Tighter LP relaxations for configuration knapsacks using extended formulations

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Configuration Knapsacks

Knapsack constraints

A frequent structure in many Mixed-Integer Programs (MIPs)

$$\begin{aligned} \min \quad & c^t x \\ \text{s.t.} \quad & Ax \le b \\ & x \in \{0,1\}^{n_b} \times \mathbb{Z}_{\ge 0}^{n_g} \times \mathbb{R}^{n_c} \end{aligned}$$

is the

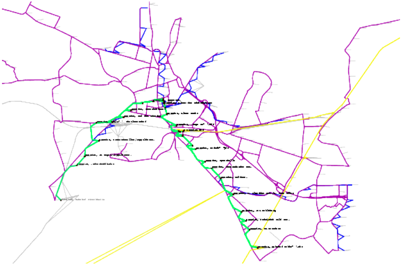
Knapsack constraint

$$\sum_j w_j x_j \le \beta \qquad \textbf{knap}(w, \beta)$$

with

- $w_j \in \mathbb{Z}_{\ge 0}$
- $w_j = 0$ for all $j > n_b$
- $\beta \in \mathbb{Z}_+$

- Input: Graph $G = (V, E)$.
- Edge demands $d_e \geq 0$
- set L of possible paths in G
- frequencies
  $F = \{f_1, \ldots, f_d\} \subset \mathbb{Z}_+$
- Operational costs $c_{l,f} > 0$.

- 

Example – Line Planning

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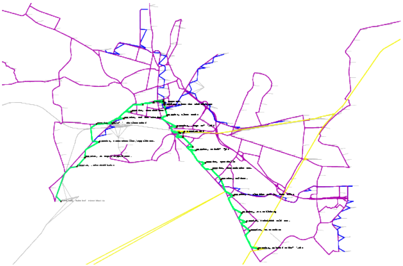
- 

Line Planning Model [Borndörfer et al., 2013]

$$
\begin{aligned}
\min \quad & \sum_{l \in L} \sum_{f \in F} c_{l,f} x_{l,f} & \\
\text{s.t.} \quad & \sum_{l \in L : e \in l} \sum_{f \in F} f \cdot x_{l,f} \geq d_e & \forall e \in E \\
& \sum_{f \in F} x_{l,f} \leq 1 & \forall l \in L \\
& x \in \{0,1\}^{L \times F} &
\end{aligned}
$$

Transformation into Knapsack

The demand inequalities can be formulated as knapsack constraint knap(w, β).

Let $e \in E$, use $\bar{x}_{l,f} = 1 - x_{l,f} \in \{0, 1\}$

$$
\begin{aligned}
& \sum_{l \in L: e \in l} \sum_{f \in F} f \cdot x_{l,f} \geq d_e \\
\Leftrightarrow \quad & \sum_{l \in L: e \in l} \sum_{f \in F} f \cdot (1 - \bar{x}_{l,f}) \geq d_e \\
\Leftrightarrow \quad & \sum_{l \in L: e \in l} \sum_{f \in F} f \cdot \bar{x}_{l,f} \leq \underbrace{\Big(\sum_{l \in L: e \in l} \sum_{f \in F} f \Big) - d_e}_{=: \beta}
\end{aligned}
$$

Central observation: Demand constraints only have "a handful" (d) different weights.

Configuration knapsack
Let $d \in \mathbb{N}$. Let $w \in \mathbb{Z}^n_{\geq 0}, \beta \in \mathbb{Z}_{\geq 0}$ define a knapsack constraint $\textbf{knap}(w, \beta)$. If there exists a partition of $[n]$ into $k \leq d$ groups $N_1, \ldots, N_k$

$$[n] = N_1 \dot\cup N_2 \dot\cup \ldots \dot\cup N_k$$

such that $i, j \in N_l \Leftrightarrow w_i = w_j (=: \omega_l)$, then $\textbf{knap}(w, \beta)$ can be written

$$\sum_{i=1}^{n} w_i x_i = \sum_{l=1}^{k} \omega_l \sum_{i \in N_l} x_i \leq \beta$$

and is called a configuration knapsack.

Cover inequalities for knapsacks

Let $\mathsf{knap}(w, \beta)$ be a knapsack constraint. Define

$$P := \{x \in \{0,1\}^n : \ w^T x \leq \beta\}$$

and

$$P^{\mathrm{LP}} := \{x \in [0,1]^n : \ w^T x \leq \beta\} \supseteq \mathsf{conv}(P)$$

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Cover Inequalities [Wolsey, 1975]

Let $C \subset [n]$ be minimal such that

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Cover Inequalities [Wolsey, 1975]

Let $C \subset [n]$ be minimal such that

$$\sum_{i \in C} w_i > \beta.$$

Then,

$$\sum_{i \in C} x_i \leq |C| - 1$$

is a valid inequality for $\mathsf{conv}(P)$.

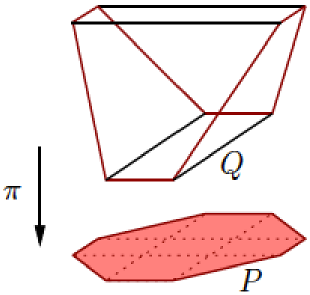
More related work

- more knapsack cutting planes:
  - Lifted cover inequalities [Balas, Gu]
  - G(eneralized) U(pper) B(ound) Inequalities [Wolsey, 1990]
  - strengthening of cover inequalities [Carr et al, 2000]
- Existence of small extended formulations for knapsack polytopes [Bienstock 2008, Bazzi et al, 2016]
- a lot of very recent work presented at this conference.

An Extended Formulation for configuration knapsacks

- Construct higher dimensional polytope Q such that the projection π into the space of x-variables is tighter, ideally $\mathbf{conv}\{P\}$

  [Borndörfer, Hoppmann, Karbstein, 2013] construct an extended formulation for the line planning problem.



Extended formulation for configuration knapsacks

Let $\mathsf{knap}(w, \beta)$ be a configuration knapsack of cardinality $k \leq d$.

$$\sum_{l=1}^{k} \omega_l \underbrace{\sum_{i \in N_l} x_i}_{=:\, y_l} \leq \beta$$

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Reformulation of knap(w, β)
Let $\mathcal{Y}$ denote all maximal points of

$$\{y : \sum \omega_l y_l \leq \beta, y_l \in \{0, \ldots, |N_l|\}, l = 1, \ldots, k\}.$$

Introduce new binary variables z_y for $y \in \mathcal{Y}$.

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Introduce new binary variables z_y for $y \in \mathcal{Y}$.

$$\begin{aligned} \sum_{i \in N_l} x_i &\leq \sum_{y \in \mathcal{Y}} y_l z_y \quad \forall l = 1, \dots, k \\ \sum_{y \in \mathcal{Y}} z_y &= 1 \\ z_y &\in \{0, 1\} \qquad\qquad y \in \mathcal{Y} \end{aligned} \tag{reform(w, β, x, z)}$$

Example

$$\underbrace{2x_1 + 2x_2 + 2x_3}_{N_1 = \{1,2,3\}} \quad + \quad \underbrace{5x_4 + 5x_5 + 5x_6}_{N_2 = \{4,5,6\}} \quad + \quad \underbrace{7x_7 + 7x_8 + 7x_9}_{N_3 = \{7,8,9\}} \quad \leq 11$$

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- three weights $\omega_1 = 2, \omega_2 = 5, \omega_3 = 7$
- weight space inequality $\sum_{l=1}^{3} \omega_l y_l \leq \beta$, $0 \leq y_l \leq 3$

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- maximal points (maximal configurations)

$$\mathcal{Y} = \left\{ \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \right\}$$

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- Reformulation: $z_y \in \{0,1\}$ for all y, $\sum_y z_y = 1$

$$\begin{array}{ccccccccccc} x_1 & + & x_2 & + & x_3 & \leq & 3z_{y_1} & + & 2z_{y_2} & & \\ x_4 & + & x_5 & + & x_6 & \leq & z_{y_1} & & & + & 2z_{y_3} \\ x_7 & + & x_9 & + & x_9 & \leq & & & z_{y_2} & & \end{array}$$

Extended formulation for configuration knapsacks

Let

- $P := \{x \in \{0,1\}^n : \; w^T x \leq \beta\}$,
- $P_1 := \{(x,z) : \; \text{reform}(w, \beta, x, z)\}$,
- $P_{1|x} := \{x : \; (x,z) \in P_1\}$

Proposition
The reformulation is valid, i.e., $P_{1|x} = P$.

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Proof:

$\subseteq$:

Since, $(x,z) \in P_1$, there exists exactly one $y' \in \mathcal{Y}$: $z_{y'} = 1$

$$\sum_i w_i x_i = \sum_l \omega_l \sum_{i \in N_l} x_i \leq \sum_l \omega_l y'_l \qquad \leq \beta$$

$\supseteq$:

For $x \in P$, there exists $y' \in \mathcal{Y}$ such that $\sum_i x_i \leq y'_l, l = 1, \ldots, k$
Set $z_{y'} = 1$, and $z_y = 0$ for all $y \neq y'$.
Then, $(x,z) \in P_1 \Rightarrow x \in P_{1|x}$

Tightness of the LP Relaxation

Proposition
The LP relaxation of the reformulation is at least as strong,

$$P_{1|x}^{\mathrm{LP}} \subseteq P^{\mathrm{LP}}$$

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Proof:

$x' \in P_{1|x}^{\mathrm{LP}} \Rightarrow \exists z' \in [0,1]^{|\mathcal{Y}|}$ such that $(x', z') \in P_1^{\mathrm{LP}}$

$$\begin{aligned}\sum_l \omega_l \sum_i x'_i &\leq \sum_l \omega_l \sum_y y_l z'_y \\ &= \sum_y z'_y \sum_l \omega_l y_l \\ &\leq \beta\end{aligned}$$

and hence $x' \in P^{\mathrm{LP}}$.

Strength of reform(w, β, x, z)

Certain types of minimal cover inequalities are implied by the reformulation.

Proposition
Let C be a minimal cover with the property $C \subseteq N_{l'}$ for some l'. Then the minimal cover inequality for C is implied by reform(w, β, x, z).

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Proof:

Every $y \in \mathcal{Y}$ satisfies $\omega^T y \leq \beta \Rightarrow y_l \leq |C| - 1$

$$\begin{aligned}\sum_{i \in C} x_i &\leq \sum_{i \in N_l} x_i \\ &\leq \sum_{y \in \mathcal{Y}} y_l z_y \\ &\leq \sum_{y \in \mathcal{Y}} (|C| - 1) z_y = |C| - 1.\end{aligned}$$

Mixed covers (intersecting several weight classes) are not necessarily implied.

- (Re-)formulation:

$$\underbrace{2x_1 + 2x_2 + 2x_3}_{N_1 = \{1,2,3\}} + \underbrace{5x_4 + 5x_5 + 5x_6}_{N_2 = \{4,5,6\}} + \underbrace{7x_7 + 7x_8 + 7x_9}_{N_3 = \{7,8,9\}} \leq 11$$

$$\begin{array}{ccccccccccc} x_1 & + & x_2 & + & x_3 & \leq & 3z_{y_1} & + & 2z_{y_2} & & \\ x_4 & + & x_5 & + & x_6 & \leq & z_{y_1} & & & + & 2z_{y_3} \\ x_7 & + & x_9 & + & x_9 & \leq & & & z_{y_2} & & \end{array}$$

- $C = \{6, 7\}$ is a cover: $w_6 + w_7 = 12 > 11$
- $x_6 + x_7 = 1 + 0.5 > |C| - 1$ is feasible for $\mathbf{reform}(w, \beta, x, z)$ $(z_{y_2} = z_{y_3} = 0.5)$

Enumeration Algorithm (Sketch)

Output: Construct $\mathcal{Y}$

- algorithm uses recursion into weight space dimension $k \leq d$
- sort $\omega_1 < \omega_2 < \cdots < \omega_k$
- set $y_l = 0,\ l = 1, \ldots, k$
- set $l = 1$
- set $y_l = \textbf{min}\{|N_l|, \lfloor \beta/\omega_1 \rfloor\}$
- recurse to next weight class with remaining capacity $\beta - y_1\omega_1$.
- decrease y_1 by 1
- repeat

Computational Results

Computational setup

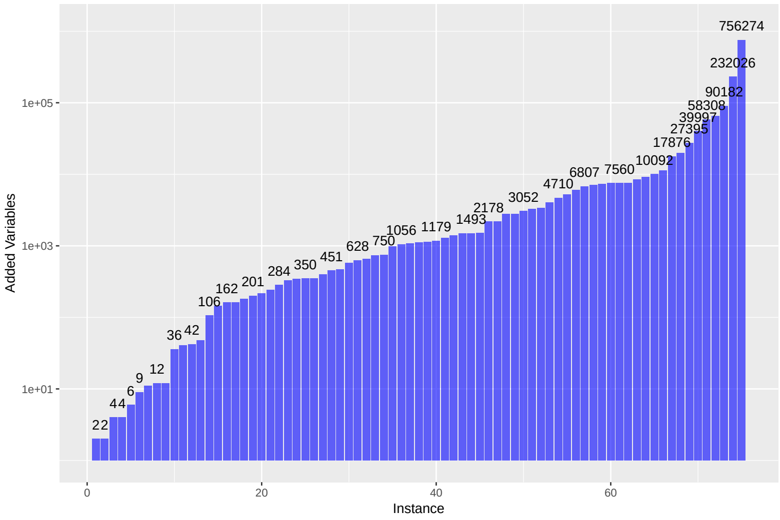
- code based on SCIP development version 5.0.1.4
    - presolver plugin written in C
    - run once during exhaustive presolving stage
    - extend every found configuration knapsack
    - skip every constraint with more than 100000 configurations
    - skip constraint if the added dimension of the reformulation exceeds 10$\times$ the number of original variables

Test Set:

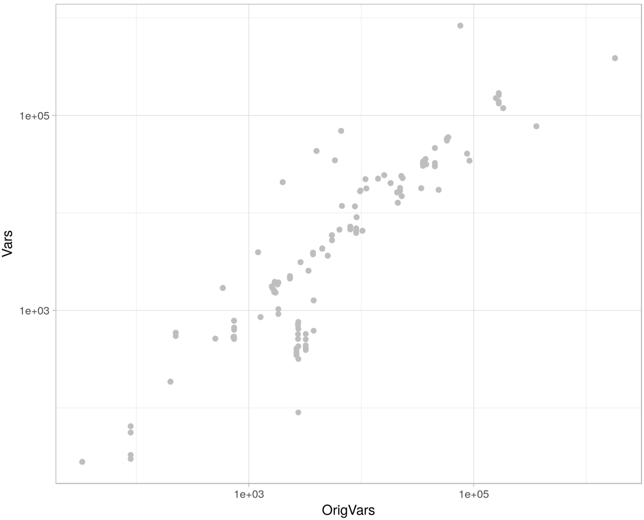
- complete MIPLIB{3,2003,2010} and Cor@l (666 instances altogether).
- 292 instances with 1 up to 522862 knapsack constraints (median: 96)
- 89 instances with configuration knapsacks
- 75 ($\approx$ 11 %) are transformed by the presolver

	Configuration type k							
Frequency	2	3	4	5	6	7	8	≥ 9
0	30	61	62	72	74	78	81	59
$[1, 10]$	17	15	16	13	10	7	6	9
$(10, 100]$	20	10	7	4	5	4	2	19
$(100, 1000]$	16	2	4					2
> 1000	6	1						

Added Variables



Comparison Original/Transformed Problem



Running Time and nodes (preliminary)

		solved		time (sec.)		nodes	
Group	Instances	config	default	config	default	config	default
all	450	305	316	485.66	478.01	1679.1	4583.5
solved by both	279	279	279	69.59	63.59	419.2	538.5
1st LP obj. better	121	77	60	801.11	2047.87	2909.0	28016.4
1st LP obj. worse	41	19	22	2252.18	1788.85	5744.6	9573.0
Root bound better	138	109	86	435.13	1170.29	2663.1	14540.9
Root bound worse	155	77	60	2610.30	1479.19	6154.6	23458.0

5h time limit, (5+default) seeds, 48 node cluster with 16 Intel Xeon Gold 5122 @ 3.60GHz, 96GB, Ubuntu 16.04, jobs nonexclusive.

Top 30 (instance,seed) pairs sorted by relative time to SCIP default

(no_config)

	ProblemName	Seed	Time	Time_noconfig	Nodes	Nodes_noconfig
185	neos-631694	3	1.28	18000.01	1	270607
186	neos-631694	4	1.35	18000.02	1	821542
182	neos-631694	0	1.36	18000.09	1	268912
184	neos-631694	2	1.42	18000.01	1	1651803
187	neos-631694	5	1.61	18000.02	2	1247056
183	neos-631694	1	1.87	18000.08	2	1496836
203	neos-631784	3	55.53	18000.02	1	3287735
202	neos-631784	2	69.78	18000	1	1224278
201	neos-631784	1	100.84	18000.02	1	945684
204	neos-631784	4	115.32	18000.01	1	416142
191	neos-631709	3	137.85	18000.01	1	82516
250	neos-885524	2	17.49	2260.41	9	6217
192	neos-631709	4	171.89	18000.03	2	141139
193	neos-631709	5	216.69	18000	1	116662
188	neos-631709	0	223.44	18000	1	47839
190	neos-631709	2	237.24	18000.01	2	49216
189	neos-631709	1	342.13	18000	10	102222
251	neos-885524	3	83.77	3293.83	98	5326
86	n3div36	0	337.44	8932.51	14825	387040
88	n3div36	2	342.15	8986.01	14677	394552
90	n3div36	4	373.98	9296.74	14087	415360
207	neos-662469	1	547.83	13434.67	7336	328913
87	n3div36	1	338.04	7688.85	13124	369837
89	n3div36	3	393.02	8659.98	16441	394077
252	neos-885524	4	16.65	334.63	9	579
249	neos-885524	1	46.52	786.35	107	981
205	neos-631784	5	117.43	1972.5	1	155566
91	n3div36	5	515.42	7962.88	19531	407377
196	neos-631710	2	1463.78	18000.04	1	1328
98	neos-1208135	0	12.9	150.51	148	8698

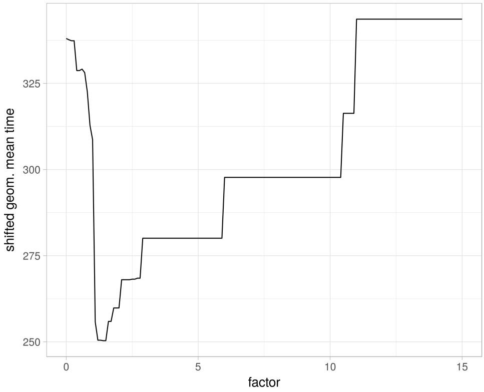
Instances with slowdown

Worst 30 (instance,seed) pairs sorted by relative time to SCIP default (no_config)

ProblemName	Seed	Time	Time_noconfig	Nodes	Nodes_noconfig
neos-913984	4	13864.82	13.32	125	1
neos-913984	2	8499.57	15.23	55	1
neos-913984	1	7733.67	16.12	12	1
neos-913984	0	8010.16	18.22	13	1
neos-913984	5	8323.97	19.71	15	1
neos-913984	3	7741.26	20.15	1	1
nsrand-ipx	1	18000.01	138.05	326	15955
bley_xl1	3	17997.99	157.23	1	1
nsrand-ipx	3	17998.9	162.98	634	21465
bley_xl1	4	17998.11	164.98	1	1
bley_xl1	5	17998.24	173.26	1	1
nsrand-ipx	5	17999.9	177.02	264	20351
bley_xl1	2	18000.04	183.75	1	1
nsrand-ipx	2	17997.25	188.09	310	25395
bley_xl1	0	17999.33	207.67	1	1
nsrand-ipx	0	17999.95	224.05	944	33928
bley_xl1	1	17998.89	237.3	1	6
neos-863472	3	2936.48	41.18	774456	33656
neos-863472	1	2522.3	37.51	706853	32159
nsrand-ipx	4	17997.68	312.92	187	40095
neos-863472	5	914.36	38.45	364263	28142
neos-863472	2	701.25	49.65	297437	40383
sp98ic	5	18000.01	1505.03	2711	120161
sp98ic	0	18000.01	1564.05	2543	119071
neos-863472	4	370.93	33.21	122712	26814
neos-863472	0	403.03	36.24	144731	33746
sp98ic	1	18000.01	1692.72	2605	117043
sp98ic	3	18000	1755.76	2327	140387
sp98ic	2	18000.01	1913.81	2520	144741
sp98ic	4	18000.01	2266.64	2227	163724

Influence of the size of the formulation

Is there a good parameter choice for the extension dimension?



Solver simulation as a function of `factor`:

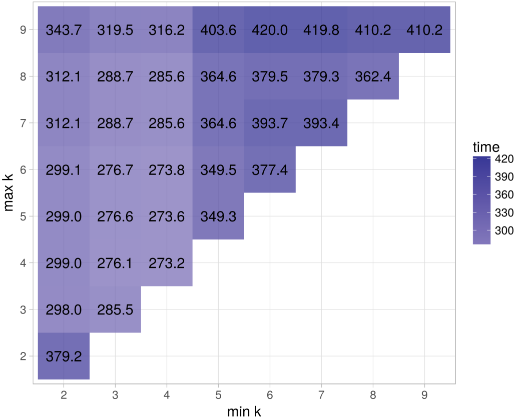
- Use configuration result if formulation dimension is below `factor` times the size of the original problem
- otherwise, use SCIP default result
- compute shifted geometric mean time

Performance with reasonable factor limit

Group	Settings	#	solved	Time abs.	Time rel.	Nodes abs.	Nodes rel.	Presolver	Iters 1st LP
	default	75	51	334.9	1.000	3208	1.000	0.000	1066.8
all	fac. 1.1	75	53	289.9	0.866	2177	0.679	0.012	1322.3
	fac. 1.2	75	52	279.7	0.835	1814	0.566	0.016	1282.2
	fac. 1.5	75	52	278.8	0.833	1482	0.462	0.025	1037.6
	fac. 2	75	50	319.7	0.955	1424	0.444	0.033	1211.4
	fac. 3	75	49	333.2	0.995	1375	0.429	0.044	1373.9
	fac. 6	75	49	340.2	1.016	1361	0.424	0.062	1411.7
	default	45	45	47.5	1.000	453	1.000	0.000	535.0
alloptimal	fac. 1.1	45	45	48.8	1.027	414	0.914	0.008	628.0
	fac. 1.2	45	45	44.4	0.934	368	0.812	0.010	751.3
	fac. 1.5	45	45	44.0	0.926	313	0.690	0.016	624.6
	fac. 2	45	45	48.0	1.009	334	0.736	0.023	752.3
	fac. 3	45	45	51.9	1.092	334	0.737	0.028	887.6
	fac. 6	45	45	54.0	1.136	381	0.841	0.032	891.8
	default	11	6	2733.5	1.000	26778	1.000	0.000	1362.9
diff-timeouts	fac. 1.1	11	8	957.5	0.350	3812	0.142	0.029	1829.9
	fac. 1.2	11	7	1041.7	0.381	2855	0.107	0.027	1283.7
	fac. 1.5	11	7	1049.4	0.384	1487	0.056	0.039	581.8
	fac. 2	11	5	1946.4	0.712	1368	0.051	0.041	823.7
	fac. 3	11	4	1957.5	0.716	1211	0.045	0.050	903.1
	fac. 6	11	4	1960.7	0.717	896	0.033	0.055	1060.1

2h time limit, only default seed, 48 node cluster with 16 Intel Xeon Gold 5122 @ 3.60GHz, 96GB, Ubuntu 16.04, jobs exclusive.

Selection of a good d to limit k



Remark: based on nonexclusive 5h experiments

Solver simulation:

- Use timing result of reformulation only on problems that have configuration knapsack constraints with $k_{\text{min}} \leq k \leq k_{\text{max}}$ weight classes.
- otherwise, use default result

Conclusion & Outlook

Conclusions

- extended formulation for configuration knapsacks
- configuration knapsacks occur in ~11 % of our benchmark set of public instances
- tighter LP relaxation in theory
- often also tighter LP relaxation in practice
- speed up of up to 16 % by simple limit on the extension dimension

Outlook

- analyze other classes of inequalities (e.g. MIR, lifted/GUB cover) w.r.t. the reformulation.

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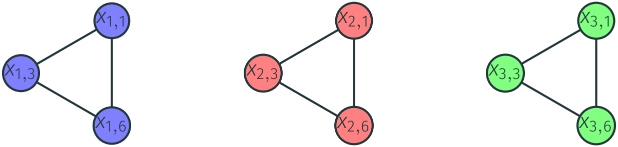
- Line planning offers a very special GUB structure to exploit.

$$\begin{array}{llll}
x_{1,1} + 3x_{1,3} + 6x_{1,6} + & x_{2,1} + 3x_{2,3} + 6x_{2,6} + & x_{3,1} + 3x_{3,3} + 6x_{3,6} & \geq 10 \\
x_{1,1} + x_{1,3} + x_{1,6} & & & \leq 1 \\
 & x_{2,1} + x_{2,3} + x_{2,6} & & \leq 1 \\
 & & x_{3,1} + x_{3,3} + x_{3,6} & \leq 1
\end{array}$$

$\Rightarrow$ GUB Conflict Graph $G = (V, E)$



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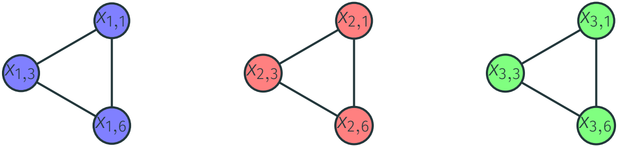

- generalize reformulation to mixed-binary linear constraints

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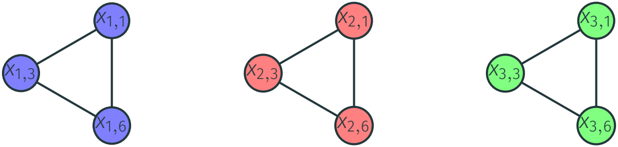

- generalize reformulation to mixed-binary linear constraints
- code will be contained in the next SCIP Release

Thank you for your attention!

in memory of our colleague and friend
Heide Hoppmann