_j + \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}))(1 - x_j^*) + \sum_{j \in L_1} y_j^* - (u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}}))x_j^* \\ - \sum_{j \in C_2} u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}})(1 - x_j^*) + \sum_{j \in L_2} \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}})x_j^* \\ - \sum_{j \in N_2 \backslash (C_2 \cup L_2)} y_j^* > b,\end{aligned}$$

where $\beta = -\frac{\lambda}{\bar{u}}$, or show that no inequality in the class of c-MIRFCIs is violated by (x^*, y^*).

In the following, we assume that a flow cover (C_1, C_2) for X^{SNF} exists, i.e., $\sum_{j \in N_1} u_j > b$.

The separation problem for the class of c-MIRFCIs is equivalent to solving a family of equality knapsack problems parameterized by λ and $\bar{u}$. However, if a flow cover (C_1, C_2) for X^{SNF} is specified and $\bar{u}$ is fixed, the remaining problem of choosing L_1 and L_2 such that the violation of the resulting c-MIRFCI is maximized can be solved by comparison. Thus, the main aspect of the separation algorithm is to choose a useful flow cover (C_1, C_2) for X^{SNF} and a useful value of $\bar{u} \in \mathbb{Q}_+ \backslash \{0\}$ with $\bar{u} > \lambda$.

In the last section, we showed that for two values of $\bar{u}$, the c-MIRFCI is at least as strong as the SGFCI if $L_1 = \emptyset$ and at least as strong as the EGFCI respectively. Therefore, as a first attempt we concentrate on those two values of $\bar{u}$. Furthermore, to find a useful flow cover, we use the same approach which has been devised by various researchers for the separation algorithm for the class of SGFCIs (see [46, 54, 55]). Note that in [55], Van Roy and Wolsey use this approach also to obtain a useful flow cover in the separation algorithm for the class of EGFCIs (see also [46]).

Let $z \in \{0,1\}^n$ be the incidence vector of the flow cover to be determined. The separation problem for the class of SGFCIs is equivalent to solving the family of equality knapsack problems

$$\begin{aligned} Z_\lambda = \max \ \{ & \sum_{j\in N_1} [y_j^* + (u_j - \lambda)^+(1 - x_j^*)]z_j - \sum_{j\in N_2} u_j z_j \\ & - \sum_{j\in N_2} \min\{\min\{u_j, \lambda\}x^*, y_j^*\}(1 - z_j) - b : \\ & \qquad \sum_{j\in N_1} u_j z_j - \sum_{j\in N_2} u_j z_j = b + \lambda, \\ & \qquad z_j \in \{0,1\} \text{ for all } j \in N\}, \end{aligned}$$

for all positive values of λ. There exists a violated SGFCI with $\lambda = \lambda^* > 0$ if and only if $Z_{\lambda^*} > 0$ (see [46, 54]). As stated in [46], there are two difficulties with this separation problem. Equality knapsack problems are hard to solve, and the function Z_λ is not well behaved as a function of λ. Therefore, in the separation heuristic for the class of SGFCI given in [55], Van Roy and Wolsey solve the knapsack problem

$$\begin{aligned} \max \ \{ & \sum_{j\in N_1} (x_j^* - 1)z_j + \sum_{j\in N_2} x_j^* z_j : \\ & \sum_{j\in N_1} u_j z_j - \sum_{j\in N_2} u_j z_j > b, \\ & z_j \in \{0,1\} \text{ for all } j \in N\} \end{aligned} \qquad (\mathrm{KP}^{SNF})$$

in order to find a useful flow cover for X^{SNF}. Here, the idea is to consider a subclass of weakenings of SGFCIs and to work with an upper bound on the violation of these inequalities. This is achieved by setting $L_2 = N_2 \backslash C_2$, decreasing $(u_j - \lambda)^+$ to $u_j - \lambda$ for all $j \in C_1$, increasing $\min\{u_j, \lambda\}$ to λ for all $j \in L_2$ and replacing y_j^* by $u_j x_j^*$ for all $j \in N$ (see also [46, 54]).

In summary, in our separation algorithm for the class of c-MIRFCIs, we first solve the knapsack problem KP^{SNF} in order to obtain a flow cover (C_1, C_2) for X^{SNF} and then fix $\bar{u}$ according to Corollary 5.5 and Corollary 5.4. For both values of $\bar{u}$, we choose the sets $L_1 \subseteq N_1 \backslash C_1$ and $L_2 \subseteq N_2 \backslash C_2$ by comparison and generate the c-MIRFCI. If we found violated c-MIRFCIs, we choose the most violated one. Note that for $\bar{u}$ chosen according to Corollary 5.5, the resulting c-MIRFCI is not necessarily at least as strong as the corresponding SGFCI since we allow $L_1 \neq \emptyset$, but the violation of the c-MIRFCI is greater than or equal to the violation of the SGFCI, since we maximize the violation by choosing L_1 by comparison.

In the next sections, we describe this procedure in more details and discuss different algorithmic aspects. Furthermore, we modify this procedure by extending the candidate set for the value of $\bar{u}$.

In Remark 5.7, we explained that c-MIRFPIs valid for X^{SNF} can be obtained as c-MIRFCIs for the relaxation X^{SNF}_{rel} of X^{SNF}. X^{SNF}_{rel} is constructed by introducing a slack variable $s \in \mathbb{R}_+$ in the conservation constraint defining X^{SNF}, multiplying the obtained equality by minus one and relaxing it to an inequality. Thus, we can apply our separation algorithm for the class of c-MIRFCIs in addition to X^{SNF}_{rel} (the introduced slack variable is handled like the real variables y_j with $j \in N_1 \backslash (C_1 \cup L_1)$). We suggest to test this approach in order to improve the performance of our cutting plane separator for the 0-1 single node flow problem.

5.3.1 Flow Cover

In the first part of the separation algorithm for the class of c-MIRFCIs, we solve the knapsack problem KP^{SNF} in order to obtain a useful flow cover for X^{SNF}.

In Section 4.3.1, we have stated Algorithm 4.1, which solves a knapsack problem in the form

$$\max\{\sum_{j \in N} p_j \tilde{z}_j : \sum_{j \in N} w_j \tilde{z}_j \leq c, \tilde{z}_j \in \{0,1\} \text{ for all } j \in N\}, \tag{KP}$$

where $p_j \geq 0$ and $w_j \in \mathbb{Z}_+ \backslash \{0\}$ for all $j \in N = \{1, \ldots, n\}$ and $c \in \mathbb{Z}_+$, exactly by dynamic programming. In addition, we stated Algorithm 4.2, which solves KP, where $p_j \geq 0$ and $w_j \in \mathbb{Q}_+ \backslash \{0\}$ for all $j \in N = \{1, \ldots, n\}$ and $c \in \mathbb{Q}_+$, approximately by solving its LP relaxation using Dantzig's method and rounding down the solution. Remember that Algorithm 4.1 has time and space complexity of $O(nc)$ and Algorithm 4.2 was suggested to reduce the time and space complexity. Note that the set N has to be ordered when applying Algorithm 4.2. Sorting can be done in $O(n \log n)$ time using, e.g., the sorting algorithm *merge sort* (see [17]).

In order to solve KP^{SNF} exactly using Algorithm 4.1 or approximately using Algorithm 4.2, we transform KP^{SNF} into the required form.

If $u_j = 0$ for $j \in N$, the violation of the resulting c-MIRFCI for fixed $\bar{u}$ and L_1 and L_2 chosen by comparison is not influenced by the decision whether to put j into the flow cover. Therefore, we set $z_j = 0$ for all $j \in N$ with $u_j = 0$. To simplify the notation we assume $u_j > 0$ for all $j \in N$. By substituting $z_j = 1 - \bar{z}_j$ for all $j \in N_1$, we obtain

$$\begin{aligned} \max \quad & \{\sum_{j \in N_1} (1 - x_j^*)\bar{z}_j + \sum_{j \in N_2} x_j^* z_j : \\ & \sum_{j \in N_1} u_j \bar{z}_j + \sum_{j \in N_2} u_j z_j < -b + \sum_{j \in N_1} u_j, \\ & \bar{z}_j \in \{0,1\} \text{ for all } j \in N_1, \\ & z_j \in \{0,1\} \text{ for all } j \in N_2\}. \end{aligned} \tag{KP^{SNF}_{rat}}$$

Since u_j are rational numbers for all $j \in N$, we can multiply the constraint in KP^{SNF}_{rat} by a suitable factor $\gamma \in \mathbb{Q}_+ \backslash \{0\}$ to obtain

$$\begin{aligned} \max \quad & \{\sum_{j \in N_1} (1 - x_j^*)\bar{z}_j + \sum_{j \in N_2} x_j^* z_j : \\ & \sum_{j \in N_1} \gamma u_j \bar{z}_j + \sum_{j \in N_2} \gamma u_j z_j \leq \tilde{b}, \\ & \bar{z}_j \in \{0,1\} \text{ for all } j \in N_1, \\ & z_j \in \{0,1\} \text{ for all } j \in N_2\}, \end{aligned} \tag{KP^{SNF}_{int}}$$

Input : $(\mathrm{KP}')\quad \max\{\sum_{j\in N} p_j z_j : \sum_{j\in N} w_j z_j < c, z_j \in \{0,1\}$ for $j \in N\}$, where $p_j \geq 0$ and $w_j \in \mathbb{Q}_+ \setminus \{0\}$ for all $j \in N$, $c \in \mathbb{Q}_+$, $n = |N|$ and N is ordered such that $\frac{p_1}{w_1} \geq \frac{p_2}{w_2} \geq \ldots \geq \frac{p_n}{w_n}$.

Output: $z^* \in \{0,1\}^n$ a feasible solution of KP'.

```
w̄ ← 0
for j ← 1 to n do
    if w̄ + w_j < c then
        z*_j ← 1
        w̄ ← w̄ + w_j
    else
        while j ≤ n do
            z*_j ← 0
            j ← j + 1
return z*
```

Algorithm 5.1: Approximate algorithm to solve a knapsack problem in maximization form with '<' constraint.

such that $\gamma u_j \in \mathbb{Z}_+ \setminus \{0\}$ for all $j \in N$, and $\tilde{b} \in \mathbb{Z}_+$ is defined by

$$\tilde{b} = \begin{cases} \lfloor \gamma(-b + \sum_{j\in N_1} u_j) \rfloor & : \quad \gamma(-b + \sum_{j\in N_1} u_j) \notin \mathbb{Z}, \\ \gamma(-b + \sum_{j\in N_1} u_j) - 1 & : \quad \text{otherwise.} \end{cases}$$

To calculate the scaling factor γ, in Scip 0.81, we use an algorithm which at first finds a rational representation of given rational numbers and chooses γ as the smallest common multiple of all denominators divided by the greatest common divisor of all nominators. KP^{SNF}_{int} has the required form for using Algorithm 4.1 and Algorithm 4.2.

If we want to solve KP^{SNF} approximately, we can avoid the effort of calculating the scaling factor γ by solving KP^{SNF}_{rat} using Algorithm 5.1. It can be shown that the flow cover obtained by solving KP^{SNF}_{int} using Algorithm 4.2 is the same as the one obtained by solving KP^{SNF}_{rat} using Algorithm 5.1 if the (required) ordering of N is the same for KP^{SNF}_{int} and KP^{SNF}_{rat}.

In our cutting plane separator for the 0-1 knapsack problem, we modified the knapsack problem which has been used to find the initial cover by fixing some of the variables in advance (see Section 4.3.1). A similar modification of KP^{SNF}_{rat} and KP^{SNF}_{int} respectively can be applied here. In order to reduce the time and space complexity required for solving these knapsack problems by one of the algorithms discussed above, we suggest to fix some of the variables $\bar{z}_j$, $j \in N_1$ and z_j, $j \in N_2$ in advance according to the following strategy.

Fixing Strategy For $j \in N_1$, set $\bar{z}_j = 0$ if $x^*_j = 1$ and $\bar{z}_j = 1$ if $x^*_j = 0$. For $j \in N_2$, set $z_j = 0$ if $x^*_j = 0$ and $z_j = 1$ if $x^*_j = 1$.

We state the motivation of this strategy. In the context of our separation algorithm, solving KP^{SNF}_{rat} and KP^{SNF}_{int} respectively can be interpreted as follows. Start with the flow cover (C_1, C_2) for X^{SNF} with $C_1 = N_1$ and $C_2 = \emptyset$. Remove some variables from the set C_1 and add some variables to the set C_2 such that the violation of the resulting c-MIRFCI with $\bar{u}$ fixed according to Corollary 5.5 and Corollary 5.4 and L_1 and L_2 chosen by comparison is greater than the violation of the c-MIRFCI corresponding to the starting flow cover. Analyzing the effect of modifying the flow cover is quite complicated; thus we used the following observation to find a useful fixing strategy. Consider the c-MIRFCI corresponding to the starting flow cover and assume $y^*_j = u_j x^*_j$ for all $j \in N$. Then, for $j \in C_1$,

$$\begin{aligned} y^*_j + (u_j + \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}))(1 - x^*_j) &= u_j x^*_j + (u_j + \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}))(1 - x^*_j) \\ &= (u_j + \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}})) - \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}})x^*. \end{aligned}$$

Since $-\lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}) > 0$, the impact of $j \in C_1$ on the violation of the c-MIRFCI is the greater the greater the value of x^*_j is. This suggests to keep all variables with $x^*_j = 1$ in the set C_1 and remove all variables with $x^*_j = 0$ from the set C_1. Furthermore, for $j \in N_2 \backslash C_2$,

$$\max\{\lambda F_{f_\beta}(-\frac{u_j}{\bar{u}})x^*_j, -y^*_j\} = \max\{\lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}), -u_j\}x^*_j.$$

Since $\max\{\lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}), -u_j\} < 0$, the impact of $j \in N_2 \backslash C_2$ on the violation of the c-MIRFCI is the greater the smaller the value of x^*_j is. This suggests to keep all variables with $x^*_j = 0$ in the set $N_2 \backslash C_2$ and add all variables with $x^*_j = 1$ to the set C_2.

5.3.2 Cut Generation Heuristic

In the last section, we discussed different methods to obtain a useful flow cover (C_1, C_2) for X^{SNF} in the first part of the separation algorithm. In the second part of the separation algorithm, the cut generation heuristic, we generate a c-MIRFCI based on the obtained flow cover, and on sets $L_1 \subseteq N_1 \backslash C_1$ and $L_2 \subseteq N_2 \backslash C_2$ and a constant $\bar{u} \in \mathbb{Q}_+ \backslash \{0\}$ with $\bar{u} > \lambda$. The latter sets and the constant are still to be chosen.

Suppose we have fixed $\bar{u}$. Then, the remaining problem of choosing sets L_1 and L_2 such that the violation of the resulting c-MIRFCI is maximized can be solved by comparison. For $j \in N_1 \backslash C_1$, the contribution to the left hand side of the c-MIRFCI is $y_j - (u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}}))x_j$ if we put j into the set L_1 and zero if we put j into the set $N_1 \backslash (C_1 \cup L_1)$. For $j \in N_2 \backslash C_2$, the contribution to the left hand side of the c-MIRFCI is $\lambda F_{f_\beta}(-\frac{u_j}{\bar{u}})x_j$ if we put j into the set L_2 and $-y_j$ if we put j into the set $N_2 \backslash (C_2 \cup L_2)$. Thus, for fixed $\bar{u}$, we choose

$$L_1 = \{j \in N_1 \backslash C_1 : y^*_j - (u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}}))x^*_j \geq 0\} \tag{5.11}$$

and

$$L_2 = \{j \in N_2 \backslash C_2 : \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}})x^*_j \geq -y^*_j\}. \tag{5.12}$$

Note that for $\bar{u} = \max_{j \in C_1 \cup L_2} u_j > \lambda$, $\lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}) = -\min\{u_j, \lambda\}$ for all $j \in L_2$ (see Corollary 5.5).

To find a useful value of $\bar{u}$, we test different candidates for the value of $\bar{u}$ and choose the one which leads to the most violated c-MIRFCI with L_1 and L_2 chosen according to (5.11) and (5.12) respectively. As explained when we stated the outline of our separation algorithm, in a first attempt, we choose $\bar{u}$ according to Corollary 5.5 and Corollary 5.4, i.e., we use the set

$$N_1^* = \{\max\{u_j : j \in C_1 \cup \tilde{L}_2 \text{ and } u_j > \lambda\}, \max\{u_j : j \in C_1 \text{ and } u_j > \lambda\}\},$$

where $\tilde{L}_2 = \{j \in N_2 \backslash C_2 : -\min\{u_j, \lambda\} x_j^* \geq -y_j^*\}$ as the candidate set for the value of $\bar{u}$. Note that N_1^* can be empty.

To ensure the generation of a c-MIRFCI (not necessarily violated), we suggest to test also the extended sets

$$N_2^* = N_1^* \cup \{\lambda + 1\},$$

and

$$N_3^* = \{u_j : j \in N \text{ and } u_j > \lambda\} \cup \{\max\{u_j : j \in N \text{ and } u_j \geq \lambda\} + 1, \lambda + 1\}$$

as candidate sets for the value of $\bar{u}$. Here, the set N_3^* is defined according to the results in our computational study for the cutting plane separator for the class of c-MIR inequalities (see Chapter 3).

In the cut generation heuristic of our cutting plane separator for the class of c-MIR inequalities, we do not further use generated c-MIR inequalities with $f_\beta <$ MINFRAC, since the violation of a c-MIR inequality with small value of f_β is probably very small (see Section 3.3.3). Here we use a similar approach. We do not further use generated c-MIRFCIs with $f_\beta <$ MINFRAC for MINFRAC $= 0.01$.

5.3.3 0-1 Single Node Flow Relaxation

In this section, we present a procedure to construct a 0-1 single node flow relaxation with some additional simple constraints of a mixed integer set.

We relax integer variables which are not binary variables to real variables with suitable simple and variable lower and upper bounds. Therefore, we do not handle them explicitly in the following, but consider a mixed 0-1 integer set given in the form

$$X^{BMI} = \{(x, y) \in \{0,1\}^n \times \mathbb{R}^n : \sum_{j \in N} a_j x_j + c_j y_j \leq a_0\},$$

where a_0, a_j and c_j are rational numbers for all $j \in N = \{1, \ldots, n\}$. In addition, let each real variable y_j, $j \in N$ be bounded by a simple and variable lower bound and by a simple and variable upper bound, defined as follows. Note that we use the SCIP 0.81 specify definition.

Definition 5.8. Let $l_j, \tilde{l}_j \in \mathbb{Q} \cup \{-\infty\}$, $d_j^l \in \mathbb{Q}$ and x_j be a binary variable. Let y_j be a real variable with $l_j \leq y_j$ and $\tilde{l}_j x_j + d_j^l \leq y_j$. Then l_j is called *simple lower bound* imposed on y_j, and $\tilde{l}_j x_j + d_j^l$ is called *variable lower bound* imposed on y_j.

Definition 5.9. Let $u_j, \tilde{u}_j \in \mathbb{Q} \cup \{\infty\}$, $d_j^u \in \mathbb{Q}$ and x_j be a binary variable. Let y_j be a real variable with $y_j \leq u_j$ and $y_j \leq \tilde{u}_j x_j + d_j^u$. Then u_j is called *simple upper bound* imposed on y_j, and $\tilde{u}_j x_j + d_j^u$ is called *variable upper bound* imposed on y_j.

Remark 5.10. Note that in Definition 5.8 and Definition 5.9, $\tilde{l}_j = -\infty$ and $\tilde{u}_j = \infty$, respectively are only allowed to simplify the notation for the variable bounds. If $\tilde{l}_j = -\infty$, then $\tilde{l}_j x_j^* + d_j^l = -\infty$ for all $x^* \in [0,1]$ and $d_j^l \in \mathbb{Q}$, and if $\tilde{u}_j = \infty$, then $\tilde{u}_j x_j^* + d_j^u = \infty$ for all $x^* \in [0,1]$ and $d_j^u \in \mathbb{Q}$.

We want to relax X^{BMI} to a 0-1 single node flow set in the form

$$\{(x, y') \in \{0,1\}^{n'} \times \mathbb{R}_+^{n'} \; : \; \sum_{j \in N_1} y'_j - \sum_{j \in N_2} y'_j \leq b, y'_j \leq u'_j x_j \text{ for all } j \in N'\},$$

where b is a rational number, u'_j are nonnegative rational numbers for all $j \in N' = \{1, \ldots, n'\}$ and (N_1, N_2) is a partition of N'. The main subject here is to ensure the nonnegativity of y'_j and u'_j for all $j \in N'$. In the following, we state ten constructions to define one or two real variables y'_j, $j \in N'$ which are bounded by $u'_j x_j$ and for which j is put into the sets N_1 or N_2. Our procedure chooses for all $j \in N$ one of these constructions depending on the coefficients c_j and a_j and on the bounds imposed on y_j.

Let $j \in N$ with $c_j > 0$. Our procedure chooses one of the following constructions (Construction 1 to 4) for which the conditions are satisfied.

Construction 1 Conditions: $l_j > -\infty$, $\tilde{u}_j < \infty$, $c_j(l_j - d_j^u) \geq 0$, $c_j(l_j - d_j^u) + a_j \geq 0$ and $c_j \tilde{u}_j + a_j \geq 0$. We use the *variable upper bound* in $l_j \leq y_j \leq \tilde{u}_j x_j + d_j^u$ to define

$$y'_j = c_j(y_j - d_j^u) + a_j x_j \text{ with } c_j(l_j - d_j^u) + a_j x_j \leq y'_j \leq (c_j \tilde{u}_j + a_j) x_j.$$

We relax the lower bound imposed on y'_j to zero an put j into the set N_1.

Construction 2 Conditions: $\tilde{l}_j > -\infty$, $u_j < \infty$, $c_j(u_j - d_j^l) \leq 0$, $c_j(u_j - d_j^l) + a_j \leq 0$ and $c_j \tilde{l}_j + a_j \leq 0$. We use the *variable lower bound* in $\tilde{l}_j x_j + d_j^l \leq y_j \leq u_j$ to define

$$y'_j = -(c_j(y_j - d_j^l) + a_j x_j) \text{ with } -(c_j(u_j - d_j^l) + a_j x_j) \leq y'_j \leq -(c_j \tilde{l}_j + a_j) x_j.$$

We relax the lower bound imposed on y'_j to zero an put j into the set N_2.

Construction 3 Conditions: $l_j > -\infty$ and $u_j < \infty$. We use the *simple upper bound* in $l_j \leq y_j \leq u_j$ to define

$$y'_j = c_j(y_j - l_j) \text{ with } 0 \leq y'_j \leq c_j(u_j - l_j) x_j \text{ and } x_j = 1.$$

and put j into the set N_1. In addition, we define a second real variable analog to Construction 9 if $a_j > 0$ and analog to Construction 10 if $a_j < 0$.

Construction 4 Conditions: $l_j > -\infty$ and $u_j < \infty$. We use the *simple lower bound* in $l_j \leq y_j \leq u_j$ to define

$$y'_j = -c_j(y_j - u_j) \text{ with } 0 \leq y'_j \leq c_j(u_j - l_j) x_j \text{ and } x_j = 1.$$

and put j into the set N_2. In addition, we define a second real variable analog to Construction 9 if $a_j > 0$ and analog to Construction 10 if $a_j < 0$.

Let $j \in N$ with $c_j < 0$. Our procedure chooses one of the following constructions (Construction 5 to 8) for which the conditions are satisfied.

Construction 5 Conditions: $l_j > -\infty$, $\tilde{u}_j < \infty$, $c_j(l_j - d_j^u) \leq 0$, $c_j(l_j - d_j^u) + a_j \leq 0$ and $c_j\tilde{u}_j + a_j \leq 0$. We use the *variable upper bound* in $l_j \leq y_j \leq \tilde{u}_j x_j + d_j^u$ to define

$$y_j^{'} = -(c_j(y_j - d_j^u) + a_j x_j) \text{ with } -(c_j(l_j - d_j^u) + a_j x_j) \leq y_j^{'} \leq -(c_j\tilde{u}_j + a_j)x_j.$$

We relax the lower bound imposed on $y_j^{'}$ to zero an put j into the set N_2.

Construction 6 Conditions: $\tilde{l}_j > -\infty$, $u_j < \infty$, $c_j(u_j - d_j^l) \geq 0$, $c_j(u_j - d_j^l) + a_j \geq 0$ and $c_j\tilde{l}_j + a_j \geq 0$. We use the *variable lower bound* in $\tilde{l}_j x_j + d_j^l \leq y_j \leq u_j$ to define

$$y_j^{'} = c_j(y_j - d_j^l) + a_j x_j \text{ with } c_j(u_j - d_j^l) + a_j x_j \leq y_j^{'} \leq (c_j\tilde{l}_j + a_j)x_j.$$

We relax the lower bound imposed on $y_j^{'}$ to zero an put j into the set N_1.

Construction 7 Conditions: $l_j > -\infty$ and $u_j < \infty$. We use the *simple upper bound* in $l_j \leq y_j \leq u_j$ to define

$$y_j^{'} = -c_j(y_j - l_j) \text{ with } 0 \leq y_j^{'} \leq -c_j(u_j - l_j)x_j \text{ and } x_j = 1.$$

and put j into the set N_2. In addition, we define a second real variable analog to Construction 9 if $a_j > 0$ and analog to Construction 10 if $a_j < 0$.

Construction 8 Conditions: $l_j > -\infty$ and $u_j < \infty$. We use the *simple lower bound* in $l_j \leq y_j \leq u_j$ to define

$$y_j^{'} = c_j(y_j - u_j) \text{ with } 0 \leq y_j^{'} \leq -c_j(u_j - l_j)x_j \text{ and } x_j = 1.$$

and put j into the set N_1. In addition, we define a second real variable analog to Construction 9 if $a_j > 0$ and analog to Construction 10 if $a_j < 0$.

Let $j \in N$ with $c_j = 0$ and $a_j > 0$. Our procedure chooses Construction 9.

Construction 9 We define $y_j^{'} = a_j x_j$ with $a_j x_j \leq y_j^{'} \leq a_j x_j$, relax the lower bound imposed on $y_j^{'}$ to zero and put j into the set N_1.

Let $j \in N$ with $c_j = 0$ and $a_j < 0$. Our procedure chooses Construction 10.

Construction 10 We define $y_j^{'} = -a_j x_j$ with $-a_j x_j \leq y_j^{'} \leq -a_j x_j$, relax the lower bound imposed on $y_j^{'}$ to zero and put j into the set N_2.

If for a real variable y_j, $j \in N$, the conditions of more than one construction are satisfied, the procedure chooses the one for which the value of the bound used within the construction is closest to y_j^*. We use this criterium since the results of our computational study for the cutting plane separator for the class of c-MIR inequalities indicate that using the closest bound leads to a good performance in practice.

Note that our procedure can not handle mixed 0-1 integer sets which involve real variables y_j, $j \in N$ with $c_j \neq 0$ and $l_j = \tilde{l}_j = -\infty$ or $u_j = \tilde{u}_j = \infty$.

5.3.4 Numerical Issues

To avoid numerical troubles for our separation algorithm, we take the following measure which we also used in the cutting plane separator for the class of c-MIR inequalities (see Section 3.3.4).

The measure concerns the cut generation heuristic. Let

$$\begin{aligned}\sum_{j\in C_1} y_j + (u_j + \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}))(1-x_j) + \sum_{j\in L_1} y_j - (u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}}))x_j \\ - \sum_{j\in C_2} u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}})(1-x_j) + \sum_{j\in L_2} \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}})x_j \\ - \sum_{j\in N_2\setminus(C_2\cup L_2)} y_j \le b,\end{aligned} \tag{5.13}$$

where $\beta = -\frac{\lambda}{\bar{u}}$, be a c-MIRFCI found within the cut generation heuristic. We do not further use this c-MIRFCI if $f_\beta >$ MAXFRAC for MAXFRAC $= 0.95$. This is done to avoid large coefficients of the integer and real variables in the generated c-MIRFCIs.

5.4 Computational Study

In Section 5.3, we described in outline a cutting plane separator for the 0-1 single node flow problem which generates violated c-MIRFCIs and theoretically analyzed different algorithmic aspects of this separation algorithm. Here, we present a computational study about the effect on the performance of the cutting plane separator when using the algorithmic and implementation choices suggested in the last section.

We divided the initial test set (see Section 2.3) into two sets; the *main test set* and the *remaining test set*.

Main test set Contains all instances of the initial test set for which the default algorithm or at least one of the different versions of the default algorithm where a single aspect is altered leads to an initial gap closed of more than zero percent.

Remaining test set Contains the remaining instances of the initial test set.

The main test set will be used to evaluate the effect on the performance of the different versions of the separation algorithm and to develop the final efficient separation algorithm. This set consists of 66 MIPs, 30 are various instances from Miplib 2003 [3], 20 are instances from Miplib 3.0 [14] and 16 are members of the MIP collection of Mittelmann [45]. Table B.46 summarizes the main characteristics of the instances in the main test set. The remaining test set will only be used to ensure that the CPU time spent in our final separation algorithm is on an acceptable level for all instances in the initial test set. Table B.47 summarizes the main characteristics of the instances in the remaining test set.

See Section 2.3, for information about the workstation on which we performed our computational experiments, about the implementation environment of the cutting plane separator and about the representation of our test sets and our computational results.

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)		Sepa Time > 60 sec (Number)		Sepa Time > 600 sec (Number)	
	Value	△	Value	△	Value	△	Value	△
Default algorithm	10.92	0.00	1617.5	0.0	7	0	0	0
Flow cover - 1. modification[1]	10.00	-0.92	239.3	-1378.2	1	-6	0	0
Flow cover - 2. modification[2]	10.87	-0.05	469.3	-1148.2	2	-5	0	0
Flow cover - 3. modification[3]	11.37	0.45	345.0	-1272.5	1	-6	0	0
Cut gen. heur. - 1. modification[4]	11.99	1.07	4047.3	2429.8	10	3	3	3
Cut gen. heur. - 2. modification[5]	13.52	2.60	5082.9	3465.4	10	3	3	3
Default algorithm (c-MIRFPIs)	12.78	1.86	5003.1	3385.6	11	4	3	3
Resulting algorithm[3 2 5]	17.30	6.38	3278.7	1661.2	6	-1	1	1

Table 5.1: Summary of the computational results for the cutting plane separator for the 0-1 single node flow problem on the main test set. *Default algorithm* (applied to all rows of a MIP), *default algorithm where a single algorithmic aspect is altered* (applied to all rows of a MIP), *default algorithm* (applied to all rows of a MIP including the separation of the class of c-MIRFPIs) and *resulting algorithm* (applied to all rows of a MIP including the separation of the class of c-MIRFPIs). (△ with respect to the default algorithm (applied to all rows of a MIP))

Default Algorithm

Our default algorithm, which separates the class of c-MIRFCIs, is given in Algorithm 5.2. Note that in our implementation in SCIP 0.81, we construct the c-MIRFCI for (C_1, C_2), L_1, L_2 and $\bar{u}$ (Line 21 and 25) by calling the bound substitution heuristic and the cut generation heuristic of our cutting plane separator for the class of c-MIR inequalities according to the construction given in the proof of Theorem 5.3. That means, we modified Algorithm 3.3 such that it can handle given bounds for the substitution and modified Algorithm 3.4 such that it can use a given partition (T, U) and a given constant δ. Afterwards, we multiplied the obtained c-MIR inequality by λ.

For each row of a MIP, we use the procedure given in Section 5.3.3 to construct a 0-1 single node flow relaxation X^{SNF} plus some additional simple constraints of the mixed integer set corresponding to the row. Then, we try to generate violated c-MIRFCIs for X^{SNF} by using our default algorithm. We call this procedure *application of the separation algorithm to the row of a MIP*. Note that the derived cuts have to be restated in terms of the original variables before they are added to the MIP.

For our main test set, the results for applying the default algorithm to all rows of a MIP are given in Table B.48 and a summary of the results is contained in Table 5.1. The initial gap closed in geometric mean is 10.92 percent and the CPU time spent in the separation routine is 1617.5 seconds in total. For seven instances in our main test set, the separation time is greater than 60 seconds of CPU time and for none of the instances in our main test set, the separation time is greater than 600 seconds of CPU time. Thus, the separation time is unacceptable high.

[1] Always solve KP^{SNF}_{rat} approximately using Algorithm 5.1.

[2] Solve KP^{SNF}_{int} exactly using Algorithm 4.1 if the calculated scaling factor γ is not greater than 1,000 and nc for KP^{SNF}_{int} is not greater than 1,000,000, and solve KP^{SNF}_{rat} approximately using Algorithm 5.1 otherwise.

[3] Apply the fixing strategy suggested in Section 5.3.1 to KP^{SNF}_{int} and KP^{SNF}_{rat}.

[4] Use $N^*_2 = N^*_1 \cup \{\lambda + 1\}$ as candidate set for the value of $\bar{u}$.

[5] Use $N^*_3 = \{u_j : j \in N \text{ and } u_j > \lambda\} \cup \{\max\{u_j : j \in N \text{ and } u_j \geq \lambda\} + 1, \lambda + 1\}$ as candidate set for the value of $\bar{u}$.

Input : 0-1 single node flow set X^{SNF} defined as in (5.1) and $(x^*, y^*) \in ([0,1]^n \backslash \{0,1\}^n) \times \mathbb{R}^n_+$ fractional vector.
Output: Violated (with respect to (x^*, y^*)) inequality from the class of c-MIRFCIs or notification that no inequality was found.

`/* Flow cover */`
if $-b + \sum_{j \in N_1} u_j \leq 0$ **then return** *No inequality found*
Find a rational representation of u_j for all $j \in N$ and define γ as the smallest common multiple of all denominators divided by the greatest common divisor of all nominators.
$\tilde{N}_1 \leftarrow \{j \in N_1 : u_j > 0\}$, $\tilde{N}_2 \leftarrow \{j \in N_2 : u_j > 0\}$ and $\tilde{N} \leftarrow \tilde{N}_1 \cup \tilde{N}_2$
if $\gamma \leq$ *1,000* **then**
  if $\gamma(-b + \sum_{j \in \tilde{N}_1} u_j) \notin \mathbb{Z}$ **then** $\tilde{b} \leftarrow \lfloor \gamma(-b + \sum_{j \in \tilde{N}_1} u_j) \rfloor$
  else $\tilde{b} \leftarrow \gamma(-b + \sum_{j \in \tilde{N}_1} u_j) - 1$
  Call Algorithm 4.1 for KP^{SNF}_{int} $\max\{\sum_{j \in \tilde{N}_1} (1 - x^*_j) \bar{z}_j + \sum_{j \in \tilde{N}_2} x^*_j z_j : \sum_{j \in \tilde{N}_1} \gamma u_j \bar{z}_j + \sum_{j \in \tilde{N}_2} \gamma u_j z_j \leq \tilde{b}, \bar{z}_j \in \{0,1\}$ for all $j \in \tilde{N}_1, z_j \in \{0,1\}$ for all $j \in \tilde{N}_2\}$. (Let $(\bar{z}^*, z^*)$ be the solution.)
else
  Sort $\tilde{N}$ by nonincreasing value of $\frac{p_j}{u_j}$, where $p_j = 1 - x^*_j$ for all $j \in \tilde{N}_1$ and $p_j = x^*_j$ for all $j \in \tilde{N}_2$.
  Call Algorithm 5.1 for KP^{SNF}_{rat} $\max\{\sum_{j \in \tilde{N}_1} (1 - x^*_j) \bar{z}_j + \sum_{j \in \tilde{N}_2} x^*_j z_j : \sum_{j \in \tilde{N}_1} u_j \bar{z}_j + \sum_{j \in \tilde{N}_2} u_j z_j < -b + \sum_{j \in \tilde{N}_1} u_j, \bar{z}_j \in \{0,1\}$ for all $j \in \tilde{N}_1, z_j \in \{0,1\}$ for all $j \in \tilde{N}_2\}$. (Let $(\bar{z}^*, z^*)$ be the solution.)
$C_1 \leftarrow \{j \in \tilde{N}_1 : \bar{z}^*_j = 0\}$ and $C_2 \leftarrow \{j \in \tilde{N}_2 : z^*_j = 1\}$
$\lambda \leftarrow -b + \sum_{j \in C_1} u_j - \sum_{j \in C_2} u_j$

`/* Cut generation heuristic */`
$\tilde{L}_2 \leftarrow \{j \in N_2 \backslash C_2 : -\min\{u_j, \lambda\} x^*_j \geq -y^*_j\}$
$N^*_1 \leftarrow \{\max\{u_j : j \in C_1 \cup \tilde{L}_2 \text{ and } u_j > \lambda\}, \max\{u_j : j \in C_1 \text{ and } u_j > \lambda\}\}$
if $N^*_1 = \emptyset$ **then return** *No inequality found*
$v_{best} \leftarrow -\infty$, $L_{1best} \leftarrow \emptyset$, $L_{2best} \leftarrow \emptyset$ and $\bar{u}_{best} \leftarrow 0$
foreach $\bar{u} \in N^*_1$ **do**
  $\beta \leftarrow -\frac{\lambda}{\bar{u}}$, **if** $f_\beta < 0.01$ ***or*** $f_\beta > 0.95$ **then continue**
  $L_1 \leftarrow \{j \in N_1 \backslash C_1 : y^*_j - (u_j - \lambda F_{f_\beta}(\frac{u_j}{\bar{u}})) x^*_j \geq 0\}$
  $L_2 \leftarrow \{j \in N_2 \backslash C_2 : \lambda F_{f_\beta}(-\frac{u_j}{\bar{u}}) x^*_j \geq -y^*_j\}$
  $v \leftarrow$ violation (with respect to (x^*, y^*)) of the c-MIRFCI (5.4) for (C_1, C_2), L_1, L_2 and $\bar{u}$
  if $v > v_{best}$ **then**
    $v_{best} \leftarrow v$, $L_{1best} \leftarrow L_1$, $L_{2best} \leftarrow L_2$ and $\bar{u}_{best} \leftarrow \bar{u}$
if $v_{best} \leq 0$ **then return** *No inequality found*
else return *c-MIRFCI (5.4) for* (C_1, C_2), L_{1best}, L_{2best} *and* $\bar{u}_{best}$

Algorithm 5.2: Separation algorithm for the class of c-MIRFCIs. *Default algorithm.*

As it will turn out, for one thing, the large amount of time spent in the separation routine is caused by the methods used in the default algorithm to find the flow cover and another, by the fact that we apply our default algorithm to all rows of a MIP. Remember that the latter point was also true for our cutting plane separator for the class of c-MIR inequalities.

We have decided to use the following approach in our computational study to develop an efficient and fast cutting plane separator. At first, for each of the different algorithmic and implementation choices suggested in Section 5.3, we test its effect on the performance of the separation algorithm with respect to the initial gap closed ignoring the separation time. Only for the different methods for finding the flow cover, we also analyze their effect on the separation time. Using the results, we develop an effective separation algorithm. Since we apply this resulting algorithm to all rows of the MIP, the separation time is still unacceptable high. At the end, we describe methods for managing the application of the resulting algorithm to the rows of a MIP such that the separation time reduces to an acceptable level for all instances in the initial test set.

Flow Cover

In our default algorithm, we obtain a flow cover by solving KP^{SNF}_{int} exactly using Algorithm 4.1 if the calculated scaling factor γ is not greater than 1,000 and by solving KP^{SNF}_{rat} approximately using Algorithm 5.1 otherwise. Furthermore, in our default algorithm, we do not apply the fixing strategy suggested in Section 5.3.1.

We have tested to obtain a flow cover by always solving KP^{SNF}_{rat} approximately using Algorithm 5.1. For our main test set, the results for applying the default algorithm with this modification to all rows of a MIP are given in Table B.49 and a summary of the results is contained in Table 5.1. On the one hand, the separation time in total reduces to 239.3 seconds of CPU time and for only one instance in our main test set (atlanta-ip), the separation time is greater than 60 seconds of CPU time in contrast to seven instances for our default algorithm. On the other hand, the initial gap closed in geometric mean reduces by 0.92 percentage points. We conclude that solving KP^{SNF}_{int} exactly using Algorithm 4.1 performs better than solving KP^{SNF}_{rat} approximately using Algorithm 5.1, but can be very time consuming. This confirms our conclusions for the cutting plane separator for the 0-1 knapsack problem (see Section 4.4).

Algorithm 4.1 has time and space complexity of $O(nc)$, where n and c are defined as in Algorithm 4.1. For our cutting plane separator for the 0-1 knapsack problem, we tested to use Algorithm 4.1 only if nc is not greater than 1,000,000 and an approximate algorithm otherwise. Here we tested a similar approach. We solved KP^{SNF}_{int} exactly using Algorithm 4.1 if the calculated scaling factor γ is not greater than 1,000 and nc for KP^{SNF}_{int} is not greater than 1,000,000, and solving KP^{SNF}_{rat} approximately using Algorithm 5.1 otherwise. We have chosen the bound on the value of nc which has already been proofed to be useful for our cutting plane separator for the 0-1 knapsack problem. The results obtained for our main test set given in Table B.50 (see also the summary of the results contained in Table 5.1) show that this version performs better than the default algorithm and that the used bound on nc is well chosen. The CPU time spent in the separation routine in total reduces to 469.3 seconds and for only two instances in our main test set (atlanta-ip and msc98-ip), it is greater than 60 seconds of CPU time in contrast to seven instances for our default algorithm. The initial gap closed in geometric mean only reduces by 0.05 percentage points.

We also tested to reduce the time spent in the separation routine by using the fixing strategy suggested in Section 5.3.1 for KP^{SNF}_{int} and KP^{SNF}_{rat}, i.e., we fixed in advance all variables $\bar{z}_j$ with $j \in N_1$ to zero if $x^*_j = 1$ and to one if $x^*_j = 0$ and all variables z_j with $j \in N_2$ to zero if $x^*_j = 0$ and to one if $x^*_j = 1$. For our main test set, the results for applying the default algorithm with this modification to all rows of a MIP are given in Table B.51 and a summary is contained in Table 5.1. The results show that our fixing strategy is very useful, since the time spent in the separation routine in total reduces to 345.0 seconds of CPU time and for only one instance in our main test set (atlanta-ip), the separation time is greater than 60 seconds of CPU time in contrast to seven instances for the default algorithm. In addition, the initial gap closed in geometric mean increases by 0.45 percentage points.

Concluding in our resulting algorithm, at first we apply the fixing strategy suggested in Section 5.3.1 to KP^{SNF}_{int} and KP^{SNF}_{rat}, and than solve KP^{SNF}_{int} exactly using Algorithm 4.1 if the calculated scaling factor γ is not greater than 1,000 and nc for KP^{SNF}_{int} is not greater than 1,000,000, and solve KP^{SNF}_{rat} approximately using Algorithm 5.1 otherwise.

Cut Generation Heuristic

In Section 5.3.2, we explained that the main aspect of the cut generation heuristic is to choose a useful value of $\bar{u}$ and we stated three candidate sets for the value of $\bar{u}$. In our default algorithm, we use the candidate set N^*_1, i.e., we choose $\bar{u}$ according to Corollary 5.5 and Corollary 5.4.

We tested to use the extended candidate set $N^*_2 = N^*_1 \cup \{\lambda + 1\}$ which guarantees the generation of a c-MIRFCI (not necessarily violated). For our main test set, the results for applying the default algorithm with this modification are given in Table B.52 and a summary of the results is contained in Table 5.1. The initial gap closed in geometric mean increases by 1.07 percentage points. Note that for the instances momentum2, neos8, rentacar and rgn, where using the default algorithm leads to gap closed of zero percent, using the modified version of the default algorithm leads to a gap closed of more than zero percent.

We also tested to use the extended candidate set $N^*_3 = \{u_j : j \in N \text{ and } u_j > \lambda\} \cup \{\max\{u_j : j \in N \text{ and } u_j \geq \lambda\} + 1, \lambda + 1\}$. For our main test set, the results for applying the default algorithm with this modification to all rows of a MIP are given in Table B.53 and a summary of the results is contained in Table 5.1. The results show that using N^*_3 performs better than using N^*_2 with respect to the initial gap closed, since the initial gap closed in geometric mean increases to 13.52 percent and for the instances momentum2, neos8, rentacar and rgn the initial gap closed is again greater than zero percent.

We conclude that the performance of the separation algorithm with respect to the initial gap closed can be significantly improved when testing additional useful candidates for the value of $\bar{u}$. In our resulting algorithm, we use the candidate set N^*_3.

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)		Sepa Time > 60 sec (Number)		Sepa Time > 600 sec (Number)	
	Value	△	Value	△	Value	△	Value	△
Resulting algorithm	17.30	0.00	3278.7	0.0	6	0	1	0
Resulting algorithm[6]	16.07	-1.23	166.7	-3112.0	0	-6	0	-1

Table 5.2: Summary of the computational results for the cutting plane separator for the 0-1 single node flow problem on the main test set. *Resulting algorithm* (applied to all rows of a MIP including the separation of the class of c-MIRFPIs) and *resulting algorithm with* MAXTESTDELTA $= 10$ (applied to the rows of a MIP (ordered by nonincreasing value of ROWSCORE^i, $i \in P$) including the separation of the class of c-MIRFPIs, where the application is limited by using MAXFAILS $= 100$, MAXCUTS $= 200$ and MAXROUNDS $= 10$). (△ with respect to the resulting algorithm (applied to all rows of a MIP including the separation of the class of c-MIRFPIs))

C-MIRFPIs

So far, we applied our default algorithm and the modified versions of our default algorithm to the 0-1 single node flow relaxation X^{SNF} of the mixed integer set corresponding to the row of a MIP. In Section 5.3, we suggested to apply the separation algorithm in addition to the relaxation X^{SNF}_{rel} of X^{SNF} in order to generate c-MIRFPIs valid for X^{SNF} (see also Remark 5.7).

We have tested this approach, i.e., for each row of a MIP, we applied our default algorithm to the 0-1 single node flow relaxation X^{SNF} of the mixed integer set corresponding to the row and to the relaxation X^{SNF}_{rel} of X^{SNF}. The results obtained for our main test set are given in Table B.54 and a summary of the results is contained in Table 5.1. The initial gap closed in geometric mean increases by 1.86 percentage points and for the instances atlanta-ip, momentum2, and rentacar, where applying the default algorithm only to X^{SNF} leads to a gap closed of zero percent, the initial gap closed is greater than zero percent.

We conclude that separating in addition the class of c-MIRFPIs improves the performance of the cutting plane separator for the 0-1 single node flow problem with respect to the initial gap closed. Therefore, we will apply our resulting algorithm to the 0-1 single node flow relaxation X^{SNF} of the mixed integer set corresponding to the row of a MIP and to the relaxation X^{SNF}_{rel} of X^{SNF}. We call this approach *application of the separation algorithm to the row of a MIP including the separation of the class of c-MIRFPIs*.

Resulting Algorithm

From the results of our computational study, we obtain the following best algorithmic and implementation choices for the cutting plane separator for the 0-1 single node flow problem.

Flow cover Apply the fixing strategy suggested in Section 5.3.1 to KP^{SNF}_{int} and KP^{SNF}_{rat}, and solve KP^{SNF}_{int} exactly using Algorithm 4.1 if the calculated scaling factor γ is not greater than 1,000 and nc for KP^{SNF}_{int} is not greater than 1,000,000, and solve KP^{SNF}_{rat} approximately using Algorithm 5.1 otherwise.

Cut generation heuristic Use N^*_3 as candidate set for the value of $\bar{u}$.

[6] Use MAXTESTDELTA $= 10$.

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)		Sepa Time > 60 sec (Number)		Sepa Time > 600 sec (Number)	
	Value	△	Value	△	Value	△	Value	△
Resulting algorithm	1.04	0.00	1527.1	0.0	3	0	1	0
Resulting algorithm[6]	1.04	0.00	34.8	-1492.3	0	-3	0	-1

Table 5.3: Summary of the computational results for the cutting plane separator for the 0-1 single node flow problem on the remaining test set. *Resulting algorithm* (applied to all rows of a MIP including the separation of the class of c-MIRFPIs) and *resulting algorithm with* MAXTESTDELTA = 10 (applied to the rows of a MIP (ordered by nonincreasing value of ROWSCORE^i, $i \in P$) including the separation of the class of c-MIRFPIs, where the application is limited by using MAXFAILS = 100, MAXCUTS = 200 and MAXROUNDS = 10). (△ with respect to the resulting algorithm (applied to all rows of a MIP including the separation of the class of c-MIRFPIs))

We call the corresponding separation algorithm *resulting algorithm*. For our main test set, the results for applying the resulting algorithm to all rows of the MIP including the separation of the class of c-MIRFPIs are given in Table B.55 and a summary of the results is contained in Table 5.1 and Table 5.2. On the one hand, the initial gap closed in geometric mean increases by 6.38 percentage points. Thus, the modifications significantly improve the performance of the cutting plane separator with respect to the initial gap closed. On the other hand, the CPU time spent in the separation routine is 3278.7 seconds in total, which is unacceptable high. For six instances in our main test set, the separation time is greater than 60 seconds of CPU time and for the instance atlanta-ip it is even greater than 600 seconds of CPU time. However, the separation time in total is smaller than the one for applying our default algorithm with N_3^* as candidate set for the value of $\bar{u}$ to all rows of a MIP (5082.9 seconds of CPU time, see Table B.53) and smaller than the one for applying our default algorithm to all rows of a MIP including the separation of the class of c-MIRFPIs (5003.1 seconds of CPU time, see Table B.54). We conclude that using the new methods for finding the flow cover help to reduce the separation time.

For our remaining test set, the results for applying the resulting algorithm to all rows of a MIP including the separation of the class of c-MIRFPIs are given in Table B.56 and a summary of the results is contained in Table 5.3. The initial gap closed is zero percent for all instances in our remaining test set except for neos632659 (14.02 percent) and pp08a (0.29 percent). That means, for these two instances, where neither the default algorithm nor the default algorithm with a single aspect altered leads to a gap closed of more than zero percent, the combination of the best aspects improves the performance of the cutting plane separator. The separation time in total is 1527.1 seconds of CPU time. For three instances in the remaining test set, the separation time is greater than 60 seconds of CPU time and for neos19 it is even greater than 600 seconds of CPU time. Thus, as for the main test set, the separation time is unacceptable high.

Since we want to implement an efficient cutting plane separator for the 0-1 single node flow problem, we have to find methods for reducing the separation time without loosing too much of the initial gap closed. We apply our resulting algorithm to all rows of a MIP. Because of our experiences for implementing an efficient cutting plane separator for the class of c-MIR inequalities, we suppose that a large amount of time spent in the separation routine here is among other things caused by a large number of rows. Therefore, we want to be more selective about the rows to which we apply the separation algorithm, i.e., we want to use only those rows which may lead to violated c-MIRFCIs or c-MIRFPIs. For that, we use an approach which is similar to the one used in our cutting plane separator for the class of c-MIR inequalities. Let P be the set of all rows of the MIP. For $i \in P$, let the mixed integer set corresponding to row i be given in the form

$$\{(x, y) \in \mathbb{Z}^n \times \mathbb{R}^m : \sum_{j \in N} a^i_j x_j + \sum_{j \in M} c^i_j y_j \leq a^i_0\},$$

where a^i_0 and a^i_j are rational numbers for all $j \in N = \{1, \ldots, n\}$ and c^i_j are rational numbers for all $j \in M = \{1, \ldots, m\}$. Furthermore, let db_i be the LP solution value of the dual variable corresponding to row $i \in P$. See [52], for the definition of the dual LP. Let $dens_i = \frac{|\{j \in N : a^i_j \neq 0\}| + |\{j \in M : c^i_j \neq 0\}|}{n+m}$ be the density of row $i \in P$ and $s^* = a^i_0 - (\sum_{j \in N} a^i_j y^*_j + \sum_{j \in M} c^i_j x^*_j)$ be the slack of row $i \in P$. For each $i \in P$, we define

$$\begin{aligned} \text{ROWSCORE}^i = \; & \max\{\tfrac{db_i}{\max\{\|(c,d)\|, 1.0\}}, 0.0001\} + 0.0001 dens_i + \\ & 0.001(1 - \tfrac{s^*}{\max\{\|(a^i, c^i)\|, 0.1\}}), \end{aligned}$$

where $(c, d) \in \mathbb{Q}^n \times \mathbb{Q}^m$ is the vector of the coefficients of all variables in the objective function of the MIP, $(a^i, c^i) \in \mathbb{Q}^n \times \mathbb{Q}^m$ is the vector of the coefficients of all variables in row i and $\| \cdot \|$ is the Euclidean norm. We select the rows by nonincreasing value of ROWSCORE^i, $i \in P$ and limit the number of rows to which we apply our resulting algorithm by the following parameter.

MAXFAILS The parameter denotes the maximum number of rows $i \in P$ per separation round for which we consecutively did not obtain a violated c-MIRFCI or c-MIRFPI. Note that in early separation rounds we increase this value up to the double value, i.e., we allow up to MAXFAILS $+$ (MAXFAILS $- 2k)^+$ consecutive fails, where k is the number of separation rounds which have already been performed at the current branch-and-bound node.

That means we prefer rows for which the corresponding dual variable has a great LP solution value, which have a large density and which are tight. Note that this approach is similar to the one used in our cutting plane separator for the class of c-MIR inequalities, except that here we prefer rows with a large density since we do not perform any aggregation.

If the separation algorithm generates cuts for nearly every row of the MIP, the parameter MAXFAILS does not help to reduce the separation time. Therefore, we suggest to use in addition the following parameters.

MAXCUTS The parameter denotes the maximum number of violated c-MIRFCIs and c-MIRFPIs generated per separation round.

MAXROUNDS The parameter denotes the maximum number of separation rounds performed at the current branch-and-bound node.

Another point which may cause a large separation time for our resulting algorithm is the fact that we test all candidates for the value of $\bar{u}$ contained in the set N_3^*. If a MIP has a large number of variables, the cardinality of N_3^* can be very large. We suggest to limit the time spent in the cut generation heuristic by the following parameter.

MAXTESTDELTA The parameter denotes the maximum number of different values of $\bar{u}$ for which we generate a c-MIRFCI in the cut generation heuristic.

Note that if we use N_3^* as candidate set for the value of $\bar{u}$, we first test the values in $N_2^* \subseteq N_3^*$ and the value $\max\{u_j : j \in N \text{ and } u_j \geq \lambda\} + 1$.

In order to find useful values for the four parameters introduced above, we selected all instances of our main and remaining test set for which the separation time in the last test was greater than 60 seconds of CPU time and some instances with small separation time. We applied our resulting algorithm to all rows (ordered by nonincreasing value of ROWSCORE^i, $i \in P$) of these MIPs including the separation of the class of c-MIRFPIs and analyzed the behavior of the separation algorithm with respect to the four parameters.

For our main test set, the results for applying our resulting algorithm with MAXTESTDELTA $= 10$ to the rows of a MIP (ordered by nonincreasing value of ROWSCORE^i, $i \in P$) including the separation of the class of c-MIRFPIs and limiting the application by using MAXFAILS $= 100$, MAXCUTS $= 200$ and MAXROUNDS $= 10$ are given in Table B.57 and a summary of the results is contained in Table 5.2. Here the $\triangle$ values are given with respect to the resulting algorithm applied to all rows of a MIP including the separation of the class of c-MIRFPIs. The initial gap closed in geometric mean reduces only by 1.23 percentage points and the separation time in total is now 166.7 seconds of CPU time. For none of the instance in our main test set, the separation time is greater than 60 seconds of CPU time. For our remaining test set, the results for the same test are given in Table B.58 and a summary of the results is contained in Table 5.3. Here the $\triangle$ values are also given with respect to the resulting algorithm applied to all rows of a MIP including the separation of the class of c-MIRFPIs. The initial gap closed does not change for any instance in our remaining test set and the separation time in total reduces to 34.8 seconds of CPU time. Thus, for both test sets, the separation time is now on an acceptable level.

Note that 10 seems to be a very small value for MAXTESTDELTA and we are close to using N_2^*, but the following table shows that for our main test set, the gap closed in geometric mean for using N_3^* and MAXTESTDELTA $= 10$ is close to the one for using N_3^* and MAXTESTDELTA $= 100$ and much higher than the one for testing only the values in N_2^*.

Candidate Set	MAXTESTDELTA	Gap Closed % (Geom. Mean)	Sepa Time sec (Total)
N_3^*	100	16.23	314.1
N_3^*	50	16.23	268.6
N_3^*	10	16.07	166.7
N_3^*	5	15.78	125.2
N_2^*	(∞)	13.61	85.3

In our final cutting plane separator for the 0-1 single node flow problem, we apply our resulting algorithm with MAXTESTDELTA $= 10$ to the rows of a MIP (ordered by nonincreasing value of ROWSCORE^i, $i \in P$) including the separation of the class of c-MIRFPIs and limit the application by using MAXFAILS $= 100$, MAXCUTS $= 200$ and MAXROUNDS $= 10$.

5.5 Conclusion

The results of our computational study indicate that the c-MIR approach is very useful for implementing an efficient cutting plane separator for the 0-1 single node flow problem.

We found out that the performance of the cutting plane separator is strongly influenced by the choice of the value of $\bar{u}$. The performance can be improved if one chooses the candidates for the value of $\bar{u}$ not only according to Corollary 5.5 and Corollary 5.4, but also selects a limited number of other useful candidates. Furthermore, we conclude that separating in addition to the class of c-MIRFCIs, the class of c-MIRFPIs significantly improves the performance of the cutting plane separator.

As for the cutting plane separator for the class of c-MIR inequalities, it is important to select the rows of a MIP to which the cutting plane separator is applied carefully since applying it to all rows of a MIP may lead to an unacceptable large amount of time spent in the separation algorithm.

Chapter 6

Further Cutting Plane Separators

Although there is a wide variety of cutting planes derived in the literature, only a limited number of the corresponding cutting plane separators have been shown to be practical useful within linear programming based branch-and-cut algorithms for solving general MIPs.

In the previous chapters, we have studied three of them in details. Here, we give a brief introduction to three further cutting plane separators which are also provided by SCIP 0.81. They will be included in the computational study presented in the next chapter.

6.1 Cutting Plane Separator for the Class of GMI Inequalities

Gomory mixed integer cuts (GMI cuts) are general cutting planes and we have already mentioned them in connection with c-MIR cuts (see Section 3.1). As we will see in the next chapter, the cutting plane separator for the class of GMI inequalities is one of the most important ones within a linear programming based branch-and-cut algorithm.

We consider a mixed integer set in the form

$$X = \{(x, y) \in \mathbb{Z}_+^n \times \mathbb{R}_+^m : \sum_{j \in N} a_j x_j + \sum_{j \in M} c_j y_j = a_0\},$$

where a_0, a_j for all $j \in N = \{1, \dots, n\}$, and c_j for all $j \in M = \{1, \dots, m\}$ are rational numbers. Furthermore, for $d \in \mathbb{R}$, let $f_d = d - \lfloor d \rfloor$. Then the GMI inequality

$$\sum_{\substack{j \in N, \\ f_{a_j} \leq f_{a_0}}} f_{a_j} x_j + \frac{f_{a_0}}{1 - f_{a_0}} \sum_{\substack{j \in N, \\ f_{a_j} > f_{a_0}}} (1 - f_{a_j}) x_j + \sum_{\substack{j \in M, \\ c_j \geq 0}} c_j y_j - \frac{f_{a_0}}{1 - f_{a_0}} \sum_{\substack{j \in M, \\ c_j < 0}} c_j y_j \geq f_{a_0}$$

is valid for X (see [28]). Thus, the GMI inequality defined for a mixed integer set which is a relaxation of the feasible region of a MIP is valid for the feasible region of the MIP.

A special case arises when we want to separate a *basic feasible solution* (where at least one integer variable is fractional) of the LP relaxation of a MIP. For an arbitrary integer variable with fractional value, consider the corresponding row of the optimal *simplex tableau*. It is well known that the GMI inequality for the mixed integer set defined by this simplex tableau row is violated by the given basic feasible solution of the LP relaxation (see [19]).

This approach is used in our cutting plane separator for the class of GMI inequalities. Given an optimal solution of the LP relaxation of the MIP, we generate for each integer variable x_i with fractional value x_i^* in the given vector, except those for which $f_{x_i^*} < 0.05$, the GMI inequality associated with the simplex tableau row corresponding to x_i.

Adding generated GMI cuts to the MIP may cause numerical instability. In SCIP 0.81, we address this problem in the following way. We scale generate GMI cuts such that all integer variables have integer coefficients in the scaled cuts. For each integer variable, the scaling algorithm first tries to find a fractional representation of the rational coefficient of the variable such that the denominator is not greater than $1,000$. If this was successful for all integer variables, it calculates the greatest common divisor of the nominators (*gcd*) and the smallest common multiple of the denominators (*scm*) and than multiplies the GMI cut by the scalar $\frac{scm}{gcd}$. But, out off the set of the scaled GMI cuts, we only allow those to be added to the MIP for which the scalar is not greater than $1,000$.

6.2 Cutting Plane Separator for the Node Packing Problem

This cutting plane separator generates strong valid inequalities for the *node packing polytope* associated with the node packing problem. More precisely, we separate the class of *clique inequalities*.

Given a graph $G = (V, E)$, where V is the set of nodes and E is the set of edges, a node packing is a subset of nodes such that *no* pair of nodes is joined by an edge (see [46]). Clique inequalities are based on the structure of a clique, which is a subset of nodes such that *each* pair of nodes is joined by an edge (see [46]). It is well known that if a clique $C \subseteq V$ is maximal with respect to node inclusion, then the corresponding clique inequality

$$\sum_{j \in C} x_j \leq 1$$

defines a facet of the node packing polytope (see [49]).

In a linear programming based branch-and-cut algorithm for solving MIPs, the cutting plane separator can be applied to the *node packing relaxation* of the feasible region of a MIP. To obtain this relaxation, we use the *conflict graph* of the MIP, which is constructed from logical implications between binary variables of the MIP derived by probing techniques (see [6, 56]). It contains a node for every binary variable of the MIP and for its complement. Furthermore, there is an edge between two nodes if at most one of the binary variables represented by the nodes can be equal to one in any feasible solution of the MIP. Thus, any feasible solution of the MIP corresponds to a node packing in the conflict graph. Therefore, the node packing polytope corresponding to the conflict graph defines a relaxation of the convex hull of the feasible region of the MIP and valid inequalities for the feasible region of the node packing problem are also valid for the feasible region of the MIP (see [6, 40]).

The separation problem for the class of clique inequalities can be formulated as a *maximum weighted clique problem*, where the weights of the nodes are defined by the given fractional vector to be separated (see [6]). This problem is known to be NP-hard (see [36]). In our cutting plane separator, we use a *branch-and-bound algorithm* to exactly solve a maximum weighted clique problem. The algorithm is due to Ralf Borndörfer and Zoltán Kormos and uses a heuristic for *coloring weighted graphs* in the bounding step.

We apply the branch-and-bound algorithm only to the subgraph of the conflict graph associated with the nodes which have nonzero weights. Then, a generated clique inequality defines a facet of the restriction of the node packing polytope to some lower dimensional space where all variables which are represented by nodes with zero weights are fixed to zero. If the inequality is violated, we use the concept of lifting (see Section 2.2) to generate a strong valid inequality for the node packing polytope in the original space (see also [46]). Here, a lifting step consists of checking whether the current clique and the node which represents to current variable still form a clique. In our cutting plane separator, we perform the lifting step only for a limited number of variables fixed to zero.

6.3 Cutting Plane Separator for the Class of Implied Bound Inequalities

In the previous section, we described how logical implications between binary variables can help to derive valid inequalities for the feasible region of a MIP. The cutting plane separator which we briefly describe here does also use logical implications derived by preprocessing and probing techniques (see [6, 56]).

Let x be a binary variable and y be a real variable with bounds $l \leq y \leq u$, where $l, u \in \mathbb{Q}$. Preprocessing and probing techniques may yield logical implications of the form

$$x = 0 \Rightarrow y \leq b, \text{ with } b \in \mathbb{Q} \text{ and } l \leq b < u, \tag{6.1}$$

$$x = 0 \Rightarrow y \geq b, \text{ with } b \in \mathbb{Q} \text{ and } l < b \leq u, \tag{6.2}$$

$$x = 1 \Rightarrow y \leq b, \text{ with } b \in \mathbb{Q} \text{ and } l \leq b < u, \tag{6.3}$$

and

$$x = 1 \Rightarrow y \geq b, \text{ with } b \in \mathbb{Q} \text{ and } l < b \leq u. \tag{6.4}$$

These implications state that if the binary variable x is fixed to zero or one, the lower and upper bound imposed on the real variable y respectively can be strengthened. Logical implications of the form (6.1), (6.2), (6.3), and (6.4) imply that the inequalities

$$y \leq b + (u - b)x,$$

$$y \geq b - (b - l)x,$$

$$y \leq u - (u - b)x,$$

and

$$y \geq l + (b - l)x,$$

respectively are valid for the feasible region of a MIP (see [56]). We denote these inequalities by *implied bound inequalities.*

Given a fractional vector, our cutting plane separator for the class of implied bound inequalities checks for each binary variable of the MIP with fractional value in the given vector, if there exists an implication of the from (6.1), (6.2), (6.3), or (6.4) for which the corresponding implied bound inequality is violated by the given vector.

Chapter 7

Computational Results

In the previous chapters, we evaluated the implementation of cutting plane separators for

- the class of c-MIR inequalities (see Chapter 3),
- the 0-1 knapsack problem (see Chapter 4), and for
- the 0-1 single node flow problem (see Chapter 5).

The main focus was to find implementations such that the separators are efficient when they are used isolated. We considered this to be a good starting point for developing cutting plane separators which are efficient with respect to the overall performance of the MIP solver in which they are embedded. Furthermore, in Chapter 6, we gave a brief introduction to the cutting plane separators for

- the class of GMI inequalities,
- the node packing problem, and for
- the class of implied bound inequalities

which are also implemented in SCIP 0.81.

When we speak of efficient cutting plane separators, the following question arises.

- Are these cutting plane separator implemented in SCIP 0.81 competitive to the ones included in other MIP solvers?

In the first part of this chapter, we will answer that question for the three separators[1] discussed in details in this thesis and for the separator for the class of GMI inequalities. The remaining two separators are based on logical implications derived by preprocessing and probing techniques. In SCIP 0.81, this information is not available anymore when we disable presolving. But, since we have to do that to ensure a fair comparison with the other MIP solvers, we do not include these two separators in our computational study. Nevertheless, as we will see in Section 7.2,

[1] The cutting plane separator for the 0-1 knapsack problem can also be applied to relaxations of the set $\{x \in \{0,1\}^n : \sum_{j \in N} a_j x_j \leq a_0\}$, where a_0 and a_j are *rational* numbers for all $j \in N = \{1, \ldots, n\}$. In all test runs for SCIP 0.81 concerning the cutting plane separator for the 0-1 knapsack problem, reported in this chapter, we also include this application.

these separators do not belong to the ones which are most important for improving the performance of Scip 0.81. The same holds for CPlex 8.0 (see [15]).

In the second part of this chapter, we remove the isolated application of the cutting plane separators and answer the following question.

- How strong is the impact of the individual cutting plane separators on the overall performance of Scip 0.81?

Here, we consider all of the six cutting plane separators mentioned above.

All computational experiments described in this chapter were performed on a Dell Precison 650 MT working station with a Dual 3.06 GHz Intel Xeon CPU (512 KB cache) and 3.6 GB RAM. In each test, we used a time limit of 3,600 seconds of CPU time and a memory limit of 512 MB for each instance contained in the considered test set. In the tables and figures which report the results of our test runs, we denote the above mentioned cutting plane separators by C-MIR, Knapsack, Flow Cover, GMI, Clique, and Impl. B. in that order.

7.1 Comparison with Cplex and Cbc

We start with answering the first question asked in the introduction of this chapter. For that, we compare the effectiveness of four cutting plane separators implemented in Scip 0.81 with the corresponding ones provided by the commercial solver Cplex 10.01, and the ones included in the Coin-Or Cut Generator Library 0.5 which is used in the non-commercial solver Coin-Or Branch and Cut solver 1.01.00 (Cbc 1.01.00) [24, 25].

In our experiments, we used Cplex 10.01 as underlying LP solver in Scip 0.81 and the Coin-Or Linear Program Solver 1.3 [26] in Cbc 1.01.00. To shorten the presentation, we will leave out the version numbers of the solvers in the remainder of this chapter and refer to them just by Scip, Cplex, and Cbc.

We applied the presolving routines of Cplex to all instances in our *initial test set* (see Section 2.3) and use the obtained presolved instances as our test set for the comparison. This was done in order to eliminate the effect of using different versions of presolvers in our test runs. A summary of the main characteristics of the instances in our test set is given in Table B.59. The summary is presented in the same way as it was done for the test sets used in the computational studies of the previous chapters (see Section 2.3, for details). Note that here z_{LP} is the value obtained by running Scip.

For each of the four cutting plane separators, we ran three tests; one for each solver[2]. In each of these tests, only one cutting plane separator was enabled in the used solver[3]. Furthermore, to make the results obtained for the different solvers comparable and to eliminate the effect of other features of the solvers, we considered only the root node of the branch-and-cut tree and called up each solver with its default settings, except the following changes.

[2]The cutting plane separator for the class of c-MIR inequalities provided by Scip is compared to the separators which generate MIR cuts in Cplex and Cbc.

[3]In Cplex, we set the associated cut indicator to 2, i.e., the cuts are generated aggressively.

Primal heuristics If a primal heuristic is successful, this might cause further dual propagations, which could lead to the generation of further cuts. Therefore, we disabled all primal heuristics.

Strong branching Every branching strategy employing strong branching can detect infeasibility of subproblems of a MIP (see [2]) and may therefore cause fixing of variables. As this influences the dual bound, we did not use strong branching, but *most infeasible branching* (see [2]).

Presolving and probing Since our test set already contains the presolved instances (obtained by the presolving routines of CPLEX), we disabled presolving in all three MIP solvers. In contrast to SCIP and CBC, CPLEX applies probing, which may cause fixing of binary variables, after the preprocessing step. Therefore, in CPLEX, in addition, we disabled probing.

MIP gap tolerance In CPLEX, we set the absolute MIP gap tolerance to 10^{-9} and the relative MIP gap tolerance to 0.0 and in CBC, we set the allowable gap to 10^{-9} and the ratio gap to 0.0, which are the corresponding values to the ones used in SCIP.

Restarts. This feature is only provided by SCIP. As it may influence the dual bound, we disabled it in SCIP.

Scaling of the problem This feature is only provided by CBC. As it may improve the dual bound, we disabled it in CBC.

To evaluate and compare the performance of the cutting plane separators of the three solvers, we use the performance measure Gap closed % which denotes the percentage of the initial gap that is closed by using the separation algorithm (see Section 2.3, for the definition). Furthermore, we report the dual bound used in the calculation of Gap closed % (denoted by Dual Bound)[4], the number of cuts generated (denoted by Cuts)[5], and the overall CPU time in seconds required by the solver (denoted by Time).

For each cutting plane separator, the results for the three solvers are reported in one table (Table B.60, B.61, B.62, and B.63). Column 2 to 5 contain the results for SCIP, Column 6 to 10 the ones for CPLEX, and Column 11 to 15 the ones for CBC. At the bottom of each table, in the row labelled Total, we give the sum of the reported values over all instances. And, in the row labelled Geom. Mean, we give the geometric mean of the reported values over all instances where individual values smaller than one were replaced by one. For CPLEX and CBC, we state, in addition, the difference to SCIP for the performance measure Gap closed %. Here, numbers in blue indicate that the value of the performance measure obtained by CPLEX and CBC respectively is better than the one obtained by SCIP and numbers

[4]In CPLEX, although we do not use strong branching, the variable selection algorithm influences the dual bound. Therefore, for CPLEX, we consider the dual bound which the solver obtains before the variable selection algorithm is called. A test run where we disabled *all* cutting plane separators in CPLEX led to Gap closed % equal to zero for all instances in the test set. We performed the same test for CBC. For some of the instances in the test set, this led to Gap closed % not equal to zero. But, the values were such small (< 0.1) that we consider the comparison to be fair.

[5]For CBC, we report the number of cuts which are active after adding rounds of cuts.

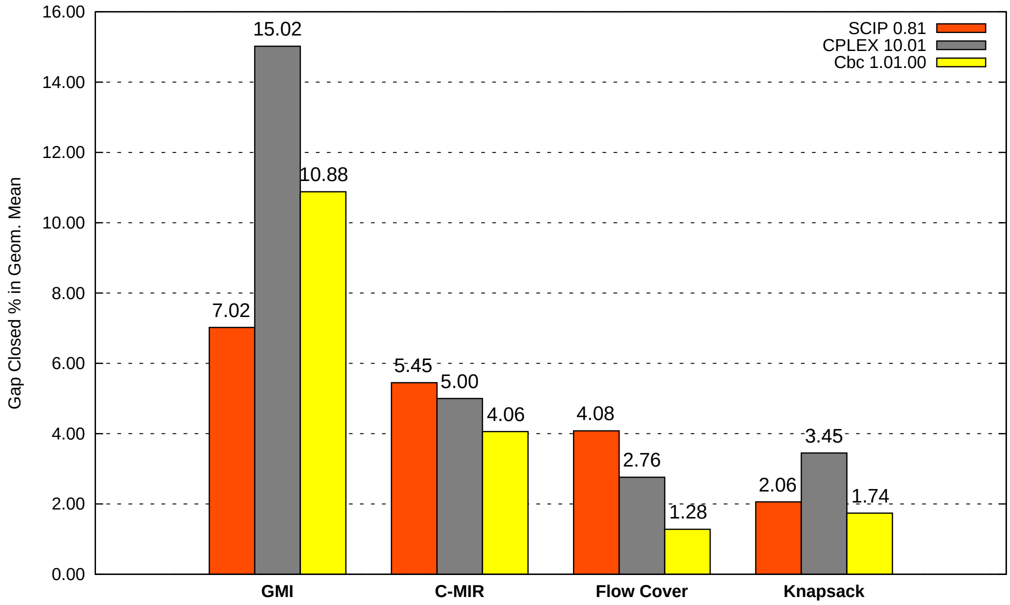

Figure 7.1: Summary of the computational results for the comparison with CPLEX and CBC.

in red indicate the opposite case. Note that the △ values for each instance, for Total, and for Geom. Mean, are given in percentage points, not in percentage.

A summary of the results of all test runs is given in Figure 7.1.

Cutting Plane Separator for the Class of GMI Inequalities

The separator provided by SCIP leads to an initial gap closed of 7.02 percent in geometric mean, in contrast to 15.02 percent for CPLEX and 10.88 percent for CBC. For only seven instances in the test set, the separator of SCIP leads to a greater value of the initial gap closed than the separators of the other two solvers, in contrast to 53 instances for CPLEX and 47 instances for CBC.

Thus, our separator is not competitive to neither the one provided by CPLEX nor the one provided by CBC. Further investigations should be carried out in order to improve the performance of this cutting plane in SCIP.

Cutting Plane Separator for the Class of C-MIR Inequalities

For this cutting plane separator, we obtained better results than for the previous one. Here, SCIP is competitive to CPLEX and CBC, in fact with respect to the initial gap closed in geometric mean and in total, it performs better than both CPLEX and CBC. The separator of SCIP leads to an initial gap closed of 5.45 percent in geometric mean, whereas the ones of CPLEX and CBC lead to 5.00 percent in geometric mean and 4.06 percent in geometric mean, respectively. Furthermore, for 31 instances in the test set, SCIP leads to a greater value of the initial gap closed than the other two solvers, in contrast to 38 instances for CPLEX and only 14 instances for CBC.

In Chapter 3, we made a great effort to reduce the computation time required by this cutting plane separator. The main aspect was our strategy for selecting the starting constraints. And, the results obtained here show that our cutting plane separator is competitive to the ones of the other two solvers with respect to the computation time. Even though, Scip generates the largest number of cuts in total, the time spent in the root node in total for Scip is similar to the one for Cplex and smaller than the one for Cbc. Note that for Cbc, the large amount of time spent in the root node in total is basically caused by the instance neos19.pre (see Table B.61). For the original instance neos19, the slow version of our resulting separation algorithm also led to a large time spent in the separation routine, whereas our fast version, i.e., the version used here, led to a much smaller separation time (see Table B.16 and B.18).

Cutting Plane Separator for the 0-1 Single Node Flow Problem

Here, again the separator of Scip is competitive to the ones of both Cplex and Cbc and it performs even better than both of them. Scip leads to an initial gap closed of 4.08 percent in geometric mean, Cplex to an initial gap closed of 2.76 percent in geometric mean, and Cbc to an initial gap closed of 1.28 percent in geometric mean. For 44 instances in the test set, Scip is the solver which leads to the greatest value of the initial gap closed and for about 60 percent of these instances both other solvers lead to an initial gap closed of zero percent. For Cplex, similar observations can be made. This solver leads for 32 instances in the test set to the greatest value of the initial gap closed and for about 60 percent of these instances none of the other solvers leads to an initial gap closed of more than zero percent. In contrast, Cbc is superior for only 5 instances in the test set with respect to the initial gap closed.

Thus, for a large number of instances where the separators of Scip and Cplex respectively lead to an improved gap, the separators of the other solvers fail to do so. This may indicate that the cutting plane separators of these two solvers use different strategies for constructing the 0-1 single node flow sets or that they separate different classes of valid inequalities for the 0-1 single node flow set (see Chapter 5).

Cutting Plane Separator for the 0-1 Knapsack Problem

For this cutting plane separator, Scip and Cbc are competitive to each other, but Cplex outperforms both. Scip and Cbc close 2.06 percent and 1.74 percent of the initial gap in geometric mean, respectively, in contrast to 3.45 percent in geometric mean obtained for Cplex. The superiorness of Cplex can also be seen in the following figures. For 59 instances in the test set, Cplex leads to a greater value of the initial gap closed than both Scip and Cbc and for about 50 percent of these instances, the other solvers close zero percent of the initial gap. Scip leads to the greatest value of the initial gap closed for only nine instances in the test set and Cbc for only five instances in the test set.

Thus, the cutting plane separator of Cplex seems to contain a feature not used in neither Scip nor Cbc. But, note that this feature might also be the reason why Cplex spends a large amount of time in the root node for some of the instances in the test set (see Table B.63). The time spent in the root node in total is more than ten times larger than the ones for both Scip and Cbc, even though Cplex produces a smaller number of cuts in total than Scip.

7.2 Impact on the Overall Performance of Scip

Cutting plane separators are a single feature out of a variety of features used in general purpose algorithms for solving MIPs. In [13] and [15], computational studies can be found which concern the impact of primal heuristics on the overall performance of Scip 0.82b and the impact of different features provided by CPlex 8.0 on the overall performance of this solver, respectively. The results of Bixby et al. [15] have shown that cutting plane separators are a very effective single feature, in fact, in CPlex 8.0, they are by far the most important one.

In all computational studies of this thesis reported so far, we have evaluated only the performance of the cutting plane separators when they are used on their own. That is, in Scip, we considered only the root node of the branch-and-cut tree, used only one type of cutting plane separator, and disabled all primal heuristics and strong branching. Now, we want to evaluate the impact of the individual cutting plane separators on the overall performance of Scip. In particular, we want to find out which of these cutting plane separators are the most important ones for improving the performance of Scip.

The interaction with other cutting plane separators may influence the performance of an individual cutting plane separator. And thus, it may also affect the impact which this cutting plane separator has on the overall performance of the MIP solver. That is why we decided to evaluate the impact of individual cutting plane separators on the overall performance of Scip when they are used for one thing

- as the only cutting plane separator in Scip (*no interaction*) and for another
- in connection with all other cutting plane separators provided by Scip (*interaction*).

How do we want to evaluate the impact of the individual cutting plane separators in these two situations? Influenced by the work of Bixby et al. [15], we compare for each cutting plane separator, the overall performance of Scip run *with* this separator with the one of Scip run *without* this separator. More precisely, we perform two kinds of tests.

- No interaction: We start with running Scip without any cutting plane separators enabled. Then we compare the performance with the one of Scip when we *enable an individual cutting plane separator*.
- Interaction: We start with running Scip with all cutting plane separator provided by Scip enabled. Then we compare the performance with the one of Scip when we *disable an individual cutting plane separator*.

There is a weakness in the version of Scip used here, i.e., in Scip 0.81, concerning the restart feature. The cuts which have been generated before a restart are not known anymore afterwards, i.e., they have to be generated once again. Thus, a performed restart may strongly influence the overall solution time. Since this weakness has been eliminated in later versions of Scip, we decided to run all tests without restarts in order to not confuse the comparison of the overall solution times of two tests.

Our test set contains all instances of the *initial test set* (see Section 2.3) except harp2 since for this instance, numerical troubles occurred in some of the test runs.

Enabling Individual Cutting Plane Separators

We start with the computational study concerning the impact of the individual cutting plane separators on the overall performance of SCIP when they are used as the only cutting plane separators.

We divided our test set into two subsets; the *solvable test set* and the *unsolvable test set*.

Solvable test set It contains all instances of our test set which can be solved to optimality within the time and memory limit by SCIP run without any cutting plane separators.

Unsolvable test set It contains the rest of the instances of our test set, i.e., all instances which cannot be solved to optimality within the time and memory limit by SCIP run without any cutting plane separators.

For the solvable test set, we use the performance measures Nodes and Time. Here, Nodes denotes the number of nodes evaluated in the branch-and-cut tree and Time is the elapsed overall CPU time in seconds. For the unsolvable test set, we use the performance measure Gap % (γ). It denotes the gap between the dual bound (db) and the primal bound (pb) both obtained when the time or memory limit is reached for an instance. It is defined as

$$\gamma = \begin{cases} 0 & : \quad db = pb, \\ \infty & : \quad (db = 0 \text{ and } db < pb) \text{ or } db \cdot pb < 0, \\ 100 \cdot \frac{pb - db}{|db|} & : \quad \text{otherwise.} \end{cases}$$

With respect to each of these performance measures, we measure the impact of an individual cutting plane separator on the overall performance of SCIP by taking the ratio

$$\frac{\begin{array}{c}\text{value of the performance measure obtained by}\\ \text{SCIP run without any cutting plane separators}\end{array}}{\begin{array}{c}\text{value of the performance measure obtained by}\\ \text{SCIP run with only one cutting plane separator enabled}\end{array}}, \tag{7.1}$$

where individual values smaller than one are replaced by one. This ratio gives us the factor by which enabling an individual separator improves the overall performance. We call it *improvement factor*. An improvement factor greater than one indicates that enabling the separator improves the overall performance and an improvement factor smaller than one indicates that enabling the separator causes a degradation in the overall performance. Therefore, an improvement factor greater than one shows that the cutting plane separator is important for improving the performance and the greater the factor (> 1) the greater is the impact of the separator.

For the solvable test set, the results for running SCIP without any cutting plane separators and for enabling individual cutting plane separators are given in Table B.64 and Table B.65. In both tables, the column headed No Cuts reports the results for running SCIP without any cutting plane separators and the columns headed Only GMI, Only C-MIR, Only Knapsack, Only Flow Cover, Only Impl. B., and Only Clique report the results for enabling individual cutting plane separators.

Cutting Plane Separator	Solvable Test Set: Not Solved to Opt. (Number)	Solvable Test Set: Nodes Improv. Factor (Geom. Mean)	Solvable Test Set: Time Improv. Factor (Geom. Mean)	Unsolvable Test Set: Solved to Opt. (Number)	Unsolvable Test Set: No Feas. Sol.[6] (Number)	Unsolvable Test Set: Gap % Improv. Factor (Geom. Mean)
C-MIR	0	3.55	1.45	6	7	1.66
Flow Cover	0	2.17	1.33	2	8	1.32
Knapsack	0	1.74	1.25	3	8	1.10
GMI	2	1.42	1.12	8	9	1.20
Impl. B.	0	1.29	1.13	5	8	1.24
Clique	0	1.08	1.01	0	7	0.99

Table 7.1: Summary of the computational results concerning the impact of individual cutting plane separators on the overall performance of SCIP. *Enabling individual cutting plane separators on the solvable and unsolvable test set.*

At the bottom of these tables, for each test run, we report the number of instances which could not be solved to optimality within the time and memory limit. These cases are indicated by the symbol '>' in front of the value of the performance measure and by the symbol '–' for the corresponding improvement factor. Furthermore, at the bottom of the tables in the row labelled Geom. Mean, we give the geometric mean of the improvement factors over all instances which could be solved to optimality in all test runs reported in the tables. Similar to the presentation of earlier results, numbers in blue indicate that the corresponding cutting plane separator is important for improving the performance of SCIP with respect to the considered performance measure. And, numbers in red indicate the opposite case.

For the unsolvable test set, the results for running SCIP without any cutting plane separators and for enabling individual cutting plane separators are given in Table B.66. They are presented in the same way as for the solvable test set, except the following changes. At the bottom of the table, we do not report the number of instances which *could not be solved to optimality* within the time and memory limit, but of course the ones which *could be solved to optimality* within the time and memory limit. Furthermore, in the row labelled No. Feas. Sol., we report the number of instances for which no feasible solution was found within the time and memory limit. For the unsolvable test set, in the columns headed Improvement Factor, the symbol '–' indicates that Gap % is infinite for using SCIP without any cutting plane separators or that Gap % is zero or infinite for enabling the corresponding cutting plane separator. For each separator, we calculate the geometric mean of the improvement factors only over those instances for which the improvement factors is given for all cutting plane separators.

A summary of the results of the computational study is given in Figure 7.2 and Table 7.1. We ordered the cutting plane separators by the magnitude of their impact on the overall performance with respect to the performance measure Nodes.

The following observations can be made.

Impact on the overall performance For all cutting plane separators, except the one for the node packing problem, enabling them significantly improves the overall performance of SCIP with respect to all three performance measures. Enabling the separator for the node packing problem slightly improves the

[6] No Feas. Sol. = 8 for SCIP run without any cutting plane separators.

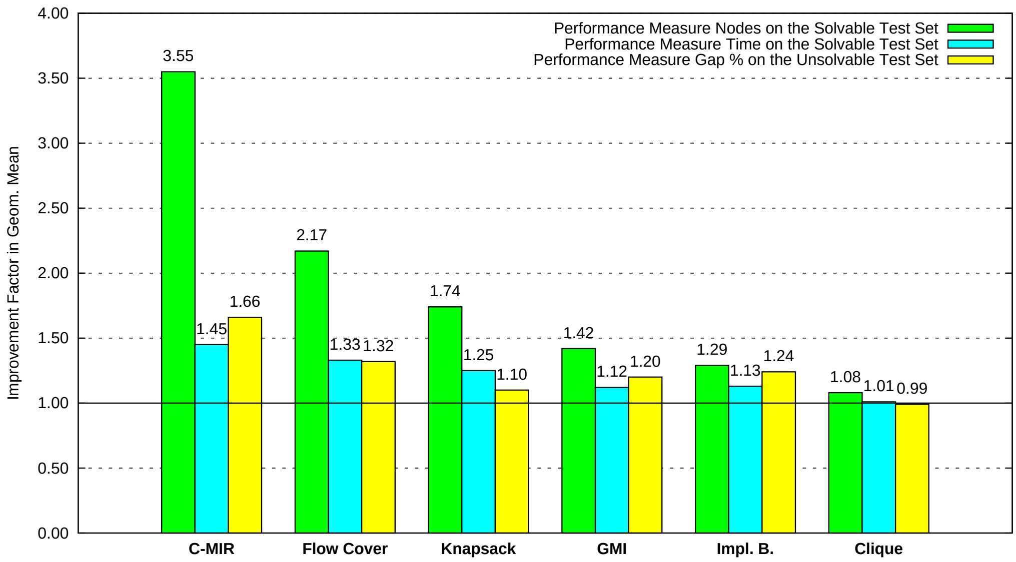

Figure 7.2: Summary of the computational results concerning the impact of individual cutting plane separators on the overall performance of Scip. *Enabling individual cutting plane separators on the solvable and unsolvable test set.*

performance on the solvable test set, but it causes a small degradation in the performance on the unsolvable test set. Thus, all separators, except the one for the node packing problem are clearly important for improving the performance of Scip when they are used as the only separators.

Ranking The cutting plane separator for the class of c-MIR inequalities is by far the most important one for improving the overall performance. It is followed by the one for the 0-1 single node flow problem, which is also very important for improving the performance. Afterwards come the ones for the 0-1 knapsack problem, for the class of GMI inequalities, and for the class of implied bound inequalities. For them, no clear ranking is possible since the ranking is different for the solvable and unsolvable test set. On the last position we find the cutting plane separator for the node packing problem.

Difference between performance measures Nodes and Time For all cutting plane separators, their impact with respect to the measure Nodes is greater than their impact with respect to the measure Time. This is not surprising since the measure Time reflects the costs of an individual cutting plane separator.

Difference between solvable and unsolvable test set The performance measures Time on the solvable test set and Gap % on the unsolvable test set both reflect the costs of an individual cutting plane separator. Thus, the results with respect to these measures are comparable. They show that for some of the separators, there is a difference in the importance of these separators between the solvable and the unsolvable test set.

On the one hand, the separators for the classes of c-MIR inequalities, GMI inequalities, and implied bound inequalities seem to be more important on the unsolvable test set than on the solvable test set. In particular, when we enable the cutting plane separator for the class of GMI inequalities, two instances in the solvable test set cannot be solved to optimality anymore within the time and memory limit, but eight instances in the unsolvable test set can now be solved to optimality. All these separators are general cutting plane separators.

On the other hand, the separator for the 0-1 knapsack problem seems to be more important on the solvable test set than on the unsolvable test set. This separator exploits knowledge about the underlying problem.

Furthermore, for the cutting plane separator for the 0-1 single node flow problem there is no significant difference between the solvable and the unsolvable test set. This separator exploits knowledge about the underlying problem, but can be applied in a quite general way (see Chapter 5).

The results seem to indicate that for the unsolvable test set, general cutting plane separators are more important than the ones which exploit knowledge about the underlying problem.

Comparison to the results of Section 7.1 The results obtained in Section 7.1 indicated that the cutting plane separator for the class of GMI inequalities might be more important for the improvement of the overall performance than the ones for the class of c-MIR inequalities and the 0-1 single node flow problem. But, the results obtained here show another trend.

Disabling Individual Cutting Plane Separators

Now, we present the results of our computational study concerning the impact of the individual cutting plane separators when they are used in connection with all other cutting plane separator provided by Scip. As we will see, in our implementation, the interaction of the cutting plane separators influences the impact which the individual separators have on the overall performance.

As for enabling individual cutting plane separators, we divided our test set into two subsets. They are also called solvable test set and unsolvable test set.

Solvable test set It contains all instances of our test set which can be solved to optimality within the time and memory limit by Scip run with all cutting plane separators enabled.

Unsolvable test set It contains the rest of the instances, i.e., all instances which cannot be solved to optimality within the time and memory limit by Scip run with all cutting plane separators enabled.

For both test sets, we use the same performance measures as for enabling individual cutting plane separators, i.e., we use the measures Nodes and Time for the solvable test set and the measure Gap % for the unsolvable test set. Furthermore, with respect to each performance measure, we measure the impact of an individual

Cutting Plane Separator	Solvable Test Set: Not Solved to Opt. (Number)	Solvable Test Set: Nodes Degrad. Factor (Geom. Mean)	Solvable Test Set: Time Degrad. Factor (Geom. Mean)	Unsolvable Test Set: Solved to Opt. (Number)	Unsolvable Test Set: No Feas. Sol.[7] (Number)	Unsolvable Test Set: Gap % Degrad. Factor (Geom. Mean)
C-MIR	4	1.85	1.13	1	9	1.29
GMI	2	1.34	1.03	0	9	1.05
Knapsack	1	1.18	1.05	1	9	1.01
Flow Cover	3	1.16	1.02	0	9	0.99
Impl. B.	1	1.05	1.03	1	7	1.05
Clique	2	1.06	0.97	1	8	1.01

Table 7.2: Summary of the computational results concerning the impact of individual cutting plane separators on the overall performance of Scip. *Disabling individual cutting plane separators on the solvable and unsolvable test set.*

cutting plane separator on the overall performance by taking the ratio

$$\frac{\begin{array}{c}\text{value of the performance measure obtained by}\\ \text{Scip run with one cutting plane separator disabled}\end{array}}{\begin{array}{c}\text{value of the performance measure obtained by}\\ \text{Scip run with all cutting plane separators enabled}\end{array}}, \tag{7.2}$$

where individual values smaller than one are replaced by one. Here, the ratio gives us the factor by which disabling an individual separator degrades the overall performance. We call it *degradation factor*. A degradation factor greater than one indicates that disabling the cutting plane separator causes a degradation in the performance and a degradation factor smaller than one indicates an improved performance for disabling the separator. Therefore, a degradation factor greater than one shows that the corresponding cutting plane separator is important for an improved overall performance and the greater the factor (> 1) the greater is the impact of the separator.

For the solvable test set, the results for running Scip with all cutting plane separators enabled and for disabling individual cutting plane separators are given in Table B.67 and Table B.68. In both tables, the column headed All Cuts reports the results for running Scip with all cutting plane separator enabled, and the columns headed No GMI, No C-MIR, No Knapsack, No Flow Cover, No Impl. B., and No Clique report the results for disabling individual cutting plane separators. Apart from that, the presentation of the results is the same as for enabling individual separators. Note that in both tables the cutting plane separators are ordered with respect to the results obtained in [15] for disabling the corresponding separators in CPlex 8.0.

For the unsolvable test set, the results for the same tests are given in Table B.69. The meaning of the columns is the same as in Table B.67 and Table B.68, and the presentation of the results is the same as for enabling individual cutting plane separators.

A summary of the results of the computational study can be found in Figure 7.3 and Table 7.2. As for enabling individual separators, we ordered the separators by their importance for improving the overall performance with respect to the performance measure Nodes.

[7] No Feas. Sol. = 8 for Scip run with all cutting plane separators enabled.

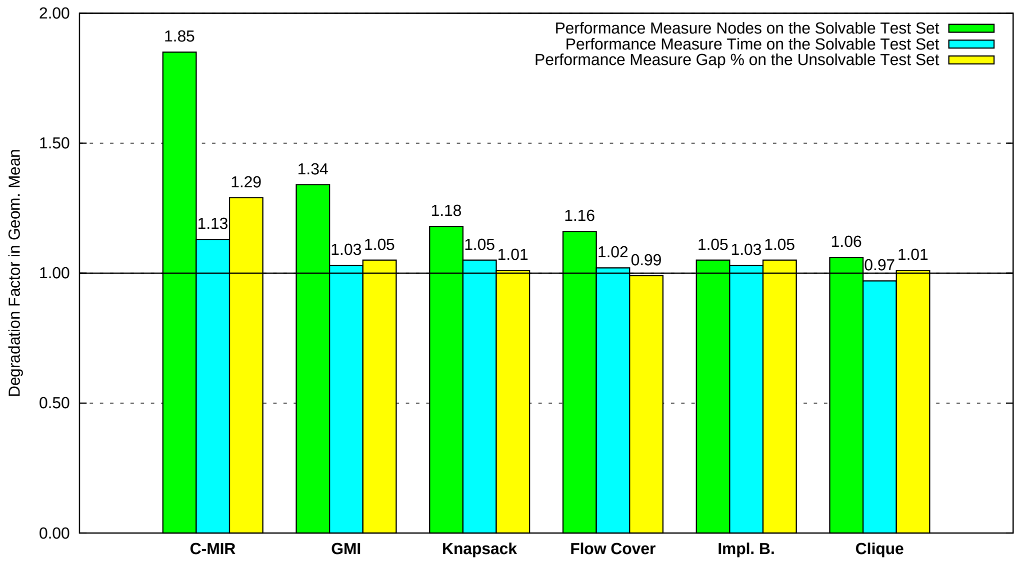

Figure 7.3: Summary of the computational results concerning the impact of individual cutting plane separators on the overall performance of SCIP. *Disabling individual cutting plane separators on the solvable and unsolvable test set.*

Here, we observe the following.

Impact on the overall performance For all cutting plane separators except the ones for the 0-1 single node flow problem and the node packing problem, disabling them causes a degradation in the overall performance with respect to all three performance measures. Disabling the separator for the 0-1 single node flow problem leads to a slight improvement in the performance with respect to the measure Gap % on the unsolvable test set. This is different to the situation where the separator was used as the only cutting plane separator (see Figure 7.2 and Table 7.1). We will give a possible explanation when we discuss the ranking of the separators. Disabling the cutting plane separator for the node packing problem causes an improved performance with respect to the measure Time on the solvable test set. This result goes along with the one obtained for using it as the only cutting plane separator (see Figure 7.2 and Table 7.1).

Ranking There is a difference in the ranking of the separators here to the one obtained for enabling individual cutting plane separators.

First of all, the cutting plane separator for the class of c-MIR inequalities is still the most important one for improving the performance when it is used in connection with all other separators provided by SCIP. And, the cutting plane separator for the node packing problem is still the least important one for improving the performance.

For the rest of the separators, it is hard to find a ranking with respect to the measures Time and Gap %. Since the measure Nodes gives a much clearer picture, we consider it for the ranking of these separators. Here, the importance of the separators for the class of GMI inequalities and the 0-1 single node flow problem interchange. After the separator for the class of c-MIR inequalities (degradation factor of 1.85 in geometric mean), the separator for the GMI inequalities is the most important one for improving the overall performance (degradation factor of 1.34 in geometric mean). It is followed by the ones for the 0-1 knapsack problem and the 0-1 single node flow problem (degradation factors of 1.18 and 1.16 in geometric mean, respectively). The impact of the cutting plane separator for the class of implied bound inequalities is similar to the one for the node packing problem with respect to the measure Nodes.

Valid inequalities for the 0-1 single node flow problem can also be obtained as c-MIR inequalities (see Chapter 5). Therefore, it is not surprising that the importance of the cutting plane separator for the 0-1 single node flow problem decreases when it is used in connection with all other cutting plane separators provided by Scip. But the results obtained here also show that this separator is still important for improving the overall performance of Scip when it is used in connection with the one for the class of c-MIR inequalities. Also note that three instances in the solvable test set cannot be solved to optimality anymore when the separator for the 0-1 single node flow problem is disabled.

Furthermore, in our implementation, the cutting plane separator for the class of GMI inequalities seems to be more important for improving the overall performance when it is used in connection with other cutting plane separators.

Difference between solvable and unsolvable test set As for enabling individual cutting plane separators, we can observe that the general cutting plane separators, i.e., the separators for the classes of c-MIR inequalities, GMI inequalities, and implied bound inequalities, seem to be more important for improving the performance of Scip on the unsolvable test set than on the solvable test set. Furthermore, the cutting plane separator for the 0-1 knapsack problem seems again to be more important for the solvable test set than for the unsolvable test set. Concerning the separator for the 0-1 single node flow problem the results are different to the ones obtained for using the separator without any other separators. Here, its importance for improving the performance of Scip on the unsolvable test set is not similar anymore to the one on the solvable test set. The separator seems to be more important for the solvable test set.

Comparison to the results of Bixby et al. [15] Bixby et al. [15] performed the following test for CPlex 8.0 on a test set of 106 instances which were solvable by CPlex 8.0 in less than 1,000 seconds and were not solvable by CPlex 5.0 (with disabled generation of clique cuts and knapsack cover cuts) in 100,000 seconds or less. For each of the eight kinds of default cutting plane separators, they disabled that one kind of separators, compared the results with the default running time, and calculated the geometric mean over these ratios. Note that in [15] the geometric mean is calculated over all instances in the test set, i.e., the instances which could not be solved to optimality within the time limit of 100,000 seconds by one of the test runs are included in the geometric mean. This may explain, why the magnitude of their degradation factors is greater than the ones obtained here. Their results show that the separator for the class of GMI inequalities is the most important one for improving the performance, followed by the one for the class of MIR inequalities. The separators for knapsack cover cuts, flow cover cuts, and implied bound cuts are in the middle field, and the one for the clique cuts is one of the least important ones for improving the performance. Thus, the main difference between our results obtain for SCIP and the ones obtained by Bixby et al. is that in SCIP the separator for the class of c-MIR inequalities is more important than the one for the class of GMI inequalities and that in CPLEX 8.0 the opposite case is true.

7.3 Conclusion

The results of our computational study concerning the *comparison with* CPLEX *and* CBC have shown the following. For two of the tested cutting plane separators, namely the ones for the class of c-MIR inequalities and the 0-1 single node flow problem, SCIP is competitive to both CPLEX and CBC and performs even better than both of them.

For the cutting plane separator for the class of GMI inequalities, SCIP is not competitive to neither CPLEX nor CBC. This should be improved in one of the next versions of SCIP since this cutting plane separator is one of the most important ones in SCIP and CPLEX for improving the overall performance of the MIP solver. Finally, our cutting plane separator for the 0-1 knapsack problem is superior to the one of CBC, but CPLEX provides a more effective separator than SCIP with respect to the initial gap closed.

From the results of our second study, where we evaluated the *impact of the individual cutting plane separators on the overall performance of* SCIP, we conclude that the cutting plane separator for the class of c-MIR inequalities is the most important one and the one for the node packing problem is the least important one.

Furthermore, each tested cutting plane separator is clearly efficient when it is used as the only cutting plane separator in SCIP. When we apply the individual cutting plane separators in connection with all other separators provided by SCIP they are still efficient with respect to the number of nodes evaluated in the branch-and-cut tree. But, their impact on improving the overall solution time decreases. These results indicate that in order to further improve the performance of SCIP with respect to the overall solution time, further investigations should be done concerning methods which manage the application of the cutting plane separators.

Finally, the results of both computational studies suggest that using the c-MIR approach for our *cutting plane separator for the 0-1 single node flow problem* leads to good results in practice. The cutting plane separator is very efficient when it is applied as the only one in SCIP. Used in connection with all other cutting plane separators, in particular, in addition to the one for the class of c-MIR inequalities, the efficiency decreases, but the separator is still important for improving the overall performance of SCIP.

Appendix A

Zusammenfassung

In dieser Diplomarbeit haben wir uns mit der Implementierung von effizienten Schnittebenenverfahren für gemischt-ganzzahlige Programme (MIPs) befasst. Da das Lösen eines MIPs NP-schwer ist, verwenden moderne MIP Löser heute auf linearer Programmierung basierende Branch-und-Cut Algorithmen. Schnittebenenverfahren sind ein wichtiger Bestandteil dieser Lösungsmethode. Die hier vorgestellten Verfahren wurden in den MIP Löser Scip integriert, welcher am Zuse Institute Berlin entwickelt wurde.

Im Hauptteil dieser Arbeit beschäftigten wir uns mit Schnittebenenverfahren für

- die Klasse der c-MIR Ungleichungen,
- das 0-1 Knapsack Problem, und
- das 0-1 Single Node Flow Problem.

Wir haben jeweils einen Literaturüberblick über die zugrundeliegende Theorie und eine Übersicht über bereits in der Literatur diskutierte Algorithmen gegeben. Desweiteren haben wir die Klassen der gültigen Ungleichungen vorgestellt, die wir separieren wollten. Für das 0-1 Knapsack Problem und das 0-1 Single Node Flow Problem kamen mehrere Klassen in Frage.

Für das erste Problem haben unsere Experimente gezeigt, dass eine Kombination von Schnittebenenverfahren für zwei der drei untersuchten Klassen zu den besten Ergebnissen in der Praxis führt.

Für das zweite Problem ist bekannt, dass verschiedene Klassen von gültigen Ungleichungen auch als c-MIR Ungleichungen generiert werden können. Die Untersuchungen in unserem Abschlussexperiment haben ergeben, dass es sich in der Praxis lohnt, neben dem allgemeinen Separierer für die Klasse der c-MIR Ungleichungen zusätzlich einen Algorithmus zu verwenden, der diese speziellen c-MIR Ungleichungen gezielt separiert.

Für die Implementierung von effizienten Schnittebenenverfahren muss eine Vielzahl an Entscheidungen getroffen werden bezüglich der verwendeten Heuristiken. Für die untersuchten Schnittebenenverfahren haben wir verschiedene Varianten zunächst theoretisch diskutiert. In einer experimentellen Untersuchung analysierten wir dann welche Aspekte der Algorithmen für die Effizienz der Schnittebenenverfahren massgebend sind und welche Heuristiken zu den besten Ergebnissen führen.

Hierbei sind wird auch auf Methoden zur Reduzierung der benötigten Rechenzeit eingegangen. In diesen Experimente wurden die einzelnen Schnittebenenverfahren ohne Interaktion mit anderen Bestandteilen des MIP Lösers verwendet.

Die Arbeit wurde durch ein Abschlussexperiment ergänzt, in welches wir drei weitere Schnittebenenverfahren aus SCIP einbezogen haben. In dieser Untersuchung haben wir gezeigt, dass unsere Schnittebenenverfahren für die Klasse der c-MIR Ungleichungen und für das 0-1 Single Node Flow Problem jeweils wettbewerbsfähig sind zu denen anderer moderner MIP Löser. Auch dieses Ergebniss bezieht sich auf die Verwendung der einzelnen Verfahren ohne Interaktion mit anderen Bestandteilen des MIP Lösers. Zum Schluss, haben wir analysiert welchen Einfluss die sechs Schnittebenenverfahren jeweils auf den Gesamtlösungsprozess von SCIP haben. Es hat sich herausgestellt, dass der Separierer für die Klasse der c-MIR Ungleichungen am wichtigsten ist, um die Leistung von SCIP zu steigern.

Appendix B

Tables

B.1 Cutting Plane Separator for the Class of C-MIR Inequalities

Name	Type	Conss	Vars	z_{LP}	z_{MIP}
a1c1s1	BMIP	3312	3648	997.529583	11566.5904
aflow30a	BMIP	479	842	983.167425	1158
aflow40b	BMIP	1442	2728	1005.66482	1168
arki001	MIP	1048	1388	7579621.83	7580814.51
atlanta-ip	MIP	21732	48738	81.2455967	95.0095497
bc1	BMIP	1913	1751	2.18877397	3.33836255
bell3a	MIP	123	133	866171.733	878430.316
bell5	MIP	91	104	8908552.45	8966406.49
bienst1	BMIP	576	505	11.7241379	46.75
bienst2	BMIP	576	505	11.7241379	54.6
binkar10_1	BMIP	1026	2298	6637.18803	6742.20002
blend2	MIP	274	353	6.91567511	7.598985
dano3_4	BMIP	3202	13873	576.23162	576.435225
dano3_5	BMIP	3202	13873	576.23162	576.924916
dano3mip	BMIP	3202	13873	576.23162	705.941176
danoint	BMIP	664	521	62.6372804	65.67
dcmulti	BMIP	290	548	184466.891	188182
egout	BMIP	98	141	511.61784	568.1007
fiber	BMIP	363	1298	198107.358	405935.18
fixnet6	BMIP	478	878	3192.042	3983
flugpl	MIP	18	18	1167185.73	1201500
gen	MIP	780	870	112271.463	112313.363
gesa2	MIP	1392	1224	25492512.1	25779856.4
gesa2-o	MIP	1248	1224	25476489.7	25779856.4
gesa3	MIP	1368	1152	27846437.5	27991042.6
gesa3_o	MIP	1224	1152	27833632.5	27991042.6
gt2	IP	29	188	20146.7613	21166
harp2	BIP	112	2993	-74325169.3	-73899597
khb05250	BMIP	101	1350	95919464	106940226
lseu	BIP	28	89	947.957237	1120
mitre	BIP	2054	10724	114782.467	115155
mkc	BMIP	3411	5325	-611.85	-563.212
mod008	BIP	6	319	290.931073	307
mod010	BIP	146	2655	6532.08333	6548
mod011	BMIP	4480	10958	-62081950.3	-54558535
modglob	BMIP	291	422	20430947.6	20740508
momentum1	BMIP	42680	5174	82424.4594	109143.493
momentum2	MIP	24237	3732	10696.1116	12314.2196
msc98-ip	MIP	15850	21143	19520966.2	23271298

continued on the next page

Name	Type	Conss	Vars	z_{LP}	z_{MIP}
neos1	BIP	5020	2112	5.6	19
neos2	BMIP	1103	2101	-4407.09724	454.864697
neos3	BMIP	1442	2747	-6158.20911	368.842751
neos616206	BMIP	534	480	787.721258	937.6
neos632659	BMIP	244	420	-119.47619	-94
neos7	MIP	1994	1556	562977.43	721934
neos8	IP	46324	23228	-3725	-3719
neos14	BMIP	552	792	32734.1148	74333.3433
neos15	BMIP	552	792	33463.7701	80851.6678
neos16	IP	1018	377	429	450
neos22	BMIP	5208	3240	777191.429	779715
neos23	BMIP	1568	477	56	137
net12	BMIP	14021	14115	68.3978758	214
nsrand-ipx	BMIP	735	6621	49667.8923	51520
p0033	BIP	16	33	2828.33136	3089
p0282	BIP	241	282	180000.3	258411
p0548	BIP	176	548	4790.57713	8691
p2756	BIP	755	2756	2701.14437	3124
pp08a	BMIP	136	240	2748.34524	7350
pp08aCUTS	BMIP	246	240	5480.60616	7350
prod1	BMIP	208	250	-84.4158719	-56
qnet1	MIP	503	1541	14274.1027	16029.6927
qnet1_o	MIP	456	1541	12557.2479	16029.6927
ran10x26	BMIP	296	520	3857.02278	4270
ran12x21	BMIP	285	504	3157.37744	3664
ran13x13	BMIP	195	338	2691.43947	3252
ran14x18_1	BMIP	284	504	3016.94435	3714
ran8x32	BMIP	296	512	4937.58453	5247
rentacar	BMIP	6803	9557	28928379.6	30356761
rgn	BMIP	24	180	48.7999986	82.1999992
roll3000	MIP	2295	1166	11097.2754	12899
set1ch	BMIP	492	712	35118.1098	54537.75
sp97ar	BIP	1761	14101	652560391	663164724
swath	BMIP	884	6805	334.496858	477.34101
timtab1	MIP	171	397	157896.037	764772
timtab2	MIP	294	675	210652.471	1184230
tr12-30	BMIP	750	1080	18124.1745	130596
vpm1	BMIP	234	378	16.4333333	20
vpm2	BMIP	234	378	10.303297	13.75

Table B.1: Summary of the main test set for the cutting plane separator for the class of c-MIR inequalities.

Name	Type	Cons	Vars	z_{LP}	z_{MIP}
10teams	BMIP	230	2025	917	924
30:70:4_5:0_5:100	BMIP	12050	10772	8.1	9
30:70:4_5:0_95:98	BMIP	12471	10990	11.5	12
air03	BIP	124	10757	338864.25	340160
air04	BIP	823	8904	55535.4364	56137
air05	BIP	426	7195	25877.6093	26374
cap6000	BIP	2176	6000	-2451537.33	-2451377
dano3_3	BMIP	3202	13873	576.23162	576.344633
ds	BIP	656	67732	57.2347263	468.645
eilD76	BIP	75	1898	680.538997	885.411847
fast0507	BIP	507	63009	172.145567	174
glass4	BMIP	396	322	800002400	1.6000134e+09
irp	BIP	39	20315	12123.5302	12159.4928
l152lav	BIP	97	1989	4656.36364	4722
liu	BMIP	2178	1156	560	1146
manna81	IP	6480	3321	-13297	-13164
markshare1	BMIP	6	62	0	1
markshare2	BMIP	7	74	0	1
mas284	BMIP	68	151	86195.863	91405.7237
mas74	BMIP	13	151	10482.7953	11801.1857
mas76	BMIP	12	151	38893.9036	40005.0541
misc03	BMIP	96	160	1910	3360
misc06	BMIP	820	1808	12841.6894	12850.8607
misc07	BMIP	212	260	1415	2810
mkc1	BMIP	3411	5325	-611.85	-607.207
mzzv11	IP	9499	10240	-22944.9875	-21718
mzzv42z	IP	10460	11717	-21622.9985	-20540
neos648910	BMIP	1491	814	16	32
neos9	BMIP	31600	81408	780	798
neos10	IP	46793	23489	-1196.33333	-1135
neos11	BMIP	2706	1220	6	9
neos12	BMIP	8317	3983	9.41161243	13
neos13	BMIP	20852	1827	-126.178378	-95.4748066
neos17	BMIP	486	535	0.000681498501	0.150002577
neos18	BIP	11402	3312	7	16
neos19	BMIP	34082	103789	-1611	-1499
neos20	MIP	2446	1165	-475	-434
neos21	BMIP	1085	614	2.21648352	7
noswot	MIP	182	128	-43	-41
nug08	BIP	912	1632	203.5	214
nw04	BIP	36	87482	16310.6667	16862
opt1217	BMIP	64	769	-20.0213904	-16
p0201	BIP	133	201	7125	7615
pk1	BMIP	45	86	0	11
protfold	BIP	2112	1835	-41.9574468	-23
qap10	BIP	1820	4150	332.566228	340
qiu	BMIP	1192	840	-931.638854	-132.873137
rout	MIP	291	556	981.864286	1077.56
seymour	BIP	4944	1372	403.846474	423
seymour1	BMIP	4944	1372	403.846474	410.763701
stein27	BIP	118	27	13	18
stein45	BIP	331	45	22	30
swath1	BMIP	884	6805	334.496858	379.071296
swath2	BMIP	884	6805	334.496858	385.199693
swath3	BMIP	884	6805	334.496858	397.761344
t1717	BIP	551	73885	134531.021	288658

Table B.2: Summary of the remaining test set for the cutting plane separator for the class of c-MIR inequalities.

Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
a1c1s1	48.34	1432	520.8	14.9
aflow30a	46.36	408	39.2	1.3
aflow40b	31.65	428	186.6	7.2
arki001	2.40	156	4.9	0.8
atlanta-ip	0.17	60	3500.6	583.4
bc1	0.00	0	0.0	0.0
bell3a	60.20	15	0.0	0.0
bell5	45.63	38	0.1	0.0
bienst1	0.00	0	0.0	0.0
bienst2	0.00	0	0.0	0.0
binkar10_1	55.01	52	0.4	0.1
blend2	11.50	20	0.4	0.0
dano3_4	0.00	0	3.3	3.3
dano3_5	0.00	0	4.5	4.5
dano3mip	0.00	1	52.3	26.2
danoint	0.51	14	1.0	0.2
dcmulti	30.07	115	0.7	0.1
egout	90.80	69	0.2	0.0
fiber	88.69	58	0.2	0.0
fixnet6	73.75	1227	35.6	0.4
flugpl	2.01	2	0.0	0.0
gen	100.00	23	0.0	0.0
gesa2	74.60	161	1.3	0.2
gesa2-o	69.13	323	4.9	0.3
gesa3	57.91	114	1.8	0.3
gesa3_o	61.15	184	2.7	0.3
gt2	48.50	18	0.0	0.0
harp2	11.11	3	0.0	0.0
khb05250	4.70	1	0.0	0.0
lseu	46.23	29	0.0	0.0
mitre	10.78	819	37.5	6.3
mkc	13.37	171	19.8	1.1
mod008	40.41	13	0.1	0.0
mod010	18.32	2	0.2	0.1
mod011	88.63	3374	465.2	7.6
modglob	27.00	109	0.1	0.0
momentum1	0.00	11	317.0	79.2
momentum2	0.02	16	732.5	146.5
msc98-ip	0.97	378	856.4	107.0
neos1	0.00	186	0.4	0.1
neos2	9.48	500	70.0	2.3
neos3	10.39	740	164.6	3.7
neos616206	0.14	78	0.4	0.1
neos632659	54.38	549	1.5	0.1
neos7	71.72	473	20.6	0.8
neos8	4.17	5	59.9	30.0
neos14	67.85	638	20.9	0.4
neos15	71.64	1164	40.3	0.8
neos16	9.52	139	0.4	0.1
neos22	94.20	30	2.7	0.9
neos23	9.49	291	2.6	0.2
net12	2.92	167	930.0	54.7
nsrand-ipx	11.71	61	8.3	1.4
p0033	63.37	22	0.0	0.0
p0282	92.80	124	0.3	0.0
p0548	88.96	125	0.1	0.0
p2756	72.98	242	0.9	0.1
pp08a	93.98	209	0.6	0.0
pp08aCUTS	89.91	115	0.3	0.0
prod1	0.85	97	0.7	0.1
qnet1	71.96	70	0.3	0.0
qnet1_o	88.98	99	0.5	0.0
ran10x26	44.85	73	0.5	0.0

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Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
ran12x21	34.49	49	0.3	0.0
ran13x13	27.10	31	0.1	0.0
ran14x18_1	33.09	75	0.6	0.0
ran8x32	42.26	95	0.9	0.1
rentacar	0.00	0	0.0	0.0
rgn	95.77	220	0.9	0.0
roll3000	57.34	266	54.8	3.4
set1ch	99.26	267	0.6	0.1
sp97ar	0.75	8	12.1	6.0
swath	0.00	0	0.2	0.2
timtab1	30.81	323	1.5	0.1
timtab2	18.37	704	7.2	0.5
tr12-30	93.60	1420	62.2	1.8
vpm1	100.00	44	0.0	0.0
vpm2	74.15	197	0.6	0.0
Total	3093.17	19740	8259.5	1099.3
Geom. Mean	16.29	59	4.5	1.9

Table B.3: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Default algorithm.*

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
a1c1s1	78.99	30.65	3406	1974	1250.5	729.7	19.8	4.9
aflow30a	45.62	-0.74	381	-27	36.0	-3.2	1.4	0.1
aflow40b	35.56	3.91	429	1	261.6	75.0	9.3	2.1
arki001	15.05	12.65	265	109	7.2	2.3	1.2	0.4
atlanta-ip	0.17	0.00	64	4	4162.9	662.3	832.6	249.2
bc1	37.93	37.93	17	17	1.3	1.3	0.2	0.2
bell3a	59.02	-1.18	15	0	0.0	0.0	0.0	0.0
bell5	47.29	1.66	48	10	0.1	0.0	0.0	0.0
bienst1	6.82	6.82	136	136	2.0	2.0	0.1	0.1
bienst2	7.57	7.57	206	206	3.0	3.0	0.2	0.2
binkar10_1	55.01	0.00	52	0	0.3	-0.1	0.1	0.0
blend2	10.49	-1.01	25	5	0.4	0.0	0.0	0.0
dano3_4	1.46	1.46	2	2	6.9	3.6	3.5	0.2
dano3_5	1.44	1.44	6	6	20.0	15.5	5.0	0.5
dano3mip	0.01	0.01	6	5	91.5	39.2	30.5	4.3
danoint	0.93	0.42	60	46	2.2	1.2	0.4	0.2
dcmulti	50.84	20.77	192	77	1.5	0.8	0.1	0.0
egout	100.00	9.20	95	26	0.1	-0.1	0.0	0.0
fiber	88.69	0.00	58	0	0.2	0.0	0.0	0.0
fixnet6	69.05	-4.70	1030	-197	44.3	8.7	0.7	0.3
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	23	0	0.1	0.1	0.0	0.0
gesa2	99.64	25.04	277	116	3.3	2.0	0.3	0.1
gesa2-o	91.26	22.13	404	81	5.4	0.5	0.4	0.1
gesa3	79.12	21.21	161	47	3.7	1.9	0.5	0.2
gesa3_o	82.73	21.58	266	82	4.8	2.1	0.5	0.2
gt2	48.50	0.00	18	0	0.0	0.0	0.0	0.0
harp2	11.11	0.00	3	0	0.0	0.0	0.0	0.0
khb05250	94.42	89.72	982	981	19.9	19.9	0.4	0.4
lseu	46.23	0.00	29	0	0.0	0.0	0.0	0.0
mitre	10.78	0.00	819	0	37.5	0.0	6.3	0.0
mkc	13.37	0.00	171	0	19.8	0.0	1.1	0.0
mod008	40.41	0.00	13	0	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.2	0.0	0.1	0.0
mod011	86.54	-2.09	2260	-1114	294.3	-170.9	5.5	-2.1
modglob	36.30	9.30	177	68	0.2	0.1	0.0	0.0
momentum1	0.00	0.00	19	8	367.3	50.3	91.8	12.6
momentum2	0.02	0.00	58	42	1157.6	425.1	192.9	46.4
msc98-ip	1.03	0.06	392	14	904.0	47.6	113.0	6.0
neos1	0.00	0.00	186	0	0.4	0.0	0.1	0.0
neos2	18.72	9.24	1425	925	825.6	755.6	6.8	4.5
neos3	13.58	3.19	1277	537	1138.4	973.8	9.4	5.7
neos616206	0.00	-0.14	125	47	0.5	0.1	0.1	0.0
neos632659	0.00	-54.38	114	-435	0.1	-1.4	0.0	-0.1
neos7	72.10	0.38	505	32	22.9	2.3	1.0	0.2
neos8	4.17	0.00	5	0	59.6	-0.3	29.8	-0.2
neos14	76.43	8.58	1370	732	64.7	43.8	0.9	0.5
neos15	80.94	9.30	2000	836	101.6	61.3	1.2	0.4
neos16	9.52	0.00	139	0	0.4	0.0	0.1	0.0
neos22	100.00	5.80	27	-3	2.7	0.0	0.9	0.0
neos23	12.28	2.79	468	177	4.0	1.4	0.2	0.0
net12	2.92	0.00	167	0	929.5	-0.5	54.7	0.0
nsrand-ipx	11.71	0.00	61	0	8.3	0.0	1.4	0.0
p0033	63.37	0.00	22	0	0.0	0.0	0.0	0.0
p0282	92.80	0.00	124	0	0.3	0.0	0.0	0.0
p0548	88.96	0.00	125	0	0.2	0.1	0.0	0.0
p2756	72.98	0.00	242	0	0.9	0.0	0.1	0.0
pp08a	97.23	3.25	366	157	3.1	2.5	0.1	0.1
pp08aCUTS	92.21	2.30	181	66	1.4	1.1	0.1	0.1
prod1	0.85	0.00	97	0	0.7	0.0	0.1	0.0
qnet1	71.96	0.00	70	0	0.3	0.0	0.0	0.0
qnet1_o	88.98	0.00	99	0	0.5	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	52.18	7.33	167	94	6.5	6.0	0.3	0.3
ran12x21	57.12	22.63	202	153	6.3	6.0	0.2	0.2
ran13x13	52.40	25.30	180	149	3.7	3.6	0.1	0.1
ran14x18_1	46.48	13.39	232	157	5.9	5.3	0.2	0.2
ran8x32	72.89	30.63	168	73	5.0	4.1	0.2	0.1
rentacar	18.10	18.10	62	62	3.4	3.4	0.1	0.1
rgn	98.80	3.03	137	-83	0.6	-0.3	0.0	0.0
roll3000	57.34	0.00	266	0	79.5	24.7	5.0	1.6
set1ch	99.90	0.64	269	2	0.4	-0.2	0.1	0.0
sp97ar	0.75	0.00	8	0	12.0	-0.1	6.0	0.0
swath	5.45	5.45	1	1	0.5	0.3	0.3	0.1
timtab1	64.43	33.62	826	503	10.4	8.9	0.3	0.2
timtab2	32.90	14.53	1540	836	33.3	26.1	1.0	0.5
tr12-30	96.11	2.51	2487	1067	165.6	103.4	3.1	1.3
vpm1	100.00	0.00	33	-11	0.0	0.0	0.0	0.0
vpm2	75.56	1.41	221	24	0.9	0.3	0.1	0.1
Total	3575.89	482.72	28563	8823	12210.5	3951.0	1441.8	342.5
Geom. Mean	21.46	5.17	107	48	7.2	2.7	2.0	0.1

Table B.4: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Aggregation heuristic.* Use Score Type 3. (△ with respect to the default algorithm)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
a1c1s1	77.42	29.08	3646	2214	1211.5	690.7	19.5	4.6
aflow30a	45.30	-1.06	396	-12	22.8	-16.4	0.9	-0.4
aflow40b	33.22	1.57	472	44	216.1	29.5	6.0	-1.2
arki001	3.28	0.88	171	15	3.0	-1.9	0.5	-0.3
atlanta-ip	0.17	0.00	58	-2	3694.9	194.3	615.8	32.4
bc1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell3a	52.67	-7.53	11	-4	0.0	0.0	0.0	0.0
bell5	45.06	-0.57	41	3	0.0	-0.1	0.0	0.0
bienst1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bienst2	0.00	0.00	0	0	0.0	0.0	0.0	0.0
binkar10_1	55.01	0.00	52	0	0.3	-0.1	0.1	0.0
blend2	9.55	-1.95	32	12	0.3	-0.1	0.0	0.0
dano3_4	0.00	0.00	2	2	5.2	1.9	2.6	-0.7
dano3_5	0.19	0.19	3	3	7.1	2.6	3.5	-1.0
dano3mip	0.03	0.03	38	37	103.5	51.2	17.3	-8.9
danoint	0.95	0.44	49	35	1.2	0.2	0.2	0.0
dcmulti	38.30	8.23	194	79	1.4	0.7	0.1	0.0
egout	100.00	9.20	30	-39	0.0	-0.2	0.0	0.0
fiber	88.69	0.00	58	0	0.2	0.0	0.0	0.0
fixnet6	68.62	-5.13	925	-302	48.2	12.6	0.6	0.2
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	22	-1	0.1	0.1	0.0	0.0
gesa2	72.69	-1.91	160	-1	1.1	-0.2	0.1	-0.1
gesa2-o	39.96	-29.17	235	-88	1.4	-3.5	0.2	-0.1
gesa3	57.89	-0.02	112	-2	1.7	-0.1	0.2	-0.1
gesa3_o	61.07	-0.08	185	1	2.5	-0.2	0.3	0.0
gt2	48.50	0.00	18	0	0.0	0.0	0.0	0.0
harp2	11.11	0.00	3	0	0.0	0.0	0.0	0.0
khb05250	4.70	0.00	1	0	0.0	0.0	0.0	0.0
lseu	46.23	0.00	29	0	0.0	0.0	0.0	0.0
mitre	10.78	0.00	819	0	38.1	0.6	6.3	0.0
mkc	13.37	0.00	171	0	21.5	1.7	1.2	0.1
mod008	40.41	0.00	13	0	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.2	0.0	0.1	0.0
mod011	56.30	-32.33	651	-2723	41.7	-423.5	1.2	-6.4
modglob	17.81	-9.19	101	-8	0.1	0.0	0.0	0.0
momentum1	0.00	0.00	13	2	332.9	15.9	66.6	-12.6
momentum2	0.02	0.00	16	0	720.0	-12.5	144.0	-2.5
msc98-ip	1.04	0.07	392	14	1196.7	340.3	119.7	12.7
neos1	0.00	0.00	186	0	0.4	0.0	0.1	0.0
neos2	0.74	-8.74	32	-468	2.2	-67.8	0.3	-2.0
neos3	1.03	-9.36	59	-681	6.1	-158.5	0.7	-3.0
neos616206	0.04	-0.10	76	-2	0.4	0.0	0.1	0.0
neos632659	28.04	-26.34	218	-331	0.3	-1.2	0.0	-0.1
neos7	71.44	-0.28	608	135	25.1	4.5	0.8	0.0
neos8	4.17	0.00	5	0	68.1	8.2	34.1	4.1
neos14	66.47	-1.38	608	-30	17.9	-3.0	0.4	0.0
neos15	65.53	-6.11	1097	-67	40.6	0.3	0.7	-0.1
neos16	9.52	0.00	139	0	0.4	0.0	0.1	0.0
neos22	24.51	-69.69	44	14	2.9	0.2	0.6	-0.3
neos23	9.49	0.00	315	24	2.5	-0.1	0.1	-0.1
net12	2.92	0.00	167	0	937.0	7.0	55.1	0.4
nsrand-ipx	11.71	0.00	61	0	8.2	-0.1	1.4	0.0
p0033	63.37	0.00	22	0	0.0	0.0	0.0	0.0
p0282	92.80	0.00	124	0	0.3	0.0	0.0	0.0
p0548	88.96	0.00	125	0	0.2	0.1	0.0	0.0
p2756	72.98	0.00	242	0	0.9	0.0	0.1	0.0
pp08a	94.35	0.37	250	41	0.7	0.1	0.0	0.0
pp08aCUTS	86.86	-3.05	147	32	0.3	0.0	0.0	0.0
prod1	0.85	0.00	97	0	0.7	0.0	0.1	0.0
qnet1	71.96	0.00	70	0	0.3	0.0	0.0	0.0
qnet1_o	88.98	0.00	99	0	0.5	0.0	0.0	0.0

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	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
ran10x26	6.07	-38.78	2	-71	0.0	-0.5	0.0	0.0
ran12x21	29.35	-5.14	21	-28	0.0	-0.3	0.0	0.0
ran13x13	27.10	0.00	31	0	0.1	0.0	0.0	0.0
ran14x18_1	19.97	-13.12	20	-55	0.1	-0.5	0.0	0.0
ran8x32	38.71	-3.55	3	-92	0.0	-0.9	0.0	-0.1
rentacar	0.00	0.00	0	0	0.0	0.0	0.0	0.0
rgn	98.88	3.11	179	-41	0.9	0.0	0.0	0.0
roll3000	57.34	0.00	266	0	56.1	1.3	3.5	0.1
set1ch	99.13	-0.13	275	8	0.6	0.0	0.1	0.0
sp97ar	0.75	0.00	8	0	12.0	-0.1	6.0	0.0
swath	5.45	5.45	1	1	0.5	0.3	0.2	0.0
timtab1	36.52	5.71	317	-6	0.8	-0.7	0.1	0.0
timtab2	20.64	2.27	706	2	4.7	-2.5	0.3	-0.2
tr12-30	95.11	1.51	1751	331	93.4	31.2	2.3	0.5
vpm1	100.00	0.00	26	-18	0.0	0.0	0.0	0.0
vpm2	74.75	0.60	211	14	1.0	0.4	0.0	0.0
Total	2887.19	-205.98	17731	-2009	8960.3	700.8	1114.6	15.3
Geom. Mean	14.69	-1.60	52	-7	4.1	-0.4	1.8	-0.1

Table B.5: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Aggregation heuristic*. Use Score Type 4. (△ with respect to the default algorithm)

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
a1c1s1	24.09	-24.25	282	-1150	43.8	-477.0	4.9	-10.0
aflow30a	31.17	-15.19	194	-214	3.0	-36.2	0.2	-1.1
aflow40b	23.93	-7.72	377	-51	19.5	-167.1	0.8	-6.4
arki001	10.48	8.08	94	-62	4.0	-0.9	0.8	0.0
atlanta-ip	0.17	0.00	66	6	2968.3	-532.3	593.7	10.3
bc1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell3a	60.20	0.00	15	0	0.0	0.0	0.0	0.0
bell5	34.49	-11.14	31	-7	0.0	-0.1	0.0	0.0
bienst1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bienst2	0.00	0.00	0	0	0.0	0.0	0.0	0.0
binkar10_1	55.01	0.00	52	0	0.3	-0.1	0.1	0.0
blend2	4.46	-7.04	13	-7	0.1	-0.3	0.0	0.0
dano3_4	0.00	0.00	0	0	2.9	-0.4	2.9	-0.4
dano3_5	0.00	0.00	0	0	3.9	-0.6	3.9	-0.6
dano3mip	0.00	0.00	0	-1	16.2	-36.1	16.2	-10.0
danoint	0.09	-0.42	6	-8	0.5	-0.5	0.1	-0.1
dcmulti	9.07	-21.00	35	-80	0.2	-0.5	0.0	-0.1
egout	13.27	-77.53	14	-55	0.0	-0.2	0.0	0.0
fiber	88.69	0.00	58	0	0.2	0.0	0.0	0.0
fixnet6	45.56	-28.19	217	-1010	2.4	-33.2	0.1	-0.3
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	34	11	0.1	0.1	0.0	0.0
gesa2	71.05	-3.55	125	-36	1.2	-0.1	0.2	0.0
gesa2-o	42.60	-26.53	181	-142	1.6	-3.3	0.2	-0.1
gesa3	56.64	-1.27	111	-3	1.6	-0.2	0.3	0.0
gesa3_o	59.47	-1.68	133	-51	2.7	0.0	0.3	0.0
gt2	48.50	0.00	18	0	0.0	0.0	0.0	0.0
harp2	11.11	0.00	3	0	0.0	0.0	0.0	0.0
khb05250	0.00	-4.70	0	-1	0.0	0.0	0.0	0.0
lseu	46.23	0.00	29	0	0.0	0.0	0.0	0.0
mitre	10.78	0.00	819	0	37.4	-0.1	6.2	-0.1
mkc	13.37	0.00	171	0	19.8	0.0	1.1	0.0
mod008	40.41	0.00	13	0	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.2	0.0	0.1	0.0
mod011	34.68	-53.95	1439	-1935	76.3	-388.9	2.5	-5.1
modglob	35.53	8.53	70	-39	0.1	0.0	0.0	0.0
momentum1	0.00	0.00	3	-8	69.0	-248.0	34.5	-44.7
momentum2	0.02	0.00	23	7	264.8	-467.7	53.0	-93.5
msc98-ip	1.02	0.05	698	320	861.5	5.1	123.1	16.1
neos1	0.00	0.00	186	0	0.4	0.0	0.1	0.0
neos2	1.12	-8.36	11	-489	0.5	-69.5	0.1	-2.2
neos3	0.83	-9.56	10	-730	0.7	-163.9	0.1	-3.6
neos616206	0.93	0.79	67	-11	0.7	0.3	0.1	0.0
neos632659	0.00	-54.38	87	-462	0.1	-1.4	0.0	-0.1
neos7	6.96	-64.76	16	-457	0.8	-19.8	0.3	-0.5
neos8	4.17	0.00	5	0	59.6	-0.3	29.8	-0.2
neos14	69.89	2.04	432	-206	6.6	-14.3	0.4	0.0
neos15	65.73	-5.91	518	-646	8.8	-31.5	0.5	-0.3
neos16	9.52	0.00	139	0	0.4	0.0	0.1	0.0
neos22	1.88	-92.32	75	45	7.7	5.0	1.1	0.2
neos23	0.00	-9.49	13	-278	0.3	-2.3	0.1	-0.1
net12	2.67	-0.25	122	-45	585.6	-344.4	48.8	-5.9
nsrand-ipx	11.71	0.00	61	0	8.2	-0.1	1.4	0.0
p0033	63.37	0.00	22	0	0.0	0.0	0.0	0.0
p0282	92.80	0.00	124	0	0.3	0.0	0.0	0.0
p0548	88.96	0.00	125	0	0.2	0.1	0.0	0.0
p2756	72.98	0.00	242	0	0.9	0.0	0.1	0.0
pp08a	95.28	1.30	243	34	0.7	0.1	0.0	0.0
pp08aCUTS	88.28	-1.63	187	72	1.1	0.8	0.1	0.1
prod1	0.85	0.00	97	0	0.7	0.0	0.1	0.0
qnet1	71.96	0.00	70	0	0.3	0.0	0.0	0.0
qnet1_o	88.98	0.00	99	0	0.5	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	40.56	-4.29	249	176	2.1	1.6	0.1	0.1
ran12x21	29.53	-4.96	366	317	3.1	2.8	0.1	0.1
ran13x13	26.90	-0.20	161	130	0.6	0.5	0.0	0.0
ran14x18_1	27.88	-5.21	582	507	6.2	5.6	0.1	0.1
ran8x32	31.25	-11.01	169	74	1.1	0.2	0.1	0.0
rentacar	0.00	0.00	0	0	0.0	0.0	0.0	0.0
rgn	100.00	4.23	119	-101	0.4	-0.5	0.0	0.0
roll3000	57.34	0.00	266	0	56.1	1.3	3.5	0.1
set1ch	96.36	-2.90	508	241	3.4	2.8	0.2	0.1
sp97ar	0.75	0.00	8	0	12.0	-0.1	6.0	0.0
swath	0.00	0.00	0	0	0.2	0.0	0.2	0.0
timtab1	16.30	-14.51	127	-196	0.4	-1.1	0.0	-0.1
timtab2	6.91	-11.46	186	-518	1.0	-6.2	0.1	-0.4
tr12-30	96.47	2.87	914	-506	17.9	-44.3	0.9	-0.9
vpm1	100.00	0.00	148	104	0.1	0.1	0.0	0.0
vpm2	67.23	-6.92	169	-28	0.4	-0.2	0.0	0.0
Total	2528.79	-564.38	12251	-7489	5191.8	-3067.7	939.7	-159.6
Geom. Mean	11.52	-4.77	44	-15	3.0	-1.5	1.7	-0.2

Table B.6: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Bound substitution heuristic*. Use Criterium F1 in the first step. (△ with respect to the default algorithm)

Name	Gap Closed % Value	Gap Closed % △	Cuts Value	Cuts △	Sepa Time Value	Sepa Time △	Average Sepa Time Value	Average Sepa Time △
a1c1s1	5.07	-43.27	88	-1344	68.2	-452.6	3.8	-11.1
aflow30a	28.55	-17.81	175	-233	16.4	-22.8	0.7	-0.6
aflow40b	27.12	-4.53	256	-172	178.5	-8.1	5.1	-2.1
arki001	2.55	0.15	130	-26	4.6	-0.3	0.8	0.0
atlanta-ip	0.17	0.00	53	-7	3712.2	211.6	618.7	35.3
bc1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell3a	39.74	-20.46	9	-6	0.0	0.0	0.0	0.0
bell5	58.29	12.66	28	-10	0.0	-0.1	0.0	0.0
bienst1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bienst2	0.00	0.00	0	0	0.0	0.0	0.0	0.0
binkar10_1	55.01	0.00	52	0	0.3	-0.1	0.1	0.0
blend2	11.50	0.00	20	0	0.4	0.0	0.0	0.0
dano3_4	0.00	0.00	0	0	3.4	0.1	3.4	0.1
dano3_5	0.00	0.00	0	0	4.7	0.2	4.7	0.2
dano3mip	0.00	0.00	0	-1	29.4	-22.9	29.4	3.2
danoint	0.53	0.02	12	-2	1.2	0.2	0.2	0.0
dcmulti	42.85	12.78	95	-20	0.8	0.1	0.1	0.0
egout	77.60	-13.20	38	-31	0.1	-0.1	0.0	0.0
fiber	88.69	0.00	58	0	0.2	0.0	0.0	0.0
fixnet6	17.17	-56.58	45	-1182	0.3	-35.3	0.0	-0.4
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	14	-9	0.0	0.0	0.0	0.0
gesa2	70.54	-4.06	147	-14	1.1	-0.2	0.1	-0.1
gesa2-o	56.52	-12.61	222	-101	2.4	-2.5	0.2	-0.1
gesa3	57.45	-0.46	112	-2	1.9	0.1	0.3	0.0
gesa3_o	58.37	-2.78	152	-32	1.6	-1.1	0.3	0.0
gt2	48.50	0.00	18	0	0.0	0.0	0.0	0.0
harp2	11.11	0.00	3	0	0.0	0.0	0.0	0.0
khb05250	4.70	0.00	1	0	0.0	0.0	0.0	0.0
lseu	46.23	0.00	29	0	0.0	0.0	0.0	0.0
mitre	10.78	0.00	819	0	37.7	0.2	6.3	0.0
mkc	13.37	0.00	171	0	20.3	0.5	1.1	0.0
mod008	40.41	0.00	13	0	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.2	0.0	0.1	0.0
mod011	69.07	-19.56	1465	-1909	134.2	-331.0	2.4	-5.2
modglob	31.20	4.20	112	3	0.1	0.0	0.0	0.0
momentum1	0.00	0.00	5	-6	239.8	-77.2	79.9	0.7
momentum2	0.01	-0.01	8	-8	449.1	-283.4	149.7	3.2
msc98-ip	0.97	0.00	333	-45	969.9	113.5	121.2	14.2
neos1	0.00	0.00	186	0	0.4	0.0	0.1	0.0
neos2	4.41	-5.07	91	-409	31.7	-38.3	1.1	-1.2
neos3	3.89	-6.50	168	-572	43.4	-121.2	1.4	-2.3
neos616206	0.04	-0.10	66	-12	0.4	0.0	0.1	0.0
neos632659	18.69	-35.69	98	-451	0.2	-1.3	0.0	-0.1
neos7	10.25	-61.47	13	-460	1.0	-19.6	0.3	-0.5
neos8	4.17	0.00	5	0	59.7	-0.2	29.8	-0.2
neos14	0.28	-67.57	16	-622	0.2	-20.7	0.1	-0.3
neos15	0.24	-71.40	16	-1148	0.3	-40.0	0.1	-0.7
neos16	9.52	0.00	139	0	0.4	0.0	0.1	0.0
neos22	94.20	0.00	21	-9	2.1	-0.6	1.0	0.1
neos23	0.00	-9.49	0	-291	0.1	-2.5	0.1	-0.1
net12	2.67	-0.25	116	-51	567.7	-362.3	51.6	-3.1
nsrand-ipx	11.71	0.00	61	0	8.2	-0.1	1.4	0.0
p0033	63.37	0.00	22	0	0.0	0.0	0.0	0.0
p0282	92.80	0.00	124	0	0.3	0.0	0.0	0.0
p0548	88.96	0.00	125	0	0.2	0.1	0.0	0.0
p2756	72.98	0.00	242	0	0.9	0.0	0.1	0.0
pp08a	92.35	-1.63	183	-26	0.4	-0.2	0.0	0.0
pp08aCUTS	89.91	0.00	114	-1	0.3	0.0	0.0	0.0
prod1	0.85	0.00	97	0	0.7	0.0	0.1	0.0
qnet1	71.96	0.00	70	0	0.3	0.0	0.0	0.0
qnet1_o	88.98	0.00	99	0	0.5	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	43.44	-1.41	71	-2	0.6	0.1	0.0	0.0
ran12x21	34.12	-0.37	54	5	0.4	0.1	0.0	0.0
ran13x13	26.91	-0.19	30	-1	0.1	0.0	0.0	0.0
ran14x18_1	30.81	-2.28	62	-13	0.4	-0.2	0.0	0.0
ran8x32	38.81	-3.45	74	-21	0.7	-0.2	0.0	-0.1
rentacar	0.00	0.00	0	0	0.0	0.0	0.0	0.0
rgn	40.68	-55.09	41	-179	0.1	-0.8	0.0	0.0
roll3000	53.63	-3.71	249	-17	51.2	-3.6	3.2	-0.2
set1ch	11.33	-87.93	7	-260	0.1	-0.5	0.0	-0.1
sp97ar	0.75	0.00	8	0	12.0	-0.1	6.0	0.0
swath	0.00	0.00	0	0	0.2	0.0	0.2	0.0
timtab1	30.47	-0.34	221	-102	0.8	-0.7	0.1	0.0
timtab2	8.97	-9.40	185	-519	1.6	-5.6	0.1	-0.4
tr12-30	2.83	-90.77	17	-1403	2.3	-59.9	0.2	-1.6
vpm1	100.00	0.00	26	-18	0.0	0.0	0.0	0.0
vpm2	72.65	-1.50	162	-35	0.3	-0.3	0.0	0.0
Total	2412.05	-681.12	7966	-11774	6669.4	-1590.1	1126.4	27.1
Geom. Mean	11.40	-4.89	31	-28	3.1	-1.4	1.8	-0.1

Table B.7: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Bound substitution heuristic.* Use Criterium S1 in the second step. (△ with respect to the default algorithm)

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
a1c1s1	41.22	-7.12	814	-618	321.1	-199.7	12.8	-2.1
aflow30a	27.11	-19.25	127	-281	7.8	-31.4	0.6	-0.7
aflow40b	23.62	-8.03	195	-233	89.2	-97.4	3.9	-3.3
arki001	6.72	4.32	158	2	5.0	0.1	0.8	0.0
atlanta-ip	0.17	0.00	58	-2	2899.4	-601.2	579.9	-3.5
bc1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell3a	60.20	0.00	13	-2	0.0	0.0	0.0	0.0
bell5	19.21	-26.42	26	-12	0.0	-0.1	0.0	0.0
bienst1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bienst2	0.00	0.00	0	0	0.0	0.0	0.0	0.0
binkar10_1	55.01	0.00	52	0	0.3	-0.1	0.1	0.0
blend2	11.50	0.00	20	0	0.4	0.0	0.0	0.0
dano3_4	0.00	0.00	0	0	3.2	-0.1	3.2	-0.1
dano3_5	0.00	0.00	0	0	4.6	0.1	4.6	0.1
dano3mip	0.00	0.00	0	-1	25.6	-26.7	25.6	-0.6
danoint	0.57	0.06	19	5	1.0	0.0	0.2	0.0
dcmulti	20.36	-9.71	96	-19	0.7	0.0	0.1	0.0
egout	42.56	-48.24	33	-36	0.1	-0.1	0.0	0.0
fiber	88.69	0.00	58	0	0.2	0.0	0.0	0.0
fixnet6	66.35	-7.40	830	-397	27.4	-8.2	0.4	0.0
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	19	-4	0.0	0.0	0.0	0.0
gesa2	72.17	-2.43	148	-13	1.2	-0.1	0.2	0.0
gesa2-o	43.65	-25.48	213	-110	2.0	-2.9	0.2	-0.1
gesa3	58.08	0.17	113	-1	2.0	0.2	0.3	0.0
gesa3_o	60.84	-0.31	176	-8	2.7	0.0	0.3	0.0
gt2	48.50	0.00	18	0	0.0	0.0	0.0	0.0
harp2	11.11	0.00	3	0	0.0	0.0	0.0	0.0
khb05250	4.70	0.00	1	0	0.0	0.0	0.0	0.0
lseu	46.23	0.00	29	0	0.0	0.0	0.0	0.0
mitre	10.78	0.00	819	0	37.5	0.0	6.2	-0.1
mkc	13.37	0.00	171	0	19.9	0.1	1.1	0.0
mod008	40.41	0.00	13	0	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.2	0.0	0.1	0.0
mod011	27.63	-61.00	385	-2989	25.9	-439.3	0.8	-6.8
modglob	26.31	-0.69	152	43	0.2	0.1	0.0	0.0
momentum1	0.00	0.00	4	-7	236.8	-80.2	79.0	-0.2
momentum2	0.01	-0.01	25	9	907.3	174.8	151.2	4.7
msc98-ip	0.36	-0.61	203	-175	779.0	-77.4	111.3	4.3
neos1	0.00	0.00	186	0	0.4	0.0	0.1	0.0
neos2	1.84	-7.64	85	-415	23.7	-46.3	1.2	-1.1
neos3	1.11	-9.28	51	-689	15.1	-149.5	1.4	-2.3
neos616206	0.00	-0.14	95	17	0.4	0.0	0.1	0.0
neos632659	60.75	6.37	371	-178	0.9	-0.6	0.0	-0.1
neos7	69.24	-2.48	330	-143	8.2	-12.4	0.6	-0.2
neos8	4.17	0.00	5	0	62.5	2.6	31.2	1.2
neos14	13.77	-54.08	154	-484	12.9	-8.0	0.2	-0.2
neos15	22.70	-48.94	235	-929	11.6	-28.7	0.2	-0.6
neos16	9.52	0.00	139	0	0.4	0.0	0.1	0.0
neos22	94.20	0.00	30	0	2.4	-0.3	0.8	-0.1
neos23	0.00	-9.49	0	-291	0.1	-2.5	0.1	-0.1
net12	2.67	-0.25	129	-38	811.7	-118.3	54.1	-0.6
nsrand-ipx	11.71	0.00	61	0	8.3	0.0	1.4	0.0
p0033	63.37	0.00	22	0	0.0	0.0	0.0	0.0
p0282	92.80	0.00	124	0	0.3	0.0	0.0	0.0
p0548	88.96	0.00	125	0	0.2	0.1	0.0	0.0
p2756	72.98	0.00	242	0	0.9	0.0	0.1	0.0
pp08a	89.64	-4.34	191	-18	0.3	-0.3	0.0	0.0
pp08aCUTS	89.91	0.00	116	1	0.3	0.0	0.0	0.0
prod1	0.85	0.00	97	0	0.7	0.0	0.1	0.0
qnet1	71.96	0.00	70	0	0.3	0.0	0.0	0.0
qnet1_o	88.98	0.00	99	0	0.5	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	46.53	1.68	87	14	1.1	0.6	0.0	0.0
ran12x21	35.09	0.60	66	17	0.6	0.3	0.0	0.0
ran13x13	27.09	-0.01	28	-3	0.1	0.0	0.0	0.0
ran14x18_1	33.65	0.56	76	1	0.5	-0.1	0.0	0.0
ran8x32	42.37	0.11	96	1	0.7	-0.2	0.0	-0.1
rentacar	0.00	0.00	0	0	0.0	0.0	0.0	0.0
rgn	98.80	3.03	107	-113	0.3	-0.6	0.0	0.0
roll3000	57.34	0.00	266	0	56.3	1.5	3.5	0.1
set1ch	7.76	-91.50	55	-212	0.5	-0.1	0.0	-0.1
sp97ar	0.75	0.00	8	0	12.0	-0.1	6.0	0.0
swath	0.00	0.00	0	0	0.2	0.0	0.2	0.0
timtab1	22.42	-8.39	223	-100	0.8	-0.7	0.1	0.0
timtab2	13.31	-5.06	432	-272	5.3	-1.9	0.3	-0.2
tr12-30	72.62	-20.98	1232	-188	48.3	-13.9	0.9	-0.9
vpm1	100.00	0.00	44	0	0.0	0.0	0.0	0.0
vpm2	74.95	0.80	182	-15	0.5	-0.1	0.0	0.0
Total	2631.61	-461.56	10854	-8886	6490.2	-1769.3	1086.0	-13.3
Geom. Mean	13.27	-3.02	43	-16	3.7	-0.8	1.7	-0.2

Table B.8: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Bound substitution heuristic.* Use Criterium S2 in the second step. (△ with respect to the default algorithm)

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
a1c1s1	6.78	-41.56	193	-1239	156.7	-364.1	4.7	-10.2
aflow30a	44.11	-2.25	320	-88	23.7	-15.5	1.0	-0.3
aflow40b	35.83	4.18	463	35	261.8	75.2	7.7	0.5
arki001	2.56	0.16	160	4	5.0	0.1	0.8	0.0
atlanta-ip	0.17	0.00	60	0	3532.0	31.4	588.7	5.3
bc1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell3a	60.20	0.00	15	0	0.0	0.0	0.0	0.0
bell5	45.63	0.00	37	-1	0.1	0.0	0.0	0.0
bienst1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bienst2	0.00	0.00	0	0	0.0	0.0	0.0	0.0
binkar10_1	55.01	0.00	52	0	0.3	-0.1	0.1	0.0
blend2	11.50	0.00	20	0	0.4	0.0	0.0	0.0
dano3_4	0.00	0.00	0	0	3.3	0.0	3.3	0.0
dano3_5	0.00	0.00	0	0	4.5	0.0	4.5	0.0
dano3mip	0.00	0.00	0	-1	26.2	-26.1	26.2	0.0
danoint	0.42	-0.09	14	0	0.9	-0.1	0.2	0.0
dcmulti	43.74	13.67	144	29	0.8	0.1	0.1	0.0
egout	78.12	-12.68	45	-24	0.1	-0.1	0.0	0.0
fiber	88.69	0.00	58	0	0.2	0.0	0.0	0.0
fixnet6	37.77	-35.98	317	-910	8.9	-26.7	0.2	-0.2
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	22	-1	0.1	0.1	0.0	0.0
gesa2	96.77	22.17	224	63	3.2	1.9	0.2	0.0
gesa2-o	69.47	0.34	267	-56	2.9	-2.0	0.2	-0.1
gesa3	57.41	-0.50	116	2	1.9	0.1	0.3	0.0
gesa3_o	59.05	-2.10	159	-25	2.4	-0.3	0.3	0.0
gt2	48.50	0.00	18	0	0.0	0.0	0.0	0.0
harp2	11.11	0.00	3	0	0.0	0.0	0.0	0.0
khb05250	4.70	0.00	1	0	0.0	0.0	0.0	0.0
lseu	46.23	0.00	29	0	0.0	0.0	0.0	0.0
mitre	10.78	0.00	819	0	37.5	0.0	6.2	-0.1
mkc	13.37	0.00	171	0	19.6	-0.2	1.1	0.0
mod008	40.41	0.00	13	0	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.2	0.0	0.1	0.0
mod011	60.00	-28.63	938	-2436	75.0	-390.2	1.7	-5.9
modglob	31.85	4.85	112	3	0.2	0.1	0.0	0.0
momentum1	0.00	0.00	10	-1	314.0	-3.0	78.5	-0.7
momentum2	0.02	0.00	16	0	745.8	13.3	149.2	2.7
msc98-ip	0.97	0.00	378	0	855.2	-1.2	106.9	-0.1
neos1	0.00	0.00	186	0	0.4	0.0	0.1	0.0
neos2	4.80	-4.68	81	-419	15.0	-55.0	0.9	-1.4
neos3	5.46	-4.93	194	-546	51.6	-113.0	1.8	-1.9
neos616206	0.14	0.00	78	0	0.4	0.0	0.1	0.0
neos632659	14.02	-40.36	71	-478	0.1	-1.4	0.0	-0.1
neos7	67.02	-4.70	425	-48	14.5	-6.1	0.5	-0.3
neos8	4.17	0.00	5	0	59.6	-0.3	29.8	-0.2
neos14	64.96	-2.89	473	-165	7.7	-13.2	0.4	0.0
neos15	73.19	1.55	1240	76	45.1	4.8	0.8	0.0
neos16	9.52	0.00	139	0	0.5	0.1	0.1	0.0
neos22	94.20	0.00	30	0	2.5	-0.2	0.8	-0.1
neos23	9.21	-0.28	301	10	2.1	-0.5	0.1	-0.1
net12	2.67	-0.25	116	-51	564.8	-365.2	51.3	-3.4
nsrand-ipx	11.71	0.00	61	0	8.2	-0.1	1.4	0.0
p0033	63.37	0.00	22	0	0.0	0.0	0.0	0.0
p0282	92.80	0.00	124	0	0.3	0.0	0.0	0.0
p0548	88.96	0.00	125	0	0.2	0.1	0.0	0.0
p2756	72.98	0.00	242	0	0.9	0.0	0.1	0.0
pp08a	93.86	-0.12	211	2	0.6	0.0	0.0	0.0
pp08aCUTS	89.91	0.00	114	-1	0.3	0.0	0.0	0.0
prod1	0.85	0.00	97	0	0.7	0.0	0.1	0.0
qnet1	71.96	0.00	70	0	0.3	0.0	0.0	0.0
qnet1_o	88.98	0.00	99	0	0.5	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	44.85	0.00	73	0	0.6	0.1	0.0	0.0
ran12x21	34.49	0.00	49	0	0.3	0.0	0.0	0.0
ran13x13	27.10	0.00	31	0	0.1	0.0	0.0	0.0
ran14x18_1	33.09	0.00	75	0	0.6	0.0	0.0	0.0
ran8x32	42.26	0.00	95	0	1.0	0.1	0.1	0.0
rentacar	0.00	0.00	0	0	0.0	0.0	0.0	0.0
rgn	95.53	-0.24	130	-90	0.4	-0.5	0.0	0.0
roll3000	57.34	0.00	266	0	55.5	0.7	3.5	0.1
set1ch	99.26	0.00	263	-4	0.6	0.0	0.1	0.0
sp97ar	0.75	0.00	8	0	12.0	-0.1	6.0	0.0
swath	0.00	0.00	0	0	0.2	0.0	0.2	0.0
timtab1	29.62	-1.19	286	-37	1.3	-0.2	0.1	0.0
timtab2	17.96	-0.41	586	-118	5.2	-2.0	0.4	-0.1
tr12-30	93.00	-0.60	1191	-229	47.6	-14.6	1.5	-0.3
vpm1	100.00	0.00	44	0	0.0	0.0	0.0	0.0
vpm2	73.95	-0.20	185	-12	0.6	0.0	0.0	0.0
Total	2955.44	-137.73	12984	-6756	6985.2	-1274.3	1082.9	-16.4
Geom. Mean	15.23	-1.06	50	-9	3.9	-0.6	1.8	-0.1

Table B.9: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Bound substitution heuristic.* Use Criterium S4 in the second step. (△ with respect to the default algorithm)

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
a1c1s1	47.77	-0.57	1300	-132	693.6	172.8	22.4	7.5
aflow30a	44.57	-1.79	380	-28	54.4	15.2	2.2	0.9
aflow40b	32.17	0.52	397	-31	319.2	132.6	12.3	5.1
arki001	2.56	0.16	158	2	7.8	2.9	1.3	0.5
atlanta-ip	0.17	0.00	60	0	5854.8	2354.2	975.8	392.4
bc1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell3a	60.20	0.00	15	0	0.0	0.0	0.0	0.0
bell5	46.72	1.09	41	3	0.1	0.0	0.0	0.0
bienst1	0.00	0.00	0	0	0.1	0.1	0.1	0.1
bienst2	0.00	0.00	0	0	0.1	0.1	0.1	0.1
binkar10_1	55.01	0.00	52	0	0.5	0.1	0.1	0.0
blend2	11.12	-0.38	25	5	1.3	0.9	0.1	0.1
dano3_4	0.00	0.00	0	0	4.9	1.6	4.9	1.6
dano3_5	0.00	0.00	0	0	6.8	2.3	6.8	2.3
dano3mip	0.00	0.00	1	0	85.9	33.6	42.9	16.7
danoint	0.51	0.00	17	3	1.4	0.4	0.2	0.0
dcmulti	29.98	-0.09	111	-4	1.2	0.5	0.1	0.0
egout	90.08	-0.72	71	2	0.3	0.1	0.0	0.0
fiber	88.69	0.00	56	-2	0.3	0.1	0.0	0.0
fixnet6	72.83	-0.92	1081	-146	46.4	10.8	0.6	0.2
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	26	3	0.1	0.1	0.0	0.0
gesa2	74.60	0.00	162	1	1.9	0.6	0.2	0.0
gesa2-o	79.19	10.06	341	18	7.7	2.8	0.5	0.2
gesa3	57.96	0.05	111	-3	2.2	0.4	0.3	0.0
gesa3_o	60.75	-0.40	179	-5	4.2	1.5	0.5	0.2
gt2	48.50	0.00	20	2	0.0	0.0	0.0	0.0
harp2	11.11	0.00	3	0	0.0	0.0	0.0	0.0
khb05250	4.70	0.00	1	0	0.0	0.0	0.0	0.0
lseu	46.23	0.00	29	0	0.0	0.0	0.0	0.0
mitre	11.92	1.14	785	-34	41.9	4.4	7.0	0.7
mkc	10.64	-2.73	157	-14	25.0	5.2	1.5	0.4
mod008	40.41	0.00	14	1	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.3	0.1	0.1	0.0
mod011	91.29	2.66	4272	898	1185.5	720.3	14.8	7.2
modglob	19.63	-7.37	91	-18	0.1	0.0	0.0	0.0
momentum1	0.00	0.00	11	0	688.3	371.3	137.7	58.5
momentum2	0.02	0.00	16	0	1321.3	588.8	264.3	117.8
msc98-ip	0.99	0.02	408	30	1439.4	583.0	143.9	36.9
neos1	0.00	0.00	184	-2	0.4	0.0	0.1	0.0
neos2	12.04	2.56	713	213	200.2	130.2	4.4	2.1
neos3	12.07	1.68	1058	318	476.1	311.5	6.2	2.5
neos616206	0.14	0.00	78	0	0.5	0.1	0.1	0.0
neos632659	0.00	-54.38	128	-421	0.2	-1.3	0.0	-0.1
neos7	71.43	-0.29	541	68	36.0	15.4	1.3	0.5
neos8	4.17	0.00	5	0	86.0	26.1	43.0	13.0
neos14	69.77	1.92	659	21	25.4	4.5	0.7	0.3
neos15	72.69	1.05	1190	26	58.1	17.8	1.4	0.6
neos16	9.52	0.00	97	-42	1.2	0.8	0.2	0.1
neos22	94.20	0.00	30	0	3.7	1.0	1.2	0.3
neos23	0.00	-9.49	175	-116	0.8	-1.8	0.1	-0.1
net12	2.88	-0.04	165	-2	944.2	14.2	72.6	17.9
nsrand-ipx	12.03	0.32	66	5	11.3	3.0	1.9	0.5
p0033	70.09	6.72	36	14	0.0	0.0	0.0	0.0
p0282	92.80	0.00	132	8	0.4	0.1	0.0	0.0
p0548	88.99	0.03	140	15	0.2	0.1	0.0	0.0
p2756	72.98	0.00	251	9	1.3	0.4	0.1	0.0
pp08a	94.26	0.28	211	2	0.8	0.2	0.0	0.0
pp08aCUTS	89.91	0.00	115	0	0.5	0.2	0.0	0.0
prod1	0.85	0.00	97	0	1.2	0.5	0.2	0.1
qnet1	75.08	3.12	82	12	0.6	0.3	0.1	0.1
qnet1_o	88.97	-0.01	98	-1	0.7	0.2	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	44.85	0.00	72	-1	0.9	0.4	0.1	0.1
ran12x21	34.49	0.00	48	-1	0.5	0.2	0.0	0.0
ran13x13	27.10	0.00	31	0	0.1	0.0	0.0	0.0
ran14x18_1	33.09	0.00	76	1	0.9	0.3	0.1	0.1
ran8x32	42.26	0.00	98	3	1.6	0.7	0.1	0.0
rentacar	0.00	0.00	0	0	0.0	0.0	0.0	0.0
rgn	100.00	4.23	212	-8	1.3	0.4	0.0	0.0
roll3000	57.77	0.43	277	11	140.1	85.3	7.0	3.6
set1ch	99.26	0.00	267	0	0.8	0.2	0.1	0.0
sp97ar	0.75	0.00	8	0	14.3	2.2	7.1	1.1
swath	0.00	0.00	0	0	0.3	0.1	0.3	0.1
timtab1	29.46	-1.35	276	-47	1.6	0.1	0.2	0.1
timtab2	18.49	0.12	773	69	12.1	4.9	0.8	0.3
tr12-30	93.32	-0.28	1243	-177	80.1	17.9	2.8	1.0
vpm1	100.00	0.00	36	-8	0.0	0.0	0.0	0.0
vpm2	73.91	-0.24	176	-21	0.8	0.2	0.1	0.1
Total	3050.27	-42.90	20239	499	13902.3	5642.8	1793.0	693.7
Geom. Mean	15.10	-1.19	58	-1	5.5	1.0	2.1	0.2

Table B.10: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Bound substitution heuristic.* Multiply the given single mixed integer constraint by minus one in addition. (△ with respect to the default algorithm)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
a1c1s1	42.00	-6.34	1160	-272	383.2	-137.6	12.0	-2.9
aflow30a	35.68	-10.68	202	-206	21.7	-17.5	0.9	-0.4
aflow40b	21.37	-10.28	173	-255	89.5	-97.1	4.5	-2.7
arki001	2.45	0.05	137	-19	4.8	-0.1	0.8	0.0
atlanta-ip	0.15	-0.02	38	-22	2255.7	-1244.9	451.1	-132.3
bc1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell3a	58.26	-1.94	14	-1	0.0	0.0	0.0	0.0
bell5	37.98	-7.65	35	-3	0.0	-0.1	0.0	0.0
bienst1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bienst2	0.00	0.00	0	0	0.0	0.0	0.0	0.0
binkar10_1	47.14	-7.87	27	-25	0.4	0.0	0.1	0.0
blend2	3.04	-8.46	8	-12	0.1	-0.3	0.0	0.0
dano3_4	0.00	0.00	0	0	3.1	-0.2	3.1	-0.2
dano3_5	0.00	0.00	0	0	4.3	-0.2	4.3	-0.2
dano3mip	0.00	0.00	1	0	50.2	-2.1	25.1	-1.1
danoint	0.41	-0.10	16	2	0.8	-0.2	0.1	-0.1
dcmulti	40.88	10.81	159	44	1.2	0.5	0.1	0.0
egout	56.12	-34.68	43	-26	0.0	-0.2	0.0	0.0
fiber	84.66	-4.03	37	-21	0.1	-0.1	0.0	0.0
fixnet6	61.76	-11.99	575	-652	26.8	-8.8	0.3	-0.1
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	27	4	0.1	0.1	0.0	0.0
gesa2	71.32	-3.28	124	-37	1.2	-0.1	0.1	-0.1
gesa2-o	44.38	-24.75	207	-116	1.8	-3.1	0.2	-0.1
gesa3	54.35	-3.56	80	-34	1.3	-0.5	0.2	-0.1
gesa3_o	55.66	-5.49	124	-60	2.8	0.1	0.3	0.0
gt2	56.43	7.93	21	3	0.0	0.0	0.0	0.0
harp2	11.11	0.00	4	1	0.0	0.0	0.0	0.0
khb05250	4.70	0.00	1	0	0.0	0.0	0.0	0.0
lseu	40.72	-5.51	14	-15	0.0	0.0	0.0	0.0
mitre	2.05	-8.73	612	-207	33.2	-4.3	5.5	-0.8
mkc	8.54	-4.83	133	-38	15.6	-4.2	1.0	-0.1
mod008	23.34	-17.07	12	-1	0.0	-0.1	0.0	0.0
mod010	18.32	0.00	2	0	0.1	-0.1	0.0	-0.1
mod011	75.51	-13.12	1588	-1786	165.8	-299.4	3.8	-3.8
modglob	19.42	-7.58	109	0	0.1	0.0	0.0	0.0
momentum1	0.00	0.00	0	-11	76.6	-240.4	76.6	-2.6
momentum2	0.03	0.01	17	1	711.5	-21.0	142.3	-4.2
msc98-ip	0.86	-0.11	274	-104	671.7	-184.7	111.9	4.9
neos1	0.00	0.00	179	-7	0.3	-0.1	0.1	0.0
neos2	10.35	0.87	467	-33	70.2	0.2	1.9	-0.4
neos3	10.81	0.42	648	-92	124.4	-40.2	3.5	-0.2
neos616206	0.14	0.00	78	0	0.3	-0.1	0.1	0.0
neos632659	0.00	-54.38	119	-430	0.1	-1.4	0.0	-0.1
neos7	62.32	-9.40	498	25	15.4	-5.2	0.7	-0.1
neos8	0.00	-4.17	0	-5	33.7	-26.2	33.7	3.7
neos14	61.49	-6.36	776	138	23.4	2.5	0.4	0.0
neos15	56.10	-15.54	760	-404	18.0	-22.3	0.6	-0.2
neos16	9.52	0.00	170	31	0.5	0.1	0.1	0.0
neos22	94.20	0.00	30	0	2.7	0.0	0.9	0.0
neos23	8.51	-0.98	242	-49	2.5	-0.1	0.1	-0.1
net12	2.91	-0.01	128	-39	721.7	-208.3	51.6	-3.1
nsrand-ipx	11.77	0.06	41	-20	6.9	-1.4	1.4	0.0
p0033	9.56	-53.81	8	-14	0.0	0.0	0.0	0.0
p0282	95.28	2.48	102	-22	0.2	-0.1	0.0	0.0
p0548	87.18	-1.78	100	-25	0.1	0.0	0.0	0.0
p2756	85.16	12.18	244	2	1.1	0.2	0.1	0.0
pp08a	89.80	-4.18	236	27	0.8	0.2	0.0	0.0
pp08aCUTS	88.55	-1.36	140	25	0.5	0.2	0.0	0.0
prod1	0.11	-0.74	3	-94	0.1	-0.6	0.0	-0.1
qnet1	22.11	-49.85	26	-44	0.2	-0.1	0.0	0.0
qnet1_o	65.14	-23.84	53	-46	0.3	-0.2	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	36.96	-7.89	34	-39	0.2	-0.3	0.0	0.0
ran12x21	31.16	-3.33	41	-8	0.2	-0.1	0.0	0.0
ran13x13	26.72	-0.38	27	-4	0.0	-0.1	0.0	0.0
ran14x18_1	29.52	-3.57	55	-20	0.2	-0.4	0.0	0.0
ran8x32	33.18	-9.08	49	-46	0.3	-0.6	0.0	-0.1
rentacar	0.00	0.00	0	0	0.0	0.0	0.0	0.0
rgn	96.04	0.27	196	-24	0.8	-0.1	0.0	0.0
roll3000	32.54	-24.80	199	-67	21.7	-33.1	2.0	-1.4
set1ch	93.42	-5.84	464	197	3.8	3.2	0.2	0.1
sp97ar	0.00	-0.75	2	-6	7.9	-4.2	4.0	-2.0
swath	0.00	0.00	0	0	0.2	0.0	0.2	0.0
timtab1	28.64	-2.17	281	-42	1.0	-0.5	0.1	0.0
timtab2	16.32	-2.05	603	-101	4.6	-2.6	0.4	-0.1
tr12-30	80.41	-13.19	2144	724	88.4	26.2	2.4	0.6
vpm1	100.00	0.00	27	-17	0.0	0.0	0.0	0.0
vpm2	67.79	-6.36	111	-86	0.2	-0.4	0.0	0.0
Total	2624.35	-468.82	15257	-4483	5677.0	-2582.5	949.2	-150.1
Geom. Mean	13.01	-3.28	42	-17	4.0	-0.5	1.8	-0.1

Table B.11: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Cut generation heuristic.* Use Procedure 2. (△ with respect to the default algorithm)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
a1c1s1	49.48	1.14	1145	-287	429.2	-91.6	12.6	-2.3
aflow30a	36.54	-9.82	284	-124	21.3	-17.9	0.9	-0.4
aflow40b	25.64	-6.01	341	-87	236.6	50.0	5.8	-1.4
arki001	8.80	6.40	137	-19	5.1	0.2	0.8	0.0
atlanta-ip	0.10	-0.07	37	-23	1814.4	-1686.2	453.6	-129.8
bc1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell3a	60.20	0.00	15	0	0.0	0.0	0.0	0.0
bell5	45.81	0.18	33	-5	0.0	-0.1	0.0	0.0
bienst1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bienst2	0.00	0.00	0	0	0.0	0.0	0.0	0.0
binkar10_1	39.33	-15.68	30	-22	0.3	-0.1	0.0	-0.1
blend2	6.88	-4.62	12	-8	0.2	-0.2	0.0	0.0
dano3_4	0.00	0.00	0	0	3.3	0.0	3.3	0.0
dano3_5	0.00	0.00	0	0	4.5	0.0	4.5	0.0
dano3mip	0.00	0.00	1	0	51.8	-0.5	25.9	-0.3
danoint	0.24	-0.27	9	-5	0.6	-0.4	0.2	0.0
dcmulti	28.98	-1.09	107	-8	0.8	0.1	0.1	0.0
egout	89.01	-1.79	75	6	0.2	0.0	0.0	0.0
fiber	87.87	-0.82	46	-12	0.1	-0.1	0.0	0.0
fixnet6	70.30	-3.45	964	-263	38.6	3.0	0.4	0.0
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	22	-1	0.1	0.1	0.0	0.0
gesa2	73.64	-0.96	177	16	1.7	0.4	0.2	0.0
gesa2-o	54.93	-14.20	280	-43	4.1	-0.8	0.3	0.0
gesa3	51.23	-6.68	71	-43	1.1	-0.7	0.2	-0.1
gesa3_o	51.79	-9.36	123	-61	1.7	-1.0	0.2	-0.1
gt2	11.96	-36.54	20	2	0.0	0.0	0.0	0.0
harp2	11.11	0.00	3	0	0.0	0.0	0.0	0.0
khb05250	4.70	0.00	1	0	0.0	0.0	0.0	0.0
lseu	43.97	-2.26	16	-13	0.0	0.0	0.0	0.0
mitre	22.22	11.44	339	-480	20.9	-16.6	2.3	-4.0
mkc	2.51	-10.86	104	-67	8.3	-11.5	0.8	-0.3
mod008	19.15	-21.26	10	-3	0.0	-0.1	0.0	0.0
mod010	18.32	0.00	2	0	0.3	0.1	0.1	0.0
mod011	90.65	2.02	2890	-484	522.4	57.2	7.5	-0.1
modglob	27.35	0.35	111	2	0.1	0.0	0.0	0.0
momentum1	0.00	0.00	15	4	460.6	143.6	76.8	-2.4
momentum2	0.00	-0.02	18	2	800.9	68.4	160.2	13.7
msc98-ip	0.75	-0.22	371	-7	764.9	-91.5	95.6	-11.4
neos1	55.22	55.22	628	442	2.4	2.0	0.1	0.0
neos2	10.05	0.57	475	-25	92.9	22.9	2.5	0.2
neos3	9.32	-1.07	586	-154	229.2	64.6	3.5	-0.2
neos616206	0.14	0.00	78	0	0.3	-0.1	0.0	-0.1
neos632659	50.65	-3.73	797	248	2.6	1.1	0.1	0.0
neos7	69.64	-2.08	388	-85	15.0	-5.6	0.6	-0.2
neos8	0.00	-4.17	6	1	51.6	-8.3	25.8	-4.2
neos14	64.61	-3.24	524	-114	11.4	-9.5	0.4	0.0
neos15	73.00	1.36	1162	-2	34.4	-5.9	0.9	0.1
neos16	9.52	0.00	109	-30	0.3	-0.1	0.1	0.0
neos22	94.20	0.00	30	0	2.6	-0.1	0.9	0.0
neos23	8.98	-0.51	276	-15	1.9	-0.7	0.1	-0.1
net12	1.17	-1.75	99	-68	529.4	-400.6	35.3	-19.4
nsrand-ipx	11.73	0.02	34	-27	6.1	-2.2	1.2	-0.2
p0033	46.48	-16.89	11	-11	0.0	0.0	0.0	0.0
p0282	94.66	1.86	98	-26	0.2	-0.1	0.0	0.0
p0548	87.27	-1.69	107	-18	0.1	0.0	0.0	0.0
p2756	59.56	-13.42	245	3	1.1	0.2	0.1	0.0
pp08a	92.01	-1.97	194	-15	0.3	-0.3	0.0	0.0
pp08aCUTS	89.97	0.06	118	3	0.2	-0.1	0.0	0.0
prod1	0.49	-0.36	66	-31	0.2	-0.5	0.1	0.0
qnet1	79.25	7.29	109	39	0.6	0.3	0.0	0.0
qnet1_o	92.04	3.06	106	7	0.6	0.1	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	43.26	-1.59	65	-8	0.4	-0.1	0.0	0.0
ran12x21	29.82	-4.67	36	-13	0.2	-0.1	0.0	0.0
ran13x13	23.68	-3.42	27	-4	0.1	0.0	0.0	0.0
ran14x18_1	31.46	-1.63	48	-27	0.2	-0.4	0.0	0.0
ran8x32	40.83	-1.43	64	-31	0.5	-0.4	0.0	-0.1
rentacar	0.00	0.00	0	0	0.0	0.0	0.0	0.0
rgn	97.66	1.89	136	-84	0.5	-0.4	0.0	0.0
roll3000	55.04	-2.30	211	-55	41.4	-13.4	2.3	-1.1
set1ch	99.34	0.08	267	0	0.6	0.0	0.1	0.0
sp97ar	0.75	0.00	8	0	12.1	0.0	6.0	0.0
swath	0.00	0.00	0	0	0.3	0.1	0.3	0.1
timtab1	29.68	-1.13	272	-51	1.1	-0.4	0.1	0.0
timtab2	18.05	-0.32	556	-148	5.8	-1.4	0.4	-0.1
tr12-30	93.11	-0.49	1200	-220	48.4	-13.8	1.6	-0.2
vpm1	100.00	0.00	42	-2	0.0	0.0	0.0	0.0
vpm2	75.64	1.49	190	-7	0.7	0.1	0.0	0.0
Total	2973.78	-119.39	17179	-2561	6290.1	-1969.4	935.5	-163.8
Geom. Mean	15.65	-0.64	51	-8	4.3	-0.2	1.8	-0.1

Table B.12: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Cut generation heuristic.* Use Procedure 3. (△ with respect to the default algorithm)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
a1c1s1	49.10	0.76	1362	-70	488.9	-31.9	16.9	2.0
aflow30a	44.81	-1.55	394	-14	29.7	-9.5	1.3	0.0
aflow40b	31.53	-0.12	421	-7	251.3	64.7	7.6	0.4
arki001	2.80	0.40	163	7	5.7	0.8	0.9	0.1
atlanta-ip	0.17	0.00	59	-1	3754.3	253.7	625.7	42.3
bc1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell3a	60.20	0.00	15	0	0.0	0.0	0.0	0.0
bell5	45.63	0.00	40	2	0.1	0.0	0.0	0.0
bienst1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bienst2	0.00	0.00	0	0	0.1	0.1	0.1	0.1
binkar10_1	56.09	1.08	53	1	0.3	-0.1	0.1	0.0
blend2	11.50	0.00	20	0	0.5	0.1	0.1	0.1
dano3_4	0.00	0.00	0	0	3.9	0.6	3.9	0.6
dano3_5	0.00	0.00	0	0	5.3	0.8	5.3	0.8
dano3mip	0.00	0.00	1	0	60.0	7.7	30.0	3.8
danoint	0.51	0.00	14	0	1.1	0.1	0.2	0.0
dcmulti	30.06	-0.01	109	-6	0.8	0.1	0.1	0.0
egout	90.60	-0.20	64	-5	0.2	0.0	0.0	0.0
fiber	88.69	0.00	62	4	0.3	0.1	0.0	0.0
fixnet6	71.68	-2.07	1143	-84	36.5	0.9	0.4	0.0
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	38	15	0.1	0.1	0.0	0.0
gesa2	71.93	-2.67	156	-5	1.5	0.2	0.2	0.0
gesa2-o	79.14	10.01	343	20	5.3	0.4	0.4	0.1
gesa3	57.91	0.00	107	-7	2.0	0.2	0.3	0.0
gesa3_o	61.15	0.00	183	-1	3.2	0.5	0.3	0.0
gt2	49.64	1.14	18	0	0.0	0.0	0.0	0.0
harp2	11.11	0.00	5	2	0.0	0.0	0.0	0.0
khb05250	4.70	0.00	1	0	0.0	0.0	0.0	0.0
lseu	44.92	-1.31	31	2	0.0	0.0	0.0	0.0
mitre	12.59	1.81	799	-20	39.6	2.1	6.6	0.3
mkc	9.26	-4.11	184	13	19.5	-0.3	1.2	0.1
mod008	31.69	-8.72	10	-3	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.2	0.0	0.1	0.0
mod011	90.63	2.00	3241	-133	513.7	48.5	8.9	1.3
modglob	26.09	-0.91	109	0	0.1	0.0	0.0	0.0
momentum1	0.00	0.00	25	14	442.8	125.8	88.6	9.4
momentum2	0.04	0.02	47	31	828.6	96.1	165.7	19.2
msc98-ip	1.04	0.07	383	5	1251.2	394.8	125.1	18.1
neos1	55.22	55.22	572	386	2.8	2.4	0.2	0.1
neos2	11.07	1.59	601	101	152.8	82.8	3.0	0.7
neos3	10.38	-0.01	734	-6	228.0	63.4	4.5	0.8
neos616206	0.14	0.00	78	0	0.4	0.0	0.1	0.0
neos632659	48.39	-5.99	687	138	2.2	0.7	0.1	0.0
neos7	82.68	10.96	553	80	25.1	4.5	0.7	-0.1
neos8	4.17	0.00	5	0	75.6	15.7	37.8	7.8
neos14	73.61	5.76	704	66	22.2	1.3	0.7	0.3
neos15	75.90	4.26	1326	162	55.2	14.9	1.1	0.3
neos16	9.52	0.00	139	0	0.5	0.1	0.1	0.0
neos22	94.20	0.00	30	0	3.2	0.5	1.1	0.2
neos23	9.44	-0.05	325	34	3.5	0.9	0.2	0.0
net12	3.10	0.18	137	-30	796.5	-133.5	61.3	6.6
nsrand-ipx	12.63	0.92	61	0	9.2	0.9	1.5	0.1
p0033	67.84	4.47	28	6	0.0	0.0	0.0	0.0
p0282	93.13	0.33	144	20	0.5	0.2	0.0	0.0
p0548	88.97	0.01	123	-2	0.2	0.1	0.0	0.0
p2756	72.98	0.00	243	1	1.1	0.2	0.1	0.0
pp08a	94.31	0.33	246	37	1.3	0.7	0.0	0.0
pp08aCUTS	89.96	0.05	119	4	0.4	0.1	0.0	0.0
prod1	0.85	0.00	98	1	0.7	0.0	0.1	0.0
qnet1	77.04	5.08	92	22	0.7	0.4	0.0	0.0
qnet1_o	85.55	-3.43	97	-2	0.4	-0.1	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	45.80	0.95	64	-9	0.5	0.0	0.0	0.0
ran12x21	33.80	-0.69	69	20	0.7	0.4	0.0	0.0
ran13x13	27.10	0.00	31	0	0.1	0.0	0.0	0.0
ran14x18_1	30.66	-2.43	61	-14	0.4	-0.2	0.0	0.0
ran8x32	42.29	0.03	98	3	1.2	0.3	0.1	0.0
rentacar	0.00	0.00	0	0	0.0	0.0	0.0	0.0
rgn	96.70	0.93	207	-13	0.9	0.0	0.0	0.0
roll3000	57.93	0.59	267	1	63.8	9.0	3.5	0.1
set1ch	99.26	0.00	270	3	0.7	0.1	0.1	0.0
sp97ar	0.75	0.00	8	0	12.8	0.7	6.4	0.4
swath	0.00	0.00	17	17	0.8	0.6	0.4	0.2
timtab1	29.58	-1.23	306	-17	1.4	-0.1	0.1	0.0
timtab2	18.30	-0.07	651	-53	7.3	0.1	0.6	0.1
tr12-30	99.55	5.95	1079	-341	33.8	-28.4	1.1	-0.7
vpm1	100.00	0.00	33	-11	0.0	0.0	0.0	0.0
vpm2	74.39	0.24	212	15	0.8	0.2	0.1	0.1
Total	3172.73	79.56	20119	379	9255.2	995.7	1215.2	115.9
Geom. Mean	17.22	0.93	65	6	4.9	0.4	1.9	0.0

Table B.13: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Cut generation heuristic.* Use the extended candidate set for the value of δ. (△ with respect to the default algorithm)

Name	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
	Value	△	Value	△	Value	△	Value	△
a1c1s1	48.08	-0.26	1521	89	588.5	67.7	9.1	-5.8
aflow30a	46.68	0.32	452	44	45.1	5.9	1.4	0.1
aflow40b	30.81	-0.84	492	64	228.0	41.4	7.6	0.4
arki001	2.40	0.00	155	-1	4.9	0.0	0.8	0.0
atlanta-ip	0.08	-0.09	34	-26	1974.2	-1526.4	394.8	-188.6
bc1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell3a	60.20	0.00	15	0	0.0	0.0	0.0	0.0
bell5	45.63	0.00	38	0	0.0	-0.1	0.0	0.0
bienst1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bienst2	0.00	0.00	0	0	0.0	0.0	0.0	0.0
binkar10_1	54.47	-0.54	47	-5	0.3	-0.1	0.0	-0.1
blend2	10.42	-1.08	30	10	0.7	0.3	0.1	0.1
dano3_4	0.00	0.00	0	0	3.1	-0.2	3.1	-0.2
dano3_5	0.00	0.00	0	0	4.2	-0.3	4.2	-0.3
dano3mip	0.00	0.00	1	0	50.7	-1.6	25.4	-0.8
danoint	0.51	0.00	14	0	0.8	-0.2	0.1	-0.1
dcmulti	44.82	14.75	170	55	0.8	0.1	0.1	0.0
egout	86.19	-4.61	78	9	0.2	0.0	0.0	0.0
fiber	88.21	-0.48	52	-6	0.2	0.0	0.0	0.0
fixnet6	74.49	0.74	1484	257	39.9	4.3	0.4	0.0
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	20	-3	0.1	0.1	0.0	0.0
gesa2	72.34	-2.26	164	3	1.1	-0.2	0.1	-0.1
gesa2-o	68.61	-0.52	311	-12	4.1	-0.8	0.2	-0.1
gesa3	57.61	-0.30	103	-11	1.4	-0.4	0.2	-0.1
gesa3_o	61.16	0.01	184	0	3.0	0.3	0.3	0.0
gt2	48.50	0.00	18	0	0.0	0.0	0.0	0.0
harp2	11.11	0.00	3	0	0.0	0.0	0.0	0.0
khb05250	4.70	0.00	1	0	0.0	0.0	0.0	0.0
lseu	46.23	0.00	29	0	0.0	0.0	0.0	0.0
mitre	10.77	-0.01	804	-15	23.6	-13.9	3.9	-2.4
mkc	9.24	-4.13	141	-30	11.6	-8.2	0.8	-0.3
mod008	39.69	-0.72	14	1	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.1	-0.1	0.0	-0.1
mod011	91.43	2.80	4414	1040	677.5	212.3	8.4	0.8
modglob	27.09	0.09	105	-4	0.1	0.0	0.0	0.0
momentum1	0.00	0.00	11	0	302.6	-14.4	75.6	-3.6
momentum2	0.01	-0.01	10	-6	439.3	-293.2	146.4	-0.1
msc98-ip	0.55	-0.42	298	-80	525.9	-330.5	87.6	-19.4
neos1	0.00	0.00	164	-22	0.2	-0.2	0.0	-0.1
neos2	8.99	-0.49	406	-94	94.0	24.0	2.1	-0.2
neos3	10.28	-0.11	591	-149	178.6	14.0	3.4	-0.3
neos616206	0.14	0.00	78	0	0.2	-0.2	0.0	-0.1
neos632659	28.04	-26.34	290	-259	0.6	-0.9	0.0	-0.1
neos7	67.46	-4.26	450	-23	18.2	-2.4	0.5	-0.3
neos8	4.17	0.00	5	0	48.5	-11.4	24.3	-5.7
neos14	67.50	-0.35	642	4	14.3	-6.6	0.4	0.0
neos15	70.30	-1.34	1031	-133	28.1	-12.2	0.8	0.0
neos16	9.52	0.00	127	-12	0.2	-0.2	0.1	0.0
neos22	94.20	0.00	30	0	2.4	-0.3	0.8	-0.1
neos23	0.00	-9.49	149	-142	0.5	-2.1	0.1	-0.1
net12	2.40	-0.52	91	-76	420.9	-509.1	35.1	-19.6
nsrand-ipx	11.71	0.00	61	0	6.5	-1.8	1.1	-0.3
p0033	68.03	4.66	36	14	0.0	0.0	0.0	0.0
p0282	93.19	0.39	118	-6	0.3	0.0	0.0	0.0
p0548	88.96	0.00	124	-1	0.1	0.0	0.0	0.0
p2756	72.58	-0.40	231	-11	0.8	-0.1	0.1	0.0
pp08a	93.84	-0.14	207	-2	0.5	-0.1	0.0	0.0
pp08aCUTS	89.91	0.00	113	-2	0.3	0.0	0.0	0.0
prod1	0.85	0.00	96	-1	0.7	0.0	0.1	0.0
qnet1	72.80	0.84	80	10	0.5	0.2	0.0	0.0
qnet1_o	82.15	-6.83	66	-33	0.2	-0.3	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	43.42	-1.43	59	-14	0.3	-0.2	0.0	0.0
ran12x21	33.81	-0.68	55	6	0.4	0.1	0.0	0.0
ran13x13	25.30	-1.80	17	-14	0.0	-0.1	0.0	0.0
ran14x18_1	33.51	0.42	73	-2	0.5	-0.1	0.0	0.0
ran8x32	42.03	-0.23	83	-12	0.8	-0.1	0.0	-0.1
rentacar	0.00	0.00	0	0	0.0	0.0	0.0	0.0
rgn	95.16	-0.61	186	-34	0.7	-0.2	0.0	0.0
roll3000	57.86	0.52	215	-51	59.4	4.6	3.1	-0.3
set1ch	99.26	0.00	263	-4	0.5	-0.1	0.1	0.0
sp97ar	0.75	0.00	8	0	6.1	-6.0	3.1	-2.9
swath	0.00	0.00	0	0	0.2	0.0	0.2	0.0
timtab1	30.35	-0.46	297	-26	1.5	0.0	0.1	0.0
timtab2	18.27	-0.10	683	-21	6.3	-0.9	0.5	0.0
tr12-30	93.84	0.24	1366	-54	60.6	-1.6	1.8	0.0
vpm1	100.00	0.00	44	0	0.0	0.0	0.0	0.0
vpm2	73.89	-0.26	181	-16	0.4	-0.2	0.0	0.0
Total	3046.82	-46.35	19933	193	5886.7	-2372.8	849.2	-250.1
Geom. Mean	15.56	-0.73	55	-4	4.2	-0.3	1.8	-0.1

Table B.14: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Cut generation heuristic.* If $f_\beta = 0$ for all $\delta \in N^*$, do not complement additional integer variables x_j, $j \in T$ lying strictly between their bounds. (△ with respect to the default algorithm)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
a1c1s1	47.47	-0.87	1407	-25	453.2	-67.6	14.6	-0.3
aflow30a	45.74	-0.62	373	-35	24.2	-15.0	1.2	-0.1
aflow40b	34.17	2.52	458	30	323.7	137.1	7.7	0.5
arki001	2.40	0.00	156	0	4.8	-0.1	0.8	0.0
atlanta-ip	0.17	0.00	61	1	3530.7	30.1	588.5	5.1
bc1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell3a	60.20	0.00	15	0	0.0	0.0	0.0	0.0
bell5	45.63	0.00	38	0	0.1	0.0	0.0	0.0
bienst1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bienst2	0.00	0.00	0	0	0.0	0.0	0.0	0.0
binkar10_1	55.56	0.55	60	8	0.4	0.0	0.1	0.0
blend2	8.25	-3.25	27	7	0.3	-0.1	0.0	0.0
dano3_4	0.00	0.00	0	0	3.3	0.0	3.3	0.0
dano3_5	0.00	0.00	0	0	4.5	0.0	4.5	0.0
dano3mip	0.00	0.00	1	0	51.8	-0.5	25.9	-0.3
danoint	0.51	0.00	14	0	0.9	-0.1	0.2	0.0
dcmulti	30.86	0.79	131	16	0.8	0.1	0.1	0.0
egout	87.06	-3.74	64	-5	0.1	-0.1	0.0	0.0
fiber	88.69	0.00	57	-1	0.2	0.0	0.0	0.0
fixnet6	74.18	0.43	1294	67	42.4	6.8	0.5	0.1
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	23	0	0.1	0.1	0.0	0.0
gesa2	74.58	-0.02	162	1	1.2	-0.1	0.2	0.0
gesa2-o	69.41	0.28	323	0	4.4	-0.5	0.3	0.0
gesa3	58.02	0.11	111	-3	1.7	-0.1	0.2	-0.1
gesa3_o	61.15	0.00	185	1	2.7	0.0	0.3	0.0
gt2	52.10	3.60	22	4	0.0	0.0	0.0	0.0
harp2	11.11	0.00	3	0	0.0	0.0	0.0	0.0
khb05250	4.70	0.00	1	0	0.0	0.0	0.0	0.0
lseu	44.42	-1.81	19	-10	0.0	0.0	0.0	0.0
mitre	12.17	1.39	819	0	37.2	-0.3	6.2	-0.1
mkc	2.92	-10.45	143	-28	10.3	-9.5	1.0	-0.1
mod008	30.75	-9.66	13	0	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.2	0.0	0.1	0.0
mod011	88.38	-0.25	3174	-200	414.5	-50.7	6.9	-0.7
modglob	30.63	3.63	134	25	0.2	0.1	0.0	0.0
momentum1	0.00	0.00	11	0	313.6	-3.4	78.4	-0.8
momentum2	0.02	0.00	16	0	732.1	-0.4	146.4	-0.1
msc98-ip	1.04	0.07	338	-40	806.2	-50.2	100.8	-6.2
neos1	0.00	0.00	186	0	0.4	0.0	0.1	0.0
neos2	11.16	1.68	553	53	158.6	88.6	2.6	0.3
neos3	9.38	-1.01	760	20	187.1	22.5	3.6	-0.1
neos616206	0.14	0.00	78	0	0.3	-0.1	0.1	0.0
neos632659	44.45	-9.93	666	117	1.7	0.2	0.1	0.0
neos7	70.88	-0.84	533	60	20.6	0.0	0.8	0.0
neos8	4.17	0.00	5	0	59.6	-0.3	29.8	-0.2
neos14	65.74	-2.11	466	-172	9.2	-11.7	0.4	0.0
neos15	71.08	-0.56	1154	-10	32.0	-8.3	0.9	0.1
neos16	9.52	0.00	139	0	0.4	0.0	0.1	0.0
neos22	94.20	0.00	30	0	2.5	-0.2	0.8	-0.1
neos23	9.49	0.00	354	63	3.2	0.6	0.2	0.0
net12	3.01	0.09	157	-10	868.9	-61.1	51.1	-3.6
nsrand-ipx	11.71	0.00	61	0	8.2	-0.1	1.4	0.0
p0033	9.56	-53.81	11	-11	0.0	0.0	0.0	0.0
p0282	93.00	0.20	140	16	0.4	0.1	0.0	0.0
p0548	89.01	0.05	137	12	0.2	0.1	0.0	0.0
p2756	72.98	0.00	241	-1	0.8	-0.1	0.1	0.0
pp08a	91.97	-2.01	166	-43	0.2	-0.4	0.0	0.0
pp08aCUTS	89.57	-0.34	113	-2	0.2	-0.1	0.0	0.0
prod1	0.88	0.03	104	7	0.6	-0.1	0.1	0.0
qnet1	68.64	-3.32	83	13	0.6	0.3	0.0	0.0
qnet1_o	88.31	-0.67	83	-16	0.2	-0.3	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	43.92	-0.93	72	-1	0.6	0.1	0.0	0.0
ran12x21	34.62	0.13	55	6	0.4	0.1	0.0	0.0
ran13x13	27.19	0.09	35	4	0.1	0.0	0.0	0.0
ran14x18_1	32.34	-0.75	74	-1	0.6	0.0	0.0	0.0
ran8x32	41.20	-1.06	86	-9	1.0	0.1	0.1	0.0
rentacar	0.00	0.00	0	0	0.0	0.0	0.0	0.0
rgn	96.78	1.01	151	-69	0.6	-0.3	0.0	0.0
roll3000	57.42	0.08	269	3	58.9	4.1	3.5	0.1
set1ch	99.26	0.00	269	2	0.5	-0.1	0.1	0.0
sp97ar	0.75	0.00	8	0	12.0	-0.1	6.0	0.0
swath	0.00	0.00	0	0	0.2	0.0	0.2	0.0
timtab1	30.95	0.14	295	-28	1.5	0.0	0.1	0.0
timtab2	18.54	0.17	672	-32	6.5	-0.7	0.5	0.0
tr12-30	93.93	0.33	1476	56	74.1	11.9	2.0	0.2
vpm1	100.00	0.00	43	-1	0.0	0.0	0.0	0.0
vpm2	74.00	-0.15	186	-11	0.5	-0.1	0.0	0.0
Total	3002.39	-90.78	19568	-172	8279.5	20.0	1092.9	-6.4
Geom. Mean	15.49	-0.80	58	-1	4.5	0.0	1.9	0.0

Table B.15: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Cut generation heuristic.* Do not try to improve the violation of the c-MIR inequality by successively complementing each integer variable x_j, $j \in T$ lying strictly between its bounds. (△ with respect to the default algorithm)

Name	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
	Value	△	Value	△	Value	△	Value	△
a1c1s1	79.84	31.50	2890	1458	1037.1	516.3	19.2	4.3
aflow30a	45.90	-0.46	370	-38	34.1	-5.1	1.2	-0.1
aflow40b	34.63	2.98	425	-3	236.1	49.5	7.4	0.2
arki001	15.26	12.86	258	102	6.6	1.7	1.1	0.3
atlanta-ip	0.17	0.00	62	2	5260.2	1759.6	876.7	293.3
bc1	32.69	32.69	12	12	0.6	0.6	0.1	0.1
bell3a	59.02	-1.18	15	0	0.0	0.0	0.0	0.0
bell5	42.02	-3.61	44	6	0.1	0.0	0.0	0.0
bienst1	6.82	6.82	138	138	2.2	2.2	0.1	0.1
bienst2	7.57	7.57	226	226	2.9	2.9	0.2	0.2
binkar10_1	56.09	1.08	53	1	0.3	-0.1	0.1	0.0
blend2	6.96	-4.54	15	-5	0.2	-0.2	0.0	0.0
dano3_4	1.46	1.46	2	2	7.4	4.1	3.7	0.4
dano3_5	1.44	1.44	5	5	14.8	10.3	5.0	0.5
dano3mip	0.01	0.01	6	5	84.4	32.1	28.1	1.9
danoint	0.92	0.41	62	48	2.1	1.1	0.3	0.1
dcmulti	52.42	22.35	196	81	2.2	1.5	0.1	0.0
egout	99.97	9.17	87	18	0.1	-0.1	0.0	0.0
fiber	88.69	0.00	62	4	0.3	0.1	0.0	0.0
fixnet6	74.36	0.61	1428	201	80.2	44.6	1.0	0.6
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	38	15	0.1	0.1	0.0	0.0
gesa2	97.84	23.24	289	128	4.5	3.2	0.3	0.1
gesa2-o	95.24	26.11	475	152	8.8	3.9	0.4	0.1
gesa3	79.24	21.33	168	54	5.3	3.5	0.5	0.2
gesa3_o	81.99	20.84	251	67	4.1	1.4	0.5	0.2
gt2	49.64	1.14	18	0	0.0	0.0	0.0	0.0
harp2	11.11	0.00	5	2	0.0	0.0	0.0	0.0
khb05250	99.65	94.95	1011	1010	25.8	25.8	0.3	0.3
lseu	44.92	-1.31	31	2	0.0	0.0	0.0	0.0
mitre	12.59	1.81	799	-20	39.6	2.1	6.6	0.3
mkc	9.26	-4.11	184	13	19.6	-0.2	1.2	0.1
mod008	31.69	-8.72	10	-3	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.2	0.0	0.1	0.0
mod011	85.99	-2.64	2105	-1269	257.4	-207.8	4.9	-2.7
modglob	30.84	3.84	143	34	0.2	0.1	0.0	0.0
momentum1	0.00	0.00	34	23	518.3	201.3	86.4	7.2
momentum2	0.04	0.02	105	89	1054.0	321.5	175.7	29.2
msc98-ip	1.00	0.03	564	186	1403.6	547.2	140.4	33.4
neos1	55.22	55.22	572	386	2.7	2.3	0.2	0.1
neos2	14.93	5.45	1057	557	502.6	432.6	5.2	2.9
neos3	3.54	-6.85	594	-146	210.3	45.7	6.2	2.5
neos616206	0.00	-0.14	114	36	0.5	0.1	0.1	0.0
neos632659	22.43	-31.95	172	-377	0.2	-1.3	0.0	-0.1
neos7	72.17	0.45	447	-26	17.1	-3.5	0.9	0.1
neos8	4.17	0.00	5	0	76.1	16.2	38.0	8.0
neos14	74.13	6.28	786	148	21.3	0.4	0.8	0.4
neos15	74.67	3.03	1211	47	44.7	4.4	1.0	0.2
neos16	9.52	0.00	139	0	0.5	0.1	0.1	0.0
neos22	100.00	5.80	27	-3	3.0	0.3	1.0	0.1
neos23	13.94	4.45	541	250	6.8	4.2	0.3	0.1
net12	3.10	0.18	137	-30	794.1	-135.9	61.1	6.4
nsrand-ipx	12.63	0.92	61	0	9.2	0.9	1.5	0.1
p0033	67.84	4.47	28	6	0.0	0.0	0.0	0.0
p0282	93.13	0.33	144	20	0.5	0.2	0.0	0.0
p0548	88.97	0.01	123	-2	0.2	0.1	0.0	0.0
p2756	72.98	0.00	243	1	1.1	0.2	0.1	0.0
pp08a	95.48	1.50	306	97	1.4	0.8	0.1	0.1
pp08aCUTS	92.47	2.56	185	70	1.1	0.8	0.1	0.1
prod1	0.85	0.00	98	1	0.7	0.0	0.1	0.0
qnet1	77.04	5.08	92	22	0.7	0.4	0.0	0.0
qnet1_o	85.55	-3.43	97	-2	0.5	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	47.78	2.93	127	54	2.8	2.3	0.2	0.2
ran12x21	56.59	22.10	214	165	5.1	4.8	0.2	0.2
ran13x13	54.56	27.46	174	143	3.1	3.0	0.1	0.1
ran14x18_1	47.08	13.99	233	158	6.3	5.7	0.2	0.2
ran8x32	57.27	15.01	118	23	2.1	1.2	0.1	0.0
rentacar	8.38	8.38	19	19	0.7	0.7	0.1	0.1
rgn	98.80	3.03	141	-79	0.6	-0.3	0.0	0.0
roll3000	57.93	0.59	267	1	72.2	17.4	4.0	0.6
set1ch	99.90	0.64	269	2	0.5	-0.1	0.1	0.0
sp97ar	0.75	0.00	8	0	13.6	1.5	6.8	0.8
swath	17.12	17.12	133	133	22.5	22.3	1.6	1.4
timtab1	60.78	29.97	905	582	9.4	7.9	0.3	0.2
timtab2	30.27	11.90	1362	658	24.9	17.7	1.0	0.5
tr12-30	99.57	5.97	1034	-386	21.6	-40.6	1.1	-0.7
vpm1	100.00	0.00	34	-10	0.0	0.0	0.0	0.0
vpm2	76.15	2.00	236	39	0.9	0.3	0.1	0.1
Total	3605.32	512.15	25043	5303	11995.6	3736.1	1494.4	395.1
Geom. Mean	22.78	6.49	112	53	6.8	2.3	1.9	0.0

Table B.16: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Resulting algorithm (slow version).* Use MAXAGGR = 5. Use Score Type 3. Use the extended candidate set for the value of δ. (△ with respect to the default algorithm)

Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
10teams	0.00	0	0.1	0.1
30:70:4_5:0_5:100	0.00	0	60.9	60.9
30:70:4_5:0_95:98	0.00	0	60.1	60.1
air03	0.00	0	0.6	0.6
air04	0.00	0	3.3	3.3
air05	0.00	0	1.8	1.8
cap6000	0.00	0	0.1	0.1
dano3_3	0.00	0	2.3	2.3
ds	0.00	0	168.7	168.7
eilD76	0.00	0	0.1	0.1
fast0507	0.00	0	26.2	26.2
glass4	0.00	215	0.1	0.0
irp	0.00	0	0.8	0.8
l152lav	0.00	0	0.1	0.1
liu	0.00	62	3.2	0.5
manna81	0.00	0	5.1	5.1
markshare1	0.00	7	0.0	0.0
markshare2	0.00	11	0.0	0.0
mas284	0.00	0	0.1	0.1
mas74	0.00	0	0.0	0.0
mas76	0.00	0	0.0	0.0
misc03	0.00	0	0.0	0.0
misc06	0.00	0	0.0	0.0
misc07	0.00	0	0.0	0.0
mkc1	0.00	23	46.3	7.7
mzzv11	0.00	2	146.1	48.7
mzzv42z	0.00	0	53.2	53.2
neos648910	0.00	496	0.7	0.1
neos9	0.00	0	21.7	21.7
neos10	0.00	10	77.7	38.9
neos11	0.00	0	13.6	13.6
neos12	0.00	0	323.2	323.2
neos13	0.00	0	21.5	21.5
neos17	0.00	0	0.1	0.1
neos18	0.00	0	1.9	1.9
neos19	0.00	18972	2628.8	525.8
neos20	0.00	45	0.7	0.2
neos21	0.00	0	0.3	0.3
noswot	0.00	51	0.0	0.0
nug08	0.00	0	0.4	0.4
nw04	0.00	0	3.2	3.2
opt1217	0.00	0	0.0	0.0
p0201	0.00	9	0.0	0.0
pk1	0.00	0	0.0	0.0
protfold	0.00	0	1.8	1.8
qap10	0.00	0	4.4	4.4
qiu	0.00	0	1.8	1.8
rout	0.00	123	0.3	0.0
seymour	0.00	0	11.6	11.6
seymour1	0.00	0	2.2	2.2
stein27	0.00	0	0.0	0.0
stein45	0.00	0	0.0	0.0
swath1	5.91	24	5.7	0.8
swath2	5.20	26	6.5	0.9
swath3	4.16	28	6.6	0.9
t1717	0.00	0	64.0	64.0
Total	15.27	20104	3778.1	1480.0
Geom. Mean	1.09	2	4.1	3.3

Table B.17: Computational results for the cutting plane separator for the class of c-MIR inequalities on the remaining test set. *Resulting algorithm (slow version).* Use MAXAGGR = 5. Use Score Type 3. Use the extended candidate set for the value of δ.

Name	Gap Closed % Value	Gap Closed % △	Cuts Value	Cuts △	Sepa Time Value	Sepa Time △	Average Sepa Time Value	Average Sepa Time △
a1c1s1	53.53	-26.31	1069	-1821	13.5	-1023.6	0.9	-18.3
aflow30a	42.84	-3.06	325	-45	10.4	-23.7	0.7	-0.5
aflow40b	27.75	-6.88	287	-138	50.9	-185.2	3.4	-4.0
arki001	16.01	0.75	246	-12	5.1	-1.5	0.9	-0.2
atlanta-ip	0.16	-0.01	16	-46	59.0	-5201.2	11.8	-864.9
bc1	32.69	0.00	12	0	0.6	0.0	0.1	0.0
bell3a	59.02	0.00	15	0	0.0	0.0	0.0	0.0
bell5	42.02	0.00	44	0	0.1	0.0	0.0	0.0
bienst1	6.82	0.00	138	0	2.1	-0.1	0.1	0.0
bienst2	7.57	0.00	222	-4	2.9	0.0	0.2	0.0
binkar10_1	56.09	0.00	53	0	0.3	0.0	0.1	0.0
blend2	6.96	0.00	15	0	0.2	0.0	0.0	0.0
dano3_4	1.46	0.00	2	0	2.2	-5.2	1.1	-2.6
dano3_5	1.47	0.03	7	2	6.5	-8.3	1.6	-3.4
dano3mip	0.01	0.00	4	-2	7.4	-77.0	2.5	-25.6
danoint	0.83	-0.09	59	-3	1.7	-0.4	0.3	0.0
dcmulti	51.90	-0.52	174	-22	1.4	-0.8	0.1	0.0
egout	85.52	-14.45	61	-26	0.1	0.0	0.0	0.0
fiber	88.68	-0.01	61	-1	0.3	0.0	0.0	0.0
fixnet6	45.53	-28.83	275	-1153	1.9	-78.3	0.1	-0.9
flugpl	2.01	0.00	2	0	0.0	0.0	0.0	0.0
gen	100.00	0.00	38	0	0.1	0.0	0.0	0.0
gesa2	97.84	0.00	287	-2	4.5	0.0	0.3	0.0
gesa2-o	93.89	-1.35	421	-54	5.7	-3.1	0.4	0.0
gesa3	79.21	-0.03	165	-3	4.1	-1.2	0.5	0.0
gesa3_o	80.92	-1.07	219	-32	3.8	-0.3	0.5	0.0
gt2	49.64	0.00	18	0	0.0	0.0	0.0	0.0
harp2	11.11	0.00	5	0	0.0	0.0	0.0	0.0
khb05250	32.53	-67.12	135	-876	0.6	-25.2	0.0	-0.3
lseu	44.92	0.00	31	0	0.0	0.0	0.0	0.0
mitre	4.99	-7.60	529	-270	15.1	-24.5	2.5	-4.1
mkc	9.26	0.00	184	0	16.7	-2.9	1.1	-0.1
mod008	31.69	0.00	10	0	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.2	0.0	0.1	0.0
mod011	60.34	-25.65	500	-1605	10.6	-246.8	0.7	-4.2
modglob	30.84	0.00	143	0	0.2	0.0	0.0	0.0
momentum1	0.00	0.00	1	-33	17.4	-500.9	8.7	-77.7
momentum2	0.03	-0.01	37	-68	33.5	-1020.5	5.6	-170.1
msc98-ip	0.36	-0.64	76	-488	47.5	-1356.1	7.9	-132.5
neos1	55.04	-0.18	554	-18	2.2	-0.5	0.1	-0.1
neos2	3.61	-11.32	253	-804	43.2	-459.4	2.9	-2.3
neos3	2.25	-1.29	313	-281	72.5	-137.8	4.8	-1.4
neos616206	0.00	0.00	114	0	0.5	0.0	0.1	0.0
neos632659	22.43	0.00	172	0	0.2	0.0	0.0	0.0
neos7	70.64	-1.53	421	-26	12.7	-4.4	0.8	-0.1
neos8	4.17	0.00	5	0	3.4	-72.7	1.7	-36.3
neos14	73.03	-1.10	691	-95	9.4	-11.9	0.6	-0.2
neos15	69.68	-4.99	765	-446	9.6	-35.1	0.6	-0.4
neos16	9.52	0.00	128	-11	0.2	-0.3	0.1	0.0
neos22	100.00	0.00	27	0	3.0	0.0	1.0	0.0
neos23	13.58	-0.36	446	-95	3.6	-3.2	0.2	-0.1
net12	2.62	-0.48	153	16	92.2	-701.9	6.1	-55.0
nsrand-ipx	12.63	0.00	61	0	8.3	-0.9	1.4	-0.1
p0033	67.84	0.00	28	0	0.0	0.0	0.0	0.0
p0282	92.93	-0.20	115	-29	0.2	-0.3	0.0	0.0
p0548	88.92	-0.05	121	-2	0.2	0.0	0.0	0.0
p2756	72.99	0.01	241	-2	1.0	-0.1	0.1	0.0
pp08a	95.45	-0.03	301	-5	1.0	-0.4	0.1	0.0
pp08aCUTS	92.20	-0.27	166	-19	1.0	-0.1	0.1	0.0
prod1	0.85	0.00	98	0	0.5	-0.2	0.1	0.0
qnet1	77.04	0.00	92	0	0.7	0.0	0.0	0.0
qnet1_o	85.55	0.00	97	0	0.5	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
ran10x26	47.59	-0.19	122	-5	1.8	-1.0	0.1	-0.1
ran12x21	50.27	-6.32	168	-46	1.9	-3.2	0.1	-0.1
ran13x13	48.29	-6.27	106	-68	0.7	-2.4	0.0	-0.1
ran14x18_1	43.14	-3.94	166	-67	1.7	-4.6	0.1	-0.1
ran8x32	56.09	-1.18	102	-16	1.1	-1.0	0.1	0.0
rentacar	8.38	0.00	19	0	0.7	0.0	0.1	0.0
rgn	98.80	0.00	141	0	0.6	0.0	0.0	0.0
roll3000	56.23	-1.70	252	-15	25.1	-47.1	1.7	-2.3
set1ch	99.89	-0.01	267	-2	0.4	-0.1	0.1	0.0
sp97ar	0.75	0.00	8	0	9.1	-4.5	4.6	-2.2
swath	17.12	0.00	133	0	22.7	0.2	1.6	0.0
timtab1	52.66	-8.12	605	-300	4.0	-5.4	0.3	0.0
timtab2	31.80	1.53	957	-405	6.9	-18.0	0.5	-0.5
tr12-30	70.58	-28.99	797	-237	5.1	-16.5	0.3	-0.8
vpm1	100.00	0.00	34	0	0.0	0.0	0.0	0.0
vpm2	76.15	0.00	236	0	0.8	-0.1	0.1	0.0
Total	3345.51	-259.81	15363	-9680	675.4	-11320.2	82.8	-1411.6
Geom. Mean	20.90	-1.88	85	-27	3.1	-3.7	1.3	-0.6

Table B.18: Computational results for the cutting plane separator for the class of c-MIR inequalities on the main test set. *Resulting algorithm (fast version).* Use MAXTESTDELTA $= 10$. Select starting constraints $i \in P$ by nonincreasing value of CONSSCORE^{i}. Use MAXFAILS $= 150$, MAXCONTS $= 20$, MAXCUTS $= 100$ and MAXROUNDS $= 15$. (△ with respect to the resulting algorithm (slow version))

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
10teams	0.00	0.00	0	0	0.1	0.0	0.1	0.0
30:70:4_5:0_5:100	0.00	0.00	0	0	3.4	-57.5	3.4	-57.5
30:70:4_5:0_95:98	0.00	0.00	0	0	3.5	-56.6	3.5	-56.6
air03	0.00	0.00	0	0	0.6	0.0	0.6	0.0
air04	0.00	0.00	0	0	1.9	-1.4	1.9	-1.4
air05	0.00	0.00	0	0	1.6	-0.2	1.6	-0.2
cap6000	0.00	0.00	0	0	0.1	0.0	0.1	0.0
dano3_3	0.00	0.00	0	0	0.8	-1.5	0.8	-1.5
ds	0.00	0.00	0	0	74.4	-94.3	74.4	-94.3
eilD76	0.00	0.00	0	0	0.1	0.0	0.1	0.0
fast0507	0.00	0.00	0	0	19.7	-6.5	19.7	-6.5
glass4	0.00	0.00	215	0	0.1	0.0	0.0	0.0
irp	0.00	0.00	0	0	0.8	0.0	0.8	0.0
l152lav	0.00	0.00	0	0	0.1	0.0	0.1	0.0
liu	0.00	0.00	51	-11	0.7	-2.5	0.1	-0.4
manna81	0.00	0.00	0	0	0.8	-4.3	0.8	-4.3
markshare1	0.00	0.00	7	0	0.0	0.0	0.0	0.0
markshare2	0.00	0.00	11	0	0.0	0.0	0.0	0.0
mas284	0.00	0.00	0	0	0.0	-0.1	0.0	-0.1
mas74	0.00	0.00	0	0	0.0	0.0	0.0	0.0
mas76	0.00	0.00	0	0	0.0	0.0	0.0	0.0
misc03	0.00	0.00	0	0	0.0	0.0	0.0	0.0
misc06	0.00	0.00	0	0	0.0	0.0	0.0	0.0
misc07	0.00	0.00	0	0	0.0	0.0	0.0	0.0
mkc1	0.00	0.00	23	0	1.3	-45.0	0.2	-7.5
mzzv11	0.00	0.00	0	-2	3.0	-143.1	3.0	-45.7
mzzv42z	0.00	0.00	0	0	3.6	-49.6	3.6	-49.6
neos648910	0.00	0.00	327	-169	0.3	-0.4	0.0	-0.1
neos9	0.00	0.00	0	0	16.8	-4.9	16.8	-4.9
neos10	0.00	0.00	10	0	4.1	-73.6	2.1	-36.8
neos11	0.00	0.00	0	0	5.0	-8.6	5.0	-8.6
neos12	0.00	0.00	0	0	85.7	-237.5	85.7	-237.5
neos13	0.00	0.00	0	0	16.3	-5.2	16.3	-5.2
neos17	0.00	0.00	0	0	0.1	0.0	0.1	0.0
neos18	0.00	0.00	0	0	0.1	-1.8	0.1	-1.8
neos19	0.00	0.00	0	-18972	7.8	-2621.0	7.8	-518.0
neos20	0.00	0.00	45	0	0.4	-0.3	0.1	-0.1
neos21	0.00	0.00	0	0	0.1	-0.2	0.1	-0.2
noswot	0.00	0.00	51	0	0.0	0.0	0.0	0.0
nug08	0.00	0.00	0	0	0.1	-0.3	0.1	-0.3
nw04	0.00	0.00	0	0	3.2	0.0	3.2	0.0
opt1217	0.00	0.00	0	0	0.0	0.0	0.0	0.0
p0201	0.00	0.00	9	0	0.0	0.0	0.0	0.0
pk1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
protfold	0.00	0.00	0	0	0.4	-1.4	0.4	-1.4
qap10	0.00	0.00	0	0	0.9	-3.5	0.9	-3.5
qiu	0.00	0.00	0	0	0.1	-1.7	0.1	-1.7
rout	0.00	0.00	123	0	0.3	0.0	0.0	0.0
seymour	0.00	0.00	0	0	0.6	-11.0	0.6	-11.0
seymour1	0.00	0.00	0	0	0.4	-1.8	0.4	-1.8
stein27	0.00	0.00	0	0	0.0	0.0	0.0	0.0
stein45	0.00	0.00	0	0	0.0	0.0	0.0	0.0
swath1	5.91	0.00	24	0	5.7	0.0	0.8	0.0
swath2	5.20	0.00	26	0	5.4	-1.1	0.8	-0.1
swath3	4.16	0.00	28	0	6.6	0.0	0.9	0.0
t1717	0.00	0.00	0	0	33.6	-30.4	33.6	-30.4
Total	15.27	0.00	950	-19154	310.8	-3467.3	291.0	-1189.0
Geom. Mean	1.09	0.00	2	0	2.0	-2.1	1.8	-1.5

Table B.19: Computational results for the cutting plane separator for the class of c-MIR inequalities on the remaining test set. *Resulting algorithm (fast version).* Use MAXTESTDELTA = 10. Select starting constraints $i \in P$ by nonincreasing value of CONSSCORE^i. Use MAXFAILS = 150, MAXCONTS = 20, MAXCUTS = 100 and MAXROUNDS = 15. (△ with respect to the resulting algorithm (slow version))

B.2 Cutting Plane Separator for the 0-1 Knapsack Problem

Name	Type	Conss	Vars	z_{LP}	z_{MIP}
cnr_dual_mip1	MIP	52170	19370	58802732.7	59803578.3
dfn3free	BIP	70668	137581	54345.2812	58019.386
dfn3orig	BIP	38134	37849	59870.6949	67143.015
dfn-stop-1	BIP	817	6116	113006094	119265705
dfn-stop-2	BIP	817	6088	101183673	102475418
ep1a	BIP	382	10255	23865985.2	33211910
ep5b	BIP	382	10255	44966343.5	48013065
rlp2	BMIP	68	451	10.2110412	19
rococoC11-010100	IP	4010	12321	8773.20654	25586
rococoC11-011100	IP	2367	6491	9024.20541	22906
tasncp285	IP	43272	12310	16074786	18594200
umts	MIP	4465	2947	29129565.2	30124085
atlanta-ip	MIP	21732	48738	81.2455967	95.0095497
cap6000	BIP	2176	6000	-2451537.33	-2451377
fiber	BMIP	363	1298	198107.358	405935.18
gen	MIP	780	870	112271.463	112313.363
harp2	BIP	112	2993	-74325169.3	-73899597
lseu	BIP	28	89	947.957237	1120
mitre	BIP	2054	10724	114782.467	115155
mkc	BMIP	3411	5325	-611.85	-563.212
mod008	BIP	6	319	290.931073	307
mod010	BIP	146	2655	6532.08333	6548
nsrand-ipx	BMIP	735	6621	49667.8923	51520
p0033	BIP	16	33	2828.33136	3089
p0282	BIP	241	282	180000.3	258411
p0548	BIP	176	548	4790.57713	8691
p2756	BIP	755	2756	2701.14437	3124
roll3000	MIP	2295	1166	11097.2754	12899
sp97ar	BIP	1761	14101	652560391	663164724

Table B.20: Summary of the main test set for the cutting plane separator for the 0-1 knapsack problem.

Name	Type	Cons	Vars	z_{LP}	z_{MIP}
tkat3	IP	14300	12552	64752921.3	66845312.1
tkatTV5	IP	2580	2332	28045750.6	28117644.2
10teams	BMIP	230	2025	917	924
a1c1s1	BMIP	3312	3648	997.529583	11566.5904
30:70:4_5:0_5:100	BMIP	12050	10772	8.1	9
30:70:4_5:0_95:98	BMIP	12471	10990	11.5	12
aflow30a	BMIP	479	842	983.167425	1158
aflow40b	BMIP	1442	2728	1005.66482	1168
air03	BIP	124	10757	338864.25	340160
air04	BIP	823	8904	55535.4364	56137
air05	BIP	426	7195	25877.6093	26374
arki001	MIP	1048	1388	7579621.83	7580814.51
bc1	BMIP	1913	1751	2.18877397	3.33836255
bell3a	MIP	123	133	866171.733	878430.316
bell5	MIP	91	104	8908552.45	8966406.49
bienst1	BMIP	576	505	11.7241379	46.75
bienst2	BMIP	576	505	11.7241379	54.6
binkar10_1	BMIP	1026	2298	6637.18803	6742.20002
blend2	MIP	274	353	6.91567511	7.598985
dano3_3	BMIP	3202	13873	576.23162	576.344633
dano3_4	BMIP	3202	13873	576.23162	576.435225
dano3_5	BMIP	3202	13873	576.23162	576.924916
dano3mip	BMIP	3202	13873	576.23162	705.941176
danoint	BMIP	664	521	62.6372804	65.67
dcmulti	BMIP	290	548	184466.891	188182
ds	BIP	656	67732	57.2347263	468.645
egout	BMIP	98	141	511.61784	568.1007
eilD76	BIP	75	1898	680.538997	885.411847
fast0507	BIP	507	63009	172.145567	174
fixnet6	BMIP	478	878	3192.042	3983
flugpl	MIP	18	18	1167185.73	1201500
gesa2	MIP	1392	1224	25492512.1	25779856.4
gesa2-o	MIP	1248	1224	25476489.7	25779856.4
gesa3	MIP	1368	1152	27846437.5	27991042.6
gesa3_o	MIP	1224	1152	27833632.5	27991042.6
glass4	BMIP	396	322	800002400	1.6000134e+09
gt2	IP	29	188	20146.7613	21166
irp	BIP	39	20315	12123.5302	12159.4928
khb05250	BMIP	101	1350	95919464	106940226
l152lav	BIP	97	1989	4656.36364	4722
liu	BMIP	2178	1156	560	1146
manna81	IP	6480	3321	-13297	-13164
markshare1	BMIP	6	62	0	1
markshare2	BMIP	7	74	0	1
mas284	BMIP	68	151	86195.863	91405.7237
mas74	BMIP	13	151	10482.7953	11801.1857
mas76	BMIP	12	151	38893.9036	40005.0541
misc03	BMIP	96	160	1910	3360
misc06	BMIP	820	1808	12841.6894	12850.8607
misc07	BMIP	212	260	1415	2810
mkc1	BMIP	3411	5325	-611.85	-607.207
mod011	BMIP	4480	10958	-62081950.3	-54558535
modglob	BMIP	291	422	20430947.6	20740508
momentum1	BMIP	42680	5174	82424.4594	109143.493
momentum2	MIP	24237	3732	10696.1116	12314.2196
msc98-ip	MIP	15850	21143	19520966.2	23271298
mzzv11	IP	9499	10240	-22944.9875	-21718
mzzv42z	IP	10460	11717	-21622.9985	-20540
neos1	BIP	5020	2112	5.6	19
neos2	BMIP	1103	2101	-4407.09724	454.864697
neos3	BMIP	1442	2747	-6158.20911	368.842751
neos616206	BMIP	534	480	787.721258	937.6
neos632659	BMIP	244	420	-119.47619	-94

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Name	Type	Conss	Vars	z_{LP}	z_{MIP}
neos648910	BMIP	1491	814	16	32
neos7	MIP	1994	1556	562977.43	721934
neos8	IP	46324	23228	-3725	-3719
neos9	BMIP	31600	81408	780	798
neos10	IP	46793	23489	-1196.33333	-1135
neos11	BMIP	2706	1220	6	9
neos12	BMIP	8317	3983	9.41161243	13
neos13	BMIP	20852	1827	-126.178378	-95.4748066
neos14	BMIP	552	792	32734.1148	74333.3433
neos15	BMIP	552	792	33463.7701	80851.6678
neos16	IP	1018	377	429	450
neos17	BMIP	486	535	0.000681498501	0.150002577
neos18	BIP	11402	3312	7	16
neos19	BMIP	34082	103789	-1611	-1499
neos20	MIP	2446	1165	-475	-434
neos21	BMIP	1085	614	2.21648352	7
neos22	BMIP	5208	3240	777191.429	779715
neos23	BMIP	1568	477	56	137
net12	BMIP	14021	14115	68.3978758	214
noswot	MIP	182	128	-43	-41
nug08	BIP	912	1632	203.5	214
nw04	BIP	36	87482	16310.6667	16862
opt1217	BMIP	64	769	-20.0213904	-16
p0201	BIP	133	201	7125	7615
pk1	BMIP	45	86	0	11
pp08a	BMIP	136	240	2748.34524	7350
pp08aCUTS	BMIP	246	240	5480.60616	7350
prod1	BMIP	208	250	-84.4158719	-56
protfold	BIP	2112	1835	-41.9574468	-23
qap10	BIP	1820	4150	332.566228	340
qiu	BMIP	1192	840	-931.638854	-132.873137
qnet1	MIP	503	1541	14274.1027	16029.6927
qnet1_o	MIP	456	1541	12557.2479	16029.6927
ran10x26	BMIP	296	520	3857.02278	4270
ran12x21	BMIP	285	504	3157.37744	3664
ran13x13	BMIP	195	338	2691.43947	3252
ran14x18_1	BMIP	284	504	3016.94435	3714
ran8x32	BMIP	296	512	4937.58453	5247
rentacar	BMIP	6803	9557	28928379.6	30356761
rgn	BMIP	24	180	48.7999986	82.1999992
rout	MIP	291	556	981.864286	1077.56
set1ch	BMIP	492	712	35118.1098	54537.75
seymour	BIP	4944	1372	403.846474	423
seymour1	BMIP	4944	1372	403.846474	410.763701
stein27	BIP	118	27	13	18
stein45	BIP	331	45	22	30
swath	BMIP	884	6805	334.496858	477.34101
swath1	BMIP	884	6805	334.496858	379.071296
swath2	BMIP	884	6805	334.496858	385.199693
swath3	BMIP	884	6805	334.496858	397.761344
t1717	BIP	551	73885	134531.021	288658
timtab1	MIP	171	397	157896.037	764772
timtab2	MIP	294	675	210652.471	1184230
tr12-30	BMIP	750	1080	18124.1745	130596
vpm1	BMIP	234	378	16.4333333	20
vpm2	BMIP	234	378	10.303297	13.75

Table B.21: Summary of the remaining test set for the cutting plane separator for the 0-1 knapsack problem.

Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
cnr_dual_mip1	1.72	5	0.1	0.0
dfn3free	26.89	149	422.2	35.2
dfn3orig	90.29	148	7.3	0.3
dfn-stop-1	52.30	575	8.9	0.3
dfn-stop-2	38.91	247	1.2	0.1
ep1a	94.89	1935	176.0	3.9
ep5b	60.48	2553	681.2	13.4
rlp2	43.11	19	0.0	0.0
rococoC11-010100	36.29	1842	0.9	0.0
rococoC11-011100	42.47	910	0.6	0.0
tasncp285	49.42	492	0.1	0.0
umts	0.00	0	0.0	0.0
atlanta-ip	0.09	12	0.1	0.0
cap6000	2.16	6	0.1	0.0
fiber	88.73	99	0.0	0.0
gen	97.02	7	0.0	0.0
harp2	20.98	50	53.0	7.6
lseu	33.11	13	0.0	0.0
mitre	0.24	1156	0.2	0.0
mkc	0.76	91	0.6	0.1
mod008	15.71	27	0.1	0.0
mod010	18.32	3	0.0	0.0
nsrand-ipx	11.96	108	2.0	0.2
p0033	28.89	9	0.0	0.0
p0282	95.68	201	0.0	0.0
p0548	85.78	137	0.0	0.0
p2756	85.67	332	0.1	0.0
roll3000	0.06	5	0.0	0.0
sp97ar	0.77	9	1.4	0.7
Total	1122.67	11140	1355.9	62.0
Geom. Mean	16.31	67	2.5	1.4

Table B.22: Computational results for the separation algorithm for the class of LMCI1 on the main test set. *Default algorithm.*

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.0	0.0
dfn3free	23.05	-3.84	130	-19	2.3	-419.9	0.2	-35.0
dfn3orig	90.17	-0.12	150	2	0.8	-6.5	0.0	-0.3
dfn-stop-1	49.27	-3.03	395	-180	9.1	0.2	0.5	0.2
dfn-stop-2	38.91	0.00	247	0	1.4	0.2	0.1	0.0
ep1a	93.32	-1.57	1315	-620	3.1	-172.9	0.1	-3.8
ep5b	49.17	-11.31	1228	-1325	2.7	-678.5	0.1	-13.3
rlp2	43.11	0.00	19	0	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1842	0	0.9	0.0	0.0	0.0
rococoC11-011100	42.47	0.00	910	0	0.6	0.0	0.0	0.0
tasncp285	49.42	0.00	492	0	0.1	0.0	0.0	0.0
umts	0.00	0.00	0	0	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	0	0.1	0.0	0.0	0.0
cap6000	2.16	0.00	6	0	0.1	0.0	0.0	0.0
fiber	88.73	0.00	99	0	0.0	0.0	0.0	0.0
gen	97.02	0.00	7	0	0.0	0.0	0.0	0.0
harp2	13.30	-7.68	38	-12	0.0	-53.0	0.0	-7.6
lseu	33.11	0.00	13	0	0.0	0.0	0.0	0.0
mitre	0.24	0.00	1156	0	0.2	0.0	0.0	0.0
mkc	0.76	0.00	91	0	0.2	-0.4	0.0	-0.1
mod008	15.71	0.00	27	0	0.1	0.0	0.0	0.0
mod010	18.32	0.00	3	0	0.0	0.0	0.0	0.0
nsrand-ipx	11.96	0.00	108	0	2.0	0.0	0.3	0.1
p0033	28.89	0.00	9	0	0.0	0.0	0.0	0.0
p0282	95.68	0.00	201	0	0.0	0.0	0.0	0.0
p0548	85.78	0.00	137	0	0.0	0.0	0.0	0.0
p2756	85.67	0.00	332	0	0.1	0.0	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	0	1.6	0.2	0.8	0.1
Total	1095.13	-27.54	8986	-2154	25.3	-1330.6	2.3	-59.7
Geom. Mean	15.81	-0.50	63	-4	1.3	-1.2	1.0	-0.4

Table B.23: Computational results for the separation algorithm for the class of LMCI1 on the main test set. *Initial cover.* Solve $KP1^{BK}_{max}$ exactly using Algorithm 4.1 if nc is not greater than 1,000,000 and approximately using Algorithm 4.2 otherwise. (△ with respect to the default algorithm)

Name	Gap Closed % Value	Gap Closed % △	Cuts Value	Cuts △	Sepa Time Value	Sepa Time △	Average Sepa Time Value	Average Sepa Time △
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.0	0.0
dfn3free	1.66	-25.23	9	-140	59.6	-362.6	19.9	-15.3
dfn3orig	91.68	1.39	68	-80	4.3	-3.0	0.4	0.1
dfn-stop-1	57.85	5.55	269	-306	8.0	-0.9	0.5	0.2
dfn-stop-2	75.68	36.77	157	-90	2.0	0.8	0.2	0.1
ep1a	55.44	-39.45	870	-1065	48.1	-127.9	1.7	-2.2
ep5b	27.26	-33.22	668	-1885	39.4	-641.8	2.5	-10.9
rlp2	43.11	0.00	17	-2	0.0	0.0	0.0	0.0
rococoC11-010100	27.15	-9.14	615	-1227	0.3	-0.6	0.0	0.0
rococoC11-011100	34.75	-7.72	403	-507	0.3	-0.3	0.0	0.0
tasncp285	47.56	-1.86	390	-102	0.1	0.0	0.0	0.0
umts	0.00	0.00	2	2	0.0	0.0	0.0	0.0
atlanta-ip	0.00	-0.09	10	-2	0.1	0.0	0.0	0.0
cap6000	0.00	-2.16	6	0	0.1	0.0	0.0	0.0
fiber	88.83	0.10	99	0	0.0	0.0	0.0	0.0
gen	98.17	1.15	8	1	0.0	0.0	0.0	0.0
harp2	31.43	10.45	88	38	90.0	37.0	6.9	-0.7
lseu	33.43	0.32	11	-2	0.0	0.0	0.0	0.0
mitre	0.88	0.64	1126	-30	0.2	0.0	0.0	0.0
mkc	1.45	0.69	103	12	0.2	-0.4	0.0	-0.1
mod008	15.36	-0.35	18	-9	0.1	0.0	0.0	0.0
mod010	18.32	0.00	3	0	0.0	0.0	0.0	0.0
nsrand-ipx	12.26	0.30	105	-3	2.4	0.4	0.3	0.1
p0033	44.70	15.81	17	8	0.0	0.0	0.0	0.0
p0282	94.22	-1.46	127	-74	0.0	0.0	0.0	0.0
p0548	87.74	1.96	135	-2	0.0	0.0	0.0	0.0
p2756	84.74	-0.93	228	-104	0.0	-0.1	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	0	1.7	0.3	0.9	0.2
Total	1076.23	-46.44	5571	-5569	256.9	-1099.0	33.3	-28.7
Geom. Mean	14.50	-1.81	50	-17	2.1	-0.4	1.2	-0.2

Table B.24: Computational results for the separation algorithm for the class of LMCI1 on the main test set. *Initial cover.* Solve $KP2^{BK}_{max}$ exactly using Algorithm 4.1. (△ with respect to the default algorithm)

Name	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
	Value	△	Value	△	Value	△	Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.1	0.1
dfn3free	7.22	-19.67	18	-131	0.9	-421.3	0.1	-35.1
dfn3orig	91.47	1.18	64	-84	0.0	-7.3	0.0	-0.3
dfn-stop-1	58.86	6.56	310	-265	0.1	-8.8	0.0	-0.3
dfn-stop-2	79.03	40.12	191	-56	0.1	-1.1	0.0	-0.1
ep1a	94.91	0.02	1993	58	0.4	-175.6	0.0	-3.9
ep5b	69.16	8.68	2687	134	0.4	-680.8	0.0	-13.4
rlp2	43.11	0.00	10	-9	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1840	-2	0.4	-0.5	0.0	0.0
rococoC11-011100	42.47	0.00	905	-5	0.2	-0.4	0.0	0.0
tasncp285	48.81	-0.61	424	-68	0.1	0.0	0.0	0.0
umts	0.00	0.00	2	2	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	0	0.0	-0.1	0.0	0.0
cap6000	0.00	-2.16	6	0	0.1	0.0	0.0	0.0
fiber	88.65	-0.08	97	-2	0.0	0.0	0.0	0.0
gen	98.17	1.15	8	1	0.0	0.0	0.0	0.0
harp2	31.75	10.77	80	30	0.0	-53.0	0.0	-7.6
lseu	33.43	0.32	11	-2	0.0	0.0	0.0	0.0
mitre	0.95	0.71	1171	15	0.2	0.0	0.0	0.0
mkc	1.59	0.83	102	11	0.0	-0.6	0.0	-0.1
mod008	15.36	-0.35	18	-9	0.0	-0.1	0.0	0.0
mod010	18.32	0.00	3	0	0.0	0.0	0.0	0.0
nsrand-ipx	12.02	0.06	107	-1	2.4	0.4	0.3	0.1
p0033	44.70	15.81	17	8	0.0	0.0	0.0	0.0
p0282	93.93	-1.75	135	-66	0.0	0.0	0.0	0.0
p0548	88.04	2.26	144	7	0.0	0.0	0.0	0.0
p2756	84.97	-0.70	232	-100	0.0	-0.1	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	0	1.6	0.2	0.8	0.1
Total	1185.86	63.19	10606	-534	7.1	-1348.8	1.4	-60.6
Geom. Mean	16.42	0.11	59	-8	1.0	-1.5	1.0	-0.4

Table B.25: Computational results for the separation algorithm for the class of LMCI1 on the main test set. *Initial cover.* Solve $KP2^{BK}_{max}$ approximately using Algorithm 4.2. (△ with respect to the default algorithm)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.1	0.1
dfn3free	31.40	4.51	164	15	507.4	85.2	39.0	3.8
dfn3orig	95.21	4.92	99	-49	6.2	-1.1	0.4	0.1
dfn-stop-1	59.40	7.10	287	-288	6.9	-2.0	0.5	0.2
dfn-stop-2	79.60	40.69	208	-39	2.4	1.2	0.2	0.1
ep1a	94.91	0.02	1930	-5	214.8	38.8	4.7	0.8
ep5b	69.63	9.15	2851	298	926.9	245.7	16.6	3.2
rlp2	43.11	0.00	10	-9	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1848	6	0.8	-0.1	0.0	0.0
rococoC11-011100	42.46	-0.01	911	1	0.7	0.1	0.0	0.0
tasncp285	49.40	-0.02	470	-22	0.1	0.0	0.0	0.0
umts	0.00	0.00	0	0	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	0	0.1	0.0	0.0	0.0
cap6000	2.07	-0.09	6	0	0.1	0.0	0.0	0.0
fiber	88.65	-0.08	99	0	0.0	0.0	0.0	0.0
gen	97.02	0.00	7	0	0.0	0.0	0.0	0.0
harp2	25.40	4.42	64	14	67.7	14.7	7.5	-0.1
lseu	33.42	0.31	12	-1	0.0	0.0	0.0	0.0
mitre	0.24	0.00	1156	0	0.2	0.0	0.0	0.0
mkc	0.76	0.00	91	0	0.2	-0.4	0.0	-0.1
mod008	13.75	-1.96	20	-7	0.1	0.0	0.0	0.0
mod010	18.32	0.00	3	0	0.0	0.0	0.0	0.0
nsrand-ipx	11.96	0.00	109	1	2.0	0.0	0.2	0.0
p0033	28.89	0.00	9	0	0.0	0.0	0.0	0.0
p0282	95.80	0.12	201	0	0.0	0.0	0.0	0.0
p0548	87.04	1.26	130	-7	0.0	0.0	0.0	0.0
p2756	85.68	0.01	237	-95	0.0	-0.1	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	0	1.6	0.2	0.8	0.1
Total	1193.05	70.38	10953	-187	1738.2	382.3	70.1	8.1
Geom. Mean	17.02	0.71	62	-5	2.7	0.2	1.4	0.0

Table B.26: Computational results for the separation algorithm for the class of LMCI1 on the main test set. *Minimal cover.* Use nonincreasing a_j as the second order criterium for removing variables. (△ with respect to the default algorithm)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.0	0.0
dfn3free	7.22	-19.67	18	-131	1.3	-420.9	0.2	-35.0
dfn3orig	91.47	1.18	64	-84	0.0	-7.3	0.0	-0.3
dfn-stop-1	58.50	6.20	285	-290	0.1	-8.8	0.0	-0.3
dfn-stop-2	77.33	38.42	168	-79	0.1	-1.1	0.0	-0.1
ep1a	94.91	0.02	1993	58	0.4	-175.6	0.0	-3.9
ep5b	69.16	8.68	2687	134	0.4	-680.8	0.0	-13.4
rlp2	43.11	0.00	20	1	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1840	-2	0.4	-0.5	0.0	0.0
rococoC11-011100	42.47	0.00	905	-5	0.2	-0.4	0.0	0.0
tasncp285	48.81	-0.61	424	-68	0.1	0.0	0.0	0.0
umts	0.00	0.00	2	2	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	0	0.0	-0.1	0.0	0.0
cap6000	0.00	-2.16	6	0	0.1	0.0	0.0	0.0
fiber	88.65	-0.08	89	-10	0.0	0.0	0.0	0.0
gen	98.17	1.15	6	-1	0.0	0.0	0.0	0.0
harp2	30.00	9.02	77	27	0.0	-53.0	0.0	-7.6
lseu	33.43	0.32	10	-3	0.0	0.0	0.0	0.0
mitre	0.95	0.71	1171	15	0.2	0.0	0.0	0.0
mkc	1.56	0.80	94	3	0.0	-0.6	0.0	-0.1
mod008	15.36	-0.35	18	-9	0.0	-0.1	0.0	0.0
mod010	18.32	0.00	2	-1	0.0	0.0	0.0	0.0
nsrand-ipx	12.02	0.06	107	-1	2.4	0.4	0.3	0.1
p0033	44.70	15.81	17	8	0.0	0.0	0.0	0.0
p0282	94.30	-1.38	143	-58	0.0	0.0	0.0	0.0
p0548	87.80	2.02	139	2	0.0	0.0	0.0	0.0
p2756	84.97	-0.70	232	-100	0.0	-0.1	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	0	1.6	0.2	0.8	0.1
Total	1182.14	59.47	10548	-592	7.4	-1348.5	1.5	-60.5
Geom. Mean	16.36	0.05	58	-9	1.1	-1.4	1.0	-0.4

Table B.27: Computational results for the separation algorithm for the class of LMCI1 on the main test set. *Resulting algorithm.* Solve $KP2^{BK}_{max}$ approximately using Algorithm 4.2. Set $C_2 = \{j \in C : x^*_j = 1\}$ and $C_1 = C \backslash C_2$. Change the partition if $|C_1| = 0$. (△ with respect to the default algorithm)

Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
cnr_dual_mip1	1.72	5	0.1	0.0
dfn3free	27.01	148	254.6	21.2
dfn3orig	87.01	333	14.0	0.5
dfn-stop-1	51.07	902	22.7	0.6
dfn-stop-2	29.40	403	2.8	0.1
ep1a	94.83	1942	184.6	4.4
ep5b	67.01	2888	976.3	15.7
rlp2	43.11	27	0.0	0.0
rococoC11-010100	36.29	1842	0.8	0.0
rococoC11-011100	42.47	910	0.7	0.0
tasncp285	49.42	595	0.1	0.0
umts	0.00	0	0.0	0.0
atlanta-ip	0.09	12	0.1	0.0
cap6000	2.16	6	0.1	0.0
fiber	88.73	92	0.0	0.0
gen	98.17	6	0.0	0.0
harp2	21.19	53	49.6	6.2
lseu	37.05	11	0.0	0.0
mitre	0.24	1156	0.2	0.0
mkc	0.76	89	0.2	0.0
mod008	18.99	26	0.1	0.0
mod010	18.32	2	0.0	0.0
nsrand-ipx	12.26	109	1.9	0.2
p0033	32.64	10	0.0	0.0
p0282	95.94	178	0.0	0.0
p0548	87.48	199	0.0	0.0
p2756	85.64	430	0.1	0.0
roll3000	0.06	5	0.0	0.0
sp97ar	0.77	9	1.5	0.8
Total	1129.83	12388	1510.3	49.9
Geom. Mean	16.45	73	2.8	1.4

Table B.28: Computational results for the separation algorithm for the class of LEWI on the main test set. *Default algorithm*

Name	Gap Closed % Value	Gap Closed % △	Cuts Value	Cuts △	Sepa Time Value	Sepa Time △	Average Sepa Time Value	Average Sepa Time △
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.0	0.0
dfn3free	9.26	-17.75	43	-105	1.3	-253.3	0.1	-21.1
dfn3orig	86.09	-0.92	316	-17	0.1	-13.9	0.0	-0.5
dfn-stop-1	51.19	0.12	913	11	0.3	-22.4	0.0	-0.6
dfn-stop-2	28.59	-0.81	368	-35	0.1	-2.7	0.0	-0.1
ep1a	93.07	-1.76	1367	-575	0.2	-184.4	0.0	-4.4
ep5b	57.17	-9.84	1528	-1360	0.4	-975.9	0.0	-15.7
rlp2	43.11	0.00	17	-10	0.0	0.0	0.0	0.0
rococoC11-010100	33.93	-2.36	1006	-836	0.3	-0.5	0.0	0.0
rococoC11-011100	40.46	-2.01	605	-305	0.1	-0.6	0.0	0.0
tasncp285	48.73	-0.69	537	-58	0.1	0.0	0.0	0.0
umts	0.00	0.00	2	2	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	9	-3	0.0	-0.1	0.0	0.0
cap6000	2.16	0.00	6	0	0.1	0.0	0.0	0.0
fiber	88.14	-0.59	85	-7	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	13.93	-7.26	40	-13	0.0	-49.6	0.0	-6.2
lseu	41.30	4.25	13	2	0.0	0.0	0.0	0.0
mitre	3.35	3.11	970	-186	0.1	-0.1	0.0	0.0
mkc	1.12	0.36	93	4	0.0	-0.2	0.0	0.0
mod008	13.57	-5.42	13	-13	0.0	-0.1	0.0	0.0
mod010	18.32	0.00	2	0	0.0	0.0	0.0	0.0
nsrand-ipx	11.83	-0.43	103	-6	2.0	0.1	0.3	0.1
p0033	41.40	8.76	13	3	0.0	0.0	0.0	0.0
p0282	94.80	-1.14	115	-63	0.0	0.0	0.0	0.0
p0548	85.30	-2.18	155	-44	0.0	0.0	0.0	0.0
p2756	85.32	-0.32	422	-8	0.0	-0.1	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	10	1	2.1	0.6	0.7	-0.1
Total	1092.94	-36.89	8767	-3621	7.3	-1503.0	1.3	-48.6
Geom. Mean	16.13	-0.32	61	-12	1.1	-1.7	1.0	-0.4

Table B.29: Computational results for the separation algorithm for the class of LEWI on the main test set. *Initial cover.* Solve $KP1^{BK}_{max}$ approximately using Algorithm 4.2. (△ with respect to the default algorithm)

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.0	0.0
dfn3free	9.26	-17.75	43	-105	1.8	-252.8	0.2	-21.0
dfn3orig	86.85	-0.16	319	-14	1.1	-12.9	0.0	-0.5
dfn-stop-1	52.19	1.12	983	81	4.1	-18.6	0.1	-0.5
dfn-stop-2	29.34	-0.06	381	-22	1.6	-1.2	0.1	0.0
ep1a	93.32	-1.51	1355	-587	2.9	-181.7	0.1	-4.3
ep5b	54.04	-12.97	1441	-1447	2.6	-973.7	0.1	-15.6
rlp2	43.11	0.00	27	0	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1842	0	0.9	0.1	0.0	0.0
rococoC11-011100	42.47	0.00	910	0	0.6	-0.1	0.0	0.0
tasncp285	49.42	0.00	595	0	0.1	0.0	0.0	0.0
umts	0.00	0.00	0	0	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	0	0.1	0.0	0.0	0.0
cap6000	2.16	0.00	6	0	0.1	0.0	0.0	0.0
fiber	88.73	0.00	92	0	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	13.93	-7.26	40	-13	0.1	-49.5	0.0	-6.2
lseu	37.05	0.00	11	0	0.0	0.0	0.0	0.0
mitre	0.24	0.00	1156	0	0.2	0.0	0.0	0.0
mkc	0.76	0.00	89	0	0.5	0.3	0.1	0.1
mod008	18.99	0.00	26	0	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.0	0.0	0.0	0.0
nsrand-ipx	12.26	0.00	109	0	1.9	0.0	0.2	0.0
p0033	32.64	0.00	10	0	0.0	0.0	0.0	0.0
p0282	95.94	0.00	178	0	0.0	0.0	0.0	0.0
p0548	87.48	0.00	199	0	0.0	0.0	0.0	0.0
p2756	85.64	0.00	430	0	0.1	0.0	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	0	1.5	0.0	0.7	-0.1
Total	1091.23	-38.60	10281	-2107	20.3	-1490.0	1.8	-48.1
Geom. Mean	15.51	-0.94	67	-6	1.2	-1.6	1.0	-0.4

Table B.30: Computational results for the separation algorithm for the class of LEWI on the main test set. *Initial cover.* Solve $KP1^{BK}_{max}$ exactly using Algorithm 4.1 if nc is not greater than 1,000,000 and approximately using Algorithm 4.2 otherwise. (△ with respect to the default algorithm)

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.1	0.1
dfn3free	0.82	-26.19	20	-128	231.8	-22.8	38.6	17.4
dfn3orig	84.95	-2.06	323	-10	18.1	4.1	0.6	0.1
dfn-stop-1	50.05	-1.02	1001	99	30.4	7.7	0.6	0.0
dfn-stop-2	29.40	0.00	386	-17	3.0	0.2	0.1	0.0
ep1a	54.23	-40.60	868	-1074	47.7	-136.9	1.5	-2.9
ep5b	25.92	-41.09	722	-2166	41.4	-934.9	2.1	-13.6
rlp2	43.11	0.00	25	-2	0.0	0.0	0.0	0.0
rococoC11-010100	28.65	-7.64	665	-1177	0.3	-0.5	0.0	0.0
rococoC11-011100	34.33	-8.14	371	-539	0.2	-0.5	0.0	0.0
tasncp285	47.70	-1.72	496	-99	0.1	0.0	0.0	0.0
umts	0.00	0.00	2	2	0.0	0.0	0.0	0.0
atlanta-ip	0.00	-0.09	10	-2	0.1	0.0	0.0	0.0
cap6000	0.00	-2.16	6	0	0.1	0.0	0.0	0.0
fiber	88.02	-0.71	87	-5	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	21.49	0.30	58	5	43.1	-6.5	5.4	-0.8
lseu	42.80	5.75	15	4	0.0	0.0	0.0	0.0
mitre	1.23	0.99	1127	-29	0.2	0.0	0.0	0.0
mkc	1.45	0.69	102	13	0.1	-0.1	0.0	0.0
mod008	18.99	0.00	24	-2	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.0	0.0	0.0	0.0
nsrand-ipx	12.26	0.00	98	-11	2.3	0.4	0.3	0.1
p0033	44.70	12.06	17	7	0.0	0.0	0.0	0.0
p0282	94.23	-1.71	187	9	0.0	0.0	0.0	0.0
p0548	86.15	-1.33	185	-14	0.0	0.0	0.0	0.0
p2756	81.96	-3.68	415	-15	0.1	0.0	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	0	1.6	0.1	0.8	0.0
Total	1011.49	-118.34	7237	-5151	420.8	-1089.5	50.2	0.3
Geom. Mean	13.78	-2.67	61	-12	2.4	-0.4	1.2	-0.2

Table B.31: Computational results for the separation algorithm for the class of LEWI on the main test set. *Initial cover.* Solve KP2$^{BK}_{max}$ exactly using Algorithm 4.1. (△ with respect to the default algorithm)

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.1	0.1
dfn3free	4.92	-22.09	9	-139	0.6	-254.0	0.2	-21.0
dfn3orig	82.56	-4.45	320	-13	0.1	-13.9	0.0	-0.5
dfn-stop-1	35.15	-15.92	242	-660	0.1	-22.6	0.0	-0.6
dfn-stop-2	23.93	-5.47	198	-205	0.1	-2.7	0.0	-0.1
ep1a	85.58	-9.25	2058	116	0.4	-184.2	0.0	-4.4
ep5b	69.24	2.23	3887	999	0.7	-975.6	0.0	-15.7
rlp2	30.36	-12.75	29	2	0.0	0.0	0.0	0.0
rococoC11-010100	36.28	-0.01	1898	56	0.4	-0.4	0.0	0.0
rococoC11-011100	42.46	-0.01	906	-4	0.2	-0.5	0.0	0.0
tasncp285	41.75	-7.67	401	-194	0.0	-0.1	0.0	0.0
umts	0.02	0.02	7	7	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	7	-5	0.0	-0.1	0.0	0.0
cap6000	0.00	-2.16	3	-3	0.1	0.0	0.0	0.0
fiber	82.15	-6.58	76	-16	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	18.95	-2.24	55	2	0.0	-49.6	0.0	-6.2
lseu	42.22	5.17	14	3	0.0	0.0	0.0	0.0
mitre	6.08	5.84	869	-287	0.2	0.0	0.0	0.0
mkc	0.50	-0.26	88	-1	0.1	-0.1	0.0	0.0
mod008	22.43	3.44	23	-3	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.0	0.0	0.0	0.0
nsrand-ipx	11.21	-1.05	53	-56	1.3	-0.6	0.3	0.1
p0033	35.56	2.92	12	2	0.0	0.0	0.0	0.0
p0282	89.80	-6.14	72	-106	0.0	0.0	0.0	0.0
p0548	81.06	-6.42	121	-78	0.0	0.0	0.0	0.0
p2756	20.93	-64.71	161	-269	0.0	-0.1	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.75	-0.02	9	0	2.4	0.9	0.8	0.0
Total	982.27	-147.56	11536	-852	6.9	-1503.4	1.4	-48.5
Geom. Mean	14.67	-1.78	56	-17	1.0	-1.8	1.0	-0.4

Table B.32: Computational results for the separation algorithm for the class of LEWI on the main test set. *Initial cover.* Solve $KP2^{BK}_{max}$ approximately using Algorithm 4.2. (△ with respect to the default algorithm)

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.1	0.1
dfn3free	22.26	-4.75	122	-26	342.1	87.5	28.5	7.3
dfn3orig	0.00	-87.01	0	-333	0.4	-13.6	0.4	-0.1
dfn-stop-1	27.81	-23.26	38	-864	1.2	-21.5	0.1	-0.5
dfn-stop-2	2.07	-27.33	9	-394	0.2	-2.6	0.0	-0.1
ep1a	0.00	-94.83	0	-1942	0.0	-184.6	0.0	-4.4
ep5b	0.00	-67.01	0	-2888	0.1	-976.2	0.1	-15.6
rlp2	24.98	-18.13	98	71	0.0	0.0	0.0	0.0
rococoC11-010100	26.24	-10.05	1385	-457	0.8	0.0	0.0	0.0
rococoC11-011100	5.50	-36.97	78	-832	0.2	-0.5	0.0	0.0
tasncp285	8.52	-40.90	168	-427	0.1	0.0	0.0	0.0
umts	0.00	0.00	0	0	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	6	-6	0.1	0.0	0.0	0.0
cap6000	0.00	-2.16	0	-6	0.0	-0.1	0.0	0.0
fiber	44.07	-44.66	8	-84	0.0	0.0	0.0	0.0
gen	70.39	-27.78	3	-3	0.0	0.0	0.0	0.0
harp2	11.18	-10.01	13	-40	31.1	-18.5	6.2	0.0
lseu	37.06	0.01	9	-2	0.0	0.0	0.0	0.0
mitre	0.00	-0.24	687	-469	0.1	-0.1	0.0	0.0
mkc	0.00	-0.76	9	-80	0.1	-0.1	0.0	0.0
mod008	17.59	-1.40	20	-6	0.1	0.0	0.0	0.0
mod010	0.00	-18.32	0	-2	0.0	0.0	0.0	0.0
nsrand-ipx	5.83	-6.43	78	-31	1.6	-0.3	0.2	0.0
p0033	0.00	-32.64	4	-6	0.0	0.0	0.0	0.0
p0282	6.67	-89.27	66	-112	0.0	0.0	0.0	0.0
p0548	0.00	-87.48	0	-199	0.0	0.0	0.0	0.0
p2756	0.00	-85.64	1	-429	0.0	-0.1	0.0	0.0
roll3000	0.02	-0.04	4	-1	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	7	-2	1.4	-0.1	0.7	-0.1
Total	312.77	-817.06	2818	-9570	379.9	-1130.4	36.6	-13.3
Geom. Mean	3.79	-12.66	10	-63	1.4	-1.4	1.2	-0.2

Table B.33: Computational results for the separation algorithm for the class of LEWI on the main test set. *Partition.* Set $T_2 = \{j \in T : x_j^* = 1\}$ and $T_1 = T \backslash T_2$. Do not change the partition if $|T_1| = 0$. (△ with respect to the default algorithm)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.0	0.0
dfn3free	26.88	-0.13	145	-3	617.5	362.9	51.5	30.3
dfn3orig	87.01	0.00	331	-2	14.3	0.3	0.5	0.0
dfn-stop-1	51.86	0.79	954	52	23.2	0.5	0.6	0.0
dfn-stop-2	29.40	0.00	403	0	2.8	0.0	0.1	0.0
ep1a	94.92	0.09	1972	30	226.9	42.3	4.5	0.1
ep5b	62.63	-4.38	2547	-341	633.8	-342.5	12.4	-3.3
rlp2	43.11	0.00	27	0	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1839	-3	0.8	0.0	0.0	0.0
rococoC11-011100	42.47	0.00	896	-14	0.6	-0.1	0.0	0.0
tasncp285	49.42	0.00	595	0	0.1	0.0	0.0	0.0
umts	0.00	0.00	0	0	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	0	0.1	0.0	0.0	0.0
cap6000	2.16	0.00	6	0	0.1	0.0	0.0	0.0
fiber	88.73	0.00	92	0	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	21.19	0.00	53	0	48.0	-1.6	6.0	-0.2
lseu	37.05	0.00	11	0	0.0	0.0	0.0	0.0
mitre	0.24	0.00	1156	0	0.2	0.0	0.0	0.0
mkc	0.76	0.00	91	2	0.2	0.0	0.0	0.0
mod008	18.99	0.00	26	0	0.1	0.0	0.0	0.0
mod010	18.32	0.00	2	0	0.0	0.0	0.0	0.0
nsrand-ipx	12.26	0.00	109	0	1.8	-0.1	0.2	0.0
p0033	32.64	0.00	10	0	0.0	0.0	0.0	0.0
p0282	95.94	0.00	178	0	0.0	0.0	0.0	0.0
p0548	86.48	-1.00	197	-2	0.0	0.0	0.0	0.0
p2756	85.64	0.00	430	0	0.1	0.0	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	0	1.4	-0.1	0.7	-0.1
Total	1125.19	-4.64	12107	-281	1572.2	61.9	76.8	26.9
Geom. Mean	16.42	-0.03	73	0	2.8	0.0	1.4	0.0

Table B.34: Computational results for the separation algorithm for the class of LEWI on the main test set. *Lifting sequence.* Do not use the restriction to lift first the variable which has been removed last from the initial cover. (△ with respect to the default algorithm)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.0	0.0
dfn3free	9.02	-17.99	46	-102	1.9	-252.7	0.2	-21.0
dfn3orig	86.87	-0.14	320	-13	0.1	-13.9	0.0	-0.5
dfn-stop-1	50.95	-0.12	883	-19	0.2	-22.5	0.0	-0.6
dfn-stop-2	29.99	0.59	396	-7	0.1	-2.7	0.0	-0.1
ep1a	94.61	-0.22	1972	30	0.4	-184.2	0.0	-4.4
ep5b	60.72	-6.29	2555	-333	0.4	-975.9	0.0	-15.7
rlp2	43.11	0.00	18	-9	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1823	-19	0.4	-0.4	0.0	0.0
rococoC11-011100	42.47	0.00	907	-3	0.3	-0.4	0.0	0.0
tasncp285	48.97	-0.45	549	-46	0.1	0.0	0.0	0.0
umts	0.00	0.00	2	2	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	9	-3	0.0	-0.1	0.0	0.0
cap6000	2.16	0.00	6	0	0.1	0.0	0.0	0.0
fiber	87.95	-0.78	82	-10	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	13.93	-7.26	40	-13	0.0	-49.6	0.0	-6.2
lseu	41.69	4.64	15	4	0.0	0.0	0.0	0.0
mitre	10.91	10.67	968	-188	0.2	0.0	0.0	0.0
mkc	1.12	0.36	98	9	0.0	-0.2	0.0	0.0
mod008	9.71	-9.28	11	-15	0.0	-0.1	0.0	0.0
mod010	18.32	0.00	2	0	0.0	0.0	0.0	0.0
nsrand-ipx	11.83	-0.43	103	-6	1.9	0.0	0.2	0.0
p0033	44.93	12.29	19	9	0.0	0.0	0.0	0.0
p0282	94.76	-1.18	135	-43	0.0	0.0	0.0	0.0
p0548	85.82	-1.66	164	-35	0.0	0.0	0.0	0.0
p2756	85.36	-0.28	425	-5	0.1	0.0	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	10	1	2.2	0.7	0.7	-0.1
Total	1112.29	-17.54	11574	-814	8.5	-1501.8	1.4	-48.5
Geom. Mean	16.79	0.34	67	-6	1.1	-1.7	1.0	-0.4

Table B.35: Computational results for the separation algorithm for the class of LEWI on the main test set. *Resulting algorithm.* Solve KP1$^{BK}_{max}$ approximately using Algorithm 4.2. Do not use the restriction to lift first the variable which has been removed last from the initial cover. (△ with respect to the default algorithm)

Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
cnr_dual_mip1	1.59	7	0.1	0.0
dfn3free	26.89	149	252.2	21.0
dfn3orig	90.05	147	8.6	0.4
dfn-stop-1	48.39	433	8.3	0.4
dfn-stop-2	37.48	247	1.5	0.1
ep1a	94.86	1923	215.2	5.0
ep5b	60.48	2553	676.6	13.3
rlp2	43.11	12	0.0	0.0
rococoC11-010100	36.29	1842	0.8	0.0
rococoC11-011100	42.47	910	0.6	0.0
tasncp285	49.42	492	0.1	0.0
umts	0.00	0	0.0	0.0
atlanta-ip	0.09	12	0.1	0.0
cap6000	2.16	6	0.1	0.0
fiber	87.19	95	0.0	0.0
gen	98.17	6	0.0	0.0
harp2	20.52	45	54.1	7.7
lseu	14.77	11	0.0	0.0
mitre	0.24	1083	0.2	0.0
mkc	1.13	76	0.2	0.0
mod008	3.17	11	0.0	0.0
mod010	18.32	1	0.0	0.0
nsrand-ipx	4.76	64	0.7	0.1
p0033	28.89	11	0.0	0.0
p0282	94.05	211	0.0	0.0
p0548	82.92	121	0.0	0.0
p2756	85.67	334	0.0	0.0
roll3000	0.06	5	0.0	0.0
sp97ar	0.08	7	0.8	0.4
Total	1073.21	10814	1220.2	48.6
Geom. Mean	14.46	59	2.5	1.4

Table B.36: Computational results for the separation algorithm for the class of LMCI2 on the main test set. *Default algorithm.*

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
cnr_dual_mip1	1.59	0.00	7	0	0.1	0.0	0.0	0.0
dfn3free	23.05	-3.84	130	-19	1.7	-250.5	0.1	-20.9
dfn3orig	89.93	-0.12	149	2	0.7	-7.9	0.0	-0.4
dfn-stop-1	48.11	-0.28	427	-6	2.7	-5.6	0.1	-0.3
dfn-stop-2	37.48	0.00	247	0	1.3	-0.2	0.1	0.0
ep1a	93.32	-1.54	1315	-608	2.9	-212.3	0.1	-4.9
ep5b	49.17	-11.31	1228	-1325	2.5	-674.1	0.1	-13.2
rlp2	43.11	0.00	12	0	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1842	0	0.8	0.0	0.0	0.0
rococoC11-011100	42.47	0.00	910	0	0.5	-0.1	0.0	0.0
tasncp285	49.42	0.00	492	0	0.0	-0.1	0.0	0.0
umts	0.00	0.00	0	0	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	0	0.1	0.0	0.0	0.0
cap6000	2.16	0.00	6	0	0.1	0.0	0.0	0.0
fiber	87.19	0.00	95	0	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	12.42	-8.10	29	-16	0.0	-54.1	0.0	-7.7
lseu	14.77	0.00	11	0	0.0	0.0	0.0	0.0
mitre	0.24	0.00	1083	0	0.2	0.0	0.0	0.0
mkc	1.13	0.00	76	0	0.2	0.0	0.0	0.0
mod008	3.17	0.00	11	0	0.0	0.0	0.0	0.0
mod010	18.32	0.00	1	0	0.0	0.0	0.0	0.0
nsrand-ipx	4.76	0.00	64	0	0.7	0.0	0.1	0.0
p0033	28.89	0.00	11	0	0.0	0.0	0.0	0.0
p0282	94.05	0.00	211	0	0.0	0.0	0.0	0.0
p0548	82.92	0.00	121	0	0.0	0.0	0.0	0.0
p2756	85.67	0.00	334	0	0.1	0.1	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.08	0.00	7	0	0.6	-0.2	0.3	-0.1
Total	1048.02	-25.19	8842	-1972	15.1	-1205.1	1.1	-47.5
Geom. Mean	14.03	-0.43	56	-3	1.1	-1.4	1.0	-0.4

Table B.37: Computational results for the separation algorithm for the class of LMCI2 on the main test set. *Initial cover.* Solve $KP1^{BK}_{max}$ exactly using Algorithm 4.1 if nc is not greater than 1,000,000 and approximately using Algorithm 4.2 otherwise. (△ with respect to the default algorithm)

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
cnr_dual_mip1	1.59	0.00	7	0	0.1	0.0	0.0	0.0
dfn3free	1.66	-25.23	9	-140	35.5	-216.7	11.8	-9.2
dfn3orig	91.44	1.39	67	-80	4.7	-3.9	0.4	0.0
dfn-stop-1	53.26	4.87	165	-268	4.0	-4.3	0.3	-0.1
dfn-stop-2	71.89	34.41	121	-126	1.0	-0.5	0.1	0.0
ep1a	55.44	-39.42	870	-1053	45.5	-169.7	1.6	-3.4
ep5b	27.26	-33.22	668	-1885	37.4	-639.2	2.3	-11.0
rlp2	43.11	0.00	21	9	0.0	0.0	0.0	0.0
rococoC11-010100	27.15	-9.14	615	-1227	0.3	-0.5	0.0	0.0
rococoC11-011100	34.75	-7.72	403	-507	0.2	-0.4	0.0	0.0
tasncp285	47.56	-1.86	390	-102	0.1	0.0	0.0	0.0
umts	0.00	0.00	0	0	0.0	0.0	0.0	0.0
atlanta-ip	0.00	-0.09	10	-2	0.1	0.0	0.0	0.0
cap6000	0.00	-2.16	6	0	0.1	0.0	0.0	0.0
fiber	86.69	-0.50	85	-10	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	26.06	5.54	46	1	50.9	-3.2	7.3	-0.4
lseu	14.68	-0.09	9	-2	0.0	0.0	0.0	0.0
mitre	0.25	0.01	1067	-16	0.2	0.0	0.0	0.0
mkc	2.73	1.60	88	12	0.3	0.1	0.0	0.0
mod008	11.28	8.11	12	1	0.0	0.0	0.0	0.0
mod010	18.32	0.00	1	0	0.0	0.0	0.0	0.0
nsrand-ipx	4.83	0.07	61	-3	0.8	0.1	0.1	0.0
p0033	38.18	9.29	14	3	0.0	0.0	0.0	0.0
p0282	92.70	-1.35	116	-95	0.0	0.0	0.0	0.0
p0548	82.34	-0.58	98	-23	0.0	0.0	0.0	0.0
p2756	84.71	-0.96	228	-106	0.0	0.0	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.08	0.00	7	0	0.7	-0.1	0.3	-0.1
Total	1016.20	-57.01	5195	-5619	181.8	-1038.4	24.5	-24.1
Geom. Mean	13.49	-0.97	41	-18	1.9	-0.6	1.2	-0.2

Table B.38: Computational results for the separation algorithm for the class of LMCI2 on the main test set. *Initial cover*. Solve $KP2^{BK}_{max}$ exactly using Algorithm 4.1. (△ with respect to the default algorithm)

Name	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
	Value	△	Value	△	Value	△	Value	△
cnr_dual_mip1	1.59	0.00	7	0	0.1	0.0	0.0	0.0
dfn3free	7.22	-19.67	18	-131	0.7	-251.5	0.1	-20.9
dfn3orig	91.20	1.15	62	-85	0.0	-8.6	0.0	-0.4
dfn-stop-1	54.82	6.43	221	-212	0.1	-8.2	0.0	-0.4
dfn-stop-2	73.26	35.78	142	-105	0.1	-1.4	0.0	-0.1
ep1a	94.90	0.04	2003	80	0.3	-214.9	0.0	-5.0
ep5b	68.99	8.51	2652	99	0.4	-676.2	0.0	-13.3
rlp2	43.11	0.00	22	10	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1840	-2	0.3	-0.5	0.0	0.0
rococoC11-011100	42.47	0.00	905	-5	0.2	-0.4	0.0	0.0
tasncp285	48.81	-0.61	424	-68	0.1	0.0	0.0	0.0
umts	0.00	0.00	0	0	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	0	0.0	-0.1	0.0	0.0
cap6000	0.00	-2.16	6	0	0.1	0.0	0.0	0.0
fiber	86.40	-0.79	88	-7	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	26.21	5.69	53	8	0.0	-54.1	0.0	-7.7
lseu	14.68	-0.09	9	-2	0.0	0.0	0.0	0.0
mitre	0.24	0.00	1088	5	0.2	0.0	0.0	0.0
mkc	1.85	0.72	86	10	0.0	-0.2	0.0	0.0
mod008	11.28	8.11	12	1	0.0	0.0	0.0	0.0
mod010	18.32	0.00	1	0	0.0	0.0	0.0	0.0
nsrand-ipx	4.76	0.00	61	-3	0.8	0.1	0.1	0.0
p0033	37.85	8.96	15	4	0.0	0.0	0.0	0.0
p0282	92.71	-1.34	132	-79	0.0	0.0	0.0	0.0
p0548	82.50	-0.42	104	-17	0.0	0.0	0.0	0.0
p2756	84.95	-0.72	231	-103	0.0	0.0	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.08	0.00	7	0	0.6	-0.2	0.3	-0.1
Total	1122.82	49.61	10212	-602	4.1	-1216.1	0.7	-47.9
Geom. Mean	15.01	0.55	51	-8	1.0	-1.5	1.0	-0.4

Table B.39: Computational results for the separation algorithm for the class of LMCI2 on the main test set. *Initial cover.* Solve $KP2^{BK}_{max}$ approximately using Algorithm 4.2. (△ with respect to the default algorithm)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
cnr_dual_mip1	1.59	0.00	7	0	0.1	0.0	0.0	0.0
dfn3free	31.40	4.51	164	15	286.2	34.0	22.0	1.0
dfn3orig	94.97	4.92	98	-49	6.2	-2.4	0.4	0.0
dfn-stop-1	56.99	8.60	298	-135	10.0	1.7	0.6	0.2
dfn-stop-2	76.29	38.81	181	-66	1.7	0.2	0.1	0.0
ep1a	94.91	0.05	1881	-42	194.8	-20.4	4.6	-0.4
ep5b	69.58	9.10	2875	322	945.4	268.8	16.6	3.3
rlp2	43.11	0.00	22	10	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1848	6	0.7	-0.1	0.0	0.0
rococoC11-011100	42.46	-0.01	911	1	0.6	0.0	0.0	0.0
tasncp285	49.40	-0.02	470	-22	0.1	0.0	0.0	0.0
umts	0.00	0.00	0	0	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	0	0.1	0.0	0.0	0.0
cap6000	2.07	-0.09	6	0	0.1	0.0	0.0	0.0
fiber	87.11	-0.08	93	-2	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	23.80	3.28	41	-4	53.5	-0.6	8.9	1.2
lseu	14.77	0.00	11	0	0.0	0.0	0.0	0.0
mitre	0.24	0.00	1083	0	0.2	0.0	0.0	0.0
mkc	1.13	0.00	76	0	0.5	0.3	0.1	0.1
mod008	11.53	8.36	14	3	0.1	0.1	0.0	0.0
mod010	18.32	0.00	1	0	0.0	0.0	0.0	0.0
nsrand-ipx	4.76	0.00	62	-2	0.8	0.1	0.1	0.0
p0033	28.89	0.00	11	0	0.0	0.0	0.0	0.0
p0282	93.99	-0.06	181	-30	0.0	0.0	0.0	0.0
p0548	82.92	0.00	114	-7	0.0	0.0	0.0	0.0
p2756	85.68	0.01	237	-97	0.0	0.0	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.08	0.00	7	0	0.7	-0.1	0.4	0.0
Total	1150.62	77.41	10715	-99	1501.8	281.6	54.0	5.4
Geom. Mean	15.83	1.37	58	-1	2.5	0.0	1.4	0.0

Table B.40: Computational results for the separation algorithm for the class of LMCI2 on the main test set. *Minimal cover.* Use nonincreasing a_j as the second order criterium for removing variables. (△ with respect to the default algorithm)

Name	Gap Closed % Value	Gap Closed % △	Cuts Value	Cuts △	Sepa Time Value	Sepa Time △	Average Sepa Time Value	Average Sepa Time △
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.1	0.1
dfn3free	10.38	1.36	59	13	1.9	0.0	0.2	0.0
dfn3orig	94.63	7.76	107	-213	0.0	-0.1	0.0	0.0
dfn-stop-1	61.13	10.18	494	-389	0.1	-0.1	0.0	0.0
dfn-stop-2	77.37	47.38	261	-135	0.1	0.0	0.0	0.0
ep1a	94.88	0.27	2110	138	0.5	0.1	0.0	0.0
ep5b	71.99	11.27	3278	723	0.7	0.3	0.0	0.0
rlp2	43.11	0.00	28	10	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1831	8	0.5	0.1	0.0	0.0
rococoC11-011100	42.46	-0.01	907	0	0.3	0.0	0.0	0.0
tasncp285	49.31	0.34	463	-86	0.1	0.0	0.0	0.0
umts	0.00	0.00	3	1	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	3	0.0	0.0	0.0	0.0
cap6000	2.07	-0.09	8	2	0.1	0.0	0.0	0.0
fiber	88.68	0.73	92	10	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	30.11	16.18	90	50	0.0	0.0	0.0	0.0
lseu	43.02	1.33	18	3	0.0	0.0	0.0	0.0
mitre	0.12	-10.79	692	-276	0.1	-0.1	0.0	0.0
mkc	2.01	0.89	112	14	0.1	0.1	0.0	0.0
mod008	19.17	9.46	26	15	0.1	0.1	0.0	0.0
mod010	18.32	0.00	2	0	0.0	0.0	0.0	0.0
nsrand-ipx	12.29	0.46	96	-7	3.8	1.9	0.5	0.3
p0033	44.70	-0.23	17	-2	0.0	0.0	0.0	0.0
p0282	94.49	-0.27	176	41	0.0	0.0	0.0	0.0
p0548	88.35	2.53	164	0	0.0	0.0	0.0	0.0
p2756	85.68	0.32	299	-126	0.1	0.0	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	-1	2.8	0.6	1.4	0.7
Total	1211.39	99.10	11370	-204	11.6	3.1	2.3	0.9
Geom. Mean	17.52	0.73	70	3	1.1	0.0	1.0	0.0

Table B.41: Computational results for the separation algorithm for the classes of LMCI1 and LEWI on the main test set. (△ with respect to the resulting separation algorithm for the class of LEWI)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.1	0.0	0.1	0.1
dfn3free	7.22	0.00	18	0	0.9	-0.4	0.2	0.0
dfn3orig	91.47	0.00	65	1	0.0	0.0	0.0	0.0
dfn-stop-1	58.59	0.09	341	56	0.1	0.0	0.0	0.0
dfn-stop-2	77.70	0.37	189	21	0.1	0.0	0.0	0.0
ep1a	94.89	-0.02	2005	12	0.5	0.1	0.0	0.0
ep5b	69.16	0.00	2739	52	0.5	0.1	0.0	0.0
rlp2	43.11	0.00	21	1	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1841	1	0.4	0.0	0.0	0.0
rococoC11-011100	42.47	0.00	905	0	0.3	0.1	0.0	0.0
tasncp285	48.80	-0.01	410	-14	0.1	0.0	0.0	0.0
umts	0.00	0.00	2	0	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	0	0.0	0.0	0.0	0.0
cap6000	0.00	0.00	6	0	0.1	0.0	0.0	0.0
fiber	88.65	0.00	98	9	0.0	0.0	0.0	0.0
gen	98.17	0.00	7	1	0.0	0.0	0.0	0.0
harp2	29.26	-0.74	79	2	0.0	0.0	0.0	0.0
lseu	33.43	0.00	10	0	0.0	0.0	0.0	0.0
mitre	0.12	-0.83	1010	-161	0.1	-0.1	0.0	0.0
mkc	2.62	1.06	133	39	0.1	0.1	0.0	0.0
mod008	15.36	0.00	18	0	0.0	0.0	0.0	0.0
mod010	18.32	0.00	3	1	0.0	0.0	0.0	0.0
nsrand-ipx	10.98	-1.04	85	-22	2.0	-0.4	0.3	0.0
p0033	44.70	0.00	21	4	0.0	0.0	0.0	0.0
p0282	94.33	0.03	165	22	0.0	0.0	0.0	0.0
p0548	88.14	0.34	153	14	0.0	0.0	0.0	0.0
p2756	84.97	0.00	232	0	0.1	0.1	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	0	1.7	0.1	0.9	0.1
Total	1181.39	-0.75	10587	39	7.2	-0.2	1.5	0.0
Geom. Mean	16.59	0.23	61	3	1.0	-0.1	1.0	0.0

Table B.42: Computational results for the separation algorithm for the classes of LMCI1 and LMCI2 on the main test set. (△ with respect to the resulting separation algorithm for the class of LMCI1)

Name	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
	Value	△	Value	△	Value	△	Value	△
cnr_dual_mip1	1.72	0.00	6	1	0.1	0.0	0.1	0.1
dfn3free	10.38	1.36	59	13	1.7	-0.2	0.2	0.0
dfn3orig	94.63	7.76	104	-216	0.0	-0.1	0.0	0.0
dfn-stop-1	58.95	8.00	480	-403	0.1	-0.1	0.0	0.0
dfn-stop-2	78.34	48.35	264	-132	0.1	0.0	0.0	0.0
ep1a	94.88	0.27	2111	139	0.5	0.1	0.0	0.0
ep5b	69.81	9.09	3114	559	0.7	0.3	0.0	0.0
rlp2	43.11	0.00	20	2	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	1831	8	0.5	0.1	0.0	0.0
rococoC11-011100	42.46	-0.01	907	0	0.3	0.0	0.0	0.0
tasncp285	49.31	0.34	463	-86	0.1	0.0	0.0	0.0
umts	0.00	0.00	2	0	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	3	0.0	0.0	0.0	0.0
cap6000	2.07	-0.09	8	2	0.1	0.0	0.0	0.0
fiber	88.66	0.71	105	23	0.0	0.0	0.0	0.0
gen	98.17	0.00	6	0	0.0	0.0	0.0	0.0
harp2	28.29	14.36	73	33	0.0	0.0	0.0	0.0
lseu	41.69	0.00	16	1	0.0	0.0	0.0	0.0
mitre	0.12	-10.79	793	-175	0.2	0.0	0.0	0.0
mkc	1.21	0.09	112	14	0.0	0.0	0.0	0.0
mod008	18.38	8.67	20	9	0.1	0.1	0.0	0.0
mod010	18.32	0.00	2	0	0.0	0.0	0.0	0.0
nsrand-ipx	11.96	0.13	124	21	2.5	0.6	0.3	0.1
p0033	44.93	0.00	23	4	0.0	0.0	0.0	0.0
p0282	94.19	-0.57	216	81	0.0	0.0	0.0	0.0
p0548	88.18	2.36	166	2	0.0	0.0	0.0	0.0
p2756	85.68	0.32	299	-126	0.0	-0.1	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	-1	1.8	-0.4	0.9	0.2
Total	1202.65	90.36	11350	-224	8.9	0.4	1.6	0.2
Geom. Mean	17.09	0.30	70	3	1.1	0.0	1.0	0.0

Table B.43: Computational results for the separation algorithm for the classes of LEWI and LMCI2 on the main test set. (△ with respect to the resulting separation algorithm for the class of LEWI)

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
cnr_dual_mip1	1.72	0.00	5	0	0.2	0.1	0.1	0.1
dfn3free	10.38	1.36	59	13	2.0	0.1	0.2	0.0
dfn3orig	94.63	7.76	108	-212	0.0	-0.1	0.0	0.0
dfn-stop-1	61.02	10.07	537	-346	0.2	0.0	0.0	0.0
dfn-stop-2	79.94	49.95	318	-78	0.1	0.0	0.0	0.0
ep1a	94.91	0.30	2155	183	0.6	0.2	0.0	0.0
ep5b	71.63	10.91	3185	630	0.8	0.4	0.0	0.0
rlp2	43.11	0.00	28	10	0.0	0.0	0.0	0.0
rococoC11-010100	36.29	0.00	2034	211	0.6	0.2	0.0	0.0
rococoC11-011100	42.46	-0.01	908	1	0.4	0.1	0.0	0.0
tasncp285	49.34	0.37	480	-69	0.1	0.0	0.0	0.0
umts	0.00	0.00	3	1	0.0	0.0	0.0	0.0
atlanta-ip	0.09	0.00	12	3	0.1	0.1	0.0	0.0
cap6000	2.07	-0.09	8	2	0.2	0.1	0.0	0.0
fiber	88.66	0.71	108	26	0.0	0.0	0.0	0.0
gen	98.17	0.00	7	1	0.0	0.0	0.0	0.0
harp2	29.49	15.56	97	57	0.0	0.0	0.0	0.0
lseu	42.80	1.11	19	4	0.0	0.0	0.0	0.0
mitre	1.18	-9.73	722	-246	0.1	-0.1	0.0	0.0
mkc	1.21	0.09	124	26	0.1	0.1	0.0	0.0
mod008	19.17	9.46	26	15	0.1	0.1	0.0	0.0
mod010	18.32	0.00	3	1	0.0	0.0	0.0	0.0
nsrand-ipx	11.37	-0.46	89	-14	3.9	2.0	0.5	0.3
p0033	44.70	-0.23	21	2	0.0	0.0	0.0	0.0
p0282	94.41	-0.35	219	84	0.0	0.0	0.0	0.0
p0548	88.35	2.53	170	6	0.0	0.0	0.0	0.0
p2756	85.68	0.32	277	-148	0.1	0.0	0.0	0.0
roll3000	0.06	0.00	5	0	0.0	0.0	0.0	0.0
sp97ar	0.77	0.00	9	-1	2.9	0.7	1.4	0.7
Total	1211.93	99.64	11736	162	12.5	4.0	2.4	1.0
Geom. Mean	17.27	0.48	74	7	1.1	0.0	1.0	0.0

Table B.44: Computational results for the separation algorithm for the classes of LMCI1, LEWI, and LMCI2 on the main test set. (△ with respect to the resulting separation algorithm for the class of LEWI)

Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
tkat3	0.00	19	0.1	0.0
tkatTV5	0.00	6	0.0	0.0
10teams	0.00	0	0.0	0.0
a1c1s1	0.00	0	0.0	0.0
30:70:4_5:0_5:100	0.00	0	0.1	0.1
30:70:4_5:0_95:98	0.00	0	0.1	0.1
aflow30a	0.00	0	0.0	0.0
aflow40b	0.00	0	0.0	0.0
air03	0.00	0	0.0	0.0
air04	0.00	0	0.0	0.0
air05	0.00	0	0.0	0.0
arki001	0.00	0	0.0	0.0
bc1	0.00	0	0.0	0.0
bell3a	0.00	0	0.0	0.0
bell5	0.00	0	0.0	0.0
bienst1	0.00	0	0.0	0.0
bienst2	0.00	0	0.0	0.0
binkar10_1	0.00	0	0.0	0.0
blend2	0.00	0	0.0	0.0
dano3_3	0.00	0	0.0	0.0
dano3_4	0.00	0	0.0	0.0
dano3_5	0.00	0	0.0	0.0
dano3mip	0.00	0	0.0	0.0
danoint	0.00	0	0.0	0.0
dcmulti	0.00	0	0.0	0.0
ds	0.00	0	0.0	0.0
egout	0.00	0	0.0	0.0
eilD76	0.00	0	0.0	0.0
fast0507	0.00	0	0.0	0.0
fixnet6	0.00	0	0.0	0.0
flugpl	0.00	0	0.0	0.0
gesa2	0.00	0	0.0	0.0
gesa2-o	0.00	0	0.0	0.0
gesa3	0.00	0	0.0	0.0
gesa3_o	0.00	0	0.0	0.0
glass4	0.00	0	0.0	0.0
gt2	0.00	0	0.0	0.0
irp	0.00	0	0.0	0.0
khb05250	0.00	0	0.0	0.0
l152lav	0.00	0	0.0	0.0
liu	0.00	0	0.0	0.0
manna81	0.00	0	0.0	0.0
markshare1	0.00	17	0.0	0.0
markshare2	0.00	13	0.0	0.0
mas284	0.00	0	0.0	0.0
mas74	0.00	0	0.0	0.0
mas76	0.00	0	0.0	0.0
misc03	0.00	0	0.0	0.0
misc06	0.00	0	0.0	0.0
misc07	0.00	0	0.0	0.0
mkc1	0.00	0	0.0	0.0
mod011	0.00	0	0.0	0.0
modglob	0.00	0	0.0	0.0
momentum1	0.00	0	0.0	0.0
momentum2	0.00	0	0.0	0.0
msc98-ip	0.00	12	0.0	0.0
mzzv11	0.00	1	0.0	0.0
mzzv42z	0.00	0	0.0	0.0
neos1	0.00	176	0.0	0.0
neos2	0.00	0	0.0	0.0
neos3	0.00	0	0.0	0.0
neos616206	0.00	0	0.0	0.0
neos632659	0.00	0	0.0	0.0

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Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
neos648910	0.00	0	0.0	0.0
neos7	0.00	0	0.0	0.0
neos8	0.00	6	0.0	0.0
neos9	0.00	0	0.0	0.0
neos10	0.00	13	0.1	0.0
neos11	0.00	0	0.0	0.0
neos12	0.00	0	0.0	0.0
neos13	0.00	0	0.0	0.0
neos14	0.00	0	0.0	0.0
neos15	0.00	0	0.0	0.0
neos16	0.00	0	0.0	0.0
neos17	0.00	0	0.0	0.0
neos18	0.00	0	0.0	0.0
neos19	0.00	0	0.0	0.0
neos20	0.00	43	0.0	0.0
neos21	0.00	0	0.0	0.0
neos22	0.00	0	0.0	0.0
neos23	0.00	0	0.0	0.0
net12	0.00	0	0.0	0.0
noswot	0.00	0	0.0	0.0
nug08	0.00	0	0.0	0.0
nw04	0.00	0	0.0	0.0
opt1217	0.00	0	0.0	0.0
p0201	0.00	10	0.0	0.0
pk1	0.00	0	0.0	0.0
pp08a	0.00	0	0.0	0.0
pp08aCUTS	0.00	0	0.0	0.0
prod1	0.00	0	0.0	0.0
protfold	0.00	0	0.0	0.0
qap10	0.00	0	0.0	0.0
qiu	0.00	0	0.0	0.0
qnet1	0.00	0	0.0	0.0
qnet1_o	0.00	0	0.0	0.0
ran10x26	0.00	0	0.0	0.0
ran12x21	0.00	0	0.0	0.0
ran13x13	0.00	0	0.0	0.0
ran14x18_1	0.00	0	0.0	0.0
ran8x32	0.00	0	0.0	0.0
rentacar	0.00	0	0.0	0.0
rgn	0.00	0	0.0	0.0
rout	0.00	0	0.0	0.0
set1ch	0.00	0	0.0	0.0
seymour	0.00	0	0.0	0.0
seymour1	0.00	0	0.0	0.0
stein27	0.00	0	0.0	0.0
stein45	0.00	0	0.0	0.0
swath	0.00	0	0.0	0.0
swath1	0.00	0	0.0	0.0
swath2	0.00	0	0.0	0.0
swath3	0.00	0	0.0	0.0
t1717	0.00	0	0.0	0.0
timtab1	0.00	0	0.0	0.0
timtab2	0.00	0	0.0	0.0
tr12-30	0.00	0	0.0	0.0
vpm1	0.00	0	0.0	0.0
vpm2	0.00	0	0.0	0.0
Total	0.00	316	0.4	0.2
Geom. Mean	1.00	1	1.0	1.0

Table B.45: Computational results for the separation algorithm for the classes of LMCI1 and LEWI on the remaining test set.

B.3 Cutting Plane Separator for the 0-1 Single Node Flow Problem

Name	Type	Conss	Vars	z_{LP}	z_{MIP}
a1c1s1	BMIP	3312	3648	997.529583	11566.5904
aflow30a	BMIP	479	842	983.167425	1158
aflow40b	BMIP	1442	2728	1005.66482	1168
arki001	MIP	1048	1388	7579621.83	7580814.51
atlanta-ip	MIP	21732	48738	81.2455967	95.0095497
bc1	BMIP	1913	1751	2.18877397	3.33836255
bienst1	BMIP	576	505	11.7241379	46.75
bienst2	BMIP	576	505	11.7241379	54.6
binkar10_1	BMIP	1026	2298	6637.18803	6742.20002
blend2	MIP	274	353	6.91567511	7.598985
cap6000	BIP	2176	6000	-2451537.33	-2451377
dano3mip	BMIP	3202	13873	576.23162	705.941176
danoint	BMIP	664	521	62.6372804	65.67
dcmulti	BMIP	290	548	184466.891	188182
egout	BMIP	98	141	511.61784	568.1007
fiber	BMIP	363	1298	198107.358	405935.18
fixnet6	BMIP	478	878	3192.042	3983
gen	MIP	780	870	112271.463	112313.363
gesa2	MIP	1392	1224	25492512.1	25779856.4
gesa2-o	MIP	1248	1224	25476489.7	25779856.4
gesa3	MIP	1368	1152	27846437.5	27991042.6
gesa3_o	MIP	1224	1152	27833632.5	27991042.6
gt2	IP	29	188	20146.7613	21166
harp2	BIP	112	2993	-74325169.3	-73899597
khb05250	BMIP	101	1350	95919464	106940226
lseu	BIP	28	89	947.957237	1120
mitre	BIP	2054	10724	114782.467	115155
mkc	BMIP	3411	5325	-611.85	-563.212
mod008	BIP	6	319	290.931073	307
mod010	BIP	146	2655	6532.08333	6548
mod011	BMIP	4480	10958	-62081950.3	-54558535
modglob	BMIP	291	422	20430947.6	20740508
momentum2	MIP	24237	3732	10696.1116	12314.2196
msc98-ip	MIP	15850	21143	19520966.2	23271298
neos616206	BMIP	534	480	787.721258	937.6
neos8	IP	46324	23228	-3725	-3719
neos14	BMIP	552	792	32734.1148	74333.3433
neos15	BMIP	552	792	33463.7701	80851.6678
neos16	IP	1018	377	429	450
neos22	BMIP	5208	3240	777191.429	779715
net12	BMIP	14021	14115	68.3978758	214
nsrand-ipx	BMIP	735	6621	49667.8923	51520
p0033	BIP	16	33	2828.33136	3089
p0282	BIP	241	282	180000.3	258411
p0548	BIP	176	548	4790.57713	8691
p2756	BIP	755	2756	2701.14437	3124
pp08aCUTS	BMIP	246	240	5480.60616	7350
prod1	BMIP	208	250	-84.4158719	-56
qnet1	MIP	503	1541	14274.1027	16029.6927
qnet1_o	MIP	456	1541	12557.2479	16029.6927
ran10x26	BMIP	296	520	3857.02278	4270
ran12x21	BMIP	285	504	3157.37744	3664
ran13x13	BMIP	195	338	2691.43947	3252
ran14x18_1	BMIP	284	504	3016.94435	3714
ran8x32	BMIP	296	512	4937.58453	5247
rentacar	BMIP	6803	9557	28928379.6	30356761
rgn	BMIP	24	180	48.7999986	82.1999992
roll3000	MIP	2295	1166	11097.2754	12899
rout	MIP	291	556	981.864286	1077.56

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Name	Type	Conss	Vars	z_{LP}	z_{MIP}
set1ch	BMIP	492	712	35118.1098	54537.75
sp97ar	BIP	1761	14101	652560391	663164724
timtab1	MIP	171	397	157896.037	764772
timtab2	MIP	294	675	210652.471	1184230
tr12-30	BMIP	750	1080	18124.1745	130596
vpm1	BMIP	234	378	16.4333333	20
vpm2	BMIP	234	378	10.303297	13.75

Table B.46: Summary of the main test set for the cutting plane separator for the 0-1 single node flow problem.

Name	Type	Conss	Vars	z_{LP}	z_{MIP}
10teams	BMIP	230	2025	917	924
30:70:4_5:0_5:100	BMIP	12050	10772	8.1	9
30:70:4_5:0_95:98	BMIP	12471	10990	11.5	12
air03	BIP	124	10757	338864.25	340160
air04	BIP	823	8904	55535.4364	56137
air05	BIP	426	7195	25877.6093	26374
bell3a	MIP	123	133	866171.733	878430.316
bell5	MIP	91	104	8908552.45	8966406.49
dano3_3	BMIP	3202	13873	576.23162	576.344633
dano3_4	BMIP	3202	13873	576.23162	576.435225
dano3_5	BMIP	3202	13873	576.23162	576.924916
ds	BIP	656	67732	57.2347263	468.645
eilD76	BIP	75	1898	680.538997	885.411847
fast0507	BIP	507	63009	172.145567	174
flugpl	MIP	18	18	1167185.73	1201500
glass4	BMIP	396	322	800002400	1.6000134e+09
irp	BIP	39	20315	12123.5302	12159.4928
l152lav	BIP	97	1989	4656.36364	4722
liu	BMIP	2178	1156	560	1146
manna81	IP	6480	3321	-13297	-13164
markshare1	BMIP	6	62	0	1
markshare2	BMIP	7	74	0	1
mas284	BMIP	68	151	86195.863	91405.7237
mas74	BMIP	13	151	10482.7953	11801.1857
mas76	BMIP	12	151	38893.9036	40005.0541
misc03	BMIP	96	160	1910	3360
misc06	BMIP	820	1808	12841.6894	12850.8607
misc07	BMIP	212	260	1415	2810
mkc1	BMIP	3411	5325	-611.85	-607.207
momentum1	BMIP	42680	5174	82424.4594	109143.493
mzzv11	IP	9499	10240	-22944.9875	-21718
mzzv42z	IP	10460	11717	-21622.9985	-20540
neos1	BIP	5020	2112	5.6	19
neos2	BMIP	1103	2101	-4407.09724	454.864697
neos3	BMIP	1442	2747	-6158.20911	368.842751
neos632659	BMIP	244	420	-119.47619	-94
neos648910	BMIP	1491	814	16	32
neos7	MIP	1994	1556	562977.43	721934
neos9	BMIP	31600	81408	780	784
neos10	IP	46793	23489	-1196.33333	-1135
neos11	BMIP	2706	1220	6	9
neos12	BMIP	8317	3983	9.41161243	13
neos13	BMIP	20852	1827	-126.178378	-95.4748066
neos17	BMIP	486	535	0.000681498501	0.150002577
neos18	BIP	11402	3312	7	16
neos19	BMIP	34082	103789	-1611	-1499
neos20	MIP	2446	1165	-475	-434
neos21	BMIP	1085	614	2.21648352	7
neos23	BMIP	1568	477	56	137
noswot	MIP	182	128	-43	-41
nug08	BIP	912	1632	203.5	214
nw04	BIP	36	87482	16310.6667	16862
opt1217	BMIP	64	769	-20.0213904	-16
p0201	BIP	133	201	7125	7615
pk1	BMIP	45	86	0	11
pp08a	BMIP	136	240	2748.34524	7350
protfold	BIP	2112	1835	-41.9574468	-23
qap10	BIP	1820	4150	332.566228	340
qiu	BMIP	1192	840	-931.638854	-132.873137
seymour	BIP	4944	1372	403.846474	423
seymour1	BMIP	4944	1372	403.846474	410.763701
stein27	BIP	118	27	13	18
stein45	BIP	331	45	22	30

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Name	Type	Conss	Vars	z_{LP}	z_{MIP}
swath	BMIP	884	6805	334.496858	477.34101
swath1	BMIP	884	6805	334.496858	379.071296
swath2	BMIP	884	6805	334.496858	385.199693
swath3	BMIP	884	6805	334.496858	397.761344
t1717	BIP	551	73885	134531.021	288658

Table B.47: Summary of the remaining test set for the cutting plane separator for the 0-1 single node flow problem.

Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
a1c1s1	27.14	176	10.5	1.5
aflow30a	33.48	90	0.4	0.0
aflow40b	23.74	59	2.2	0.2
arki001	3.54	72	75.1	12.5
atlanta-ip	0.00	16	254.9	51.0
bc1	32.02	14	405.2	50.7
bienst1	6.60	57	0.1	0.0
bienst2	7.43	84	0.1	0.0
binkar10_1	26.14	28	0.5	0.1
blend2	4.59	10	25.7	5.1
cap6000	0.00	0	0.1	0.1
dano3mip	0.01	4	42.0	8.4
danoint	0.58	22	0.3	0.1
dcmulti	21.96	38	0.1	0.0
egout	94.73	13	0.1	0.0
fiber	88.23	74	0.1	0.0
fixnet6	41.06	79	0.8	0.0
gen	98.17	7	0.0	0.0
gesa2	60.53	47	11.1	2.2
gesa2-o	7.46	26	5.2	1.7
gesa3	23.34	14	4.6	2.3
gesa3_o	6.98	11	3.9	1.9
gt2	33.85	6	0.0	0.0
harp2	0.00	0	0.5	0.5
khb05250	97.96	75	9.3	0.8
lseu	36.50	19	0.0	0.0
mitre	6.77	903	5.5	0.9
mkc	0.05	59	69.2	11.5
mod008	29.76	14	1.4	0.1
mod010	18.32	1	0.1	0.1
mod011	49.19	257	167.1	10.4
modglob	24.86	40	1.2	0.2
momentum2	0.00	11	41.6	10.4
msc98-ip	0.74	287	81.3	11.6
neos616206	3.06	195	2.8	0.3
neos8	0.00	6	2.9	1.4
neos14	45.78	76	0.2	0.1
neos15	40.02	76	0.2	0.1
neos16	9.52	132	0.1	0.0
neos22	3.96	111	5.0	0.8
net12	2.67	89	21.0	1.9
nsrand-ipx	4.79	69	1.4	0.2
p0033	9.34	9	0.0	0.0
p0282	93.98	121	0.1	0.0
p0548	73.05	86	0.1	0.0
p2756	72.39	228	1.0	0.1
pp08aCUTS	0.55	4	0.0	0.0
prod1	0.11	12	0.7	0.1
qnet1	29.16	26	0.1	0.0
qnet1_o	54.30	33	0.1	0.0
ran10x26	38.53	49	0.1	0.0
ran12x21	39.45	55	0.1	0.0
ran13x13	39.42	42	0.0	0.0
ran14x18_1	39.21	70	0.0	0.0
ran8x32	62.60	47	0.1	0.0
rentacar	0.00	4	356.7	178.3
rgn	0.00	0	0.0	0.0
roll3000	0.15	64	1.9	0.5
rout	0.27	58	0.3	0.1
set1ch	37.74	127	0.1	0.0
sp97ar	0.08	7	1.5	0.8
timtab1	7.56	40	0.0	0.0
timtab2	5.29	91	0.1	0.0

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Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
tr12-30	38.34	226	0.9	0.2
vpm1	64.95	22	0.0	0.0
vpm2	69.61	72	0.0	0.0
Total	1791.60	4860	1617.5	369.5
Geom. Mean	10.92	32	2.8	1.7

Table B.48: Computational results for the cutting plane separator for the 0-1 single node flow problem on the main test set. *Default algorithm.* Application to all rows of a MIP.

Name	Gap Closed % Value	Gap Closed % △	Cuts Value	Cuts △	Sepa Time Value	Sepa Time △	Average Sepa Time Value	Average Sepa Time △
a1c1s1	17.45	-9.69	124	-52	4.1	-6.4	0.6	-0.9
aflow30a	31.80	-1.68	66	-24	0.1	-0.3	0.0	0.0
aflow40b	19.08	-4.66	45	-14	0.6	-1.6	0.1	-0.1
arki001	3.54	0.00	85	13	0.2	-74.9	0.0	-12.5
atlanta-ip	0.00	0.00	16	0	71.6	-183.3	23.9	-27.1
bc1	30.58	-1.44	30	16	1.9	-403.3	0.2	-50.5
bienst1	6.61	0.01	58	1	0.1	0.0	0.0	0.0
bienst2	7.44	0.01	86	2	0.1	0.0	0.0	0.0
binkar10_1	28.49	2.35	29	1	0.1	-0.4	0.0	-0.1
blend2	3.63	-0.96	3	-7	0.0	-25.7	0.0	-5.1
cap6000	0.00	0.00	0	0	0.1	0.0	0.1	0.0
dano3mip	0.01	0.00	4	0	5.2	-36.8	1.0	-7.4
danoint	0.65	0.07	19	-3	0.0	-0.3	0.0	-0.1
dcmulti	21.96	0.00	38	0	0.0	-0.1	0.0	0.0
egout	91.46	-3.27	11	-2	0.0	-0.1	0.0	0.0
fiber	87.34	-0.89	50	-24	0.1	0.0	0.0	0.0
fixnet6	41.53	0.47	78	-1	0.1	-0.7	0.0	0.0
gen	97.57	-0.60	13	6	0.1	0.1	0.0	0.0
gesa2	57.85	-2.68	38	-9	0.2	-10.9	0.1	-2.1
gesa2-o	7.46	0.00	29	3	0.1	-5.1	0.0	-1.7
gesa3	22.99	-0.35	14	0	0.1	-4.5	0.1	-2.2
gesa3_o	6.98	0.00	11	0	0.1	-3.8	0.0	-1.9
gt2	33.21	-0.64	5	-1	0.0	0.0	0.0	0.0
harp2	0.00	0.00	0	0	0.0	-0.5	0.0	-0.5
khb05250	97.96	0.00	75	0	0.1	-9.2	0.0	-0.8
lseu	36.50	0.00	19	0	0.0	0.0	0.0	0.0
mitre	9.43	2.66	709	-194	2.8	-2.7	0.5	-0.4
mkc	0.00	-0.05	46	-13	0.7	-68.5	0.1	-11.4
mod008	29.00	-0.76	9	-5	0.0	-1.4	0.0	-0.1
mod010	0.00	-18.32	0	-1	0.0	-0.1	0.0	-0.1
mod011	48.47	-0.72	167	-90	5.6	-161.5	0.3	-10.1
modglob	24.90	0.04	37	-3	0.0	-1.2	0.0	-0.2
momentum2	0.00	0.00	11	0	50.3	8.7	12.6	2.2
msc98-ip	0.74	0.00	205	-82	57.4	-23.9	9.6	-2.0
neos616206	3.01	-0.05	199	4	0.1	-2.7	0.0	-0.3
neos8	0.00	0.00	6	0	1.3	-1.6	0.6	-0.8
neos14	14.05	-31.73	23	-53	0.0	-0.2	0.0	-0.1
neos15	12.16	-27.86	23	-53	0.0	-0.2	0.0	-0.1
neos16	9.52	0.00	132	0	0.1	0.0	0.0	0.0
neos22	3.96	0.00	111	0	5.3	0.3	0.9	0.1
net12	1.73	-0.94	66	-23	25.9	4.9	2.0	0.1
nsrand-ipx	4.78	-0.01	62	-7	1.2	-0.2	0.2	0.0
p0033	9.10	-0.24	12	3	0.0	0.0	0.0	0.0
p0282	92.78	-1.20	78	-43	0.0	-0.1	0.0	0.0
p0548	72.12	-0.93	66	-20	0.1	0.0	0.0	0.0
p2756	72.13	-0.26	240	12	0.7	-0.3	0.1	0.0
pp08aCUTS	0.00	-0.55	0	-4	0.0	0.0	0.0	0.0
prod1	0.21	0.10	12	0	0.0	-0.7	0.0	-0.1
qnet1	3.98	-25.18	7	-19	0.0	-0.1	0.0	0.0
qnet1_o	38.25	-16.05	16	-17	0.1	0.0	0.0	0.0
ran10x26	38.81	0.28	31	-18	0.0	-0.1	0.0	0.0
ran12x21	45.11	5.66	56	1	0.0	-0.1	0.0	0.0
ran13x13	39.88	0.46	51	9	0.0	0.0	0.0	0.0
ran14x18_1	37.96	-1.25	63	-7	0.1	0.1	0.0	0.0
ran8x32	64.60	2.00	47	0	0.1	0.0	0.0	0.0
rentacar	0.00	0.00	4	0	0.2	-356.5	0.1	-178.2
rgn	57.49	57.49	32	32	0.0	0.0	0.0	0.0
roll3000	0.15	0.00	64	0	0.3	-1.6	0.1	-0.4
rout	0.00	-0.27	28	-30	0.0	-0.3	0.0	-0.1
set1ch	37.74	0.00	127	0	0.0	-0.1	0.0	0.0
sp97ar	0.00	-0.08	6	-1	1.6	0.1	0.8	0.0
timtab1	7.56	0.00	40	0	0.0	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
timtab2	5.31	0.02	93	2	0.0	-0.1	0.0	0.0
tr12-30	12.83	-25.51	74	-152	0.1	-0.8	0.0	-0.2
vpm1	64.95	0.00	22	0	0.0	0.0	0.0	0.0
vpm2	68.88	-0.73	68	-4	0.0	0.0	0.0	0.0
Total	1683.65	-107.95	3989	-871	239.3	-1378.2	54.3	-315.2
Geom. Mean	10.00	-0.92	28	-4	1.5	-1.3	1.1	-0.6

Table B.49: Computational results for the cutting plane separator for the 0-1 single node flow problem on the main test set. *Flow cover.* Always solve KP^{SNF}_{rat} approximately using Algorithm 5.1. Application to all rows of a MIP. (△ with respect to the default algorithm (applied to all rows of a MIP))

Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
a1c1s1	27.14	0.00	176	0	8.1	-2.4	1.2	-0.3
aflow30a	33.48	0.00	90	0	0.4	0.0	0.0	0.0
aflow40b	23.74	0.00	59	0	2.6	0.4	0.2	0.0
arki001	3.54	0.00	85	13	2.8	-72.3	0.5	-12.0
atlanta-ip	0.00	0.00	16	0	182.3	-72.6	45.6	-5.4
bc1	32.02	0.00	14	0	8.3	-396.9	1.0	-49.7
bienst1	6.60	0.00	57	0	0.1	0.0	0.0	0.0
bienst2	7.43	0.00	84	0	0.1	0.0	0.0	0.0
binkar10_1	26.34	0.20	25	-3	0.6	0.1	0.1	0.0
blend2	3.63	-0.96	3	-7	0.0	-25.7	0.0	-5.1
cap6000	0.00	0.00	0	0	0.1	0.0	0.1	0.0
dano3mip	0.01	0.00	4	0	27.4	-14.6	5.5	-2.9
danoint	0.58	0.00	22	0	0.3	0.0	0.1	0.0
dcmulti	21.96	0.00	38	0	0.1	0.0	0.0	0.0
egout	94.73	0.00	13	0	0.0	-0.1	0.0	0.0
fiber	88.23	0.00	74	0	0.1	0.0	0.0	0.0
fixnet6	41.06	0.00	79	0	1.1	0.3	0.0	0.0
gen	98.17	0.00	7	0	0.0	0.0	0.0	0.0
gesa2	60.53	0.00	45	-2	2.3	-8.8	0.5	-1.7
gesa2-o	7.46	0.00	29	3	0.2	-5.0	0.1	-1.6
gesa3	23.34	0.00	14	0	0.7	-3.9	0.4	-1.9
gesa3_o	6.98	0.00	11	0	0.1	-3.8	0.1	-1.8
gt2	33.85	0.00	6	0	0.0	0.0	0.0	0.0
harp2	0.00	0.00	0	0	0.0	-0.5	0.0	-0.5
khb05250	97.96	0.00	75	0	1.5	-7.8	0.1	-0.7
lseu	36.50	0.00	19	0	0.0	0.0	0.0	0.0
mitre	6.49	-0.28	892	-11	5.1	-0.4	0.9	0.0
mkc	0.05	0.00	64	5	1.0	-68.2	0.2	-11.3
mod008	29.55	-0.21	10	-4	0.3	-1.1	0.0	-0.1
mod010	18.32	0.00	1	0	0.0	-0.1	0.0	-0.1
mod011	49.19	0.00	257	0	21.8	-145.3	1.4	-9.0
modglob	24.86	0.00	40	0	1.6	0.4	0.3	0.1
momentum2	0.00	0.00	11	0	55.2	13.6	13.8	3.4
msc98-ip	0.74	0.00	287	0	96.4	15.1	13.8	2.2
neos616206	3.06	0.00	195	0	4.5	1.7	0.4	0.1
neos8	0.00	0.00	6	0	4.3	1.4	2.2	0.8
neos14	45.78	0.00	76	0	0.2	0.0	0.1	0.0
neos15	40.02	0.00	76	0	0.2	0.0	0.1	0.0
neos16	9.52	0.00	132	0	0.1	0.0	0.0	0.0
neos22	3.96	0.00	111	0	5.6	0.6	0.9	0.1
net12	2.67	0.00	89	0	24.3	3.3	2.2	0.3
nsrand-ipx	4.79	0.00	69	0	1.5	0.1	0.3	0.1
p0033	9.34	0.00	9	0	0.0	0.0	0.0	0.0
p0282	93.98	0.00	121	0	0.1	0.0	0.0	0.0
p0548	73.05	0.00	86	0	0.1	0.0	0.0	0.0
p2756	72.39	0.00	228	0	1.0	0.0	0.1	0.0
pp08aCUTS	0.55	0.00	4	0	0.0	0.0	0.0	0.0
prod1	0.11	0.00	12	0	1.0	0.3	0.2	0.1
qnet1	29.16	0.00	26	0	0.1	0.0	0.0	0.0
qnet1_o	54.30	0.00	33	0	0.1	0.0	0.0	0.0
ran10x26	38.53	0.00	49	0	0.1	0.0	0.0	0.0
ran12x21	39.45	0.00	55	0	0.1	0.0	0.0	0.0
ran13x13	39.42	0.00	42	0	0.0	0.0	0.0	0.0
ran14x18_1	39.21	0.00	70	0	0.0	0.0	0.0	0.0
ran8x32	62.60	0.00	47	0	0.1	0.0	0.0	0.0
rentacar	0.00	0.00	4	0	1.5	-355.2	0.8	-177.5
rgn	0.00	0.00	0	0	0.0	0.0	0.0	0.0
roll3000	0.15	0.00	64	0	0.4	-1.5	0.1	-0.4
rout	0.27	0.00	58	0	0.0	-0.3	0.0	-0.1
set1ch	37.74	0.00	127	0	0.0	-0.1	0.0	0.0
sp97ar	0.08	0.00	7	0	1.7	0.2	0.8	0.0
timtab1	7.56	0.00	40	0	0.0	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
timtab2	5.29	0.00	91	0	0.1	0.0	0.0	0.0
tr12-30	38.34	0.00	226	0	1.0	0.1	0.3	0.1
vpm1	64.95	0.00	22	0	0.0	0.0	0.0	0.0
vpm2	69.61	0.00	72	0	0.0	0.0	0.0	0.0
Total	1790.36	-1.24	4854	-6	469.3	-1148.2	94.3	-275.2
Geom. Mean	10.87	-0.05	31	-1	1.8	-1.0	1.2	-0.5

Table B.50: Computational results for the cutting plane separator for the 0-1 single node flow problem on the main test set. *Flow cover*. Solve KP^{SNF}_{int} exactly using Algorithm 4.1 if the calculated scaling factor γ is not greater than 1,000 and nc for KP^{SNF}_{int} is not greater than 1,000,000, and solve KP^{SNF}_{rat} approximately using Algorithm 5.1 otherwise. Application to all rows of a MIP. (△ with respect to the default algorithm (applied to all rows of a MIP))

Name	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
	Value	△	Value	△	Value	△	Value	△
a1c1s1	25.91	-1.23	152	-24	1.9	-8.6	0.3	-1.2
aflow30a	33.48	0.00	89	-1	0.1	-0.3	0.0	0.0
aflow40b	26.59	2.85	76	17	1.1	-1.1	0.1	-0.1
arki001	2.48	-1.06	56	-16	0.3	-74.8	0.0	-12.5
atlanta-ip	0.00	0.00	16	0	189.4	-65.5	37.9	-13.1
bc1	32.02	0.00	14	0	1.1	-404.1	0.1	-50.6
bienst1	6.60	0.00	59	2	0.1	0.0	0.0	0.0
bienst2	7.42	-0.01	89	5	0.1	0.0	0.0	0.0
binkar10_1	26.14	0.00	29	1	0.1	-0.4	0.0	-0.1
blend2	4.59	0.00	10	0	3.4	-22.3	0.7	-4.4
cap6000	3.03	3.03	1	1	0.2	0.1	0.1	0.0
dano3mip	0.01	0.00	4	0	2.5	-39.5	0.5	-7.9
danoint	0.58	0.00	22	0	0.0	-0.3	0.0	-0.1
dcmulti	21.70	-0.26	34	-4	0.0	-0.1	0.0	0.0
egout	94.73	0.00	13	0	0.0	-0.1	0.0	0.0
fiber	88.23	0.00	74	0	0.0	-0.1	0.0	0.0
fixnet6	38.22	-2.84	77	-2	0.1	-0.7	0.0	0.0
gen	98.17	0.00	7	0	0.0	0.0	0.0	0.0
gesa2	60.05	-0.48	45	-2	0.1	-11.0	0.0	-2.2
gesa2-o	7.46	0.00	26	0	0.0	-5.2	0.0	-1.7
gesa3	23.34	0.00	14	0	0.0	-4.6	0.0	-2.3
gesa3_o	6.98	0.00	11	0	0.0	-3.9	0.0	-1.9
gt2	33.85	0.00	6	0	0.0	0.0	0.0	0.0
harp2	8.04	8.04	8	8	48.5	48.0	8.1	7.6
khb05250	97.93	-0.03	75	0	0.1	-9.2	0.0	-0.8
lseu	36.50	0.00	19	0	0.0	0.0	0.0	0.0
mitre	6.77	0.00	901	-2	3.2	-2.3	0.5	-0.4
mkc	0.05	0.00	75	16	6.1	-63.1	1.0	-10.5
mod008	29.76	0.00	14	0	0.1	-1.3	0.0	-0.1
mod010	18.32	0.00	2	1	0.0	-0.1	0.0	-0.1
mod011	48.81	-0.38	238	-19	11.9	-155.2	0.7	-9.7
modglob	24.86	0.00	40	0	0.1	-1.1	0.0	-0.2
momentum2	0.00	0.00	12	1	11.3	-30.3	2.8	-7.6
msc98-ip	0.74	0.00	245	-42	33.5	-47.8	5.6	-6.0
neos616206	3.15	0.09	205	10	0.2	-2.6	0.0	-0.3
neos8	0.00	0.00	6	0	0.8	-2.1	0.4	-1.0
neos14	45.78	0.00	76	0	0.0	-0.2	0.0	-0.1
neos15	40.02	0.00	76	0	0.0	-0.2	0.0	-0.1
neos16	9.52	0.00	132	0	0.1	0.0	0.0	0.0
neos22	3.96	0.00	103	-8	2.5	-2.5	0.4	-0.4
net12	2.67	0.00	99	10	22.0	1.0	2.0	0.1
nsrand-ipx	4.79	0.00	69	0	1.1	-0.3	0.2	0.0
p0033	9.34	0.00	9	0	0.0	0.0	0.0	0.0
p0282	93.98	0.00	120	-1	0.1	0.0	0.0	0.0
p0548	73.05	0.00	86	0	0.0	-0.1	0.0	0.0
p2756	72.39	0.00	228	0	0.3	-0.7	0.0	-0.1
pp08aCUTS	0.55	0.00	4	0	0.0	0.0	0.0	0.0
prod1	0.11	0.00	12	0	0.2	-0.5	0.0	-0.1
qnet1	29.16	0.00	26	0	0.1	0.0	0.0	0.0
qnet1_o	54.33	0.03	34	1	0.1	0.0	0.0	0.0
ran10x26	38.22	-0.31	48	-1	0.1	0.0	0.0	0.0
ran12x21	39.45	0.00	55	0	0.0	-0.1	0.0	0.0
ran13x13	39.42	0.00	42	0	0.0	0.0	0.0	0.0
ran14x18_1	39.21	0.00	70	0	0.0	0.0	0.0	0.0
ran8x32	62.60	0.00	47	0	0.0	-0.1	0.0	0.0
rentacar	0.00	0.00	4	0	0.1	-356.6	0.0	-178.3
rgn	0.00	0.00	0	0	0.0	0.0	0.0	0.0
roll3000	0.14	-0.01	44	-20	0.2	-1.7	0.0	-0.5
rout	0.27	0.00	65	7	0.0	-0.3	0.0	-0.1
set1ch	37.74	0.00	127	0	0.0	-0.1	0.0	0.0
sp97ar	0.08	0.00	7	0	1.4	-0.1	0.7	-0.1
timtab1	7.00	-0.56	36	-4	0.0	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
timtab2	5.10	-0.19	80	-11	0.1	0.0	0.0	0.0
tr12-30	38.35	0.01	228	2	0.3	-0.6	0.1	-0.1
vpm1	64.95	0.00	22	0	0.0	0.0	0.0	0.0
vpm2	68.41	-1.20	69	-3	0.0	0.0	0.0	0.0
Total	1797.09	5.49	4782	-78	345.0	-1272.5	62.6	-306.9
Geom. Mean	11.37	0.45	33	1	1.5	-1.3	1.1	-0.6

Table B.51: Computational results for the cutting plane separator for the 0-1 single node flow problem on the main test set. *Flow cover.* Apply the fixing strategy suggested in Section 5.3.1 to KP^{SNF}_{int} and KP^{SNF}_{rat}. Application to all rows of a MIP. (△ with respect to the default algorithm (applied to all rows of a MIP))

Name	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
	Value	△	Value	△	Value	△	Value	△
a1c1s1	27.28	0.14	250	74	30.6	20.1	2.8	1.3
aflow30a	33.48	0.00	90	0	0.7	0.3	0.0	0.0
aflow40b	23.74	0.00	59	0	3.9	1.7	0.3	0.1
arki001	3.54	0.00	74	2	97.0	21.9	16.2	3.7
atlanta-ip	0.00	0.00	16	0	641.0	386.1	128.2	77.2
bc1	32.02	0.00	20	6	759.2	354.0	69.0	18.3
bienst1	6.60	0.00	57	0	0.3	0.2	0.0	0.0
bienst2	7.43	0.00	84	0	0.3	0.2	0.0	0.0
binkar10_1	26.14	0.00	28	0	1.0	0.5	0.2	0.1
blend2	4.59	0.00	10	0	50.7	25.0	10.1	5.0
cap6000	0.00	0.00	0	0	0.8	0.7	0.8	0.7
dano3mip	0.01	0.00	4	0	90.0	48.0	18.0	9.6
danoint	0.58	0.00	22	0	0.6	0.3	0.1	0.0
dcmulti	21.96	0.00	38	0	0.1	0.0	0.1	0.1
egout	91.46	-3.27	12	-1	0.1	0.0	0.0	0.0
fiber	88.23	0.00	74	0	0.3	0.2	0.0	0.0
fixnet6	41.06	0.00	79	0	1.7	0.9	0.1	0.1
gen	98.17	0.00	7	0	0.1	0.1	0.0	0.0
gesa2	61.15	0.62	48	1	22.5	11.4	4.5	2.3
gesa2-o	7.58	0.12	30	4	20.8	15.6	3.5	1.8
gesa3	23.34	0.00	14	0	9.2	4.6	4.6	2.3
gesa3_o	6.98	0.00	11	0	7.6	3.7	3.8	1.9
gt2	33.85	0.00	6	0	0.0	0.0	0.0	0.0
harp2	0.00	0.00	0	0	0.8	0.3	0.8	0.3
khb05250	97.96	0.00	75	0	17.8	8.5	1.5	0.7
lseu	36.50	0.00	19	0	0.0	0.0	0.0	0.0
mitre	6.77	0.00	917	14	16.1	10.6	2.7	1.8
mkc	0.05	0.00	59	0	123.1	53.9	20.5	9.0
mod008	29.76	0.00	14	0	2.0	0.6	0.1	0.0
mod010	18.32	0.00	1	0	0.2	0.1	0.1	0.0
mod011	49.10	-0.09	238	-19	273.6	106.5	17.1	6.7
modglob	24.86	0.00	40	0	2.1	0.9	0.4	0.2
momentum2	0.04	0.04	15	4	138.4	96.8	34.6	24.2
msc98-ip	0.74	0.00	298	11	286.9	205.6	47.8	36.2
neos616206	3.06	0.00	195	0	3.7	0.9	0.3	0.0
neos8	4.17	4.17	7	1	33.1	30.2	16.6	15.2
neos14	45.78	0.00	76	0	0.2	0.0	0.1	0.0
neos15	40.02	0.00	76	0	0.2	0.0	0.1	0.0
neos16	9.52	0.00	132	0	0.2	0.1	0.1	0.1
neos22	3.96	0.00	111	0	20.1	15.1	3.4	2.6
net12	2.67	0.00	98	9	301.2	280.2	27.4	25.5
nsrand-ipx	12.13	7.34	80	11	3.9	2.5	0.5	0.3
p0033	9.34	0.00	9	0	0.0	0.0	0.0	0.0
p0282	93.98	0.00	121	0	0.2	0.1	0.0	0.0
p0548	77.75	4.70	89	3	0.2	0.1	0.0	0.0
p2756	72.39	0.00	228	0	1.9	0.9	0.2	0.1
pp08aCUTS	0.55	0.00	4	0	0.0	0.0	0.0	0.0
prod1	0.11	0.00	12	0	1.1	0.4	0.2	0.1
qnet1	31.52	2.36	33	7	0.4	0.3	0.0	0.0
qnet1_o	53.89	-0.41	42	9	0.3	0.2	0.0	0.0
ran10x26	38.47	-0.06	49	0	0.2	0.1	0.0	0.0
ran12x21	45.04	5.59	82	27	0.2	0.1	0.0	0.0
ran13x13	46.03	6.61	58	16	0.1	0.1	0.0	0.0
ran14x18_1	39.45	0.24	89	19	0.2	0.2	0.0	0.0
ran8x32	66.19	3.59	58	11	0.2	0.1	0.0	0.0
rentacar	0.69	0.69	6	2	1069.4	712.7	356.5	178.2
rgn	20.02	20.02	110	110	0.7	0.7	0.1	0.1
roll3000	0.15	0.00	65	1	2.8	0.9	0.7	0.2
rout	0.27	0.00	58	0	0.5	0.2	0.1	0.0
set1ch	37.74	0.00	127	0	0.1	0.0	0.0	0.0
sp97ar	1.39	1.31	9	2	5.0	3.5	2.5	1.7
timtab1	7.51	-0.05	43	3	0.0	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
timtab2	5.57	0.28	147	56	0.2	0.1	0.0	0.0
tr12-30	38.34	0.00	226	0	1.4	0.5	0.4	0.2
vpm1	64.95	0.00	22	0	0.0	0.0	0.0	0.0
vpm2	69.61	0.00	72	0	0.0	0.0	0.0	0.0
Total	1845.54	53.94	5243	383	4047.3	2429.8	797.1	427.6
Geom. Mean	11.99	1.07	37	5	4.3	1.5	2.2	0.5

Table B.52: Computational results for the cutting plane separator for the 0-1 single node flow problem on the main test set. *Cut generation heuristic*. Use $N_2^* = N_1^* \cup \{\lambda + 1\}$ as candidate set for the value of $\bar{u}$. Application to all rows of a MIP. (△ with respect to the default algorithm (applied to all rows of a MIP))

Name	Gap Closed % Value	Gap Closed % △	Cuts Value	Cuts △	Sepa Time Value	Sepa Time △	Average Sepa Time Value	Average Sepa Time △
a1c1s1	29.18	2.04	257	81	44.0	33.5	4.9	3.4
aflow30a	35.05	1.57	96	6	1.8	1.4	0.1	0.1
aflow40b	28.49	4.75	80	21	8.9	6.7	0.7	0.5
arki001	3.54	0.00	78	6	129.1	54.0	21.5	9.0
atlanta-ip	0.15	0.15	40	24	1320.6	1065.7	264.1	213.1
bc1	34.38	2.36	14	0	982.7	577.5	98.3	47.6
bienst1	6.59	-0.01	55	-2	0.3	0.2	0.0	0.0
bienst2	7.43	0.00	91	7	0.5	0.4	0.0	0.0
binkar10_1	22.55	-3.59	42	14	1.0	0.5	0.2	0.1
blend2	6.93	2.34	6	-4	31.4	5.7	7.8	2.7
cap6000	0.00	0.00	0	0	0.7	0.6	0.7	0.6
dano3mip	0.01	0.00	4	0	97.0	55.0	19.4	11.0
danoint	0.62	0.04	23	1	0.7	0.4	0.1	0.0
dcmulti	21.96	0.00	38	0	0.1	0.0	0.1	0.1
egout	91.46	-3.27	12	-1	0.1	0.0	0.0	0.0
fiber	89.38	1.15	100	26	0.6	0.5	0.1	0.1
fixnet6	46.53	5.47	110	31	1.9	1.1	0.1	0.1
gen	98.17	0.00	23	16	0.2	0.2	0.0	0.0
gesa2	61.31	0.78	55	8	22.6	11.5	4.5	2.3
gesa2-o	9.21	1.75	37	11	13.8	8.6	3.4	1.7
gesa3	33.26	9.92	18	4	18.4	13.8	4.6	2.3
gesa3_o	6.98	0.00	11	0	7.6	3.7	3.8	1.9
gt2	33.85	0.00	6	0	0.0	0.0	0.0	0.0
harp2	0.00	0.00	0	0	0.7	0.2	0.7	0.2
khb05250	98.16	0.20	87	12	12.9	3.6	1.3	0.5
lseu	42.88	6.38	37	18	0.0	0.0	0.0	0.0
mitre	21.27	14.50	1031	128	41.3	35.8	6.9	6.0
mkc	2.36	2.31	143	84	228.5	159.3	28.6	17.1
mod008	35.48	5.72	18	4	3.8	2.4	0.2	0.1
mod010	17.28	-1.04	3	2	0.3	0.2	0.1	0.0
mod011	49.34	0.15	258	1	274.5	107.4	16.1	5.7
modglob	25.21	0.35	36	-4	1.6	0.4	0.3	0.1
momentum2	0.04	0.04	16	5	378.1	336.5	94.5	84.1
msc98-ip	0.84	0.10	306	19	432.0	350.7	72.0	60.4
neos616206	3.04	-0.02	195	0	4.9	2.1	0.4	0.1
neos8	4.17	4.17	7	1	47.9	45.0	23.9	22.5
neos14	55.43	9.65	92	16	0.3	0.1	0.1	0.0
neos15	48.48	8.46	92	16	0.3	0.1	0.1	0.0
neos16	9.52	0.00	132	0	0.3	0.2	0.1	0.1
neos22	3.96	0.00	111	0	22.2	17.2	3.7	2.9
net12	2.67	0.00	96	7	208.0	187.0	26.0	24.1
nsrand-ipx	12.13	7.34	89	20	7.5	6.1	0.9	0.7
p0033	9.56	0.22	15	6	0.0	0.0	0.0	0.0
p0282	95.61	1.63	133	12	0.4	0.3	0.0	0.0
p0548	87.37	14.32	174	88	0.6	0.5	0.0	0.0
p2756	85.78	13.39	259	31	3.3	2.3	0.4	0.3
pp08aCUTS	0.55	0.00	4	0	0.0	0.0	0.0	0.0
prod1	0.13	0.02	13	1	1.8	1.1	0.3	0.2
qnet1	41.25	12.09	56	30	0.5	0.4	0.1	0.1
qnet1_o	59.05	4.75	55	22	0.4	0.3	0.0	0.0
ran10x26	40.37	1.84	53	4	0.4	0.3	0.0	0.0
ran12x21	45.68	6.23	59	4	0.2	0.1	0.0	0.0
ran13x13	46.55	7.13	54	12	0.1	0.1	0.0	0.0
ran14x18_1	44.73	5.52	110	40	0.5	0.5	0.0	0.0
ran8x32	66.63	4.03	61	14	0.5	0.4	0.0	0.0
rentacar	0.69	0.69	10	6	708.7	352.0	236.2	57.9
rgn	57.49	57.49	32	32	0.2	0.2	0.1	0.1
roll3000	6.14	5.99	107	43	6.5	4.6	0.9	0.4
rout	0.27	0.00	62	4	1.3	1.0	0.2	0.1
set1ch	37.78	0.04	128	1	0.1	0.0	0.0	0.0
sp97ar	0.77	0.69	9	2	6.7	5.2	3.3	2.5
timtab1	7.51	-0.05	44	4	0.1	0.1	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
timtab2	5.63	0.34	116	25	0.2	0.1	0.0	0.0
tr12-30	59.69	21.35	343	117	1.2	0.3	0.4	0.2
vpm1	64.67	-0.28	22	0	0.0	0.0	0.0	0.0
vpm2	71.28	1.67	71	-1	0.1	0.1	0.0	0.0
Total	2034.49	242.89	5935	1075	5082.9	3465.4	953.0	583.5
Geom. Mean	13.52	2.60	42	10	4.9	2.1	2.4	0.7

Table B.53: Computational results for the cutting plane separator for the 0-1 single node flow problem on the main test set. *Cut generation heuristic.* Use $N_3^* = \{u_j : j \in N \text{ and } u_j > \lambda\} \cup \{\max\{u_j : j \in N \text{ and } u_j \geq \lambda\} + 1, \lambda + 1\}$ as candidate set for the value of $\bar{u}$. Application to all rows of a MIP. (△ with respect to the default algorithm (applied to all rows of a MIP))

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
a1c1s1	30.41	3.27	295	119	21.2	10.7	2.6	1.1
aflow30a	37.76	4.28	129	39	1.2	0.8	0.1	0.1
aflow40b	30.39	6.65	94	35	9.6	7.4	0.5	0.3
arki001	2.13	-1.41	104	32	203.3	128.2	33.9	21.4
atlanta-ip	0.09	0.09	38	22	522.6	267.7	130.7	79.7
bc1	30.93	-1.09	10	-4	1132.6	727.4	188.8	138.1
bienst1	6.64	0.04	112	55	0.2	0.1	0.0	0.0
bienst2	7.45	0.02	168	84	0.2	0.1	0.0	0.0
binkar10_1	27.82	1.68	38	10	1.3	0.8	0.2	0.1
blend2	4.59	0.00	15	5	84.3	58.6	16.9	11.8
cap6000	0.00	0.00	1	1	0.5	0.4	0.2	0.1
dano3mip	0.01	0.00	4	0	135.6	93.6	27.1	18.7
danoint	0.66	0.08	41	19	0.7	0.4	0.1	0.0
dcmulti	21.96	0.00	54	16	0.1	0.0	0.1	0.1
egout	98.57	3.84	23	10	0.1	0.0	0.0	0.0
fiber	88.81	0.58	79	5	0.2	0.1	0.0	0.0
fixnet6	52.42	11.36	147	68	2.6	1.8	0.1	0.1
gen	98.17	0.00	7	0	0.1	0.1	0.0	0.0
gesa2	60.69	0.16	66	19	41.4	30.3	8.3	6.1
gesa2-o	9.03	1.57	41	15	21.8	16.6	7.3	5.6
gesa3	23.34	0.00	14	0	13.9	9.3	6.9	4.6
gesa3_o	6.98	0.00	12	1	12.2	8.3	6.1	4.2
gt2	33.85	0.00	7	1	0.0	0.0	0.0	0.0
harp2	0.00	0.00	0	0	2.5	2.0	2.5	2.0
khb05250	98.01	0.05	102	27	14.6	5.3	1.3	0.5
lseu	39.28	2.78	19	0	0.0	0.0	0.0	0.0
mitre	6.78	0.01	966	63	15.5	10.0	2.6	1.7
mkc	0.76	0.71	83	24	130.2	61.0	21.7	10.2
mod008	29.76	0.00	15	1	309.6	308.2	22.1	22.0
mod010	18.32	0.00	1	0	0.2	0.1	0.1	0.0
mod011	49.56	0.37	319	62	622.6	455.5	36.6	26.2
modglob	50.75	25.89	107	67	4.8	3.6	0.8	0.6
momentum2	0.05	0.05	29	18	88.9	47.3	22.2	11.8
msc98-ip	0.74	0.00	322	35	288.5	207.2	48.1	36.5
neos616206	3.25	0.19	219	24	5.3	2.5	0.5	0.2
neos8	0.00	0.00	6	0	6.7	3.8	3.4	2.0
neos14	45.78	0.00	80	4	0.4	0.2	0.2	0.1
neos15	40.02	0.00	80	4	0.4	0.2	0.2	0.1
neos16	9.52	0.00	165	33	0.3	0.2	0.0	0.0
neos22	3.96	0.00	114	3	11.7	6.7	1.9	1.1
net12	2.67	0.00	92	3	55.2	34.2	4.6	2.7
nsrand-ipx	12.13	7.34	94	25	43.1	41.7	5.4	5.2
p0033	37.34	28.00	21	12	0.0	0.0	0.0	0.0
p0282	96.13	2.15	146	25	0.3	0.2	0.0	0.0
p0548	85.45	12.40	158	72	0.5	0.4	0.0	0.0
p2756	85.04	12.65	348	120	2.4	1.4	0.3	0.2
pp08aCUTS	0.88	0.33	6	2	0.0	0.0	0.0	0.0
prod1	0.82	0.71	108	96	13.0	12.3	2.2	2.1
qnet1	69.47	40.31	89	63	0.4	0.3	0.0	0.0
qnet1_o	71.86	17.56	91	58	0.3	0.2	0.0	0.0
ran10x26	41.46	2.93	66	17	0.1	0.0	0.0	0.0
ran12x21	49.64	10.19	113	58	0.1	0.0	0.0	0.0
ran13x13	49.63	10.21	105	63	0.1	0.1	0.0	0.0
ran14x18_1	42.79	3.58	165	95	0.2	0.2	0.0	0.0
ran8x32	63.47	0.87	70	23	0.1	0.0	0.0	0.0
rentacar	3.65	3.65	6	2	1146.6	789.9	573.3	395.0
rgn	0.00	0.00	0	0	0.1	0.1	0.1	0.1
roll3000	20.77	20.62	105	41	14.1	12.2	2.0	1.5
rout	0.00	-0.27	82	24	0.6	0.3	0.1	0.0
set1ch	37.74	0.00	244	117	0.1	0.0	0.0	0.0
sp97ar	0.77	0.69	8	1	13.7	12.2	6.9	6.1
timtab1	7.56	0.00	41	1	0.0	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
timtab2	5.29	0.00	89	-2	0.1	0.0	0.0	0.0
tr12-30	41.56	3.22	257	31	3.9	3.0	0.7	0.5
vpm1	78.97	14.02	37	15	0.0	0.0	0.0	0.0
vpm2	69.36	-0.25	91	19	0.0	0.0	0.0	0.0
Total	2043.68	252.08	6758	1898	5003.1	3385.6	1189.9	820.4
Geom. Mean	12.78	1.86	46	14	5.5	2.7	2.6	0.9

Table B.54: Computational results for the cutting plane separator for the 0-1 single node flow problem on the main test set. *Default algorithm.* Application to all rows of a MIP including the separation of the class of c-MIRFPIs. (△ with respect to the default algorithm (applied to all rows of a MIP))

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
a1c1s1	30.51	3.37	353	177	14.7	4.2	2.1	0.6
aflow30a	37.44	3.96	124	34	3.5	3.1	0.2	0.2
aflow40b	31.66	7.92	131	72	20.8	18.6	1.1	0.9
arki001	2.48	-1.06	98	26	1.6	-73.5	0.3	-12.2
atlanta-ip	0.18	0.18	54	38	1675.8	1420.9	279.3	228.3
bc1	34.38	2.36	14	0	50.6	-354.6	5.1	-45.6
bienst1	6.64	0.04	115	58	0.4	0.3	0.0	0.0
bienst2	7.45	0.02	166	82	0.6	0.5	0.1	0.1
binkar10_1	45.99	19.85	49	21	0.5	0.0	0.1	0.0
blend2	10.86	6.27	23	13	0.9	-24.8	0.1	-5.0
cap6000	26.95	26.95	9	9	388.7	388.6	64.8	64.7
dano3mip	0.01	0.00	4	0	18.6	-23.4	3.7	-4.7
danoint	0.66	0.08	42	20	0.3	0.0	0.1	0.0
dcmulti	21.70	-0.26	52	14	0.0	-0.1	0.0	0.0
egout	95.30	0.57	23	10	0.0	-0.1	0.0	0.0
fiber	90.34	2.11	103	29	0.6	0.5	0.0	0.0
fixnet6	53.02	11.96	141	62	0.6	-0.2	0.0	0.0
gen	98.13	-0.04	15	8	0.1	0.1	0.0	0.0
gesa2	61.32	0.79	80	33	0.9	-10.2	0.2	-2.0
gesa2-o	9.34	1.88	43	17	0.3	-4.9	0.1	-1.6
gesa3	33.26	9.92	18	4	0.6	-4.0	0.2	-2.1
gesa3_o	6.98	0.00	13	2	0.2	-3.7	0.1	-1.8
gt2	33.85	0.00	6	0	0.0	0.0	0.0	0.0
harp2	9.78	9.78	8	8	0.3	-0.2	0.1	-0.4
khb05250	98.16	0.20	102	27	0.3	-9.0	0.0	-0.8
lseu	50.32	13.82	44	25	0.1	0.1	0.0	0.0
mitre	25.08	18.31	1128	225	72.5	67.0	12.1	11.2
mkc	7.48	7.43	259	200	28.4	-40.8	2.0	-9.5
mod008	37.25	7.49	20	6	2.8	1.4	0.1	0.0
mod010	18.32	0.00	3	2	0.1	0.0	0.0	-0.1
mod011	49.31	0.12	371	114	13.0	-154.1	0.9	-9.5
modglob	59.67	34.81	112	72	0.3	-0.9	0.0	-0.2
momentum2	0.05	0.05	30	19	156.6	115.0	39.2	28.8
msc98-ip	0.97	0.23	393	106	490.7	409.4	70.1	58.5
neos616206	3.29	0.23	238	43	0.7	-2.1	0.1	-0.2
neos8	4.17	4.17	7	1	35.1	32.2	17.6	16.2
neos14	55.43	9.65	143	67	0.1	-0.1	0.1	0.0
neos15	48.48	8.46	143	67	0.1	-0.1	0.1	0.0
neos16	9.52	0.00	135	3	0.5	0.4	0.1	0.1
neos22	3.96	0.00	106	-5	13.6	8.6	1.9	1.1
net12	2.67	0.00	99	10	242.0	221.0	30.2	28.3
nsrand-ipx	12.49	7.70	105	36	13.6	12.2	1.5	1.3
p0033	67.84	58.50	32	23	0.0	0.0	0.0	0.0
p0282	96.58	2.60	167	46	0.6	0.5	0.0	0.0
p0548	88.87	15.82	191	105	0.7	0.6	0.0	0.0
p2756	85.78	13.39	348	120	3.4	2.4	0.4	0.3
pp08aCUTS	0.98	0.43	6	2	0.0	0.0	0.0	0.0
prod1	0.82	0.71	107	95	3.7	3.0	0.6	0.5
qnet1	41.25	12.09	56	30	0.3	0.2	0.0	0.0
qnet1_o	59.60	5.30	61	28	0.2	0.1	0.0	0.0
ran10x26	45.85	7.32	97	48	0.4	0.3	0.0	0.0
ran12x21	55.91	16.46	117	62	0.2	0.1	0.0	0.0
ran13x13	50.14	10.72	129	87	0.2	0.2	0.0	0.0
ran14x18_1	49.10	9.89	206	136	0.6	0.6	0.0	0.0
ran8x32	68.61	6.01	127	80	0.5	0.4	0.0	0.0
rentacar	14.60	14.60	34	30	3.0	-353.7	0.2	-178.1
rgn	57.49	57.49	48	48	0.0	0.0	0.0	0.0
roll3000	28.97	28.82	117	53	1.5	-0.4	0.2	-0.3
rout	0.15	-0.12	97	39	0.3	0.0	0.0	-0.1
set1ch	37.78	0.04	245	118	0.1	0.0	0.0	0.0
sp97ar	0.77	0.69	9	2	9.9	8.4	5.0	4.2
timtab1	7.14	-0.42	42	2	0.1	0.1	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
timtab2	5.48	0.19	114	23	0.2	0.1	0.0	0.0
tr12-30	59.75	21.41	698	472	1.6	0.7	0.3	0.1
vpm1	78.69	13.74	38	16	0.0	0.0	0.0	0.0
vpm2	74.50	4.89	106	34	0.1	0.1	0.0	0.0
Total	2311.49	519.89	8314	3454	3278.7	1661.2	540.5	171.0
Geom. Mean	17.30	6.38	64	32	2.9	0.1	1.7	0.0

Table B.55: Computational results for the cutting plane separator for the 0-1 single node flow problem on the main test set. *Resulting algorithm.* Apply the fixing strategy suggested in Section 5.3.1 to KP^{SNF}_{int} and KP^{SNF}_{rat}. Solve KP^{SNF}_{int} exactly using Algorithm 4.1 if the calculated scaling factor γ is not greater than 1,000 and nc for KP^{SNF}_{int} is not greater than 1,000,000, and solve KP^{SNF}_{rat} approximately using Algorithm 5.1 otherwise. Use N_3^* as candidate set for the value of $\bar{u}$. Application to all rows of a MIP including the separation of the class of c-MIRFPIs. (△ with respect to the default algorithm (applied to all rows of a MIP))

Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
10teams	0.00	0	0.1	0.1
30:70:4_5:0_5:100	0.00	0	15.8	15.8
30:70:4_5:0_95:98	0.00	0	16.7	16.7
air03	0.00	0	0.2	0.2
air04	0.00	0	0.5	0.5
air05	0.00	0	0.2	0.2
bell3a	0.00	0	0.0	0.0
bell5	0.00	0	0.0	0.0
dano3_3	0.00	1	4.1	2.1
dano3_4	0.00	1	4.4	2.2
dano3_5	0.00	1	4.5	2.2
ds	0.00	0	9.7	9.7
eilD76	0.00	0	0.0	0.0
fast0507	0.00	0	4.5	4.5
flugpl	0.00	0	0.0	0.0
glass4	0.00	22	0.0	0.0
irp	0.00	0	0.4	0.4
l152lav	0.00	0	0.1	0.1
liu	0.00	0	0.0	0.0
manna81	0.00	0	9.6	9.6
markshare1	0.00	10	0.0	0.0
markshare2	0.00	5	0.1	0.0
mas284	0.00	0	0.2	0.2
mas74	0.00	0	0.1	0.1
mas76	0.00	0	0.1	0.1
misc03	0.00	0	0.0	0.0
misc06	0.00	0	0.0	0.0
misc07	0.00	0	0.0	0.0
mkc1	0.00	55	10.9	1.8
momentum1	0.00	17	118.3	29.6
mzzv11	0.00	4	25.8	12.9
mzzv42z	0.00	0	15.8	15.8
neos1	0.00	322	0.7	0.1
neos2	0.00	0	0.1	0.1
neos3	0.00	0	0.2	0.2
neos632659	14.02	227	0.2	0.0
neos648910	0.00	130	0.3	0.1
neos7	0.00	1	0.3	0.2
neos9	0.00	0	161.2	161.2
neos10	0.00	13	39.6	19.8
neos11	0.00	0	0.3	0.3
neos12	0.00	0	2.5	2.5
neos13	0.00	0	12.9	12.9
neos17	0.00	0	0.1	0.1
neos18	0.00	0	0.9	0.9
neos19	0.00	8542	1048.2	349.4
neos20	0.00	53	0.5	0.1
neos21	0.00	0	0.1	0.1
neos23	0.00	0	0.1	0.1
noswot	0.00	29	0.0	0.0
nug08	0.00	0	0.2	0.2
nw04	0.00	0	2.3	2.3
opt1217	0.00	0	0.0	0.0
p0201	0.00	8	0.0	0.0
pk1	0.00	0	0.0	0.0
pp08a	0.29	6	0.0	0.0
protfold	0.00	0	0.3	0.3
qap10	0.00	0	0.9	0.9
qiu	0.00	0	0.1	0.1
seymour	0.00	0	3.5	3.5
seymour1	0.00	0	1.5	1.5
stein27	0.00	0	0.0	0.0
stein45	0.00	0	0.0	0.0

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Name	Gap Closed %	Cuts	Sepa Time	Average Sepa Time
swath	0.00	0	0.1	0.1
swath1	0.00	0	0.2	0.2
swath2	0.00	0	0.2	0.2
swath3	0.00	0	0.2	0.2
t1717	0.00	0	7.3	7.3
Total	14.30	9447	1527.1	689.7
Geom. Mean	1.04	2	2.2	1.9

Table B.56: Computational results for the cutting plane separator for the 0-1 single node flow problem on the remaining test set. *Resulting algorithm.* Apply the fixing strategy suggested in Section 5.3.1 to KP_{int}^{SNF} and KP_{rat}^{SNF}. Solve KP_{int}^{SNF} exactly using Algorithm 4.1 if the calculated scaling factor γ is not greater than 1,000 and nc for KP_{int}^{SNF} is not greater than 1,000,000, and solve KP_{rat}^{SNF} approximately using Algorithm 5.1 otherwise. Use N_3^* as candidate set for the value of $\bar{u}$. Application to all rows of a MIP including the separation of the class of c-MIRFPIs.

	Gap Closed %		Cuts		Sepa Time		Average Sepa Time	
Name	Value	△	Value	△	Value	△	Value	△
a1c1s1	27.58	-2.93	316	-37	2.8	-11.9	0.4	-1.7
aflow30a	33.29	-4.15	96	-28	0.6	-2.9	0.1	-0.1
aflow40b	28.65	-3.01	93	-38	2.7	-18.1	0.3	-0.8
arki001	2.48	0.00	97	-1	1.2	-0.4	0.2	-0.1
atlanta-ip	0.00	-0.18	0	-54	1.2	-1674.6	1.2	-278.1
bc1	34.82	0.44	16	2	5.5	-45.1	0.6	-4.5
bienst1	6.64	0.00	115	0	0.4	0.0	0.0	0.0
bienst2	7.45	0.00	166	0	0.5	-0.1	0.1	0.0
binkar10_1	32.43	-13.56	64	15	0.4	-0.1	0.1	0.0
blend2	8.38	-2.48	21	-2	0.3	-0.6	0.0	-0.1
cap6000	26.95	0.00	10	1	2.3	-386.4	0.4	-64.4
dano3mip	0.00	-0.01	0	-4	0.5	-18.1	0.5	-3.2
danoint	0.66	0.00	42	0	0.3	0.0	0.0	-0.1
dcmulti	21.70	0.00	52	0	0.0	0.0	0.0	0.0
egout	95.30	0.00	23	0	0.0	0.0	0.0	0.0
fiber	89.84	-0.50	103	0	0.4	-0.2	0.0	0.0
fixnet6	49.95	-3.07	118	-23	0.2	-0.4	0.0	0.0
gen	98.13	0.00	15	0	0.1	0.0	0.0	0.0
gesa2	61.32	0.00	76	-4	0.8	-0.1	0.1	-0.1
gesa2-o	9.34	0.00	43	0	0.3	0.0	0.1	0.0
gesa3	33.26	0.00	18	0	0.5	-0.1	0.1	-0.1
gesa3_o	6.98	0.00	13	0	0.2	0.0	0.1	0.0
gt2	33.85	0.00	6	0	0.0	0.0	0.0	0.0
harp2	9.59	-0.19	8	0	0.2	-0.1	0.0	-0.1
khb05250	98.16	0.00	100	-2	0.2	-0.1	0.0	0.0
lseu	46.91	-3.41	28	-16	0.0	-0.1	0.0	0.0
mitre	9.02	-16.06	743	-385	14.3	-58.2	2.4	-9.7
mkc	7.40	-0.08	209	-50	10.2	-18.2	1.0	-1.0
mod008	28.63	-8.62	15	-5	0.1	-2.7	0.0	-0.1
mod010	18.32	0.00	3	0	0.1	0.0	0.0	0.0
mod011	48.54	-0.77	348	-23	8.1	-4.9	0.8	-0.1
modglob	59.67	0.00	112	0	0.3	0.0	0.0	0.0
momentum2	0.04	-0.01	8	-22	6.8	-149.8	1.4	-37.8
msc98-ip	0.97	0.00	207	-186	59.8	-430.9	10.0	-60.1
neos616206	3.29	0.00	235	-3	0.5	-0.2	0.1	0.0
neos8	4.17	0.00	7	0	1.8	-33.3	0.9	-16.7
neos14	55.43	0.00	143	0	0.1	0.0	0.1	0.0
neos15	48.48	0.00	143	0	0.1	0.0	0.1	0.0
neos16	9.52	0.00	157	22	0.3	-0.2	0.0	-0.1
neos22	0.00	-3.96	0	-106	0.2	-13.4	0.2	-1.7
net12	1.92	-0.75	82	-17	15.8	-226.2	1.8	-28.4
nsrand-ipx	12.49	0.00	104	-1	11.5	-2.1	1.3	-0.2
p0033	63.53	-4.31	25	-7	0.0	0.0	0.0	0.0
p0282	95.69	-0.89	100	-67	0.1	-0.5	0.0	0.0
p0548	88.49	-0.38	177	-14	0.3	-0.4	0.0	0.0
p2756	85.79	0.01	338	-10	2.5	-0.9	0.3	-0.1
pp08aCUTS	0.98	0.00	6	0	0.0	0.0	0.0	0.0
prod1	0.82	0.00	107	0	1.9	-1.8	0.3	-0.3
qnet1	41.25	0.00	56	0	0.3	0.0	0.0	0.0
qnet1_o	59.60	0.00	61	0	0.2	0.0	0.0	0.0
ran10x26	45.64	-0.21	81	-16	0.2	-0.2	0.0	0.0
ran12x21	54.75	-1.16	117	0	0.2	0.0	0.0	0.0
ran13x13	50.03	-0.11	120	-9	0.1	-0.1	0.0	0.0
ran14x18_1	48.54	-0.56	185	-21	0.2	-0.4	0.0	0.0
ran8x32	66.79	-1.82	102	-25	0.2	-0.3	0.0	0.0
rentacar	14.60	0.00	32	-2	1.7	-1.3	0.2	0.0
rgn	57.49	0.00	48	0	0.0	0.0	0.0	0.0
roll3000	28.28	-0.69	121	4	1.2	-0.3	0.1	-0.1
rout	0.13	-0.02	95	-2	0.1	-0.2	0.0	0.0
set1ch	37.78	0.00	245	0	0.1	0.0	0.0	0.0
sp97ar	0.77	0.00	9	0	5.0	-4.9	2.5	-2.5
timtab1	7.14	0.00	42	0	0.1	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
timtab2	5.47	-0.01	113	-1	0.1	-0.1	0.0	0.0
tr12-30	32.79	-26.96	395	-303	0.2	-1.4	0.1	-0.2
vpm1	78.69	0.00	38	0	0.0	0.0	0.0	0.0
vpm2	74.50	0.00	106	0	0.1	0.0	0.0	0.0
Total	2211.08	-100.41	6874	-1440	166.7	-3112.0	28.1	-512.4
Geom. Mean	16.07	-1.23	50	-14	1.5	-1.4	1.1	-0.6

Table B.57: Computational results for the cutting plane separator for the 0-1 single node flow problem on the main test set. *Resulting algorithm with* MAXTESTDELTA $= 10$. Application to the rows of a MIP (ordered by nonincreasing value of ROWSCORE^i, $i \in P$) including the separation of the class of c-MIRFPIs. Limit the application by using MAXFAILS $= 100$, MAXCUTS $= 200$ and MAXROUNDS $= 10$. (△ with respect to the resulting algorithm (applied to all rows of a MIP including the separation of the class of c-MIRFPIs))

Name	Gap Closed % Value	Gap Closed % △	Cuts Value	Cuts △	Sepa Time Value	Sepa Time △	Average Sepa Time Value	Average Sepa Time △
10teams	0.00	0.00	0	0	0.0	-0.1	0.0	-0.1
30:70:4_5:0_5:100	0.00	0.00	0	0	0.6	-15.2	0.6	-15.2
30:70:4_5:0_95:98	0.00	0.00	0	0	0.6	-16.1	0.6	-16.1
air03	0.00	0.00	0	0	0.2	0.0	0.2	0.0
air04	0.00	0.00	0	0	0.2	-0.3	0.2	-0.3
air05	0.00	0.00	0	0	0.2	0.0	0.2	0.0
bell3a	0.00	0.00	0	0	0.0	0.0	0.0	0.0
bell5	0.00	0.00	0	0	0.0	0.0	0.0	0.0
dano3_3	0.00	0.00	0	-1	0.5	-3.6	0.5	-1.6
dano3_4	0.00	0.00	0	-1	0.5	-3.9	0.5	-1.7
dano3_5	0.00	0.00	0	-1	0.5	-4.0	0.5	-1.7
ds	0.00	0.00	0	0	2.8	-6.9	2.8	-6.9
eilD76	0.00	0.00	0	0	0.0	0.0	0.0	0.0
fast0507	0.00	0.00	0	0	1.9	-2.6	1.9	-2.6
flugpl	0.00	0.00	0	0	0.0	0.0	0.0	0.0
glass4	0.00	0.00	22	0	0.0	0.0	0.0	0.0
irp	0.00	0.00	0	0	0.4	0.0	0.4	0.0
l152lav	0.00	0.00	0	0	0.0	-0.1	0.0	-0.1
liu	0.00	0.00	0	0	0.0	0.0	0.0	0.0
manna81	0.00	0.00	0	0	0.4	-9.2	0.4	-9.2
markshare1	0.00	0.00	6	-4	0.0	0.0	0.0	0.0
markshare2	0.00	0.00	4	-1	0.0	-0.1	0.0	0.0
mas284	0.00	0.00	0	0	0.0	-0.2	0.0	-0.2
mas74	0.00	0.00	0	0	0.1	0.0	0.1	0.0
mas76	0.00	0.00	0	0	0.0	-0.1	0.0	-0.1
misc03	0.00	0.00	0	0	0.0	0.0	0.0	0.0
misc06	0.00	0.00	0	0	0.0	0.0	0.0	0.0
misc07	0.00	0.00	0	0	0.0	0.0	0.0	0.0
mkc1	0.00	0.00	59	4	3.7	-7.2	0.6	-1.2
momentum1	0.00	0.00	1	-16	2.9	-115.4	1.4	-28.2
mzzv11	0.00	0.00	0	-4	0.5	-25.3	0.5	-12.4
mzzv42z	0.00	0.00	0	0	0.7	-15.1	0.7	-15.1
neos1	0.00	0.00	322	0	0.3	-0.4	0.1	0.0
neos2	0.00	0.00	0	0	0.1	0.0	0.1	0.0
neos3	0.00	0.00	0	0	0.1	-0.1	0.1	-0.1
neos632659	14.02	0.00	213	-14	0.2	0.0	0.0	0.0
neos648910	0.00	0.00	129	-1	0.2	-0.1	0.1	0.0
neos7	0.00	0.00	1	0	0.1	-0.2	0.1	-0.1
neos9	0.00	0.00	0	0	4.4	-156.8	4.4	-156.8
neos10	0.00	0.00	13	0	2.3	-37.3	1.1	-18.7
neos11	0.00	0.00	0	0	0.0	-0.3	0.0	-0.3
neos12	0.00	0.00	0	0	0.2	-2.3	0.2	-2.3
neos13	0.00	0.00	0	0	2.1	-10.8	2.1	-10.8
neos17	0.00	0.00	0	0	0.0	-0.1	0.0	-0.1
neos18	0.00	0.00	0	0	0.1	-0.8	0.1	-0.8
neos19	0.00	0.00	0	-8542	2.6	-1045.6	2.6	-346.8
neos20	0.00	0.00	53	0	0.2	-0.3	0.1	0.0
neos21	0.00	0.00	0	0	0.0	-0.1	0.0	-0.1
neos23	0.00	0.00	0	0	0.0	-0.1	0.0	-0.1
noswot	0.00	0.00	29	0	0.0	0.0	0.0	0.0
nug08	0.00	0.00	0	0	0.0	-0.2	0.0	-0.2
nw04	0.00	0.00	0	0	1.7	-0.6	1.7	-0.6
opt1217	0.00	0.00	0	0	0.0	0.0	0.0	0.0
p0201	0.00	0.00	8	0	0.0	0.0	0.0	0.0
pk1	0.00	0.00	0	0	0.0	0.0	0.0	0.0
pp08a	0.29	0.00	6	0	0.0	0.0	0.0	0.0
protfold	0.00	0.00	0	0	0.1	-0.2	0.1	-0.2
qap10	0.00	0.00	0	0	0.1	-0.8	0.1	-0.8
qiu	0.00	0.00	0	0	0.0	-0.1	0.0	-0.1
seymour	0.00	0.00	0	0	0.1	-3.4	0.1	-3.4
seymour1	0.00	0.00	0	0	0.1	-1.4	0.1	-1.4
stein27	0.00	0.00	0	0	0.0	0.0	0.0	0.0

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Name	Gap Closed % Value	△	Cuts Value	△	Sepa Time Value	△	Average Sepa Time Value	△
stein45	0.00	0.00	0	0	0.0	0.0	0.0	0.0
swath	0.00	0.00	0	0	0.1	0.0	0.1	0.0
swath1	0.00	0.00	0	0	0.2	0.0	0.2	0.0
swath2	0.00	0.00	0	0	0.2	0.0	0.2	0.0
swath3	0.00	0.00	0	0	0.2	0.0	0.2	0.0
t1717	0.00	0.00	0	0	2.2	-5.1	2.2	-5.1
Total	14.30	0.00	866	-8581	34.8	-1492.3	28.2	-661.5
Geom. Mean	1.04	0.00	1	-1	1.1	-1.1	1.1	-0.8

Table B.58: Computational results for the cutting plane separator for the 0-1 single node flow problem on the remaining test set. *Resulting algorithm with* MAXTESTDELTA = 10. Application to the rows of a MIP (ordered by nonincreasing value of ROWSCORE^i, $i \in P$) including the separation of the class of c-MIRFPIs. Limit the application by using MAXFAILS = 100, MAXCUTS = 200 and MAXROUNDS = 10. (△ with respect to the resulting algorithm (applied to all rows of a MIP including the separation of the class of c-MIRFPIs))

B.4 Comparison with Cplex and Cbc

Name	Type	Conss	Vars	z_{LP}	z_{MIP}
10teams.pre	BIP	210	1600	897	904
30:70:4_5:0_5:100.pre	BIP	12012	10751	5337.1	5338
30:70:4_5:0_95:98.pre	BIP	12392	10910	5969.5	5970
a1c1s1.pre	BMIP	2283	2619	2209.5717	11566.5904
aflow30a.pre	BMIP	455	818	91.1674253	266
aflow40b.pre	BMIP	1405	2691	-578.335183	-416
air03.pre	BIP	122	8457	311406.25	312702
air04.pre	BIP	614	7564	54030.4364	54632
air05.pre	BIP	343	6139	25877.6093	26374
arki001.pre	MIP	776	959	7009391.43	7010584.11
atlanta-ip.pre	MIP	19446	17343	81.2553981	95.0073127
bc1.pre	BMIP	1337	1043	2.18877635	3.33836255
bell3a.pre	MIP	86	100	862116.583	874375.166
bell5.pre	MIP	77	94	8341834.36	8699822.9
bienst1.pre	BMIP	520	449	11.7241379	46.75
bienst2.pre	BMIP	520	449	11.7241379	54.6
binkar10_1.pre	BMIP	823	2019	5728.13803	5833.15003
blend2.pre	MIP	184	334	6.91567511	7.598985
cap6000.pre	BIP	1891	4689	-2403390.33	-2403230
dano3_3.pre	BMIP	3151	13837	576.23162	576.344633
dano3_4.pre	BMIP	3151	13837	576.23162	576.435225
dano3_5.pre	BMIP	3151	13837	576.23162	576.924916
dano3mip.pre	BMIP	3151	13837	576.23162	705.941176
danoint.pre	BMIP	600	457	62.6372804	65.67
dcmulti.pre	BMIP	239	515	184034.377	188182
ds.pre	BIP	656	67732	57.2345653	468.645
egout.pre	BMIP	35	47	242.52422	299.00708
eilD76.pre	BIP	75	1893	680.538997	885.411847
fast0507.pre	BIP	472	62167	158.145567	160
fiber.pre	BMIP	363	1298	156082.518	405935.18
fixnet6.pre	BMIP	477	877	3190.042	3981
flugpl.pre	MIP	13	14	726875.166	760500
gen.pre	BMIP	384	543	58319.9441	58349.0903
gesa2-o.pre	MIP	1176	1152	18717600.8	19020967.5
gesa2.pre	MIP	1344	1176	25502855	25779856.4
gesa3.pre	MIP	1296	1080	27884380.4	27991042.6
gesa3_o.pre	MIP	1104	1032	12274783.2	12432193.4
glass4.pre	BMIP	392	317	800002400	1.6000134e+09
gt2.pre	IP	28	180	20146.7613	21166
harp2.pre	BIP	92	1035	-74232132.3	-73806560
irp.pre	BIP	39	19370	12123.5302	12159.4928
khb05250.pre	BMIP	100	1299	95919464	106940226
l152lav.pre	BIP	97	1988	4656.36364	4722
liu.pre	BMIP	2178	1154	346	1146
lseu.pre	BIP	28	85	949.518722	1120
manna81.pre	IP	6480	3321	-13297	-13164
markshare1.pre	BMIP	6	56	0	1
markshare2.pre	BMIP	7	67	0	1
mas284.pre	MIP	68	148	86195.863	91405.7237
mas74.pre	MIP	13	148	10482.7953	11801.1857
mas76.pre	MIP	12	148	38893.9036	40005.0541
misc03.pre	BIP	95	153	1910	3360
misc06.pre	BMIP	511	1373	12841.6894	12850.8607
misc07.pre	BIP	211	253	1415	2810
mitre.pre	BIP	1471	8469	114908.999	115155
mkc.pre	IP	1286	3223	-611.437978	-563.212
mkc1.pre	MIP	2830	4851	-611.85	-607.207
mod008.pre	BIP	6	319	290.931073	307
mod010.pre	BIP	144	2569	6532.08333	6548
mod011.pre	BMIP	1404	7022	-61678103.8	-54558535

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Name	Type	Conss	Vars	z_{LP}	z_{MIP}
modglob.pre	BMIP	286	384	19790205.8	20099766.2
momentum1.pre	BMIP	11633	3579	47128.5523	83478.2514
momentum2.pre	BMIP	19116	3306	1225.6443	6314.21959
msc98-ip.pre	MIP	15024	12798	18473197.7	22213598
mzzv11.pre	IP	8272	8775	-22773.75	-21718
mzzv42z.pre	IP	9951	11291	-21446.2397	-20540
neos1.pre	BIP	1018	1728	12	19
neos10.pre	IP	3743	1892	-1198.33333	-1137
neos11.pre	BMIP	2566	1190	6	9
neos12.pre	BMIP	5377	3780	9.41161243	13
neos13.pre	BMIP	19278	1826	-126.178378	-95.4748066
neos14.pre	BMIP	436	675	28644.7958	70244.0244
neos15.pre	BMIP	459	698	28570.7845	75958.6821
neos16.pre	IP	850	377	429	450
neos17.pre	BMIP	486	511	0.000681498501	0.150002577
neos18.pre	BIP	3060	760	5.33333333	13
neos19.pre	BMIP	23094	78344	-1611	-1499
neos2.pre	BMIP	1103	2101	-4717.66685	454.864697
neos20.pre	MIP	1298	735	-475	-434
neos21.pre	BIP	1074	592	2.21648352	7
neos22.pre	BMIP	4300	2786	777191.429	779715
neos23.pre	BMIP	1162	435	56	137
neos3.pre	BMIP	1442	2747	-6571.62916	368.842751
neos616206.pre	BMIP	534	480	787.721258	937.6
neos632659.pre	BMIP	170	290	-109.714286	-94
neos648910.pre	BMIP	650	273	16	32
neos7.pre	MIP	1715	1407	571556.49	714163
neos8.pre	IP	3303	1660	-3725	-3719
neos9.pre	BMIP	31600	81408	780	798
net12.pre	BMIP	13757	13843	26.7541667	214
noswot.pre	MIP	172	120	-43	-41
nsrand-ipx.pre	BIP	535	4073	49667.8923	51520
nug08.pre	BIP	912	1632	203.5	214
nw04.pre	BIP	36	46190	16310.6667	16862
opt1217.pre	BMIP	64	759	-20.0213904	-16
p0033.pre	BIP	13	28	2262.54674	2513
p0201.pre	BIP	107	183	7125	7615
p0282.pre	BIP	160	200	179990.3	258401
p0548.pre	BIP	129	409	5194.43615	8691
p2756.pre	IP	642	2371	2701.75	3124
pk1.pre	BMIP	45	86	0	11
pp08a.pre	BMIP	133	234	2748.34524	7350
pp08aCUTS.pre	BMIP	239	235	5280.60616	7150
prod1.pre	BIP	75	117	8.44984888	26
protfold.pre	BIP	2110	1835	-41.9574468	-23
qap10.pre	BIP	1820	4150	332.566228	340
qiu.pre	BMIP	1192	840	-931.638853	-132.873137
qnet1.pre	IP	363	1417	14274.1027	16029.6927
qnet1_o.pre	IP	245	1330	12907.7792	16029.6927
ran10x26.pre	BMIP	296	520	3857.02278	4270
ran12x21.pre	BMIP	285	504	3157.37744	3664
ran13x13.pre	BMIP	195	338	2691.43947	3252
ran14x18_1.pre	BMIP	284	504	3016.94435	3714
ran8x32.pre	BMIP	296	512	4937.58453	5247
rentacar.pre	BMIP	1010	2789	-3996349.94	-2567968.57
rgn.pre	BMIP	24	180	48.7999986	82.1999992
roll3000.pre	MIP	1109	948	11098.1402	12899
rout.pre	MIP	290	555	-1393.38571	-1297.69
set1ch.pre	BMIP	423	643	30269.8598	49689.5
seymour.pre	BIP	4624	1085	239.469492	257
seymour1.pre	BMIP	4634	1085	238.351528	244.763701
sp97ar.pre	BIP	1670	14085	648138794	658743127
stein27.pre	BIP	118	27	13	18
stein45.pre	BIP	331	45	22	30

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Name	Type	Conss	Vars	z_{LP}	z_{MIP}
swath.pre	BMIP	482	6260	334.496858	477.34101
swath1.pre	BMIP	482	6260	334.496858	379.071296
swath2.pre	BMIP	482	6260	334.496858	385.199693
swath3.pre	BMIP	482	6260	334.496858	397.761344
t1717.pre	BIP	551	16428	134531.021	288658
timtab1.pre	MIP	166	378	29032	765110
timtab2.pre	MIP	287	648	68068	1168706
tr12-30.pre	BMIP	722	1052	13924.1745	126396
vpm1.pre	BMIP	128	188	16.4333333	20
vpm2.pre	BMIP	128	188	11.1356321	13.75

Table B.59: Summary of the test set for the comparison with CPLEX and CBC.

	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
Name	Time Value	Dual Bound Value	Gap Closed % Value	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value
10teams.pre	0.7	904	100.00	3	10.5	904	100.00	0.00	5	0.1	904	100.00	0.00	2
30:70:4_5:0_5:100.pre	160.6	5337.66667	62.96	101	202.1	5338	100.00	37.04	28	251.4	5338	100.00	37.04	6
30:70:4_5:0_95:98.pre	159.0	5970	100.00	150	249.8	5970	100.00	0.00	16	284.1	5970	100.00	0.00	1
a1c1s1.pre	0.2	3882.02402	17.87	124	0.5	5246.98275	32.46	14.59	198	1.3	5660.69	36.88	19.01	117
aflow30a.pre	0.2	116.572474	14.53	34	0.1	121.507755	17.35	2.82	36	0.3	140.479	28.21	13.68	20
aflow40b.pre	0.3	-564.903867	8.27	14	0.7	-552.215342	16.09	7.82	43	1.1	-548.068	18.64	10.37	17
air03.pre	0.8	312702	100.00	16	0.3	312702	100.00	0.00	3	0.2	312702	100.00	0.00	1
air04.pre	3.9	54030.4364	0.00	0	15.1	54100.6265	11.67	11.67	8	0.5	54030.9	0.08	0.08	0
air05.pre	2.2	25877.6093	0.00	0	12.1	25913.4025	7.21	7.21	9	0.3	25877.6	0.00	0.00	0
arki001.pre	0.2	7009391.43	0.00	6	0.4	7009904.05	42.98	42.98	54	1.4	7009990	50.19	50.19	62
atlanta-ip.pre	63.1	81.2553981	0.00	0	72.5	81.3443459	0.65	0.65	4	2.8	81.2554	0.00	0.00	0
bc1.pre	0.8	2.18877635	0.00	0	2.0	2.21294248	2.10	2.10	2	0.8	2.2088	1.74	1.74	1
bell3a.pre	0.0	869109.014	57.04	11	0.0	869736.909	62.16	5.12	14	0.1	869188	57.69	0.65	7
bell5.pre	0.0	8387030.13	12.62	12	0.0	8406938.89	18.19	5.57	13	0.2	8656910	88.01	75.39	18
bienst1.pre	0.1	12.7710654	2.99	29	0.2	15.1029368	9.65	6.66	14	1.9	36.2447	70.01	67.02	16
bienst2.pre	0.1	12.7710654	2.44	36	0.2	14.4504909	6.36	3.92	25	3.1	36.303	57.33	54.89	13
binkar10_1.pre	0.1	5728.13803	0.00	0	0.5	5742.88349	14.04	14.04	24	0.1	5739.32	10.65	10.65	5
blend2.pre	0.0	6.91567511	0.00	0	0.0	7.0246493	15.95	15.95	2	0.9	7.1499	34.28	34.28	3
cap6000.pre	0.5	-2403390.33	0.00	0	0.1	-2403323.55	41.65	41.65	1	0.1	-2403390	0.00	0.00	0
dano3_3.pre	30.5	576.23162	0.00	0	99.5	576.234134	2.22	2.22	1	1.4	576.232	0.00	0.00	0
dano3_4.pre	35.4	576.23162	0.00	0	84.6	576.234953	1.64	1.64	3	1.6	576.232	0.00	0.00	0
dano3_5.pre	44.6	576.23162	0.00	0	85.5	576.249479	2.58	2.58	3	1.6	576.232	0.00	0.00	0
dano3mip.pre	38.3	576.23162	0.00	0	72.3	576.265696	0.03	0.03	8	0.5	576.232	0.00	0.00	0
danoint.pre	0.1	62.6372804	0.00	0	0.1	62.6460503	0.29	0.29	6	9.3	62.667	0.98	0.98	23
dcmulti.pre	0.1	185089.562	25.44	39	0.1	186877.886	68.56	43.12	47	3.3	187087	73.60	48.16	43
ds.pre	61.1	57.2345653	0.00	0	139.0	57.4064195	0.04	0.04	17	6.6	57.2346	0.00	0.00	0
egout.pre	0.0	266.004562	41.57	12	0.0	299.00708	100.00	58.43	17	0.2	298.39	98.91	57.34	12
eilD76.pre	0.2	680.538997	0.00	0	1.3	684.407956	1.89	1.89	6	0.0	680.539	0.00	0.00	0
fast0507.pre	44.0	158.145567	0.00	0	173.6	158.206927	3.31	3.31	6	0.8	158.146	0.02	0.02	0
fiber.pre	0.1	375159.832	87.68	44	0.1	386475.798	92.21	4.53	27	0.3	380808	89.94	2.26	0
fixnet6.pre	0.2	3415.06515	28.45	69	0.1	3408.57167	27.63	-0.82	27	0.1	3483.74	37.13	8.68	11
flugpl.pre	0.0	730596.276	11.07	11	0.0	732049.37	15.39	4.32	7	0.1	732518	16.78	5.71	6
gen.pre	0.0	58330.3843	35.82	10	0.0	58324.7035	16.33	-19.49	12	0.1	58336.4	56.46	20.64	7
gesa2-o.pre	0.1	18717643.9	0.01	2	0.1	18936542.9	72.17	72.16	89	4.7	18951800	77.20	77.19	73
gesa2.pre	0.1	25502888.7	0.01	2	0.1	25635171.3	47.77	47.76	68	2.6	25738000	84.89	84.88	55

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
	Time	Dual Bound	Gap Closed %	Cuts	Time	Dual Bound	Gap Closed %		Cuts	Time	Dual Bound	Gap Closed %		Cuts
Name	Value	Value	Value	Value	Value	Value	Value	△	Value	Value	Value	Value	△	Value
gesa3.pre	0.1	27900545.8	15.16	11	0.2	27927267.8	40.21	25.05	83	2.0	27926400	39.40	24.24	21
gesa3_o.pre	0.1	12301408.1	16.91	10	0.1	12373619.3	62.79	45.88	73	0.6	12376200	64.43	47.52	45
glass4.pre	0.0	800002436	0.00	147	0.1	800003020	0.00	0.00	193	0.0	800002000	0.00	0.00	6
gt2.pre	0.0	20370.9727	22.00	21	0.0	21148.4712	98.28	76.28	12	0.1	21166	100.00	78.00	7
harp2.pre	0.0	-74232132.3	0.00	0	0.1	-74100100.6	31.02	31.02	25	0.4	-74082800	35.09	35.09	10
irp.pre	1.8	12138.6623	42.08	6	3.9	12145.1809	60.20	18.12	10	0.2	12123.5	-0.08	-42.16	0
khb05250.pre	0.0	100189855	38.75	13	0.0	104598848	78.75	40.00	20	0.1	105786000	89.53	50.78	30
l152lav.pre	0.3	4662.84234	9.87	20	0.8	4672.56818	24.69	14.82	5	0.0	4656.36	0.00	-9.87	0
liu.pre	0.1	560	26.75	4	0.5	560	26.75	0.00	105	0.1	560	26.75	0.00	30
lseu.pre	0.0	1006.6416	33.51	7	0.0	1021.83424	42.42	8.91	7	0.1	1036.67	51.12	17.61	7
manna81.pre	0.7	-13164	100.00	272	0.4	-13164	100.00	0.00	274	0.3	-13164	100.00	0.00	269
markshare1.pre	0.0	0	0.00	0	0.0	0	0.00	0.00	1	0.1	0	0.00	0.00	0
markshare2.pre	0.0	0	0.00	0	0.0	0	0.00	0.00	0	0.2	0	0.00	0.00	0
mas284.pre	0.0	86195.863	0.00	0	0.1	86298.9993	1.98	1.98	5	1.1	86344.7	2.86	2.86	5
mas74.pre	0.0	10482.7953	0.00	0	0.0	10590.3726	8.16	8.16	7	0.2	10589.2	8.07	8.07	9
mas76.pre	0.0	38893.9036	0.00	0	0.0	38973.1845	7.14	7.14	5	0.2	38979.6	7.71	7.71	6
misc03.pre	0.0	2087	12.21	24	0.0	2185	18.97	6.76	19	1.1	2270.35	24.85	12.64	7
misc06.pre	0.1	12844.0321	25.54	4	0.1	12845.7447	44.22	18.68	15	0.1	12846.5	52.45	26.91	15
misc07.pre	0.1	1415	0.00	18	0.1	1425	0.72	0.72	7	1.9	1477.61	4.49	4.49	4
mitre.pre	0.7	115127.365	88.77	365	0.7	115155	100.00	11.23	297	1.4	115137	92.68	3.91	228
mkc.pre	0.3	-596.606142	30.75	17	0.5	-590.151765	44.14	13.39	31	0.6	-584.281	56.31	25.56	20
mkc1.pre	0.5	-611.85	0.00	4	0.6	-610.166266	36.26	36.26	36	0.2	-611.85	0.00	0.00	13
mod008.pre	0.0	290.931073	0.00	0	0.0	297.883042	43.26	43.26	7	0.5	298.255	45.58	45.58	8
mod010.pre	0.3	6548	100.00	6	0.2	6548	100.00	0.00	24	0.0	6532.08	0.00	-100.00	0
mod011.pre	0.4	-60798245.9	12.36	14	0.4	-59675911.9	28.12	15.76	16	0.1	-61356700	4.51	-7.85	6
modglob.pre	0.0	19790205.8	0.00	0	0.0	19830691.6	13.08	13.08	23	2.0	19920100	41.96	41.96	27
momentum1.pre	2.9	70575.7824	64.50	24	3.0	70575.7824	64.50	0.00	14	2.3	70575.8	64.50	0.00	6
momentum2.pre	7.9	3216.14744	39.12	14	16.6	3223.92267	39.27	0.15	13	81.8	3258.88	39.96	0.84	20
msc98-ip.pre	215.7	18595662	3.27	224	321.8	18635744	4.35	1.08	54	545.2	18635700	4.34	1.07	54
mzzv11.pre	93.2	-22357.9802	39.38	122	82.2	-22382.5471	37.05	-2.33	33	5.0	-22456.2	30.08	-9.30	23
mzzv42z.pre	49.7	-21291.9253	17.03	40	21.3	-21126.5956	35.27	18.24	39	4.8	-21268.5	19.61	2.58	29
neos1.pre	4.7	14.5077375	35.82	217	0.7	12.8863786	12.66	-23.16	38	0.2	12.16	2.29	-33.53	12
neos10.pre	2.5	-1181.10579	28.09	145	0.7	-1175.82872	36.69	8.60	21	0.4	-1181	28.26	0.17	7
neos11.pre	1.4	6	0.00	10	1.3	6	0.00	0.00	1	2.3	6	0.00	0.00	7
neos12.pre	21.3	9.41843537	0.19	20	22.9	9.45164166	1.12	0.93	41	15.6	9.42116	0.27	0.08	15

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
Name	Time Value	Dual Bound Value	Gap Closed % Value	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value
neos13.pre	5.0	-126.178378	0.00	7	7.6	-103.188882	74.88	74.88	221	2.5	-115.557	34.59	34.59	250
neos14.pre	0.0	46125.3396	42.02	96	0.1	54008.3624	60.97	18.95	105	1.1	59884.7	75.10	33.08	68
neos15.pre	0.0	47312.3703	39.55	119	0.1	53137.9649	51.84	12.29	107	9.6	65245.8	77.39	37.84	135
neos16.pre	0.3	431	9.52	231	0.2	431	9.52	0.00	51	14.8	432	14.29	4.77	77
neos17.pre	0.0	0.000681498501	0.00	0	0.2	0.0422573392	27.84	27.84	76	0.0	0.000681499	0.00	0.00	0
neos18.pre	3.7	8	34.78	146	0.9	7	21.74	-13.04	21	3.4	8	34.78	0.00	11
neos19.pre	83.7	-1611	0.00	2761	91.1	-1611	0.00	0.00	381	1580.8	-1611	0.00	0.00	1642
neos2.pre	0.1	-4574.09499	2.78	26	0.2	-3942.17298	14.99	12.21	41	0.8	-3857.13	16.64	13.86	34
neos20.pre	0.4	-475	0.00	70	0.6	-472	7.32	7.32	49	0.5	-475	0.00	0.00	36
neos21.pre	0.3	2.69870075	10.08	5	0.5	2.76110357	11.39	1.31	7	1.0	2.80011	12.20	2.12	5
neos22.pre	0.5	777227.429	1.43	141	0.5	777191.429	0.00	-1.43	109	0.3	777191	0.00	-1.43	4
neos23.pre	0.1	56	0.00	58	0.1	56.3608865	0.45	0.45	25	0.0	56	0.00	0.00	5
neos3.pre	0.2	-6404.06184	2.41	30	0.6	-5680.68576	12.84	10.43	59	1.4	-5339.72	17.75	15.34	19
neos616206.pre	0.1	787.721258	0.00	31	0.3	788.427308	0.47	0.47	28	20.1	795.413	5.13	5.13	38
neos632659.pre	0.0	-109.714286	0.00	47	0.0	-109.714286	0.00	0.00	22	0.4	-109.714	0.00	0.00	5
neos648910.pre	0.0	16	0.00	135	0.1	16	0.00	0.00	99	0.3	16	0.00	0.00	15
neos7.pre	0.4	605261.04	23.63	61	0.4	626562.405	38.57	14.94	144	0.6	624725	37.28	13.65	47
neos8.pre	1.0	-3719	100.00	61	0.1	-3719	100.00	0.00	8	0.1	-3719	100.00	0.00	1
neos9.pre	182.2	797.125	95.14	467	49.6	796.5	91.67	-3.47	260	9.9	794	77.78	-17.36	36
net12.pre	58.4	53.9043395	14.50	211	35.4	63.1463499	19.44	4.94	56	30.8	51.8409	13.40	-1.10	46
noswot.pre	0.0	-43	0.00	11	0.0	-43	0.00	0.00	24	0.4	-43	0.00	0.00	4
nsrand-ipx.pre	1.5	50137.2882	25.34	28	3.2	50453.539	42.42	17.08	39	0.5	50421.1	40.67	15.33	35
nug08.pre	9.0	204.380311	8.38	18	14.9	207.759215	40.56	32.18	7	10.4	205.347	17.59	9.21	6
nw04.pre	2.2	16310.6667	0.00	0	9.2	16790.8372	87.09	87.09	18	0.7	16310.7	0.01	0.01	0
opt1217.pre	0.1	-19.0332732	24.57	16	0.2	-18.2470811	44.12	19.55	18	0.1	-19.4516	14.17	-10.40	6
p0033.pre	0.0	2377.325	45.83	8	0.0	2388.6257	50.34	4.51	6	0.0	2277.25	5.87	-39.96	1
p0201.pre	0.1	7159.91472	7.13	9	0.1	7219.85889	19.36	12.23	6	2.8	7329.94	41.82	34.69	9
p0282.pre	0.0	185845.592	7.47	29	0.0	190091.732	12.88	5.41	17	0.9	212167	41.04	33.57	17
p0548.pre	0.0	8136.7595	84.15	46	0.0	8611.89426	97.74	13.59	50	1.2	8626.08	98.14	13.99	43
p2756.pre	0.2	3113.70411	97.56	138	0.1	3114.77295	97.81	0.25	114	0.1	3113.93	97.62	0.06	125
pk1.pre	0.0	0	0.00	1	0.0	0	0.00	0.00	1	0.3	0	0.00	0.00	0
pp08a.pre	0.0	4677.95289	41.93	34	0.0	6619.42493	84.12	42.19	76	4.2	6648.3	84.75	42.82	72
pp08aCUTS.pre	0.0	5716.26377	23.30	17	0.0	6319.12632	55.55	32.25	45	3.4	6466.21	63.42	40.12	37
prod1.pre	0.0	8.44984888	0.00	0	0.0	9.78335012	7.60	7.60	8	1.6	10.018	8.94	8.94	9
protfold.pre	22.3	-40.0953097	9.82	8	112.2	-40.2202211	9.16	-0.66	21	4.7	-40.8958	5.60	-4.22	3

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
Name	Time Value	Dual Bound Value	Gap Closed % Value	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	△	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	△	Cuts Value
qap10.pre	39.7	332.566228	0.00	0	124.9	333.814171	16.79	16.79	11	2.0	332.566	0.00	0.00	0
qiu.pre	0.1	-931.638853	0.00	0	0.3	-918.745571	1.61	1.61	20	0.9	-892.203	4.94	4.94	7
qnet1.pre	0.7	14693.1207	23.87	15	0.2	14556.3415	16.08	-7.79	6	0.2	14689.4	23.66	-0.21	8
qnet1_o.pre	0.3	14117.0404	38.73	24	0.2	14622.6349	54.93	16.20	18	0.1	14172.2	40.50	1.77	9
ran10x26.pre	0.0	3967.66673	26.79	20	0.1	3962.92193	25.64	-1.15	19	0.3	3969.47	27.23	0.44	13
ran12x21.pre	0.1	3280.95242	24.39	50	0.1	3264.04822	21.06	-3.33	16	1.0	3277.83	23.78	-0.61	16
ran13x13.pre	0.1	2858.79229	29.85	32	0.1	2814.00424	21.86	-7.99	24	2.1	2888.16	35.09	5.24	22
ran14x18_1.pre	0.1	3189.11482	24.70	37	0.1	3161.41308	20.73	-3.97	24	0.5	3134.7	16.89	-7.81	14
ran8x32.pre	0.1	5027.11899	28.94	44	0.1	5048.62415	35.89	6.95	18	0.9	5047.42	35.50	6.56	18
rentacar.pre	0.2	-3995303.15	0.07	2	0.2	-3918333.31	5.46	5.39	6	0.0	-3996350	0.00	-0.07	2
rgn.pre	0.0	55.305972	19.48	24	0.0	59.3720968	31.65	12.17	20	0.4	70.4828	64.92	45.44	28
roll3000.pre	2.5	12119.3943	56.71	108	1.2	11499.4687	22.29	-34.42	50	2.6	11910.7	45.12	-11.59	48
rout.pre	0.1	-1392.57181	0.85	17	0.1	-1388.01643	5.61	4.76	10	0.1	-1392.38	1.05	0.20	4
set1ch.pre	0.1	37861.3146	39.09	125	0.1	47918.0653	90.88	51.79	100	1.6	46881.8	85.54	46.45	101
seymour.pre	7.2	242.217415	15.68	18	8.0	242.252067	15.87	0.19	21	7.1	241.917	13.96	-1.72	11
seymour1.pre	3.9	238.883793	8.30	4	5.8	239.628851	19.92	11.62	15	2.3	239.164	12.67	4.37	4
sp97ar.pre	11.1	648639048	4.72	3	32.8	649848971	16.13	11.41	19	1.6	649046000	8.56	3.84	7
stein27.pre	0.0	13	0.00	20	0.0	13	0.00	0.00	5	0.3	13	0.00	0.00	5
stein45.pre	0.0	22	0.00	18	0.1	22	0.00	0.00	7	2.0	22	0.00	0.00	1
swath.pre	1.5	380.300953	32.07	102	2.8	380.879185	32.47	0.40	125	0.1	335.226	0.51	-31.56	1
swath1.pre	0.1	334.496858	0.00	0	0.7	340.856381	14.27	14.27	32	0.0	334.497	0.00	0.00	0
swath2.pre	0.2	340.567288	11.97	8	1.0	341.08784	13.00	1.03	38	0.1	334.497	0.00	-11.97	0
swath3.pre	0.2	338.711236	6.66	4	0.9	341.849661	11.62	4.96	38	0.1	334.497	0.00	-6.66	0
t1717.pre	21.2	134531.021	0.00	0	53.8	134593.682	0.04	0.04	12	1.6	134531	0.00	0.00	0
timtab1.pre	0.1	278266.46	33.86	227	0.1	310539.264	38.24	4.38	90	5.0	331824	41.14	7.28	63
timtab2.pre	0.3	386989.422	28.98	361	0.2	431176.364	32.99	4.01	138	23.4	461205	35.72	6.74	106
tr12-30.pre	0.1	56451.0344	37.81	207	0.2	97202.5757	74.04	36.23	372	6.3	110660	86.01	48.20	358
vpm1.pre	0.0	18.6821138	63.05	57	0.0	17.537037	30.94	-32.11	14	1.0	18.8137	66.74	3.69	26
vpm2.pre	0.0	11.657079	19.95	20	0.0	12.160145	39.19	19.24	26	2.0	12.3667	47.09	27.14	26
Total	1514.5		2838.71	9117	2265.8		4294.52	1455.81	6010	3030.2		4179.20	1340.49	5204
Geom. Mean	2.1		7.02	13	2.4		15.02	8.00	20	2.0		10.88	3.86	8

Table B.60: Computational results for the comparison with CPLEX and CBC. *Cutting plane separator for the class of GMI inequalities.* (△ with respect to SCIP)

	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
Name	Time Value	Dual Bound Value	Gap Closed % Value	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value
10teams.pre	0.4	897	0.00	0	0.1	897	0.00	0.00	0	0.0	897	0.00	0.00	0
30:70:4_5:0_5:100.pre	13.7	5337.1	0.00	0	12.4	5337.1	0.00	0.00	0	125.6	5337.1	0.00	0.00	0
30:70:4_5:0_95:98.pre	20.2	5969.5	0.00	0	13.6	5969.5	0.00	0.00	0	114.1	5969.5	0.00	0.00	0
a1c1s1.pre	12.8	5632.68511	36.58	806	1.9	7327.01796	54.69	18.11	486	0.5	5057.69	30.44	-6.14	207
aflow30a.pre	4.1	162.700587	40.92	186	0.1	152.273802	34.95	-5.97	49	0.2	145.92	31.32	-9.60	50
aflow40b.pre	21.1	-529.896163	29.84	107	0.3	-555.78942	13.89	-15.95	16	1.8	-531.634	28.77	-1.07	64
air03.pre	1.0	311406.25	0.00	0	0.2	311406.25	0.00	0.00	0	0.1	311406	0.00	0.00	0
air04.pre	3.3	54030.4364	0.00	0	1.9	54030.4364	0.00	0.00	0	0.5	54030.9	0.08	0.08	0
air05.pre	2.1	25877.6093	0.00	0	0.5	25877.6093	0.00	0.00	0	0.3	25877.6	0.00	0.00	0
arki001.pre	5.6	7009591.54	16.78	279	0.2	7009683.49	24.49	7.71	48	0.2	7009530	11.62	-5.16	6
atlanta-ip.pre	83.0	81.2652543	0.07	15	68.1	81.2762858	0.15	0.08	61	8.2	81.2717	0.12	0.05	29
bc1.pre	3.5	2.62695156	38.12	18	2.6	2.4449267	22.28	-15.84	1	1.9	2.53504	30.12	-8.00	12
bell3a.pre	0.0	869351.381	59.02	15	0.0	869039.674	56.48	-2.54	11	0.0	865166	24.88	-34.14	6
bell5.pre	0.0	8355467.02	3.81	19	0.0	8396554.78	15.29	11.48	14	0.0	8348300	1.81	-2.00	5
bienst1.pre	1.8	14.1153965	6.83	138	0.4	14.8514859	8.93	2.10	137	0.3	14.0421	6.62	-0.21	59
bienst2.pre	3.1	15.0064524	7.66	234	0.6	15.586573	9.01	1.35	181	0.5	14.9129	7.44	-0.22	83
binkar10_1.pre	0.7	5787.08201	56.13	56	0.0	5728.13803	0.00	-56.13	0	0.0	5728.16	0.02	-56.11	0
blend2.pre	0.1	6.96975772	7.91	25	0.0	7.01490511	14.52	6.61	7	0.0	7.04686	19.20	11.29	9
cap6000.pre	0.6	-2403390.33	0.00	0	0.1	-2403390.33	0.00	0.00	0	0.0	-2403390	0.00	0.00	0
dano3_3.pre	32.0	576.23207	0.00	1	44.4	576.243836	10.81	10.81	12	1.4	576.232	0.00	0.00	0
dano3_4.pre	39.7	576.23461	1.47	3	132.2	576.256062	12.00	10.53	22	1.6	576.232	0.00	-1.47	0
dano3_5.pre	51.9	576.240343	1.26	6	70.2	576.296058	9.29	8.03	30	1.6	576.232	0.00	-1.26	0
dano3mip.pre	82.3	576.284231	0.04	78	140.0	576.496184	0.20	0.16	150	0.6	576.232	0.00	-0.04	0
danoint.pre	1.8	62.6682179	1.02	78	0.2	62.696834	1.96	0.94	73	0.2	62.6581	0.69	-0.33	36
dcmulti.pre	1.1	185649.672	38.95	148	0.1	185528.93	36.03	-2.92	84	0.0	184217	4.40	-34.55	13
ds.pre	96.8	57.2345653	0.00	0	37.4	57.2345653	0.00	0.00	0	6.7	57.2346	0.00	0.00	0
egout.pre	0.1	274.908482	57.33	62	0.0	299.00708	100.00	42.67	19	-0.0	295.216	93.29	35.96	11
eilD76.pre	0.4	680.538997	0.00	0	0.1	680.538997	0.00	0.00	0	0.0	680.539	0.00	0.00	0
fast0507.pre	39.1	158.145567	0.00	0	23.7	158.145567	0.00	0.00	0	1.0	158.146	0.02	0.02	0
fiber.pre	0.7	367252.067	84.52	61	0.1	385167.597	91.69	7.17	50	0.1	295912	55.96	-28.56	20
fixnet6.pre	2.3	3535.744	43.71	266	0.2	3568.59339	47.86	4.15	40	0.2	3557.35	46.44	2.73	47
flugpl.pre	0.0	726875.166	0.00	0	0.0	726875.166	0.00	0.00	0	-0.0	726875	0.00	0.00	0
gen.pre	0.1	58349.0903	100.00	17	0.0	58325.1316	17.80	-82.20	8	0.0	58349.1	100.00	0.00	9
gesa2-o.pre	6.2	19018510	99.19	385	0.2	18972569.1	84.05	-15.14	152	0.1	18822400	34.55	-64.64	80
gesa2.pre	3.8	25778619.1	99.55	265	0.1	25730996.3	82.36	-17.19	87	0.1	25602300	35.90	-63.65	58

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
	Time	Dual Bound	Gap Closed %	Cuts	Time	Dual Bound	Gap Closed %		Cuts	Time	Dual Bound	Gap Closed %		Cuts
Name	Value	Value	Value	Value	Value	Value	Value	△	Value	Value	Value	Value	△	Value
gesa3.pre	2.6	27961999.2	72.77	118	0.1	27943159.7	55.11	-17.66	65	0.1	27919000	32.46	-40.31	23
gesa3_o.pre	3.6	12405891.3	83.29	204	0.1	12382018.6	68.12	-15.17	101	0.1	12358900	53.44	-29.85	53
glass4.pre	0.1	800002400	0.00	32	0.0	800002400	0.00	0.00	90	0.4	800003000	0.00	0.00	230
gt2.pre	0.0	20690.8105	53.38	19	0.0	20652.7281	49.64	-3.74	14	0.0	20350.9	20.03	-33.35	14
harp2.pre	0.0	-74232132.3	0.00	0	0.0	-74232132.3	0.00	0.00	0	0.2	-73996700	55.32	55.32	55
irp.pre	1.3	12123.5302	0.00	0	0.3	12123.5302	0.00	0.00	0	0.2	12123.5	-0.08	-0.08	0
khb05250.pre	0.5	106901468	99.65	132	0.1	106709067	97.90	-1.75	103	0.1	106720000	98.00	-1.65	52
l152lav.pre	0.2	4656.36364	0.00	0	0.0	4656.36364	0.00	0.00	0	0.0	4656.36	0.00	0.00	0
liu.pre	3.2	385	4.88	256	0.6	560	26.75	21.87	298	0.9	385	4.88	0.00	258
lseu.pre	0.0	1026.0419	44.89	18	0.0	949.518722	0.00	-44.89	0	0.0	1022	42.52	-2.37	8
manna81.pre	1.1	-13297	0.00	0	0.1	-13297	0.00	0.00	0	0.1	-13297	0.00	0.00	0
markshare1.pre	0.0	0	0.00	5	0.0	0	0.00	0.00	3	0.0	0	0.00	0.00	0
markshare2.pre	0.0	0	0.00	6	0.0	0	0.00	0.00	1	0.0	0	0.00	0.00	0
mas284.pre	0.1	86195.863	0.00	0	0.1	86749.5829	10.63	10.63	11	0.0	86195.9	0.00	0.00	0
mas74.pre	0.0	10482.7953	0.00	0	0.0	10577.1057	7.15	7.15	10	0.0	10482.8	0.00	0.00	0
mas76.pre	0.0	38893.9036	0.00	0	0.0	38974.1624	7.22	7.22	10	0.0	38893.9	0.00	0.00	0
misc03.pre	0.0	1910	0.00	0	0.0	1910	0.00	0.00	0	0.0	1910	0.00	0.00	0
misc06.pre	0.1	12841.6894	0.00	0	0.0	12842.147	4.99	4.99	11	0.0	12841.7	0.00	0.00	0
misc07.pre	0.0	1415	0.00	0	0.0	1415	0.00	0.00	0	0.0	1415	0.00	0.00	0
mitre.pre	10.8	114942.769	13.73	549	0.1	114908.999	0.00	-13.73	0	1.3	115072	66.26	52.53	396
mkc.pre	7.7	-608.349021	6.41	158	0.1	-611.437978	0.00	-6.41	3	0.1	-611.438	0.00	-6.41	8
mkc1.pre	0.8	-611.85	0.00	4	0.2	-611.85	0.00	0.00	8	0.2	-611.85	0.00	0.00	18
mod008.pre	0.1	294.586091	22.75	8	0.0	290.931073	0.00	-22.75	0	0.1	301.609	66.45	43.70	14
mod010.pre	0.4	6535	18.32	2	0.1	6532.08333	0.00	-18.32	0	0.0	6532.08	0.00	-18.32	0
mod011.pre	13.7	-57280568.9	61.77	566	11.9	-57607910.3	57.17	-4.60	439	0.3	-60719800	13.46	-48.31	24
modglob.pre	1.3	19987402	63.70	390	0.1	19940170.8	48.44	-15.26	74	0.1	19993700	65.74	2.04	50
momentum1.pre	8.8	47128.5523	0.00	0	1.7	47128.5824	0.00	0.00	4	0.4	47128.6	0.00	0.00	16
momentum2.pre	5.6	1225.6443	0.00	0	15.1	1225.6443	0.00	0.00	0	78.3	1225.64	0.00	0.00	1
msc98-ip.pre	77.7	18478536.3	0.14	29	210.2	18503353.9	0.81	0.67	142	60.4	18473800	0.02	-0.12	25
mzzv11.pre	55.8	-22773.75	0.00	0	68.2	-22773.75	0.00	0.00	4	0.6	-22773.8	0.00	0.00	0
mzzv42z.pre	30.8	-21446.2397	0.00	0	16.6	-21446.2397	0.00	0.00	0	0.3	-21446.2	0.00	0.00	0
neos1.pre	5.0	17.6	80.00	510	0.1	12	0.00	-80.00	0	0.1	12	0.00	-80.00	38
neos10.pre	2.9	-1198.33333	0.00	143	0.1	-1198.33333	0.00	0.00	0	0.1	-1198.33	0.01	0.01	0
neos11.pre	2.7	6	0.00	0	0.4	6	0.00	0.00	0	0.2	6	0.00	0.00	0
neos12.pre	10.1	9.41161243	0.00	0	12.0	9.41161243	0.00	0.00	4	4.8	9.41161	0.00	0.00	0

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
	Time	Dual Bound	Gap Closed %	Cuts	Time	Dual Bound	Gap Closed %		Cuts	Time	Dual Bound	Gap Closed %		Cuts
Name	Value	Value	Value	Value	Value	Value	Value	△	Value	Value	Value	Value	△	Value
neos13.pre	19.5	-126.178378	0.00	0	144.4	-126.178378	0.00	0.00	554	0.7	-126.178	0.00	0.00	0
neos14.pre	6.0	57837.5873	70.18	542	0.1	57082.008	68.36	-1.82	215	0.0	51701.9	55.43	-14.75	92
neos15.pre	6.6	61872.0344	70.27	769	0.1	58370.5232	62.88	-7.39	231	0.0	51545.7	48.48	-21.79	92
neos16.pre	0.5	431	9.52	72	0.1	431	9.52	0.00	34	0.0	429	0.00	-9.52	0
neos17.pre	0.1	0.000681498501	0.00	0	0.1	0.000681498501	0.00	0.00	0	0.0	0.000681499	0.00	0.00	0
neos18.pre	0.5	5.33333333	0.00	0	0.2	5.33333333	0.00	0.00	0	0.9	5.33333	0.00	0.00	0
neos19.pre	113.4	-1611	0.00	458	48.9	-1611	0.00	0.00	1105	2862.3	-1611	0.00	0.00	8034
neos2.pre	4.6	-4315.97183	7.77	77	0.1	-4684.5176	0.64	-7.13	31	0.0	-4717.67	0.00	-7.77	0
neos20.pre	2.0	-475	0.00	118	0.1	-475	0.00	0.00	0	0.1	-475	0.00	0.00	6
neos21.pre	0.2	2.21648352	0.00	0	0.1	2.21648352	0.00	0.00	0	0.2	2.21648	0.00	0.00	0
neos22.pre	6.7	777291.429	3.96	39	0.2	779200.714	79.62	75.66	174	0.2	777291	3.95	-0.01	72
neos23.pre	0.3	56	0.00	15	0.1	59.1663745	3.91	3.91	42	0.0	56	0.00	0.00	0
neos3.pre	8.6	-6109.46882	6.66	85	0.1	-6526.70362	0.65	-6.01	46	0.0	-6571.63	0.00	-6.66	0
neos616206.pre	0.7	787.813105	0.06	107	0.2	792.712673	3.33	3.27	101	0.4	792.56	3.23	3.17	55
neos632659.pre	0.1	-109.714286	0.00	53	0.0	-109.714286	0.00	0.00	19	0.1	-109.714	0.00	0.00	7
neos648910.pre	0.8	16	0.00	273	0.1	16	0.00	0.00	115	0.2	16	0.00	0.00	40
neos7.pre	0.5	571556.49	0.00	0	0.2	632065.382	42.43	42.43	108	0.1	572964	0.99	0.99	4
neos8.pre	7.7	-3724.75	4.17	108	0.1	-3724.75	4.17	0.00	1	0.1	-3725	0.00	-4.17	0
neos9.pre	26.8	780	0.00	0	5.7	780	0.00	0.00	0	1.4	780	0.00	0.00	0
net12.pre	84.5	44.3882086	9.42	115	14.1	45.0004787	9.74	0.32	94	13.9	43.2207	8.79	-0.63	57
noswot.pre	0.1	-43	0.00	61	0.0	-43	0.00	0.00	12	0.0	-43	0.00	0.00	25
nsrand-ipx.pre	8.0	49915.9785	13.39	81	0.1	49667.8923	0.00	-13.39	0	1.1	49999.2	17.89	4.50	47
nug08.pre	3.7	203.5	0.00	0	2.5	203.5	0.00	0.00	0	0.7	203.5	0.00	0.00	0
nw04.pre	3.1	16310.6667	0.00	0	0.8	16310.6667	0.00	0.00	0	0.7	16310.7	0.01	0.01	0
opt1217.pre	0.0	-20.0213904	0.00	0	0.0	-19.8930481	3.19	3.19	10	0.0	-20.0214	0.00	0.00	0
p0033.pre	0.0	2277.25	5.87	2	0.0	2262.54674	0.00	-5.87	0	0.0	2361.83	39.64	33.77	7
p0201.pre	0.0	7125	0.00	14	0.0	7125	0.00	0.00	0	0.0	7125	0.00	0.00	3
p0282.pre	0.3	254927.021	95.57	127	0.0	179990.3	0.00	-95.57	0	0.2	254593	95.14	-0.43	43
p0548.pre	0.1	8679.49453	99.67	95	0.0	5194.43616	0.00	-99.67	0	0.1	6085.01	25.47	-74.20	61
p2756.pre	1.3	3063.89685	85.77	231	0.0	2701.75	0.00	-85.77	0	0.1	2703	0.30	-85.47	43
pk1.pre	0.0	0	0.00	0	0.0	0	0.00	0.00	1	0.0	0	0.00	0.00	0
pp08a.pre	1.4	7157.81046	95.82	219	0.0	7097.42678	94.51	-1.31	120	0.0	5562.26	61.15	-34.67	59
pp08aCUTS.pre	0.9	6981.37326	90.98	153	0.1	6844.68413	83.67	-7.31	96	0.0	5448.34	8.97	-82.01	11
prod1.pre	0.2	8.71674344	1.52	54	0.0	8.44984888	0.00	-1.52	0	0.1	8.73797	1.64	0.12	9
protfold.pre	4.0	-41.9574468	0.00	0	3.0	-41.9574468	0.00	0.00	0	1.6	-41.9574	0.00	0.00	0

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
Name	Time Value	Dual Bound Value	Gap Closed % Value	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value
qap10.pre	38.7	332.566228	0.00	0	50.0	332.566228	0.00	0.00	0	1.8	332.566	0.00	0.00	0
qiu.pre	0.2	-931.638853	0.00	0	0.1	-931.638857	0.00	0.00	0	0.2	-931.639	0.00	0.00	0
qnet1.pre	1.4	15644.2144	78.04	97	0.2	15497.2223	69.67	-8.37	42	0.2	15003.6	41.55	-36.49	27
qnet1_o.pre	0.6	15628.6883	87.16	83	0.2	15661.1841	88.20	1.04	57	0.4	15783.1	92.10	4.94	43
ran10x26.pre	1.8	4052.97912	47.45	109	0.1	4055.682	48.10	0.65	41	0.2	4052.31	47.29	-0.16	38
ran12x21.pre	2.3	3412.23773	50.31	183	0.1	3431.15832	54.04	3.73	70	0.2	3421.59	52.15	1.84	42
ran13x13.pre	0.8	2962.14008	48.29	106	0.1	2991.0242	53.44	5.15	68	0.1	2971.87	50.03	1.74	31
ran14x18_1.pre	2.0	3313.33414	42.52	174	0.1	3341.78949	46.60	4.08	78	0.2	3333.61	45.43	2.91	55
ran8x32.pre	1.1	5116.89978	57.95	114	0.1	5140.10951	65.45	7.50	40	0.1	5122.73	59.84	1.89	32
rentacar.pre	0.5	-3980759.58	1.09	10	0.3	-3860106.14	9.54	8.45	16	0.5	-3666090	23.12	22.03	10
rgn.pre	0.5	81.7999992	98.80	109	0.0	77.6052165	86.24	-12.56	47	0.1	68	57.49	-41.31	3
roll3000.pre	20.1	12142.6253	58.00	146	0.9	12126.4995	57.10	-0.90	97	1.0	12045.8	52.62	-5.38	63
rout.pre	0.3	-1393.38571	0.00	94	0.0	-1393.38571	0.00	0.00	7	0.1	-1393.39	0.00	0.00	19
set1ch.pre	1.1	49632.6342	99.71	377	0.1	46922.1723	85.75	-13.96	165	0.0	43372.2	67.47	-32.24	120
seymour.pre	3.9	239.469492	0.00	0	3.4	239.469492	0.00	0.00	0	0.2	239.469	0.00	0.00	0
seymour1.pre	3.6	238.351528	0.00	0	5.0	239.222039	13.58	13.58	24	0.1	238.352	0.01	0.01	0
sp97ar.pre	12.2	648218654	0.75	7	1.6	648138794	0.00	-0.75	0	4.3	648344000	1.94	1.19	18
stein27.pre	0.0	13	0.00	0	0.0	13	0.00	0.00	0	0.0	13	0.00	0.00	0
stein45.pre	0.0	22	0.00	0	0.0	22	0.00	0.00	0	0.0	22	0.00	0.00	0
swath.pre	24.7	359.072887	17.20	123	0.1	373.882794	27.57	10.37	25	0.1	334.497	0.00	-17.20	11
swath1.pre	1.9	338.681683	9.39	23	0.1	338.681683	9.39	0.00	7	0.1	334.497	0.00	-9.39	6
swath2.pre	2.3	338.681683	8.25	21	0.1	343.090343	16.95	8.70	9	0.1	334.497	0.00	-8.25	6
swath3.pre	2.1	337.131796	4.16	26	0.1	343.090343	13.58	9.42	11	0.1	334.497	0.00	-4.16	6
t1717.pre	14.2	134531.021	0.00	0	7.9	134531.021	0.00	0.00	0	1.5	134531	0.00	0.00	0
timtab1.pre	4.8	407987.21	51.48	628	0.2	436050.574	55.30	3.82	181	0.0	194195	22.44	-29.04	52
timtab2.pre	8.5	452986.501	34.97	891	0.6	555537.655	44.29	9.32	299	0.1	232830	14.97	-20.00	90
tr12-30.pre	4.6	106094.34	81.95	875	0.2	110898.299	86.22	4.27	640	0.1	82083.7	60.60	-21.35	382
vpm1.pre	0.0	20	100.00	30	0.0	19.5	85.98	-14.02	37	0.0	18.7917	66.12	-33.88	17
vpm2.pre	0.9	13.069534	73.97	165	0.0	12.9509292	69.44	-4.53	50	0.0	12.9179	68.17	-5.80	32
Total	1250.9		3178.42	14676	1183.3		2735.74	-442.68	8253	3315.2		2276.56	-901.86	11957
Geom. Mean	3.0		5.45	17	1.8		5.00	-0.45	9	1.4		4.06	-1.39	7

Table B.61: Computational results for the comparison with CPLEX and CBC. *Cutting plane separator for the class of c-MIR inequalities.* (△ with respect to SCIP)

	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
Name	Time Value	Dual Bound Value	Gap Closed % Value	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	△	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	△	Cuts Value
10teams.pre	0.2	897	0.00	0	0.1	897	0.00	0.00	0	0.0	897	0.00	0.00	0
30:70:4_5:0_5:100.pre	11.0	5337.1	0.00	0	12.6	5337.1	0.00	0.00	0	125.4	5337.1	0.00	0.00	0
30:70:4_5:0_95:98.pre	17.7	5969.5	0.00	0	13.5	5969.5	0.00	0.00	0	113.3	5969.5	0.00	0.00	0
a1c1s1.pre	2.6	4218.23237	21.47	219	2.8	7546.34732	57.04	35.57	741	0.0	2209.57	0.00	-21.47	0
aflow30a.pre	0.6	145.116431	30.86	89	0.1	139.540889	27.67	-3.19	34	0.0	117.865	15.27	-15.59	29
aflow40b.pre	2.5	-536.264574	25.92	89	0.2	-567.225842	6.84	-19.08	4	0.2	-553.165	15.51	-10.41	31
air03.pre	0.7	311406.25	0.00	0	0.2	311406.25	0.00	0.00	0	0.1	311406	0.00	0.00	0
air04.pre	2.0	54030.4364	0.00	0	1.9	54030.4364	0.00	0.00	0	0.5	54030.9	0.08	0.08	0
air05.pre	0.8	25877.6093	0.00	0	0.5	25877.6093	0.00	0.00	0	0.2	25877.6	0.00	0.00	0
arki001.pre	1.4	7009421.06	2.48	88	0.2	7009391.43	0.00	-2.48	11	0.1	7009390	0.00	-2.48	0
atlanta-ip.pre	55.3	81.2553981	0.00	0	62.3	81.2553981	0.00	0.00	0	0.8	81.2554	0.00	0.00	0
bc1.pre	14.0	2.62391195	37.85	75	2.2	2.18877635	0.00	-37.85	0	0.6	2.18878	0.00	-37.85	0
bell3a.pre	0.0	862116.583	0.00	0	0.0	865459.981	27.27	27.27	4	0.0	862117	0.00	0.00	0
bell5.pre	0.0	8341834.36	0.00	0	0.0	8409924.34	19.02	19.02	6	0.0	8341830	0.00	0.00	0
bienst1.pre	0.4	14.048581	6.64	123	0.3	14.3815535	7.59	0.95	134	0.1	13.9873	6.46	-0.18	28
bienst2.pre	0.5	14.9186794	7.45	164	0.4	15.0419751	7.74	0.29	221	0.2	14.8477	7.29	-0.16	37
binkar10_1.pre	1.1	5789.42858	58.37	73	0.0	5728.13803	0.00	-58.37	0	0.0	5728.16	0.02	-58.35	0
blend2.pre	0.3	6.98015488	9.44	22	0.0	6.91567511	0.00	-9.44	6	0.0	6.91568	0.00	-9.44	0
cap6000.pre	2.4	-2403346.9	27.09	8	0.1	-2403390.33	0.00	-27.09	0	0.0	-2403390	0.00	-27.09	0
dano3_3.pre	31.1	576.23162	0.00	0	75.8	576.253185	19.08	19.08	23	1.4	576.232	0.00	0.00	0
dano3_4.pre	36.3	576.23162	0.00	0	79.1	576.262521	15.18	15.18	30	1.7	576.232	0.00	0.00	0
dano3_5.pre	45.9	576.23162	0.00	0	96.4	576.302996	10.30	10.30	53	1.7	576.232	0.00	0.00	0
dano3mip.pre	37.1	576.23162	0.00	0	120.8	576.532826	0.23	0.23	231	0.6	576.232	0.00	0.00	0
danoint.pre	0.5	62.6574167	0.66	42	0.2	62.7087387	2.36	1.70	102	0.1	62.6456	0.27	-0.39	15
dcmulti.pre	0.0	184145.514	2.68	17	0.1	185632.345	38.53	35.85	93	0.0	184044	0.23	-2.45	4
ds.pre	43.9	57.2345653	0.00	0	37.4	57.2345653	0.00	0.00	0	6.6	57.2346	0.00	0.00	0
egout.pre	0.0	296.951731	96.36	39	0.0	297.744372	97.76	1.40	11	0.0	253.703	19.79	-76.57	10
eilD76.pre	0.2	680.538997	0.00	0	0.1	680.538997	0.00	0.00	0	0.0	680.539	0.00	0.00	0
fast0507.pre	26.8	158.145567	0.00	0	23.7	158.145567	0.00	0.00	0	0.8	158.146	0.02	0.02	0
fiber.pre	0.4	383911.863	91.19	77	0.1	336929.514	72.38	-18.81	30	0.0	156083	0.00	-91.19	0
fixnet6.pre	0.3	3585.14122	49.95	123	0.1	3529.00999	42.86	-7.09	40	0.1	3318.92	16.29	-33.66	29
flugpl.pre	0.0	726875.166	0.00	0	0.0	726875.166	0.00	0.00	0	0.0	726875	0.00	0.00	0
gen.pre	0.1	58349.0903	100.00	20	0.0	58319.9441	0.00	-100.00	0	0.0	58320	0.00	-100.00	0
gesa2-o.pre	0.4	18732799.5	5.01	29	0.1	18831108.3	37.42	32.41	55	0.1	18810600	30.66	25.65	73
gesa2.pre	0.8	25668916.2	59.95	71	0.1	25567672.6	23.40	-36.55	28	0.1	25503300	0.16	-59.79	4

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
	Time	Dual Bound	Gap Closed %	Cuts	Time	Dual Bound	Gap Closed %		Cuts	Time	Dual Bound	Gap Closed %		Cuts
Name	Value	Value	Value	Value	Value	Value	Value	△	Value	Value	Value	Value	△	Value
gesa3.pre	0.4	27907701.2	21.86	21	0.0	27884504.5	0.12	-21.74	2	0.0	27884500	0.11	-21.75	2
gesa3_o.pre	0.1	12274783.2	0.00	0	0.0	12289361.6	9.26	9.26	16	0.0	12288300	8.59	8.59	20
glass4.pre	0.2	800002400	0.00	22	0.0	800002400	0.00	0.00	87	0.0	800003000	0.00	0.00	0
gt2.pre	0.0	20530.1857	37.62	8	0.0	20146.7613	0.00	-37.62	0	0.0	20146.8	0.00	-37.62	0
harp2.pre	0.2	-74232132.2	0.00	5	0.0	-74232132.3	0.00	0.00	0	0.0	-74232100	0.00	0.00	0
irp.pre	1.0	12123.5302	0.00	0	0.3	12123.5302	0.00	0.00	0	0.2	12123.5	-0.08	-0.08	0
khb05250.pre	0.3	106734430	98.13	95	0.1	106788996	98.63	0.50	86	0.0	96188100	2.44	-95.69	17
l152lav.pre	0.1	4656.36364	0.00	0	0.1	4656.36364	0.00	0.00	0	0.0	4656.36	0.00	0.00	0
liu.pre	0.1	346	0.00	0	0.2	346	0.00	0.00	220	0.1	346	0.00	0.00	500
lseu.pre	0.0	1025.48296	44.56	24	0.0	949.518722	0.00	-44.56	0	0.0	949.519	0.00	-44.56	0
manna81.pre	0.7	-13297	0.00	0	0.1	-13297	0.00	0.00	0	0.1	-13297	0.00	0.00	0
markshare1.pre	0.0	0	0.00	13	0.0	0	0.00	0.00	1	0.1	0	0.00	0.00	0
markshare2.pre	0.0	0	0.00	8	0.0	0	0.00	0.00	0	0.0	0	0.00	0.00	0
mas284.pre	0.1	86195.863	0.00	0	0.0	86196.4819	0.01	0.01	1	0.0	86195.9	0.00	0.00	0
mas74.pre	0.1	10482.7953	0.00	0	0.0	10506.1796	1.77	1.77	7	0.0	10482.8	0.00	0.00	0
mas76.pre	0.1	38893.9036	0.00	0	0.0	38908.0216	1.27	1.27	5	0.0	38893.9	0.00	0.00	0
misc03.pre	0.0	1910	0.00	0	0.0	1910	0.00	0.00	0	0.0	1910	0.00	0.00	0
misc06.pre	0.1	12841.6894	0.00	0	0.0	12841.6894	0.00	0.00	0	0.0	12841.7	0.00	0.00	0
misc07.pre	0.0	1415	0.00	0	0.0	1415	0.00	0.00	0	0.0	1415	0.00	0.00	0
mitre.pre	12.8	114935.078	10.60	756	0.1	114908.999	0.00	-10.60	0	0.1	114909	0.00	-10.60	0
mkc.pre	4.8	-607.955378	7.22	165	0.1	-611.437978	0.00	-7.22	2	0.1	-611.438	0.00	-7.22	4
mkc1.pre	3.3	-611.85	0.00	33	0.1	-611.85	0.00	0.00	5	0.1	-611.85	0.00	0.00	17
mod008.pre	0.2	295.872654	30.75	11	0.0	290.931073	0.00	-30.75	0	0.0	290.932	0.01	-30.74	0
mod010.pre	0.2	6535	18.32	3	0.1	6532.08333	0.00	-18.32	0	0.0	6532.08	0.00	-18.32	0
mod011.pre	2.4	-60720457	13.45	43	2.5	-59437366.2	31.47	18.02	496	0.1	-61675400	0.04	-13.41	0
modglob.pre	0.8	20002089.1	68.45	137	0.1	20070624	90.59	22.14	134	0.1	19901900	36.08	-32.37	68
momentum1.pre	3.7	47128.5685	0.00	2	1.4	47128.5523	0.00	0.00	0	0.3	47128.6	0.00	0.00	0
momentum2.pre	7.8	1226.55766	0.02	3	14.7	1225.6443	0.00	-0.02	0	79.7	1225.64	0.00	-0.02	0
msc98-ip.pre	101.9	18485051	0.32	125	216.8	18491081.1	0.48	0.16	116	65.1	18491100	0.48	0.16	80
mzzv11.pre	54.2	-22773.75	0.00	0	69.2	-22773.75	0.00	0.00	0	0.4	-22773.7	0.00	0.00	0
mzzv42z.pre	29.2	-21446.2397	0.00	0	16.7	-21446.2397	0.00	0.00	0	0.3	-21446.2	0.00	0.00	0
neos1.pre	1.2	14.6933333	38.48	426	0.1	12	0.00	-38.48	0	0.0	12	0.00	-38.48	0
neos10.pre	1.6	-1198.33333	0.00	105	0.2	-1198.33333	0.00	0.00	0	0.1	-1198.33	0.01	0.01	0
neos11.pre	0.5	6	0.00	0	0.4	6	0.00	0.00	0	0.3	6	0.00	0.00	0
neos12.pre	8.9	9.41161243	0.00	0	11.4	9.41161243	0.00	0.00	0	4.7	9.41161	0.00	0.00	0

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
Name	Time Value	Dual Bound Value	Gap Closed % Value	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value
neos13.pre	4.3	-126.178378	0.00	0	143.3	-126.178378	0.00	0.00	384	0.7	-126.178	0.00	0.00	0
neos14.pre	0.1	51701.9199	55.43	143	0.1	57523.5845	69.42	13.99	299	0.0	28645.2	0.00	-55.43	0
neos15.pre	0.1	51545.6642	48.48	143	0.1	59488.361	65.24	16.76	335	0.0	28571.1	0.00	-48.48	0
neos16.pre	0.3	431	9.52	129	0.0	431	9.52	0.00	18	0.1	431	9.52	0.00	37
neos17.pre	0.1	0.000681498501	0.00	0	0.1	0.000681498501	0.00	0.00	0	0.0	0.000681499	0.00	0.00	0
neos18.pre	0.3	5.33333333	0.00	0	0.3	5.33333333	0.00	0.00	0	0.9	5.33333	0.00	0.00	0
neos19.pre	71.8	-1611	0.00	707	116.0	-1611	0.00	0.00	4929	88.7	-1611	0.00	0.00	0
neos2.pre	0.1	-4407.09724	6.00	12	0.1	-4717.66685	0.00	-6.00	8	0.0	-4717.67	0.00	-6.00	0
neos20.pre	0.6	-475	0.00	97	0.1	-475	0.00	0.00	10	0.0	-475	0.00	0.00	0
neos21.pre	0.1	2.21648352	0.00	0	0.1	2.21648352	0.00	0.00	0	0.2	2.21648	0.00	0.00	0
neos22.pre	0.3	777191.429	0.00	0	0.2	777291.429	3.96	3.96	6	0.1	777191	0.00	0.00	0
neos23.pre	0.0	56	0.00	0	0.1	56.2352941	0.29	0.29	63	0.0	56	0.00	0.00	0
neos3.pre	0.2	-6158.20911	5.96	20	0.1	-6561.50713	0.15	-5.81	13	0.0	-6571.63	0.00	-5.96	0
neos616206.pre	0.7	792.485486	3.18	228	0.1	789.074798	0.90	-2.28	49	0.0	787.917	0.13	-3.05	9
neos632659.pre	0.0	-109.714286	0.00	19	0.0	-109.714286	0.00	0.00	36	0.0	-109.714	0.00	0.00	0
neos648910.pre	0.2	16	0.00	194	0.0	16	0.00	0.00	111	0.0	16.0141	0.09	0.09	0
neos7.pre	0.1	571556.49	0.00	0	0.2	631218.795	41.84	41.84	147	0.0	571556	0.00	0.00	0
neos8.pre	1.9	-3724.75	4.17	135	0.1	-3725	0.00	-4.17	0	0.0	-3725	0.00	-4.17	0
neos9.pre	10.9	780	0.00	0	5.7	780	0.00	0.00	0	1.6	780	0.00	0.00	0
net12.pre	55.8	45.312416	9.91	90	13.7	43.4909551	8.94	-0.97	53	1.2	26.7542	0.00	-9.91	0
noswot.pre	0.0	-43	0.00	19	0.0	-43	0.00	0.00	6	0.0	-43	0.00	0.00	14
nsrand-ipx.pre	9.3	49921.7162	13.70	114	0.2	49667.8923	0.00	-13.70	0	0.1	49667.9	0.00	-13.70	0
nug08.pre	3.4	203.5	0.00	0	2.7	203.5	0.00	0.00	0	0.7	203.5	0.00	0.00	0
nw04.pre	2.6	16310.6667	0.00	0	0.8	16310.6667	0.00	0.00	0	0.7	16310.7	0.01	0.01	0
opt1217.pre	0.0	-20.0213904	0.00	0	0.0	-20.0213904	0.00	0.00	0	0.0	-20.0214	0.00	0.00	0
p0033.pre	0.0	2406.01781	57.28	22	0.0	2262.54674	0.00	-57.28	0	0.0	2262.55	0.00	-57.28	0
p0201.pre	0.0	7125	0.00	13	0.0	7125	0.00	0.00	0	0.0	7125	0.00	0.00	0
p0282.pre	0.2	255021.562	95.69	99	0.0	179990.3	0.00	-95.69	0	0.0	179990	0.00	-95.69	0
p0548.pre	0.2	8658.29748	99.06	133	0.0	5194.43616	0.00	-99.06	0	0.0	5194.44	0.00	-99.06	0
p2756.pre	2.0	3063.95023	85.78	310	0.0	2701.75	0.00	-85.78	0	0.0	2701.75	0.00	-85.78	0
pk1.pre	0.0	0	0.00	0	0.0	0	0.00	0.00	0	0.0	0	0.00	0.00	0
pp08a.pre	0.0	2763.36769	0.33	10	0.1	7103.71324	94.65	94.32	136	0.0	2748.35	0.00	-0.33	0
pp08aCUTS.pre	0.0	5298.94251	0.98	6	0.1	6806.16671	81.61	80.63	102	0.0	5280.61	0.00	-0.98	0
prod1.pre	0.8	8.72341513	1.56	48	0.0	8.44984888	0.00	-1.56	0	0.0	8.44985	0.00	-1.56	0
protfold.pre	3.4	-41.9574468	0.00	0	3.0	-41.9574468	0.00	0.00	0	1.7	-41.9574	0.00	0.00	0

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
	Time	Dual Bound	Gap Closed %	Cuts	Time	Dual Bound	Gap Closed %		Cuts	Time	Dual Bound	Gap Closed %		Cuts
Name	Value	Value	Value	Value	Value	Value	Value	△	Value	Value	Value	Value	△	Value
qap10.pre	38.7	332.566228	0.00	0	49.8	332.566228	0.00	0.00	0	1.9	332.566	0.00	0.00	0
qiu.pre	0.1	-931.638853	0.00	0	0.1	-931.638857	0.00	0.00	0	0.2	-931.639	0.00	0.00	0
qnet1.pre	0.5	15058.9949	44.71	54	0.1	14274.1034	0.00	-44.71	1	0.0	14274.1	0.00	-44.71	0
qnet1_o.pre	0.3	14969.617	66.04	60	0.0	12907.7792	0.00	-66.04	0	0.0	12907.8	0.00	-66.04	0
ran10x26.pre	0.2	4047.92469	46.23	88	0.0	4004.56705	35.73	-10.50	27	0.0	3857.02	0.00	-46.23	0
ran12x21.pre	0.3	3436.22974	55.04	122	0.0	3313.18152	30.75	-24.29	34	0.0	3157.38	0.00	-55.04	0
ran13x13.pre	0.2	2972.3949	50.12	121	0.0	2920.84099	40.92	-9.20	23	0.0	2691.44	0.00	-50.12	0
ran14x18_1.pre	0.3	3354.66557	48.45	183	0.0	3148.71179	18.90	-29.55	33	0.0	3088.59	10.28	-38.17	26
ran8x32.pre	0.2	5144.49571	66.87	99	0.0	5099.29977	52.26	-14.61	16	0.0	4937.58	0.00	-66.87	0
rentacar.pre	0.9	-3759915.64	16.55	10	0.3	-3581347.76	29.05	12.50	28	0.0	-3868360	8.96	-7.59	2
rgn.pre	0.0	67.9999988	57.49	41	0.0	48.7999986	0.00	-57.49	0	0.0	48.9538	0.46	-57.03	12
roll3000.pre	3.5	12001.9899	50.19	111	0.2	11098.7402	0.03	-50.16	10	0.1	11098.1	0.00	-50.19	0
rout.pre	0.2	-1393.32143	0.07	105	0.0	-1393.38571	0.00	-0.07	0	0.0	-1393.39	0.00	-0.07	9
set1ch.pre	0.2	42417.9556	62.56	261	0.1	46877.9452	85.52	22.96	181	0.0	30270	0.00	-62.56	0
seymour.pre	3.4	239.469492	0.00	0	3.4	239.469492	0.00	0.00	0	0.1	239.469	0.00	0.00	0
seymour1.pre	3.2	238.351528	0.00	0	2.9	238.351528	0.00	0.00	0	0.1	238.352	0.01	0.01	0
sp97ar.pre	11.6	648220123	0.77	8	1.6	648138794	0.00	-0.77	0	0.9	648139000	0.00	-0.77	0
stein27.pre	0.0	13	0.00	0	0.0	13	0.00	0.00	0	0.0	13	0.00	0.00	0
stein45.pre	0.0	22	0.00	0	0.0	22	0.00	0.00	0	0.0	22	0.00	0.00	0
swath.pre	0.2	334.496858	0.00	0	0.1	334.496858	0.00	0.00	0	0.1	334.497	0.00	0.00	16
swath1.pre	0.2	334.496858	0.00	0	0.1	334.496858	0.00	0.00	0	0.1	334.497	0.00	0.00	7
swath2.pre	0.2	334.496858	0.00	0	0.1	334.496858	0.00	0.00	0	0.1	334.497	0.00	0.00	7
swath3.pre	0.2	334.496858	0.00	0	0.1	334.496858	0.00	0.00	0	0.1	334.497	0.00	0.00	9
t1717.pre	8.6	134531.021	0.00	0	7.7	134531.021	0.00	0.00	0	1.5	134531	0.00	0.00	0
timtab1.pre	0.1	195639.169	22.63	142	0.0	222145.595	26.24	3.61	96	0.0	29032	0.00	-22.63	0
timtab2.pre	0.2	233810.779	15.06	271	0.1	279543.494	19.21	4.15	160	0.0	68068	0.00	-15.06	0
tr12-30.pre	0.2	51017.9242	32.98	400	0.2	112401.489	87.56	54.58	827	0.0	13924.2	0.00	-32.98	0
vpm1.pre	0.0	19.2397959	78.69	63	0.0	17.3333333	25.23	-53.46	7	0.0	16.4333	0.00	-78.69	0
vpm2.pre	0.1	12.9105196	67.89	75	0.0	11.4521333	12.11	-55.78	13	0.0	11.2089	2.80	-65.09	3
Total	818.2		2302.50	8278	1220.3		1586.30	-716.20	11196	509.7		191.99	-2110.51	1119
Geom. Mean	2.0		4.08	10	1.8		2.76	-1.32	5	1.2		1.28	-2.80	1

Table B.62: Computational results for the comparison with Cplex and Cbc. *Cutting plane separator for the 0-1 single node flow problem.* (△ with respect to Scip)

	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
Name	Time Value	Dual Bound Value	Gap Closed % Value	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value
10teams.pre	0.2	897	0.00	0	5.6	904	100.00	100.00	6	0.0	897	0.00	0.00	0
30:70:4_5:0_5:100.pre	10.6	5337.1	0.00	0	86.7	5337.1	0.00	0.00	0	125.9	5337.1	0.00	0.00	0
30:70:4_5:0_95:98.pre	18.4	5969.5	0.00	0	73.7	5969.5	0.00	0.00	0	113.8	5969.5	0.00	0.00	0
a1c1s1.pre	0.1	2234.5717	0.27	3	140.9	3426.55867	13.01	12.74	14	0.0	2209.57	0.00	-0.27	0
aflow30a.pre	0.0	91.1674253	0.00	0	6.9	161.098539	40.00	40.00	33	0.0	91.1674	0.00	0.00	0
aflow40b.pre	0.1	-578.335183	0.00	0	10.9	-543.626763	21.38	21.38	30	0.0	-578.281	0.03	0.03	0
air03.pre	0.6	311406.25	0.00	0	3.7	312702	100.00	100.00	8	0.1	311406	0.00	0.00	0
air04.pre	1.8	54030.4364	0.00	0	3.0	54031.4352	0.17	0.17	1	0.6	54030.9	0.08	0.08	0
air05.pre	0.6	25877.6093	0.00	0	5.5	25896.8055	3.87	3.87	11	0.3	25877.6	0.00	0.00	0
arki001.pre	0.2	7009391.43	0.00	3	1.7	7009391.43	0.00	0.00	10	0.2	7009390	0.00	0.00	7
atlanta-ip.pre	65.4	81.2578061	0.02	18	67.0	81.2755788	0.15	0.13	32	5.5	81.2578	0.02	0.00	48
bc1.pre	1.0	2.21088114	1.92	14	7.0	2.37008595	15.77	13.85	31	0.5	2.18878	0.00	-1.92	0
bell3a.pre	0.0	862116.583	0.00	0	0.0	862116.583	0.00	0.00	0	0.0	862117	0.00	0.00	0
bell5.pre	0.0	8341834.36	0.00	0	0.0	8639843.32	83.25	83.25	1	0.0	8341830	0.00	0.00	0
bienst1.pre	0.0	11.7241379	0.00	0	1.0	11.7241379	0.00	0.00	0	0.0	11.7241	0.00	0.00	0
bienst2.pre	0.0	11.7241379	0.00	0	1.8	11.7241379	0.00	0.00	0	0.0	11.7241	0.00	0.00	0
binkar10_1.pre	0.1	5737.89666	9.29	18	0.5	5761.35761	31.63	22.34	17	0.1	5757.41	27.87	18.58	10
blend2.pre	0.0	6.94616147	4.46	4	0.1	6.96943468	7.87	3.41	14	0.0	6.91568	0.00	-4.46	1
cap6000.pre	0.7	-2403387.01	2.07	10	0.2	-2403388.53	0.00	-2.07	2	0.0	-2403390	0.00	-2.07	0
dano3_3.pre	30.2	576.23162	0.00	0	197.0	576.23162	0.00	0.00	0	1.4	576.232	0.00	0.00	0
dano3_4.pre	35.5	576.23162	0.00	0	1297.2	576.23162	0.00	0.00	0	1.7	576.232	0.00	0.00	0
dano3_5.pre	44.5	576.23162	0.00	0	2052.2	576.23162	0.00	0.00	0	1.7	576.232	0.00	0.00	0
dano3mip.pre	36.5	576.23162	0.00	0	535.7	576.23162	0.00	0.00	0	0.5	576.232	0.00	0.00	0
danoint.pre	0.1	62.6372804	0.00	0	3.5	62.6372804	0.00	0.00	0	0.0	62.6374	0.00	0.00	0
dcmulti.pre	0.0	184034.377	0.00	0	1.3	185515.998	35.72	35.72	7	0.0	184034	0.00	0.00	0
ds.pre	40.4	57.2345653	0.00	0	146.1	57.2966841	0.02	0.02	15	6.6	57.2346	0.00	0.00	0
egout.pre	0.0	268.476501	45.95	6	0.0	293.018964	89.40	43.45	9	0.0	273.85	55.46	9.51	8
eilD76.pre	0.1	680.538997	0.00	0	12.3	685.939282	2.64	2.64	28	0.0	680.539	0.00	0.00	0
fast0507.pre	24.7	158.145567	0.00	0	27.0	158.145567	0.00	0.00	0	0.8	158.146	0.02	0.02	0
fiber.pre	0.1	381828.264	90.35	80	0.2	382542.799	90.64	0.29	62	0.2	365536	83.83	-6.52	55
fixnet6.pre	0.0	3190.042	0.00	0	0.7	3377.64064	23.72	23.72	14	0.0	3190.53	0.06	0.06	1
flugpl.pre	0.0	726875.166	0.00	0	0.0	726875.166	0.00	0.00	0	0.0	726875	0.00	0.00	0
gen.pre	0.0	58348.3235	97.37	7	0.0	58348.3235	97.37	0.00	5	0.0	58349.1	100.00	2.63	4
gesa2-o.pre	0.1	18752813.7	11.61	4	0.2	18785808.8	22.48	10.87	13	0.0	18752800	11.60	-0.01	4
gesa2.pre	0.1	25560447.9	20.79	14	0.2	25560072.9	20.66	-0.13	13	0.0	25551100	17.42	-3.37	12

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
	Time	Dual Bound	Gap Closed %	Cuts	Time	Dual Bound	Gap Closed %		Cuts	Time	Dual Bound	Gap Closed %		Cuts
Name	Value	Value	Value	Value	Value	Value	Value	△	Value	Value	Value	Value	△	Value
gesa3.pre	0.1	27884504.5	0.12	2	0.1	27884380.4	0.00	-0.12	0	0.0	27884500	0.11	-0.01	1
gesa3_o.pre	0.0	12274783.2	0.00	0	0.1	12274783.2	0.00	0.00	0	0.0	12275500	0.46	0.46	0
glass4.pre	0.0	800002400	0.00	0	0.7	800002400	0.00	0.00	7	0.0	800003000	0.00	0.00	0
gt2.pre	0.0	20146.7613	0.00	0	0.0	20146.7613	0.00	0.00	0	0.0	20146.8	0.00	0.00	0
harp2.pre	0.1	-74098504.9	31.40	77	0.1	-74116262.9	27.23	-4.17	46	0.0	-74186100	10.82	-20.58	7
irp.pre	0.7	12123.5302	0.00	0	5.7	12123.5526	0.06	0.06	2	0.2	12123.5	-0.08	-0.08	0
khb05250.pre	0.0	96437115	4.70	1	0.3	96437115	4.70	0.00	1	0.0	95919500	0.00	-4.70	0
l152lav.pre	0.1	4656.36364	0.00	0	0.8	4659.86945	5.34	5.34	6	0.0	4656.36	0.00	0.00	0
liu.pre	0.1	346	0.00	0	0.2	346	0.00	0.00	0	0.0	346	0.00	0.00	0
lseu.pre	0.0	1026.76335	45.31	21	0.1	1083.41177	78.54	33.23	15	0.0	1015.53	38.72	-6.59	7
manna81.pre	0.2	-13297	0.00	0	0.1	-13297	0.00	0.00	0	0.1	-13297	0.00	0.00	0
markshare1.pre	0.0	0	0.00	0	0.0	0	0.00	0.00	2	0.0	0	0.00	0.00	0
markshare2.pre	0.0	0	0.00	0	0.0	0	0.00	0.00	1	0.0	0	0.00	0.00	0
mas284.pre	0.0	86195.863	0.00	0	0.0	86195.863	0.00	0.00	0	0.0	86195.9	0.00	0.00	0
mas74.pre	0.0	10482.7953	0.00	0	0.0	10482.7953	0.00	0.00	0	0.0	10482.8	0.00	0.00	0
mas76.pre	0.0	38893.9036	0.00	0	0.0	38893.9036	0.00	0.00	0	0.0	38893.9	0.00	0.00	0
misc03.pre	0.0	1910	0.00	0	0.0	1910	0.00	0.00	2	0.0	1910	0.00	0.00	0
misc06.pre	0.0	12841.6894	0.00	0	0.1	12841.6894	0.00	0.00	0	0.0	12841.7	0.00	0.00	0
misc07.pre	0.0	1415	0.00	0	0.1	1417	0.14	0.14	6	0.0	1415	0.00	0.00	0
mitre.pre	0.6	114913.463	1.81	711	0.4	115137.5	92.89	91.08	447	0.3	114909	0.00	-1.81	183
mkc.pre	0.3	-609.512038	3.99	99	0.2	-609.618018	3.77	-0.22	45	0.1	-611.066	0.77	-3.22	13
mkc1.pre	0.2	-611.85	0.00	4	0.1	-611.85	0.00	0.00	3	0.1	-611.85	0.00	0.00	0
mod008.pre	0.1	293.062222	13.26	27	0.1	295.099502	25.94	12.68	15	0.0	290.932	0.01	-13.25	0
mod010.pre	0.1	6535	18.32	2	0.2	6535	18.32	0.00	4	0.0	6532.08	0.00	-18.32	0
mod011.pre	0.2	-61678103.8	0.00	0	0.2	-61678103.8	0.00	0.00	0	0.1	-61675400	0.04	0.04	0
modglob.pre	0.0	19790205.8	0.00	0	0.0	19790205.8	0.00	0.00	0	0.0	19790200	0.00	0.00	0
momentum1.pre	1.6	47128.5523	0.00	0	111.6	48736.8557	4.42	4.42	13	0.3	47128.6	0.00	0.00	0
momentum2.pre	3.1	1225.6443	0.00	0	334.2	2132.84566	17.83	17.83	10	79.8	1225.64	0.00	0.00	0
msc98-ip.pre	99.1	18481818.4	0.23	40	288.1	18504401.6	0.83	0.60	201	56.2	18484500	0.30	0.07	86
mzzv11.pre	54.2	-22773.75	0.00	1	77.2	-22773.75	0.00	0.00	10	0.6	-22773.8	0.00	0.00	1
mzzv42z.pre	28.4	-21446.2397	0.00	0	45.9	-21446.2397	0.00	0.00	12	0.5	-21446.2	0.00	0.00	0
neos1.pre	0.5	18	85.71	351	0.1	12	0.00	-85.71	68	0.1	12	0.00	-85.71	34
neos10.pre	0.2	-1198.33333	0.00	257	0.2	-1198.33333	0.00	0.00	15	0.1	-1198.33	0.01	0.01	0
neos11.pre	0.5	6	0.00	0	24.7	6	0.00	0.00	8	0.2	6	0.00	0.00	0
neos12.pre	8.6	9.41161243	0.00	0	59.9	9.41161243	0.00	0.00	2	4.8	9.41161	0.00	0.00	0

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
			Gap											
	Time	Dual Bound	Closed %	Cuts	Time	Dual Bound	Gap Closed %		Cuts	Time	Dual Bound	Gap Closed %		Cuts
Name	Value	Value	Value	Value	Value	Value	Value	△	Value	Value	Value	Value	△	Value
neos13.pre	1.5	-126.178378	0.00	0	9.3	-126.178378	0.00	0.00	0	0.6	-126.178	0.00	0.00	0
neos14.pre	0.0	28644.7958	0.00	0	7.3	29363.7341	1.73	1.73	6	0.0	28645.2	0.00	0.00	0
neos15.pre	0.0	28570.7845	0.00	0	3.3	28942.8512	0.79	0.79	2	0.0	28571.1	0.00	0.00	0
neos16.pre	0.0	429	0.00	0	3.9	429	0.00	0.00	6	0.0	429	0.00	0.00	0
neos17.pre	0.1	0.000681498501	0.00	0	85.5	0.0702052614	46.56	46.56	79	0.0	0.000681499	0.00	0.00	0
neos18.pre	0.2	5.33333333	0.00	0	1.6	5.33333333	0.00	0.00	9	0.9	5.33333	0.00	0.00	0
neos19.pre	9.2	-1611	0.00	0	355.6	-1611	0.00	0.00	0	88.1	-1611	0.00	0.00	0
neos2.pre	0.1	-4717.66685	0.00	0	0.4	-4717.66685	0.00	0.00	6	0.0	-4717.67	0.00	0.00	0
neos20.pre	0.1	-475	0.00	114	0.1	-475	0.00	0.00	72	0.1	-475	0.00	0.00	36
neos21.pre	0.1	2.21648352	0.00	0	34.7	3.77260456	32.53	32.53	25	0.2	2.21648	0.00	0.00	0
neos22.pre	0.2	777191.429	0.00	0	0.6	777191.429	0.00	0.00	0	0.1	777191	0.00	0.00	0
neos23.pre	0.0	56	0.00	0	0.3	56	0.00	0.00	3	0.0	56	0.00	0.00	0
neos3.pre	0.1	-6571.62916	0.00	0	0.5	-6571.62916	0.00	0.00	7	0.0	-6571.63	0.00	0.00	0
neos616206.pre	0.0	787.721258	0.00	10	0.0	787.721258	0.00	0.00	8	0.0	787.721	0.00	0.00	0
neos632659.pre	0.0	-109.714286	0.00	0	0.1	-104	36.36	36.36	5	0.0	-109.714	0.00	0.00	0
neos648910.pre	0.0	16	0.00	0	1.5	16	0.00	0.00	8	0.0	16.0141	0.09	0.09	0
neos7.pre	0.1	571556.49	0.00	0	1.1	571556.49	0.00	0.00	1	0.0	571556	0.00	0.00	0
neos8.pre	0.2	-3725	0.00	230	0.1	-3725	0.00	0.00	2	0.0	-3725	0.00	0.00	0
neos9.pre	3.8	780	0.00	0	2570.83	780	0.00	0.00	0	1.4	780	0.00	0.00	0
net12.pre	22.8	44.2774911	9.36	89	289.5	44.935471	9.71	0.35	59	8.1	44.497	9.48	0.12	55
noswot.pre	0.0	-43	0.00	0	0.0	-43	0.00	0.00	5	0.0	-43	0.00	0.00	1
nsrand-ipx.pre	3.6	49900.3607	12.55	108	0.5	49899.4806	12.50	-0.05	54	0.1	49725.8	3.13	-9.42	5
nug08.pre	3.2	203.5	0.00	0	3.3	203.5	0.00	0.00	1	0.7	203.5	0.00	0.00	0
nw04.pre	1.9	16310.6667	0.00	0	22.8	16312.8	0.39	0.39	5	0.7	16310.7	0.01	0.01	0
opt1217.pre	0.0	-20.0213904	0.00	0	0.0	-20.0213904	0.00	0.00	0	0.0	-20.0214	0.00	0.00	0
p0033.pre	0.0	2405.2433	56.98	15	0.0	2405.2433	56.98	0.00	11	0.0	2360.24	39.01	-17.97	9
p0201.pre	0.0	7125	0.00	16	0.0	7125	0.00	0.00	22	0.0	7125	0.00	0.00	0
p0282.pre	0.1	254081.161	94.49	176	0.1	255450.484	96.24	1.75	60	0.0	252468	92.43	-2.06	36
p0548.pre	0.0	8656.29142	99.01	134	0.1	8650.02647	98.83	-0.18	79	0.1	8593.13	97.20	-1.81	88
p2756.pre	0.1	3063.92405	85.77	284	0.2	3053.71794	83.36	-2.41	248	0.1	3057.11	84.16	-1.61	206
pk1.pre	0.0	0	0.00	0	0.0	0	0.00	0.00	0	0.0	0	0.00	0.00	0
pp08a.pre	0.0	2748.34524	0.00	0	0.4	2777.33333	0.63	0.63	1	0.0	2748.35	0.00	0.00	0
pp08aCUTS.pre	0.0	5280.60616	0.00	0	1.5	5289.43896	0.47	0.47	1	0.0	5280.61	0.00	0.00	0
prod1.pre	0.1	8.48145852	0.18	30	1.8	9.96205125	8.62	8.44	18	0.0	8.47756	0.16	-0.02	7
protfold.pre	3.2	-41.9574468	0.00	0	93.0	-41.9574468	0.00	0.00	4	1.6	-41.9574	0.00	0.00	0

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	SCIP 0.81				CPLEX 10.01					Cbc 1.01.00				
Name	Time Value	Dual Bound Value	Gap Closed % Value	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value	Time Value	Dual Bound Value	Gap Closed % Value	Gap Closed % △	Cuts Value
qap10.pre	37.5	332.566228	0.00	0	107.6	332.699319	1.79	1.79	15	1.9	332.566	0.00	0.00	0
qiu.pre	0.1	-931.638853	0.00	0	38.2	-874.621854	7.14	7.14	5	0.2	-931.639	0.00	0.00	0
qnet1.pre	0.1	14274.1027	0.00	0	0.1	14274.1034	0.00	0.00	1	0.0	14274.1	0.00	0.00	1
qnet1_o.pre	0.1	12918.3142	0.34	1	0.0	12918.3142	0.34	0.00	1	0.0	12918.3	0.34	0.00	1
ran10x26.pre	0.1	4012.09524	37.55	79	0.1	4027.82924	41.36	3.81	27	0.1	3978.21	29.34	-8.21	28
ran12x21.pre	0.0	3310.53114	30.23	42	0.3	3433.43552	54.49	24.26	38	0.1	3375.41	43.04	12.81	32
ran13x13.pre	0.0	2907.43593	38.53	29	0.6	2983.76788	52.15	13.62	34	0.0	2906.28	38.33	-0.20	15
ran14x18_1.pre	0.0	3212.54243	28.06	38	0.2	3335.1436	45.65	17.59	48	0.1	3298.91	40.45	12.39	32
ran8x32.pre	0.0	5011.94879	24.03	61	0.0	5102.25656	53.22	29.19	22	0.1	5095.69	51.10	27.07	19
rentacar.pre	0.2	-3996349.94	0.00	0	2.3	-3898837.01	6.83	6.83	2	0.0	-3996350	0.00	0.00	0
rgn.pre	0.0	48.7999986	0.00	0	0.0	48.7999986	0.00	0.00	0	0.0	48.8	0.00	0.00	0
roll3000.pre	0.2	11099.1529	0.06	6	2.4	11101.6573	0.20	0.14	20	0.1	11098.4	0.01	-0.05	3
rout.pre	0.1	-1393.07714	0.32	34	0.0	-1393.38571	0.00	-0.32	6	0.1	-1393.08	0.32	0.00	10
set1ch.pre	0.0	37418.5597	36.81	13	0.0	38976.0165	44.83	8.02	15	0.0	30270	0.00	-36.81	0
seymour.pre	3.2	239.469492	0.00	0	12.3	239.469492	0.00	0.00	0	0.1	239.469	0.00	0.00	0
seymour1.pre	3.2	238.351528	0.00	0	11.8	238.351528	0.00	0.00	0	0.1	238.352	0.01	0.01	0
sp97ar.pre	9.2	648220123	0.77	8	2.3	648235402	0.91	0.14	12	1.1	648139000	0.00	-0.77	2
stein27.pre	0.0	13	0.00	0	0.0	13	0.00	0.00	0	0.0	13	0.00	0.00	0
stein45.pre	0.0	22	0.00	0	0.4	22	0.00	0.00	0	0.0	22	0.00	0.00	0
swath.pre	0.2	334.496858	0.00	0	0.9	335.306288	0.57	0.57	6	0.1	334.497	0.00	0.00	0
swath1.pre	0.1	334.496858	0.00	0	9.3	340.044897	12.45	12.45	7	0.0	334.497	0.00	0.00	0
swath2.pre	0.1	334.496858	0.00	0	1.5	338.451546	7.80	7.80	4	0.0	334.497	0.00	0.00	0
swath3.pre	0.2	334.496858	0.00	0	0.8	334.496997	0.00	0.00	2	0.1	334.497	0.00	0.00	0
t1717.pre	8.1	134531.021	0.00	0	28.9	134556.912	0.02	0.02	10	1.5	134531	0.00	0.00	0
timtab1.pre	0.0	29032	0.00	0	0.2	37936	1.21	1.21	4	0.0	29032	0.00	0.00	0
timtab2.pre	0.0	68068	0.00	0	0.5	84334	1.48	1.48	13	0.0	68068	0.00	0.00	0
tr12-30.pre	0.0	16712.383	2.48	4	24.3	17512.7541	3.19	0.71	12	0.0	13924.2	0.00	-2.48	0
vpm1.pre	0.0	19.5	85.98	12	0.0	20	100.00	14.02	15	0.2	17.9167	41.59	-44.39	8
vpm2.pre	0.0	12.2403221	42.25	14	0.4	12.7329969	61.10	18.85	26	0.0	12.4036	48.50	6.25	14
Total	624.7		1174.71	3321	9376.93		2062.13	887.42	2469	513.4		966.24	-208.47	1090
Geom. Mean	1.7		2.06	3	3.4		3.45	1.39	5	1.3		1.74	-0.32	1

Table B.63: Computational results for the comparison with CPLEX and CBC. *Cutting plane separator for the 0-1 knapsack problem.* (△ with respect to SCIP)

B.5 Impact on the Overall Performance of SCIP

	No Cuts	**Only GMI**		**Only C-MIR**		**Only Knapsack**		**Only Flow Cover**		**Only Impl. B.**		**Only Clique**	
	Nodes	Nodes		Nodes		Nodes		Nodes		Nodes		Nodes	
Name	Value	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor
10teams	347	1358	0.26	347	1.00	347	1.00	347	1.00	347	1.00	5059	0.07
30:70:4_5:0_95:98	16	256	0.06	16	1.00	16	1.00	16	1.00	16	1.00	16	1.00
aflow30a	196780	270959	0.73	50971	3.86	65746	2.99	65873	2.99	19473	10.11	117488	1.67
air03	2	2	1.00	2	1.00	2	1.00	2	1.00	2	1.00	2	1.00
air04	130	130	1.00	130	1.00	130	1.00	130	1.00	130	1.00	11	11.82
air05	291	291	1.00	291	1.00	291	1.00	291	1.00	291	1.00	171	1.70
bc1	19556	19556	1.00	2124	9.21	16443	1.19	6112	3.20	19556	1.00	19556	1.00
bell3a	46706	47499	0.98	47125	0.99	46706	1.00	46706	1.00	48048	0.97	46706	1.00
bell5	6794	1084	6.27	21070	0.32	6794	1.00	6794	1.00	5935	1.14	6794	1.00
bienst1	29047	8690	3.34	9819	2.96	29047	1.00	8658	3.35	29047	1.00	29047	1.00
bienst2	117138	96898	1.21	73091	1.60	117138	1.00	89647	1.31	117138	1.00	117138	1.00
blend2	10179	6628	1.54	8902	1.14	9071	1.12	5310	1.92	6174	1.65	10179	1.00
cap6000	3550	3550	1.00	3550	1.00	3004	1.18	2779	1.28	3550	1.00	3550	1.00
dano3_3	22	22	1.00	20	1.10	22	1.00	22	1.00	22	1.00	22	1.00
dano3_4	29	29	1.00	36	0.81	29	1.00	29	1.00	29	1.00	29	1.00
dano3_5	169	169	1.00	195	0.87	169	1.00	169	1.00	169	1.00	169	1.00
dcmulti	1252	203	6.17	349	3.59	1237	1.01	893	1.40	857	1.46	1327	0.94
egout	57	18	3.17	4	14.25	36	1.58	2	28.50	1	57.00	57	1.00
eilD76	12372	12372	1.00	12372	1.00	12372	1.00	12372	1.00	12372	1.00	4414	2.80
fiber	8116	504	16.10	114	71.19	98	82.82	40	202.90	8116	1.00	8116	1.00
fixnet6	271	241	1.12	357	0.76	271	1.00	95	2.85	281	0.96	271	1.00
flugpl	1131	240	4.71	1135	1.00	1131	1.00	1131	1.00	1131	1.00	1131	1.00
gen	31	74	0.42	1	31.00	1	31.00	1	31.00	31	1.00	31	1.00
gesa2	57313	69038	0.83	70	818.76	52320	1.10	45756	1.25	57357	1.00	57313	1.00
gesa2-o	114630	104680	1.10	1635	70.11	88355	1.30	90601	1.27	83482	1.37	114630	1.00
gesa3	4649	1574	2.95	34	136.74	4900	0.95	1660	2.80	4695	0.99	4649	1.00
gesa3_o	7219	6882	1.05	274	26.35	7233	1.00	6016	1.20	5593	1.29	7219	1.00
gt2	489	368	1.33	2	244.50	489	1.00	1262	0.39	489	1.00	489	1.00
irp	3347	3347	1.00	3347	1.00	3347	1.00	3347	1.00	3347	1.00	486	6.89
khb05250	1896	562	3.37	385	4.92	1854	1.02	23	82.43	32	59.25	1896	1.00
l152lav	67	53	1.26	67	1.00	67	1.00	67	1.00	67	1.00	82	0.82
lseu	6463	1231	5.25	176	36.72	101	63.99	192	33.66	6463	1.00	6463	1.00
mas284	15406	15406	1.00	15406	1.00	15406	1.00	15406	1.00	15406	1.00	15406	1.00
mas74	4209005	4209005	1.00	4209005	1.00	4209005	1.00	4209005	1.00	4209005	1.00	4209005	1.00
mas76	616407	616407	1.00	616407	1.00	616407	1.00	616407	1.00	616407	1.00	616407	1.00

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Name	No Cuts Nodes Value	Only GMI Nodes Value	Only GMI Improv. Factor	Only C-MIR Nodes Value	Only C-MIR Improv. Factor	Only Knapsack Nodes Value	Only Knapsack Improv. Factor	Only Flow Cover Nodes Value	Only Flow Cover Improv. Factor	Only Impl. B. Nodes Value	Only Impl. B. Improv. Factor	Only Clique Nodes Value	Only Clique Improv. Factor
misc03	148	187	0.79	148	1.00	148	1.00	148	1.00	148	1.00	148	1.00
misc06	34	23	1.48	34	1.00	34	1.00	34	1.00	34	1.00	34	1.00
misc07	37707	31525	1.20	37707	1.00	35257	1.07	37707	1.00	37707	1.00	37707	1.00
mitre	115	11	10.45	66	1.74	99	1.16	1134	0.10	62	1.85	63	1.83
mod008	1031	1031	1.00	228	4.52	188	5.48	401	2.57	1031	1.00	1031	1.00
mod010	34	2	17.00	4	8.50	18	1.89	4	8.50	34	1.00	34	1.00
mod011	21611	21611	1.00	2683	8.05	21611	1.00	4171	5.18	3113	6.94	21611	1.00
neos2	38588	40437	0.95	101961	0.38	91563	0.42	38588	1.00	54917	0.70	48419	0.80
neos648910	3588	>3832892	–	422	8.50	3588	1.00	34177	0.10	456393	0.01	1421883	0.00
neos8	3	1	3.00	3	1.00	3	1.00	3	1.00	3	1.00	2	1.50
neos10	17	7	2.43	5	3.40	16	1.06	16	1.06	8	2.12	16	1.06
neos11	12711	12711	1.00	12711	1.00	13551	0.94	12711	1.00	12711	1.00	12711	1.00
neos13	2668	391	6.82	2668	1.00	2668	1.00	2668	1.00	2668	1.00	2668	1.00
neos20	15283	13548	1.13	7312	2.09	6770	2.26	6836	2.24	9539	1.60	7762	1.97
neos21	1740	1613	1.08	1740	1.00	1740	1.00	1740	1.00	1740	1.00	1740	1.00
neos22	135930	18282	7.44	5	27186.00	133792	1.02	135930	1.00	56779	2.39	135930	1.00
neos23	1659312	>1880899	–	987076	1.68	1659312	1.00	1659312	1.00	1281595	1.29	1659312	1.00
nug08	3	3	1.00	3	1.00	3	1.00	3	1.00	3	1.00	3	1.00
nw04	3	3	1.00	3	1.00	3	1.00	3	1.00	3	1.00	5	0.60
p0033	98	93	1.05	3	32.67	14	7.00	8	12.25	110	0.89	110	0.89
p0201	55	122	0.45	111	0.50	134	0.41	83	0.66	55	1.00	55	1.00
p0282	148	119	1.24	55	2.69	86	1.72	96	1.54	118	1.25	148	1.00
p0548	3198	259	12.35	105	30.46	124	25.79	136	23.51	6075	0.53	3198	1.00
pk1	240849	442058	0.54	240849	1.00	240849	1.00	240849	1.00	240849	1.00	240849	1.00
prod1	61076	61076	1.00	61967	0.99	61415	0.99	60622	1.01	61076	1.00	61076	1.00
qap10	8	8	1.00	8	1.00	8	1.00	8	1.00	8	1.00	3	2.67
qiu	9865	9865	1.00	9865	1.00	9865	1.00	9865	1.00	9865	1.00	9865	1.00
qnet1	254	162	1.57	82	3.10	254	1.00	22	11.55	254	1.00	254	1.00
qnet1_o	1011	343	2.95	9	112.33	1011	1.00	124	8.15	1011	1.00	1011	1.00
ran10x26	267429	116519	2.30	65789	4.06	34413	7.77	41370	6.46	267429	1.00	267429	1.00
ran12x21	1405207	986844	1.42	171287	8.20	282414	4.98	131999	10.65	1405207	1.00	1405207	1.00
ran13x13	403238	170527	2.36	178599	2.26	148657	2.71	93726	4.30	403238	1.00	403238	1.00
ran8x32	46488	56801	0.82	21665	2.15	20072	2.32	21751	2.14	46488	1.00	46488	1.00
rentacar	68	59	1.15	31	2.19	68	1.00	25	2.72	4	17.00	68	1.00
rgn	421	1157	0.36	111	3.79	421	1.00	1245	0.34	516	0.82	421	1.00

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	No Cuts	Only GMI		Only C-MIR		Only Knapsack		Only Flow Cover		Only Impl. B.		Only Clique	
	Nodes	Nodes		Nodes		Nodes		Nodes		Nodes		Nodes	
Name	Value	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor
rout	138027	184843	0.75	339100	0.41	21648	6.38	196265	0.70	138027	1.00	138027	1.00
seymour1	7252	4229	1.71	7252	1.00	7252	1.00	7252	1.00	7252	1.00	4561	1.59
stein27	3749	4299	0.87	3749	1.00	3749	1.00	3749	1.00	3749	1.00	3749	1.00
stein45	52883	51621	1.02	52883	1.00	52883	1.00	52883	1.00	52883	1.00	52883	1.00
swath1	12347	12347	1.00	4409	2.80	12427	0.99	12347	1.00	12347	1.00	12347	1.00
swath2	27074	27074	1.00	23884	1.13	23305	1.16	27074	1.00	27074	1.00	27074	1.00
swath3	98989	184244	0.54	105206	0.94	125092	0.79	98989	1.00	98989	1.00	98989	1.00
vpm1	93941	19802	4.74	3	31313.67	1	93941.00	5	18788.20	93941	1.00	93941	1.00
vpm2	689595	393100	1.75	9625	71.65	76325	9.03	17848	38.64	226543	3.04	787553	0.88
Geom. Mean (77/79)			1.42		3.55		1.74		2.17		1.29		1.08
Not Solved to Opt.	0		2		0		0		0		0		0

Table B.64: Computational results concerning the impact of individual cutting plane separators on the overall performance of SCIP. *Enabling individual cutting plane separators on the solvable test set.* Improv. Factor is defined by (7.1). It gives the factor by which enabling a separator improves the performance with respect to the performance measure Nodes.

Name	No Cuts Time Value	Only GMI Time Value	Only GMI Time Improv. Factor	Only C-MIR Time Value	Only C-MIR Time Improv. Factor	Only Knapsack Time Value	Only Knapsack Time Improv. Factor	Only Flow Cover Time Value	Only Flow Cover Time Improv. Factor	Only Impl. B. Time Value	Only Impl. B. Time Improv. Factor	Only Clique Time Value	Only Clique Time Improv. Factor
10teams	41.0	29.3	1.40	41.2	0.99	41.0	1.00	40.7	1.01	41.1	1.00	130.1	0.31
30:70:4_5:0_95:98	411.5	523.5	0.79	416.2	0.99	406.2	1.01	401.3	1.03	408.4	1.01	408.7	1.01
aflow30a	505.9	656.1	0.77	313.8	1.61	229.7	2.20	265.0	1.91	76.5	6.61	321.8	1.57
air03	27.4	27.4	1.00	27.9	0.98	27.0	1.02	27.3	1.01	27.5	1.00	31.5	0.87
air04	236.7	238.9	0.99	238.2	0.99	234.7	1.01	234.8	1.01	237.2	1.00	119.3	1.98
air05	88.0	90.0	0.98	89.5	0.98	87.8	1.00	87.1	1.01	88.0	1.00	96.2	0.91
bc1	740.4	744.7	0.99	172.3	4.30	675.1	1.10	343.2	2.16	742.6	1.00	748.1	0.99
bell3a	64.6	125.0	0.52	131.5	0.49	64.3	1.00	64.8	1.00	49.7	1.30	64.2	1.01
bell5	4.4	1.6	2.85	14.3	0.31	4.4	1.00	4.5	0.98	3.8	1.17	4.5	0.98
bienst1	85.5	40.2	2.13	89.7	0.95	85.3	1.00	66.9	1.28	84.9	1.01	86.2	0.99
bienst2	373.3	470.3	0.79	612.9	0.61	375.2	0.99	653.7	0.57	370.0	1.01	374.4	1.00
blend2	15.8	11.4	1.39	16.3	0.97	14.5	1.09	10.4	1.52	9.1	1.74	15.8	1.00
cap6000	91.2	91.4	1.00	91.0	1.00	79.9	1.14	82.5	1.11	91.0	1.00	91.6	1.00
dano3_3	291.1	291.3	1.00	274.5	1.06	291.4	1.00	294.7	0.99	291.9	1.00	293.6	0.99
dano3_4	291.4	294.5	0.99	315.7	0.92	288.1	1.01	291.1	1.00	292.3	1.00	291.0	1.00
dano3_5	663.9	669.4	0.99	750.1	0.89	665.5	1.00	665.7	1.00	670.7	0.99	668.3	0.99
dcmulti	3.2	3.1	1.02	3.9	0.82	3.1	1.03	2.6	1.23	3.0	1.07	3.1	1.01
egout	0.0	0.1	1.00	0.1	1.00	0.1	1.00	0.0	1.00	0.0	1.00	0.1	1.00
eilD76	101.7	102.7	0.99	103.5	0.98	102.2	0.99	101.6	1.00	101.8	1.00	129.6	0.78
fiber	33.2	6.9	4.83	3.4	9.83	3.8	8.74	2.1	16.12	32.7	1.01	32.8	1.01
fixnet6	2.4	4.3	0.57	6.7	0.36	2.4	1.01	1.8	1.31	2.5	0.98	2.4	1.01
flugpl	0.8	0.4	1.00	0.8	1.00	0.8	1.00	0.8	1.00	0.8	1.00	0.8	1.00
gen	0.5	0.8	1.00	0.2	1.00	0.1	1.00	0.2	1.00	0.5	1.00	0.5	1.00
gesa2	294.3	351.3	0.84	9.1	32.31	293.0	1.00	268.1	1.10	289.8	1.02	294.4	1.00
gesa2-o	526.8	490.9	1.07	23.9	22.09	430.8	1.22	434.6	1.21	413.2	1.27	525.7	1.00
gesa3	38.9	19.1	2.03	8.6	4.54	36.8	1.06	19.3	2.02	38.6	1.01	39.0	1.00
gesa3_o	49.4	43.2	1.15	13.6	3.63	47.3	1.05	44.8	1.10	42.8	1.15	49.1	1.01
gt2	1.1	0.7	1.15	0.0	1.15	1.2	0.99	1.4	0.82	1.1	1.01	1.2	0.98
irp	338.3	338.0	1.00	338.3	1.00	337.8	1.00	336.3	1.01	338.6	1.00	113.3	2.99
khb05250	4.6	2.7	1.68	3.5	1.30	4.5	1.00	0.6	4.57	0.4	4.57	4.6	0.99
l152lav	6.6	6.9	0.96	6.6	1.01	6.5	1.01	6.5	1.01	6.5	1.02	5.8	1.14
lseu	4.2	1.7	2.47	0.5	4.20	0.3	4.20	0.5	4.20	4.2	0.99	4.2	0.99
mas284	21.2	20.9	1.01	21.2	1.00	21.3	0.99	21.2	1.00	21.0	1.01	21.0	1.01
mas74	1582.2	1576.5	1.00	1567.4	1.01	1606.8	0.98	1603.3	0.99	1592.3	0.99	1605.4	0.99
mas76	251.2	247.2	1.02	247.0	1.02	256.5	0.98	254.1	0.99	251.0	1.00	254.3	0.99

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	No Cuts	Only GMI		Only C-MIR		Only Knapsack		Only Flow Cover		Only Impl. B.		Only Clique	
	Time	Time		Time		Time		Time		Time		Time	
Name	Value	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor
misc03	1.2	2.9	0.44	1.3	0.98	1.2	1.02	1.3	0.98	1.2	1.02	1.3	0.98
misc06	0.7	0.9	1.00	0.7	1.00	0.7	1.00	0.8	1.00	0.7	1.00	0.7	1.00
misc07	66.6	63.2	1.05	68.2	0.98	64.2	1.04	66.4	1.00	67.6	0.98	67.6	0.99
mitre	85.5	12.0	7.14	116.5	0.73	69.0	1.24	220.9	0.39	77.9	1.10	86.9	0.98
mod008	1.9	1.9	1.00	1.6	1.21	1.4	1.33	2.0	0.93	1.9	0.98	1.9	0.98
mod010	5.3	4.6	1.16	4.2	1.27	4.6	1.17	4.2	1.28	5.3	1.00	5.3	1.01
mod011	408.8	407.8	1.00	352.3	1.16	403.0	1.01	160.8	2.54	151.5	2.70	410.4	1.00
neos2	202.5	234.7	0.86	522.4	0.39	387.6	0.52	205.9	0.98	285.8	0.71	240.7	0.84
neos648910	12.8	>3600.0	–	5.5	2.31	13.1	0.98	44.8	0.29	534.0	0.02	1354.3	0.01
neos8	258.3	259.1	1.00	259.0	1.00	257.3	1.00	258.3	1.00	252.9	1.02	255.8	1.01
neos10	326.7	310.7	1.05	314.5	1.04	312.5	1.05	326.1	1.00	327.2	1.00	325.2	1.00
neos11	1679.3	1691.2	0.99	1722.9	0.97	1844.7	0.91	1660.0	1.01	1700.8	0.99	1668.8	1.01
neos13	390.1	250.7	1.56	409.5	0.95	392.5	0.99	393.1	0.99	395.1	0.99	395.6	0.99
neos20	74.5	72.2	1.03	42.8	1.74	45.5	1.64	43.8	1.70	47.9	1.56	40.0	1.86
neos21	62.1	55.0	1.13	60.8	1.02	61.2	1.01	62.2	1.00	61.3	1.01	61.3	1.01
neos22	1679.6	252.3	6.66	6.7	249.20	1619.7	1.04	1657.7	1.01	657.2	2.56	1649.7	1.02
neos23	3162.1	>3600.0	–	2144.2	1.47	3178.2	0.99	3196.6	0.99	2588.9	1.22	3099.7	1.02
nug08	31.9	53.3	0.60	32.4	0.99	31.8	1.00	32.9	0.97	32.3	0.99	48.5	0.66
nw04	93.9	95.8	0.98	97.0	0.97	94.3	1.00	95.9	0.98	93.8	1.00	103.9	0.90
p0033	0.1	0.1	1.00	0.1	1.00	0.1	1.00	0.0	1.00	0.1	1.00	0.1	1.00
p0201	0.6	1.4	0.73	1.2	0.83	1.2	0.81	1.0	0.99	0.6	1.00	0.6	1.00
p0282	0.7	0.8	1.00	0.7	1.00	1.0	1.00	0.9	1.00	0.6	1.00	0.6	1.00
p0548	7.3	1.7	4.27	1.9	3.84	1.1	6.48	1.8	4.06	10.4	0.70	7.2	1.00
pk1	121.5	222.6	0.55	115.4	1.05	117.1	1.04	117.4	1.04	123.2	0.99	116.7	1.04
prod1	67.9	67.8	1.00	72.6	0.94	67.6	1.01	76.1	0.89	68.2	1.00	71.4	0.95
qap10	260.6	264.1	0.99	262.0	0.99	263.1	0.99	259.2	1.01	259.6	1.00	269.7	0.97
qiu	217.2	214.9	1.01	218.2	1.00	217.8	1.00	218.7	0.99	214.4	1.01	218.1	1.00
qnet1	7.0	7.7	0.91	6.2	1.14	6.8	1.03	2.5	2.75	7.0	1.00	6.9	1.02
qnet1_o	8.2	9.4	0.87	2.0	4.18	8.3	0.99	6.4	1.28	8.2	1.00	8.2	1.01
ran10x26	274.9	162.3	1.69	141.1	1.95	96.6	2.85	94.8	2.90	276.3	1.00	281.6	0.98
ran12x21	1268.9	1228.9	1.03	412.6	3.08	459.0	2.76	286.1	4.44	1297.8	0.98	1306.5	0.97
ran13x13	277.8	207.2	1.34	203.8	1.36	149.9	1.85	137.0	2.03	277.1	1.00	278.6	1.00
ran8x32	72.3	112.7	0.64	66.0	1.10	55.8	1.30	64.6	1.12	72.3	1.00	72.7	0.99
rentacar	24.9	25.3	0.98	24.7	1.01	25.1	0.99	20.9	1.19	5.6	4.44	24.8	1.00
rgn	0.7	1.2	0.81	1.1	0.88	0.7	1.00	1.3	0.79	0.8	1.00	0.7	1.00

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Name	No Cuts Time Value	Only GMI Time Value	Only GMI Time Improv. Factor	Only C-MIR Time Value	Only C-MIR Time Improv. Factor	Only Knapsack Time Value	Only Knapsack Time Improv. Factor	Only Flow Cover Time Value	Only Flow Cover Time Improv. Factor	Only Impl. B. Time Value	Only Impl. B. Time Improv. Factor	Only Clique Time Value	Only Clique Time Improv. Factor
rout	256.2	333.0	0.77	556.4	0.46	57.6	4.45	409.7	0.63	256.5	1.00	256.9	1.00
seymour1	1998.1	1202.3	1.66	2110.1	0.95	2023.0	0.99	2030.4	0.98	2009.9	0.99	1284.1	1.56
stein27	2.7	3.1	0.85	2.7	0.99	2.7	0.99	2.7	0.98	2.7	0.99	2.7	0.98
stein45	60.5	61.4	0.99	60.7	1.00	61.0	0.99	60.1	1.01	59.3	1.02	60.8	1.00
swath1	194.4	195.8	0.99	130.9	1.48	195.5	0.99	193.9	1.00	193.1	1.01	195.5	0.99
swath2	382.3	383.8	1.00	334.9	1.14	337.8	1.13	384.1	1.00	382.3	1.00	389.1	0.98
swath3	1030.4	1810.7	0.57	1076.8	0.96	1309.5	0.79	1027.6	1.00	1021.4	1.01	1018.1	1.01
vpm1	45.8	13.7	3.35	0.0	45.84	0.0	45.84	0.1	45.84	45.0	1.02	45.9	1.00
vpm2	438.8	308.0	1.42	14.6	30.00	57.8	7.60	19.4	22.68	153.6	2.86	518.4	0.85
Geom. Mean (77/79)			1.12		1.45		1.25		1.33		1.13		1.01
Not Solved to Opt.	0		2		0		0		0		0		0

Table B.65: Computational results concerning the impact of individual cutting plane separators on the overall performance of SCIP. *Enabling individual cutting plane separators on the solvable test set.* Improv. Factor is defined by (7.1). It gives the factor by which enabling a separator improves the performance with respect to the performance measure Time.

Name	No Cuts Gap % Value	Only GMI Gap % Value	Only GMI Improv. Factor	Only C-MIR Gap % Value	Only C-MIR Improv. Factor	Only Knapsack Gap % Value	Only Knapsack Improv. Factor	Only Flow Cover Gap % Value	Only Flow Cover Improv. Factor	Only Impl. B. Gap % Value	Only Impl. B. Improv. Factor	Only Clique Gap % Value	Only Clique Improv. Factor
a1c1s1	348.98	293.08	1.19	76.98	4.53	229.76	1.52	133.82	2.61	167.47	2.08	348.92	1.00
30:70:4_5:0_5:100	11.11	0.00	–	22.22	0.50	11.11	1.00	11.11	1.00	11.11	1.00	55.11	0.20
aflow40b	26.15	20.45	1.28	19.98	1.31	18.21	1.44	12.37	2.11	37.40	0.70	20.08	1.30
arki001	0.03	0.03	1.00	0.04	1.00	0.03	1.00	0.03	1.00	0.04	1.00	0.03	1.00
atlanta-ip	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
binkar10_1	0.71	0.71	1.00	0.33	1.00	0.35	1.00	0.37	1.00	0.71	1.00	0.71	1.00
dano3mip	26.95	26.95	1.00	25.26	1.07	27.23	0.99	26.95	1.00	26.95	1.00	26.95	1.00
danoint	4.55	4.55	1.00	3.08	1.47	4.19	1.08	4.03	1.13	4.37	1.04	4.56	1.00
ds	447.28	447.59	1.00	447.59	1.00	447.28	1.00	447.28	1.00	447.28	1.00	531.03	0.84
fast0507	4.43	4.43	1.00	4.43	1.00	4.43	1.00	4.43	1.00	4.43	1.00	4.43	1.00
glass4	137.50	168.75	0.81	162.50	0.85	137.50	1.00	175.00	0.79	152.08	0.90	168.75	0.81
liu	305.00	305.00	1.00	388.93	0.78	305.00	1.00	305.00	1.00	305.00	1.00	305.00	1.00
manna81	0.96	0.00	–	0.96	1.00	0.96	1.00	0.96	1.00	0.96	1.00	0.96	1.00
markshare1	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
markshare2	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
mkc	8.99	11.17	0.80	5.75	1.56	7.23	1.24	3.45	2.60	10.13	0.89	15.59	0.58
mkc1	0.16	0.15	1.00	0.26	1.00	0.09	1.00	0.10	1.00	0.16	1.00	0.05	1.00
modglob	0.22	0.22	1.00	0.18	1.00	0.22	1.00	0.00	–	0.22	1.00	0.22	1.00
momentum1	∞	∞	–	∞	–	30.46	–	∞	–	∞	–	37.46	–
momentum2	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
msc98-ip	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
mzzv11	3.94	3.22	1.22	3.93	1.00	6.16	0.64	3.91	1.01	0.00	–	1.70	2.32
mzzv42z	4.31	0.00	–	4.33	0.99	4.35	0.99	4.31	1.00	0.00	–	1.88	2.30
neos1	147.83	141.53	1.04	30.52	4.84	115.64	1.28	131.71	1.12	131.71	1.12	49.02	3.02
neos3	64.57	80.81	0.80	30.66	2.11	0.00	–	64.59	1.00	0.00	–	45.78	1.41
neos616206	6.53	13.97	0.47	7.96	0.82	5.10	1.28	12.35	0.53	6.53	1.00	14.74	0.44
neos632659	10.84	0.00	–	0.00	–	10.84	1.00	8.99	1.21	3.66	2.96	10.84	1.00
neos7	0.28	0.84	1.00	0.00	–	0.00	–	0.28	1.00	0.00	–	0.28	1.00
neos9	2.31	0.00	–	2.31	1.00	2.31	1.00	2.31	1.00	2.31	1.00	2.31	1.00
neos12	13.49	12.70	1.06	13.70	0.99	13.70	0.98	13.74	0.98	13.74	0.98	13.54	1.00
neos14	89.00	16.24	5.48	12.38	7.19	89.00	1.00	21.42	4.15	21.04	4.23	89.00	1.00
neos15	104.53	20.47	5.11	14.76	7.08	104.53	1.00	28.16	3.71	28.74	3.64	104.53	1.00
neos16	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
neos17	390.42	390.58	1.00	390.63	1.00	390.66	1.00	390.38	1.00	390.79	1.00	390.52	1.00
neos18	6.67	6.67	1.00	6.67	1.00	6.67	1.00	6.67	1.00	6.67	1.00	6.67	1.00

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	No Cuts	Only GMI		Only C-MIR		Only Knapsack		Only Flow Cover		Only Impl. B.		Only Clique	
	Gap %	Gap %		Gap %		Gap %		Gap %		Gap %		Gap %	
Name	Value	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor	Value	Improv. Factor
neos19	15.21	17.32	0.88	15.21	1.00	16.20	0.94	15.21	1.00	8.19	1.86	16.14	0.94
net12	63.89	∞	–	166.87	0.38	∞	–	132.82	0.48	142.55	0.45	124.46	0.51
noswot	4.65	0.00	–	4.65	1.00	4.65	1.00	4.65	1.00	0.00	–	4.65	1.00
nsrand-ipx	11.66	8.64	1.35	7.47	1.56	10.04	1.16	7.82	1.49	11.65	1.00	11.65	1.00
opt1217	19.87	16.83	1.18	19.87	1.00	19.87	1.00	19.87	1.00	19.87	1.00	19.87	1.00
p2756	1.76	0.00	–	0.00	–	0.00	–	0.00	–	1.49	1.18	0.63	1.76
pp08a	33.56	13.02	2.58	0.00	–	33.56	1.00	32.61	1.03	17.33	1.94	33.56	1.00
pp08aCUTS	6.41	0.00	–	0.00	–	6.41	1.00	5.48	1.17	6.41	1.00	6.41	1.00
protfold	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
ran14x18_1	11.16	7.64	1.46	4.97	2.25	6.95	1.61	5.17	2.16	11.16	1.00	11.16	1.00
roll3000	15.45	12.29	1.26	4.98	3.10	13.27	1.16	11.23	1.38	14.49	1.07	15.45	1.00
set1ch	40.55	4.20	9.65	0.00	–	26.21	1.55	16.98	2.39	17.12	2.37	40.55	1.00
seymour	3.90	3.18	1.23	3.90	1.00	3.90	1.00	3.90	1.00	3.90	1.00	3.16	1.24
sp97ar	5.14	5.81	0.88	5.76	0.89	4.49	1.15	3.85	1.34	5.14	1.00	5.14	1.00
swath	43.63	33.49	1.30	35.07	1.24	47.77	0.91	43.63	1.00	43.63	1.00	43.63	1.00
t1717	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
timtab1	98.24	80.64	1.22	22.02	4.46	72.72	1.35	74.71	1.32	88.89	1.11	98.24	1.00
timtab2	∞	∞	–	148.50	0.01	∞	–	∞	–	∞	–	∞	–
tr12-30	597.27	213.51	2.80	27.85	21.45	533.00	1.12	152.85	3.91	56.27	10.61	597.27	1.00
Geom. Mean (29/54)			1.20		1.66		1.10		1.32		1.24		0.99
No Feas. Sol.	8		9		7		8		8		8		7
Solved to Opt.	0		8		6		3		2		5		0

Table B.66: Computational results concerning the impact of individual cutting plane separators on the overall performance of Scip. *Enabling individual cutting plane separators on the unsolvable test set.* Improv. Factor is defined by (7.1). It gives the factor by which enabling a separator improves the performance with respect to the performance measure Gap %.

	All Cuts	**No GMI**		**No C-MIR**		**No Knapsack**		**No Flow Cover**		**No Impl. B.**		**No Clique**	
	Nodes	Nodes		Nodes		Nodes		Nodes		Nodes		Nodes	
Name	Value	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor
10teams	588	456	0.78	588	1.00	588	1.00	588	1.00	588	1.00	125	0.21
30:70:4_5:0_5:100	276	378	1.37	276	1.00	276	1.00	276	1.00	276	1.00	338	1.22
30:70:4_5:0_95:98	200	129	0.65	200	1.00	200	1.00	200	1.00	200	1.00	200	1.00
aflow30a	7132	8686	1.22	10400	1.46	18663	2.62	8091	1.13	15877	2.23	7109	1.00
air03	2	2	1.00	2	1.00	2	1.00	2	1.00	2	1.00	2	1.00
air04	85	85	1.00	85	1.00	85	1.00	85	1.00	85	1.00	148	1.74
air05	196	237	1.21	196	1.00	196	1.00	196	1.00	196	1.00	518	2.64
bc1	2232	2232	1.00	4764	2.13	10699	4.79	2119	0.95	2232	1.00	2232	1.00
bell3a	50005	42600	0.85	48012	0.96	50005	1.00	50005	1.00	45191	0.90	50005	1.00
bell5	1722	27309	15.86	1148	0.67	1722	1.00	1722	1.00	1459	0.85	1722	1.00
bienst1	7907	11759	1.49	9348	1.18	8251	1.04	8227	1.04	7907	1.00	7907	1.00
bienst2	76567	81082	1.06	76892	1.00	76567	1.00	86344	1.13	76567	1.00	76567	1.00
blend2	7699	7699	1.00	10092	1.31	8229	1.07	8734	1.13	9560	1.24	7699	1.00
cap6000	2817	2817	1.00	2817	1.00	2779	0.99	4142	1.47	2817	1.00	2817	1.00
dano3_3	20	20	1.00	22	1.10	20	1.00	20	1.00	20	1.00	20	1.00
dano3_4	36	36	1.00	29	0.81	36	1.00	36	1.00	36	1.00	36	1.00
dano3_5	195	195	1.00	169	0.87	195	1.00	195	1.00	195	1.00	195	1.00
dcmulti	129	393	3.05	230	1.78	70	0.54	103	0.80	83	0.64	134	1.04
egout	1	1	1.00	2	2.00	1	1.00	1	1.00	2	2.00	1	1.00
eilD76	4414	4414	1.00	4414	1.00	4414	1.00	4414	1.00	4414	1.00	12372	2.80
fiber	37	41	1.11	25	0.68	30	0.81	140	3.78	37	1.00	37	1.00
fixnet6	54	55	1.02	49	0.91	54	1.00	90	1.67	47	0.87	54	1.00
flugpl	361	1135	3.14	240	0.66	361	1.00	361	1.00	361	1.00	361	1.00
gen	1	1	1.00	1	1.00	1	1.00	1	1.00	1	1.00	1	1.00
gesa2	27	27	1.00	11479	425.15	280	10.37	19	0.70	51	1.89	27	1.00
gesa2-o	632	794	1.26	26429	41.82	1416	2.24	1380	2.18	554	0.88	632	1.00
gesa3	214	36	0.17	1247	5.83	32	0.15	110	0.51	84	0.39	214	1.00
gesa3_o	25	25	1.00	6058	242.32	143	5.72	25	1.00	26	1.04	25	1.00
gt2	20	639	31.95	169	8.45	20	1.00	20	1.00	20	1.00	20	1.00
irp	311	183	0.59	311	1.00	311	1.00	311	1.00	311	1.00	3347	10.76
khb05250	19	19	1.00	20	1.05	19	1.00	23	1.21	25	1.32	19	1.00
l152lav	53	56	1.06	53	1.00	53	1.00	53	1.00	53	1.00	49	0.92
lseu	180	520	2.89	137	0.76	49	0.27	201	1.12	180	1.00	180	1.00
manna81	1	1	1.00	1	1.00	1	1.00	1	1.00	1	1.00	1	1.00
mas284	15406	15406	1.00	15406	1.00	15406	1.00	15406	1.00	15406	1.00	15406	1.00

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Name	All Cuts Nodes Value	No GMI Nodes Value	No GMI Degrad. Factor	No C-MIR Nodes Value	No C-MIR Degrad. Factor	No Knapsack Nodes Value	No Knapsack Degrad. Factor	No Flow Cover Nodes Value	No Flow Cover Degrad. Factor	No Impl. B. Nodes Value	No Impl. B. Degrad. Factor	No Clique Nodes Value	No Clique Degrad. Factor
mas74	4209005	4209005	1.00	4209005	1.00	4209005	1.00	4209005	1.00	4209005	1.00	4209005	1.00
mas76	616407	616407	1.00	616407	1.00	616407	1.00	616407	1.00	616407	1.00	616407	1.00
misc03	89	124	1.39	57	0.64	89	1.00	89	1.00	89	1.00	81	0.91
misc06	23	34	1.48	23	1.00	23	1.00	23	1.00	23	1.00	23	1.00
misc07	44880	47285	1.05	44880	1.00	45038	1.00	44880	1.00	44880	1.00	44880	1.00
mitre	2	38	19.00	1	0.50	56	28.00	1	0.50	16	8.00	2	1.00
mod008	218	218	1.00	219	1.00	378	1.73	188	0.86	218	1.00	218	1.00
mod010	1	1	1.00	1	1.00	1	1.00	1	1.00	1	1.00	1	1.00
mod011	1145	1145	1.00	3168	2.77	1145	1.00	1372	1.20	1506	1.32	1145	1.00
modglob	27695	27695	1.00	33937	1.23	27695	1.00	>395893	–	27695	1.00	27695	1.00
mzzv11	3610	2935	0.81	3610	1.00	4999	1.38	3610	1.00	>12768	–	6250	1.73
mzzv42z	1314	2105	1.60	1314	1.00	1407	1.07	1314	1.00	1657	1.26	3441	2.62
neos1	1	>494053	–	>479664	–	>487410	–	>496032	–	2	2.00	>483794	–
neos2	41287	74458	1.80	36657	0.89	120635	2.92	34015	0.82	77591	1.88	67122	1.63
neos3	326050	576317	1.77	>747408	–	655812	2.01	>543128	–	501152	1.54	>602463	–
neos632659	811	5212	6.43	52569	64.82	811	1.00	1630	2.01	2477	3.05	811	1.00
neos648910	60585	116955	1.93	3297	0.05	60585	1.00	579623	9.57	1613	0.03	72494	1.20
neos7	43402	31933	0.74	27643	0.64	44663	1.03	52058	1.20	50720	1.17	43402	1.00
neos8	1	1	1.00	1	1.00	1	1.00	1	1.00	1	1.00	1	1.00
neos9	35	>1937	–	35	1.00	35	1.00	35	1.00	35	1.00	35	1.00
neos10	7	5	0.71	7	1.00	9	1.29	7	1.00	4	0.57	4	0.57
neos11	13551	13551	1.00	13551	1.00	12711	0.94	13551	1.00	13551	1.00	13551	1.00
neos13	391	2668	6.82	391	1.00	391	1.00	391	1.00	391	1.00	391	1.00
neos20	12912	10967	0.85	13591	1.05	18609	1.44	6970	0.54	9547	0.74	8204	0.64
neos21	1325	3152	2.38	1325	1.00	1325	1.00	1325	1.00	1325	1.00	1325	1.00
neos22	1	5	5.00	27317	27317.00	1	1.00	19	19.00	1	1.00	1	1.00
nug08	3	5	1.67	3	1.00	3	1.00	3	1.00	3	1.00	5	1.67
nw04	5	5	1.00	5	1.00	5	1.00	5	1.00	5	1.00	3	0.60
p0033	1	1	1.00	1	1.00	2	2.00	1	1.00	1	1.00	1	1.00
p0201	56	341	6.09	198	3.54	64	1.14	106	1.89	56	1.00	56	1.00
p0282	101	75	0.74	103	1.02	162	1.60	124	1.23	101	1.00	101	1.00
p0548	32	39	1.22	32	1.00	27	0.84	35	1.09	85	2.66	32	1.00
p2756	29	247	8.52	83	2.86	168	5.79	80	2.76	91	3.14	92	3.17
pk1	223941	240849	1.08	442058	1.97	223941	1.00	223941	1.00	223941	1.00	223941	1.00
pp08a	1310	967	0.74	>305126	–	1310	1.00	883	0.67	1197	0.91	1310	1.00

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	All Cuts	No GMI		No C-MIR		No Knapsack		No Flow Cover		No Impl. B.		No Clique	
	Nodes	Nodes		Nodes		Nodes		Nodes		Nodes		Nodes	
Name	Value	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor
pp08aCUTS	953	1284	1.35	162921	170.96	953	1.00	615	0.65	953	1.00	953	1.00
prod1	60511	60511	1.00	64686	1.07	64661	1.07	60949	1.01	60511	1.00	60511	1.00
qap10	5	5	1.00	5	1.00	5	1.00	5	1.00	5	1.00	5	1.00
qiu	9865	9865	1.00	9865	1.00	9865	1.00	9865	1.00	9865	1.00	9865	1.00
qnet1	90	71	0.79	67	0.74	90	1.00	52	0.58	30	0.33	90	1.00
qnet1_o	18	51	2.83	152	8.44	18	1.00	70	3.89	18	1.00	18	1.00
ran10x26	30417	27422	0.90	32603	1.07	27294	0.90	27320	0.90	30417	1.00	30417	1.00
ran12x21	64936	70555	1.09	104418	1.61	76775	1.18	81236	1.25	64936	1.00	64936	1.00
ran13x13	30739	43957	1.43	61692	2.01	23907	0.78	64387	2.09	30739	1.00	30739	1.00
ran8x32	8684	9915	1.14	15170	1.75	9693	1.12	17891	2.06	8684	1.00	8684	1.00
rentacar	4	4	1.00	4	1.00	4	1.00	4	1.00	32	8.00	4	1.00
rgn	43	53	1.23	1505	35.00	43	1.00	123	2.86	73	1.70	43	1.00
rout	18746	56113	2.99	70432	3.76	174116	9.29	103110	5.50	18746	1.00	18746	1.00
set1ch	29	54	1.86	>899123	–	64	2.21	21	0.72	42	1.45	29	1.00
seymour1	3994	3994	1.00	3994	1.00	3994	1.00	3994	1.00	3994	1.00	4229	1.06
stein27	4158	4237	1.02	4158	1.00	4158	1.00	4158	1.00	4158	1.00	4158	1.00
stein45	54006	52815	0.98	54006	1.00	54006	1.00	54006	1.00	54006	1.00	54006	1.00
swath1	8678	4944	0.57	12427	1.43	7922	0.91	8678	1.00	8678	1.00	8678	1.00
swath2	25389	19277	0.76	23305	0.92	27204	1.07	19148	0.75	25389	1.00	25389	1.00
swath3	129854	99495	0.77	285224	2.20	150364	1.16	119504	0.92	129854	1.00	129854	1.00
vpm1	2	3	1.50	47	23.50	2	1.00	1	0.50	2	1.00	2	1.00
vpm2	7308	5402	0.74	8602	1.18	6812	0.93	5692	0.78	5177	0.71	7308	1.00
Geom. Mean (85/92)			1.34		1.85		1.18		1.16		1.05		1.06
Not Solved to Opt.	0		2		4		1		3		1		2

Table B.67: Computational results concerning the impact of individual cutting plane separators on the overall performance of SCIP. *Disabling individual cutting plane separators on the solvable test set.* Degrad. Factor is defined by (7.2). It gives the factor by which disabling a separator degrades the performance with respect to the performance measure Nodes.

	All Cuts	No GMI		No C-MIR		No Knapsack		No Flow Cover		No Impl. B.		No Clique	
	Time	Time		Time		Time		Time		Time		Time	
Name	Value	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor
10teams	52.4	52.0	0.99	47.5	0.91	51.9	0.99	50.3	0.96	51.9	0.99	33.3	0.64
30:70:4_5:0_5:100	1355.6	655.4	0.48	1332.7	0.98	1353.9	1.00	1355.9	1.00	1344.4	0.99	677.8	0.50
30:70:4_5:0_95:98	595.2	488.1	0.82	573.7	0.96	594.3	1.00	598.6	1.01	594.8	1.00	599.2	1.01
aflow30a	71.3	76.3	1.07	64.5	0.90	137.1	1.92	81.5	1.14	133.9	1.88	75.3	1.06
air03	42.9	40.7	0.95	37.9	0.88	43.2	1.01	41.1	0.96	42.6	0.99	29.1	0.68
air04	237.1	219.2	0.92	225.4	0.95	236.9	1.00	238.0	1.00	236.5	1.00	182.8	0.77
air05	213.0	128.4	0.60	199.7	0.94	213.1	1.00	213.0	1.00	212.0	1.00	108.0	0.51
bc1	211.8	212.9	1.01	271.7	1.28	518.7	2.45	197.4	0.93	212.8	1.00	214.3	1.01
bell3a	64.9	56.9	0.88	68.0	1.05	66.0	1.02	65.4	1.01	112.4	1.73	64.9	1.00
bell5	2.1	17.8	8.52	1.6	0.77	2.1	1.00	2.1	1.00	1.9	0.90	2.1	1.00
bienst1	59.4	93.8	1.58	53.9	0.91	60.5	1.02	70.9	1.19	59.6	1.00	58.5	0.99
bienst2	661.3	683.0	1.03	538.1	0.81	664.1	1.00	761.1	1.15	650.7	0.98	659.1	1.00
blend2	15.8	15.6	0.99	15.5	0.98	16.2	1.03	13.2	0.84	19.7	1.25	15.7	1.00
cap6000	85.5	85.3	1.00	83.0	0.97	84.5	0.99	104.1	1.22	84.7	0.99	84.6	0.99
dano3_3	278.6	280.2	1.01	294.9	1.06	277.0	0.99	281.9	1.01	277.5	1.00	280.4	1.01
dano3_4	319.1	320.5	1.00	292.0	0.92	319.5	1.00	321.2	1.01	319.2	1.00	328.4	1.03
dano3_5	757.5	756.2	1.00	672.3	0.89	764.4	1.01	764.2	1.01	759.0	1.00	762.7	1.01
dcmulti	5.5	3.9	0.71	3.8	0.70	5.1	0.93	4.8	0.88	5.7	1.05	6.0	1.09
egout	0.0	0.0	1.00	0.0	1.00	0.0	1.00	0.0	1.00	0.0	1.00	0.0	1.00
eilD76	145.8	138.0	0.95	139.2	0.95	145.8	1.00	144.7	0.99	149.3	1.02	103.0	0.71
fiber	4.4	3.1	0.70	2.1	0.47	2.5	0.57	5.2	1.18	4.5	1.02	4.4	1.00
fixnet6	7.2	5.7	0.79	2.1	0.29	7.1	0.98	7.0	0.97	7.5	1.04	7.5	1.03
flugpl	0.5	0.8	1.00	0.5	1.00	0.6	1.00	0.5	1.00	0.5	1.00	0.5	1.00
gen	0.4	0.3	1.00	0.2	1.00	0.4	1.00	0.2	1.00	0.4	1.00	0.4	1.00
gesa2	7.1	6.7	0.94	75.0	10.53	12.2	1.71	5.4	0.76	7.5	1.05	6.9	0.97
gesa2-o	19.1	17.2	0.90	146.1	7.65	23.2	1.22	20.4	1.07	16.9	0.88	19.6	1.03
gesa3	14.4	9.7	0.67	17.2	1.20	8.3	0.58	11.3	0.79	13.6	0.95	14.4	1.00
gesa3_o	9.2	9.0	0.98	42.2	4.57	13.0	1.41	7.8	0.84	10.3	1.11	9.4	1.02
gt2	0.2	1.5	1.51	0.4	1.00	0.2	1.00	0.2	1.00	0.2	1.00	0.2	1.00
irp	122.4	109.5	0.89	111.7	0.91	121.1	0.99	118.9	0.97	121.6	0.99	343.7	2.81
khb05250	1.8	1.7	0.97	0.7	0.56	1.8	0.98	1.6	0.90	1.7	0.95	1.8	0.99
l152lav	11.5	10.4	0.91	9.3	0.81	11.3	0.99	9.8	0.85	11.3	0.98	10.6	0.92
lseu	0.6	0.9	1.00	0.4	1.00	0.4	1.00	0.6	1.00	0.6	1.00	0.6	1.00
manna81	6.9	7.4	1.07	4.7	0.68	7.0	1.00	5.5	0.80	6.9	1.00	7.0	1.00
mas284	21.4	21.7	1.01	21.5	1.01	21.4	1.00	21.4	1.00	21.5	1.01	21.5	1.01

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	All Cuts	No GMI		No C-MIR		No Knapsack		No Flow Cover		No Impl. B.		No Clique	
	Time	Time		Time		Time		Time		Time		Time	
Name	Value	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor
mas74	1644.4	1634.3	0.99	1648.7	1.00	1565.5	0.95	1645.8	1.00	1647.9	1.00	1658.0	1.01
mas76	260.2	259.7	1.00	257.0	0.99	250.2	0.96	260.5	1.00	258.0	0.99	261.8	1.01
misc03	2.1	2.0	0.98	2.1	0.99	2.0	0.97	2.0	0.94	2.0	0.97	1.8	0.87
misc06	1.0	0.8	0.98	1.0	0.98	1.0	1.01	0.9	0.98	1.0	0.98	1.0	1.02
misc07	80.9	84.7	1.05	80.4	0.99	81.3	1.01	79.9	0.99	79.5	0.98	79.8	0.99
mitre	94.5	88.8	0.94	41.1	0.43	90.1	0.95	42.7	0.45	88.4	0.94	85.8	0.91
mod008	1.7	1.7	1.01	1.7	0.98	2.2	1.27	1.5	0.90	1.8	1.04	1.8	1.02
mod010	4.5	5.5	1.23	4.3	0.97	4.5	1.00	4.3	0.97	6.8	1.53	4.6	1.03
mod011	265.2	262.1	0.99	171.6	0.65	263.7	0.99	332.1	1.25	369.6	1.39	266.3	1.00
modglob	73.7	74.2	1.01	94.7	1.28	74.0	1.00	>624.6	–	74.0	1.00	74.1	1.01
mzzv11	2127.9	1443.1	0.68	2070.3	0.97	2370.7	1.11	2175.9	1.02	>3600.2	–	2021.3	0.95
mzzv42z	904.1	1052.1	1.16	839.7	0.93	746.2	0.83	884.5	0.98	1433.7	1.59	1526.0	1.69
neos1	32.6	>3155.9	–	>3600.0	–	>3300.9	–	>3499.2	–	39.0	1.19	>3156.2	–
neos2	281.8	409.4	1.45	200.8	0.71	570.1	2.02	244.2	0.87	417.1	1.48	489.4	1.74
neos3	2206.6	3124.3	1.42	>3600.0	–	3576.1	1.62	>3600.0	–	3092.2	1.40	>3600.0	–
neos632659	4.3	8.9	2.07	51.8	12.07	4.2	0.99	5.8	1.36	7.8	1.83	4.3	1.00
neos648910	88.7	156.9	1.77	12.9	0.15	88.7	1.00	700.5	7.90	9.5	0.11	95.7	1.08
neos7	536.8	422.5	0.79	300.6	0.56	574.1	1.07	599.2	1.12	701.0	1.31	546.6	1.02
neos8	260.1	258.8	0.99	258.1	0.99	258.4	0.99	261.0	1.00	257.0	0.99	258.9	1.00
neos9	967.3	>3600.0	–	456.5	0.47	971.7	1.00	912.1	0.94	973.3	1.01	972.2	1.00
neos10	359.7	362.1	1.01	328.4	0.91	351.8	0.98	356.7	0.99	347.0	0.96	361.1	1.00
neos11	1755.7	1780.9	1.01	1790.6	1.02	1696.7	0.97	1784.1	1.02	1764.7	1.01	1756.0	1.00
neos13	354.9	411.5	1.16	266.2	0.75	354.3	1.00	344.4	0.97	354.7	1.00	357.2	1.01
neos20	101.8	73.1	0.72	104.4	1.03	92.7	0.91	49.5	0.49	62.3	0.61	56.2	0.55
neos21	44.1	80.0	1.82	42.9	0.97	44.4	1.01	43.6	0.99	44.5	1.01	45.6	1.03
neos22	9.4	7.4	0.78	351.6	37.24	9.2	0.97	13.4	1.42	9.0	0.96	9.2	0.97
nug08	116.6	144.7	1.24	109.1	0.94	118.6	1.02	116.7	1.00	117.0	1.00	151.2	1.30
nw04	254.1	235.2	0.93	224.6	0.88	247.0	0.97	142.3	0.56	244.9	0.96	101.5	0.40
p0033	0.1	0.1	1.00	0.1	1.00	0.1	1.00	0.1	1.00	0.1	1.00	0.1	1.00
p0201	2.2	3.3	1.53	3.3	1.51	2.2	1.01	1.9	0.89	2.1	0.98	2.2	1.00
p0282	1.5	1.5	0.96	1.7	1.12	1.7	1.11	1.4	0.93	1.5	0.99	1.5	1.00
p0548	0.6	0.8	1.00	0.5	1.00	0.6	1.00	0.4	1.00	1.7	1.68	0.6	1.00
p2756	12.1	20.0	1.65	16.3	1.34	19.5	1.61	15.6	1.28	14.4	1.19	14.1	1.16
pk1	103.3	117.5	1.14	229.1	2.22	101.7	0.99	103.0	1.00	102.7	0.99	103.9	1.01
pp08a	5.6	4.8	0.85	>380.1	–	5.6	1.00	4.5	0.81	6.2	1.10	5.6	1.00

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	All Cuts	No GMI		No C-MIR		No Knapsack		No Flow Cover		No Impl. B.		No Clique	
	Time	Time		Time		Time		Time		Time		Time	
Name	Value	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor	Value	Degrad. Factor
pp08aCUTS	5.2	6.0	1.15	301.4	58.07	5.2	1.01	4.9	0.94	5.1	0.99	5.1	0.99
prod1	79.8	79.4	0.99	79.0	0.99	78.8	0.99	74.0	0.93	79.5	1.00	78.7	0.99
qap10	602.2	574.8	0.95	586.6	0.97	591.9	0.98	598.5	0.99	599.3	1.00	356.5	0.59
qiu	214.7	215.2	1.00	214.6	1.00	215.6	1.00	219.9	1.02	219.6	1.02	220.9	1.03
qnet1	12.3	7.1	0.58	9.0	0.73	12.1	0.98	9.8	0.80	10.8	0.88	12.6	1.03
qnet1_o	8.1	5.8	0.72	9.8	1.20	8.0	0.98	7.1	0.87	8.1	0.99	8.1	0.99
ran10x26	100.7	90.1	0.89	92.4	0.92	91.7	0.91	95.6	0.95	100.3	1.00	101.7	1.01
ran12x21	187.5	212.7	1.13	254.8	1.36	219.6	1.17	233.3	1.24	187.7	1.00	189.4	1.01
ran13x13	75.9	102.7	1.35	113.8	1.50	61.3	0.81	111.9	1.47	76.0	1.00	75.4	0.99
ran8x32	37.8	40.8	1.08	53.1	1.41	40.6	1.08	73.3	1.94	37.5	0.99	37.5	0.99
rentacar	8.8	8.6	0.97	7.0	0.79	8.7	0.98	7.0	0.79	26.8	3.03	8.8	0.99
rgn	0.6	0.7	1.00	1.3	1.29	0.6	1.00	1.1	1.09	0.7	1.00	0.6	1.00
rout	56.9	141.3	2.48	175.7	3.09	361.9	6.37	255.3	4.49	56.7	1.00	57.1	1.00
set1ch	1.6	1.5	0.96	>2181.3	–	1.8	1.16	1.1	0.70	1.5	0.98	1.5	0.97
seymour1	1300.6	1346.5	1.04	1248.3	0.96	1310.6	1.01	1255.0	0.96	1250.3	0.96	1193.7	0.92
stein27	2.9	3.1	1.06	3.0	1.04	3.0	1.02	3.0	1.04	3.0	1.05	2.9	1.01
stein45	63.5	61.4	0.97	69.0	1.09	64.1	1.01	63.9	1.01	64.2	1.01	65.4	1.03
swath1	202.0	138.2	0.68	198.6	0.98	187.0	0.93	199.9	0.99	201.3	1.00	200.8	0.99
swath2	404.5	317.3	0.78	340.0	0.84	411.4	1.02	323.7	0.80	406.2	1.00	407.0	1.01
swath3	1465.8	1100.6	0.75	2571.9	1.75	1544.5	1.05	1316.3	0.90	1472.9	1.00	1463.0	1.00
vpm1	0.1	0.1	1.00	0.4	1.00	0.1	1.00	0.0	1.00	0.1	1.00	0.1	1.00
vpm2	12.2	9.8	0.80	10.0	0.82	10.8	0.89	8.9	0.73	9.8	0.80	12.2	1.00
Geom. Mean (85/92)			1.03		1.13		1.05		1.02		1.03		0.97
Not Solved to Opt.	0		2		4		1		3		1		2

Table B.68: Computational results concerning the impact of individual cutting plane separators on the overall performance of SCIP. *Disabling individual cutting plane separators on the solvable test set.* Degrad. Factor is defined by (7.2). It gives the factor by which disabling a separator degrades the performance with respect to the performance measure Time.

Name	All Cuts Gap % Value	No GMI Gap % Value	No GMI Degrad. Factor	No C-MIR Gap % Value	No C-MIR Degrad. Factor	No Knapsack Gap % Value	No Knapsack Degrad. Factor	No Flow Cover Gap % Value	No Flow Cover Degrad. Factor	No Impl. B. Gap % Value	No Impl. B. Degrad. Factor	No Clique Gap % Value	No Clique Degrad. Factor
a1c1s1	44.93	47.16	1.05	80.86	1.80	40.23	0.90	44.27	0.99	52.36	1.17	44.89	1.00
aflow40b	17.40	17.41	1.00	10.50	0.60	16.63	0.96	8.41	0.48	15.20	0.87	17.09	0.98
arki001	0.03	0.03	1.00	0.02	1.00	0.02	1.00	0.03	1.00	0.04	1.00	0.03	1.00
atlanta-ip	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
binkar10_1	0.36	0.36	1.00	0.27	1.00	0.45	1.00	0.36	1.00	0.36	1.00	0.36	1.00
dano3mip	23.32	23.32	1.00	27.23	1.17	25.26	1.08	23.32	1.00	23.32	1.00	23.32	1.00
danoint	4.34	4.34	1.00	4.34	1.00	4.18	0.97	5.31	1.22	4.32	1.00	4.37	1.01
ds	532.00	531.68	1.00	531.29	1.00	532.00	1.00	532.00	1.00	532.00	1.00	447.76	0.84
fast0507	4.43	4.43	1.00	4.43	1.00	4.43	1.00	4.43	1.00	4.43	1.00	4.43	1.00
glass4	150.00	150.00	1.00	141.67	0.94	150.00	1.00	159.38	1.06	168.75	1.13	135.94	0.91
liu	381.79	388.93	1.02	305.00	0.80	381.79	1.00	381.79	1.00	381.79	1.00	381.79	1.00
markshare1	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
markshare2	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
mkc	2.42	3.56	1.47	3.02	1.25	1.46	0.61	3.53	1.46	4.22	1.75	3.23	1.34
mkc1	0.02	0.05	1.00	0.01	1.00	0.21	1.00	0.01	1.00	0.05	1.00	0.03	1.00
momentum1	∞	∞	–	∞	–	∞	–	44.32	–	99.89	–	∞	–
momentum2	∞	∞	–	∞	–	∞	–	∞	–	43.96	–	∞	–
msc98-ip	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
neos616206	9.97	11.37	1.14	8.52	0.85	7.44	0.75	6.23	0.63	9.96	1.00	10.69	1.07
neos12	12.71	13.49	1.06	12.70	1.00	12.72	1.00	12.75	1.00	12.84	1.01	12.72	1.00
neos14	2.02	4.87	2.41	4.89	2.42	2.02	1.00	0.97	0.49	2.29	1.13	2.02	1.00
neos15	7.23	6.47	0.89	16.25	2.25	7.23	1.00	6.51	0.90	7.42	1.03	7.23	1.00
neos16	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
neos17	390.62	390.47	1.00	390.62	1.00	390.57	1.00	390.68	1.00	390.52	1.00	390.68	1.00
neos18	6.67	6.67	1.00	6.67	1.00	6.67	1.00	6.67	1.00	6.67	1.00	6.67	1.00
neos19	∞	15.33	–	∞	–	∞	–	∞	–	∞	–	∞	–
neos23	1.48	7.87	5.31	0.00	–	0.00	–	1.48	1.00	0.00	–	0.00	–
net12	151.88	∞	–	113.85	0.75	47.44	0.31	∞	–	∞	–	147.97	0.97
noswot	4.65	4.65	1.00	4.65	1.00	4.65	1.00	4.65	1.00	4.65	1.00	4.65	1.00
nsrand-ipx	7.78	7.47	0.96	6.41	0.82	11.12	1.43	9.61	1.24	7.78	1.00	7.78	1.00
opt1217	16.83	19.87	1.18	16.83	1.00	16.83	1.00	16.83	1.00	16.83	1.00	16.83	1.00
protfold	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
ran14x18_1	3.80	3.81	1.00	4.31	1.14	4.02	1.06	3.88	1.02	3.80	1.00	3.80	1.00
roll3000	2.85	3.58	1.25	5.18	1.82	4.28	1.50	5.47	1.92	2.25	0.79	2.75	0.96
seymour	2.78	2.77	1.00	2.77	1.00	2.79	1.00	2.78	1.00	2.78	1.00	3.49	1.26

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Name	All Cuts Gap % Value	No GMI Gap % Value	No GMI Degrad. Factor	No C-MIR Gap % Value	No C-MIR Degrad. Factor	No Knapsack Gap % Value	No Knapsack Degrad. Factor	No Flow Cover Gap % Value	No Flow Cover Degrad. Factor	No Impl. B. Gap % Value	No Impl. B. Degrad. Factor	No Clique Gap % Value	No Clique Degrad. Factor
sp97ar	6.99	3.34	0.48	6.99	1.00	5.56	0.80	6.99	1.00	6.99	1.00	6.99	1.00
swath	37.47	40.15	1.07	28.76	0.77	29.61	0.79	38.03	1.02	37.47	1.00	37.47	1.00
t1717	∞	∞	–	∞	–	∞	–	∞	–	∞	–	∞	–
timtab1	20.01	26.84	1.34	79.54	3.98	41.05	2.05	21.29	1.06	39.16	1.96	20.01	1.00
timtab2	175.73	∞	–	∞	–	∞	–	∞	–	149.54	0.85	175.73	1.00
tr12-30	0.12	0.07	1.00	43.74	43.74	0.12	1.00	0.15	1.00	0.42	1.00	0.12	1.00
Geom. Mean (28/41)			1.05		1.29		1.01		0.99		1.05		1.01
No Feas. Sol.	8		9		9		9		9		7		8
Solved to Opt.	0		0		1		1		0		1		1

Table B.69: Computational results concerning the impact of individual cutting plane separators on the overall performance of Scip. *Disabling individual cutting plane separators on the unsolvable test set.* Degrad. Factor is defined by (7.2). It gives the factor by which disabling a separator degrades the performance with respect to the performance measure Gap %.

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