



Using SCIP to Solve Your Favorite Integer Optimization Problem

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October 5, 2018

SCIP (Solving Constraint Integer Programs) ...

- provides a full-scale MIP and MINLP solver,
- is constraint based,
- incorporates
 - MIP features (cutting planes, LP relaxation), and
 - MINLP features (spatial branch-and-bound, OBBT)
 - CP features (domain propagation),
 - SAT-solving features (conflict analysis, restarts),
- is a branch-cut-and-price framework,
- has a modular structure via plugins,
- is free for academic purposes,
- and is available in source-code under <http://scip.zib.de> !

31 active developers

- 7 running Bachelor and Master projects
- 16 running PhD projects
- 8 postdocs and professors

4 development centers in Germany

- ZIB: SCIP, SoPlex, UG, ZIMPL
- TU Darmstadt: SCIP and SCIP-SDP
- FAU Erlangen-Nürnberg: SCIP
- RWTH Aachen: GCG



Many international contributors and users

- more than 10 000 downloads per year from 100+ countries

Careers

- 10 awards for Masters and PhD theses: MOS, EURO, GOR, DMV
- 7 former developers are now building commercial optimization software at CPLEX, FICO Xpress, Gurobi, MOSEK, and GAMS

SCIP – Solving Constraint Integer Programs

<http://scip.zib.de>

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- Toolbox for [generating](#) and [solving](#) constraint integer programs, in particular Mixed Integer (Non-)Linear Programs.
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ZIMPL

- model and generate LPs, MIPs, and MINLPs

SCIP

- MIP, MINLP and CIP solver, branch-cut-and-price framework

SoPlex

- revised primal and dual simplex algorithm

GCG

- generic branch-cut-and-price solver

UG

- framework for parallelization of MIP and MINLP solvers

- **C code and documentation**
 - more than 800 000 lines of C code, 20% documentation
 - over 50 000 assertions and 5 000 debug messages
 - HowTos: plugins types, debugging, automatic testing
 - 10 examples illustrating the use of SCIP
 - 5 problem specific SCIP applications to solve Coloring, Steiner Tree, or Multiobjective problems.
- **Interface and usability**
 - Cross-platform availability due to CMake
 - user-friendly interactive shell
 - C++ wrapper classes
 - LP solvers: SoPlex, CPLEX, Gurobi, Xpress, CLP, MOSEK, QSOPT
 - NLP solvers: IPOPT, FilterSQP, WORHP
 - over 2300 parameters and 15 emphasis settings

- interactive shell supports 10 different input formats
→ cip, cnf, flatzinc, rlp, lp, mps, opb, pip, wbo, zimpl
- C API/callable library
- C++ wrapper classes
- Python interface
- Java JNI interface
- AMPL
- GAMS
- Matlab (see also OPTI toolbox,
<http://www.i2c2.aut.ac.nz/Wiki/OPTI/>)

If you should ever get stuck, you can ...

1. type `help` in the interactive shell
2. read the documentation <http://scip.zib.de/doc/html>
→ FAQ, HowTos for each plugin type, debugging, automatic testing, ...
3. active mailing list scip@zib.de (350+ members)
 - search the mailing list archive (append `site:listserv/pipermail/scip`)
 - register <http://listserv.zib.de/mailman/listinfo/scip/> and post
4. search or post on [Stack Overflow](#) using the tag `scip` (more than 100 questions already answered)

SCIP – Solving Constraint Integer Programs

<http://scip.zib.de>

What is a good diet?

Product	Servings	Energy(kj)	Protein(g)	Calcium(g)	Price(ct)
Oatmeal	4	110	4	2	3
Chicken	3	205	32	12	24
Eggs	2	160	13	54	13
Milk	8	160	8	284	9
Pie	2	420	4	22	20
Pork	2	260	14	80	19

Needed per day:

- Energy: 2000 kj
- Protein: 55g
- Calcium: 800 mg

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Task: Find a diet satisfying all nutritional requirements at minimum cost!

The Diet LP

Find a diet satisfying all nutritional requirements at minimum cost!

Our nutrition plan should be expressed as $x \in \mathbb{R}_{\geq 0}^6$.

- Energy: 2000 kj
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$$\min 3x_{\text{oat}} + 24x_{\text{Chi}} + 13x_{\text{Egg}} + 9x_{\text{Mil}} + 20x_{\text{Pie}} + 19x_{\text{Por}}$$

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- Energy: 2000 kj

$$110x_{\text{oat}} + 205x_{\text{Chi}} + 160x_{\text{Egg}} + 160x_{\text{Mil}} + 420x_{\text{Pie}} + 260x_{\text{Por}} \geq 2000$$

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- Protein: 55g

$$4x_{\text{oat}} + 32x_{\text{Chi}} + 13x_{\text{Egg}} + 8x_{\text{Mil}} + 4x_{\text{Pie}} + 14x_{\text{Por}} \geq 55$$

- Calcium: 800 mg

$$2x_{\text{oat}} + 12x_{\text{Chi}} + 54x_{\text{Egg}} + 284x_{\text{Mil}} + 22x_{\text{Pie}} + 80x_{\text{Por}} \geq 800$$

Diet Problem

Only an integer vector $x \in \mathbb{Z}_{\geq 0}^6$ is truly feasible, because food such as eggs are unsplittable.

$$\begin{array}{ll}
 \min & (3 \quad 24 \quad 13 \quad 9 \quad 20 \quad 19) \quad x \\
 \text{s.t.} & \begin{pmatrix} 110 & 205 & 160 & 160 & 420 & 260 \\ 4 & 32 & 13 & 8 & 4 & 14 \\ 2 & 12 & 54 & 284 & 22 & 80 \end{pmatrix} x \geq \begin{pmatrix} 2000 \\ 55 \\ 800 \end{pmatrix} \\
 & x \geq 0 \\
 & x_i \in \mathbb{Z} \quad \forall i \in \{1, \dots, 6\}
 \end{array}$$

Diet Problem

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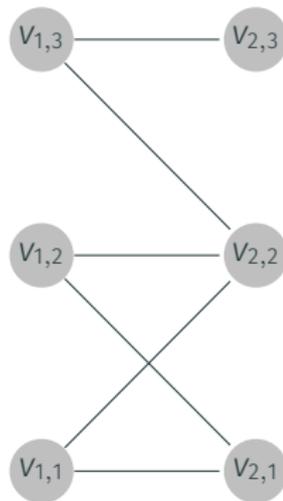
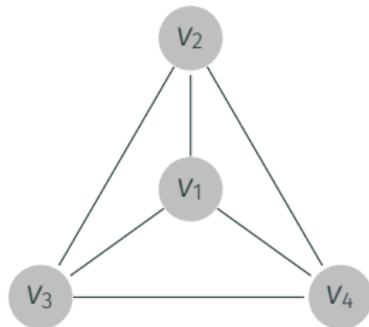
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 \end{array}$$

General MIP

$$\begin{array}{ll}
 \min & c^T x \\
 \text{s.t.} & Ax \geq b \\
 & x \geq 0 \\
 & x_i \in \mathbb{Z} \quad \forall i \in \mathcal{I} \subseteq \{1, \dots, n\}
 \end{array}
 \quad \text{(Mixed Integer Program)}$$

Simple graphs

- A **simple graph** G is a tuple (V, E) of
 - $|V| = n$ nodes and
 - $m = |E|$ edges connecting pairs of nodes, i.e. $E \subseteq \binom{V}{2}$.
 - nodes may have weights/demands $w : V \rightarrow \mathbb{R}$
 - edges may have associated costs $c : E \rightarrow \mathbb{R}$



Most of the classical optimization problems on graphs can be formulated as Mixed Integer Programs.

- Max. stable set
- Min. vertex cover
- Max. matching
- Graph coloring
- Traveling salesman problem
- Steiner tree problem
- ...

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Maximum stable set: Find a set of vertices of maximum cardinality without a connecting edge.

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- Max. stable set
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- Graph coloring
- Traveling salesman problem
- Steiner tree problem
- ...

Maximum stable set: Find a set of vertices of maximum cardinality without a connecting edge.

Integer Program:

$$\begin{array}{ll} \max & \mathbf{1}^T x \\ \text{s.t.} & x_v + x_w \leq 1 \quad \forall \{v, w\} \in E \\ \text{s.t.} & x_v \in \{0, 1\} \quad \forall v \in V \end{array}$$

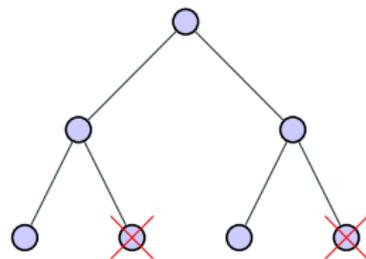
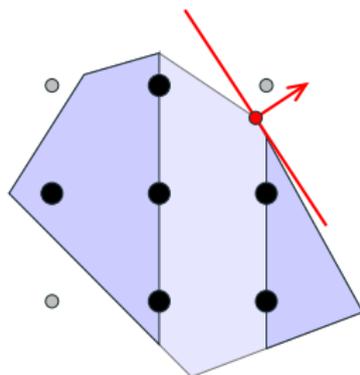
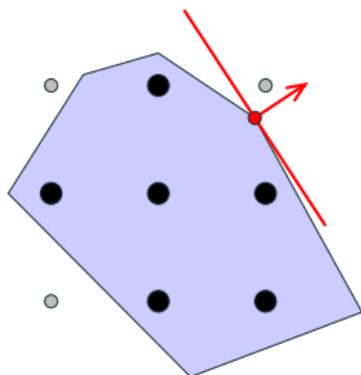
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Branch-and-bound

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & Ax \geq b \\ & x \geq 0 \\ & x_i \in \mathbb{Z} \quad \forall i \in \mathcal{I} \subseteq \{1, \dots, n\} \end{aligned}$$

(Mixed Integer Program)



Basic Workflow

```
read ../check/instances/MIP/bell5.mps
optimize
write solution mysolution.sol
quit
```

Displaying information

Use the `display ...` command to enter the menu and

- obtain `solution` information
- print the current `transproblem` to the console
- display plugin information, e.g., list all available branching rules

Changing Settings

Use the `set ...` command to list the settings menu.

Important Parameters

Numerical parameters

These must be set **before** reading a problem.

- `numerics/feastol`, default 10^{-6}
- `numerics/epsilon`, default 10^{-9}
- `numerics/infinity`, default 10^{20}

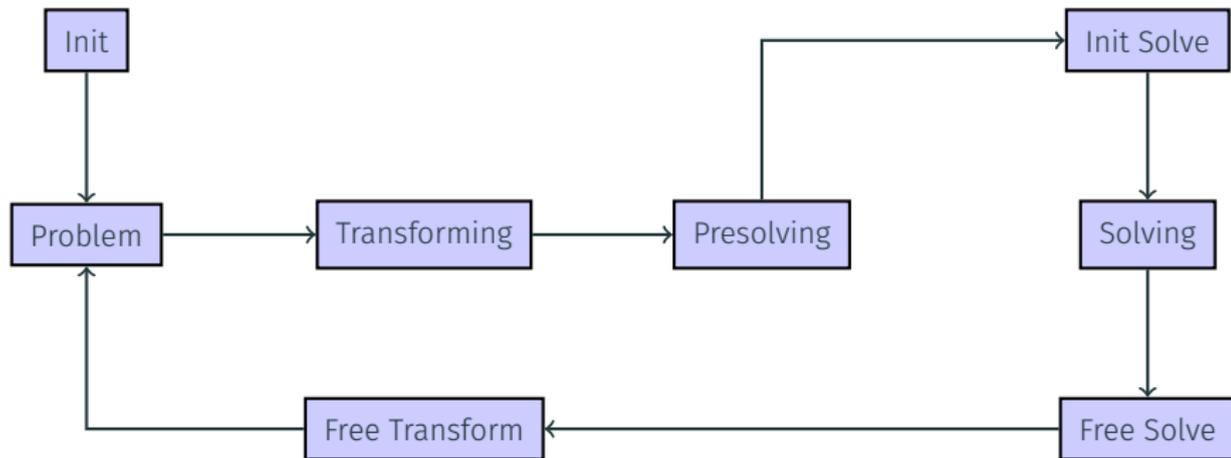
Limits

- `limits/time`
- `limits/nodes`
- `limits/gap`

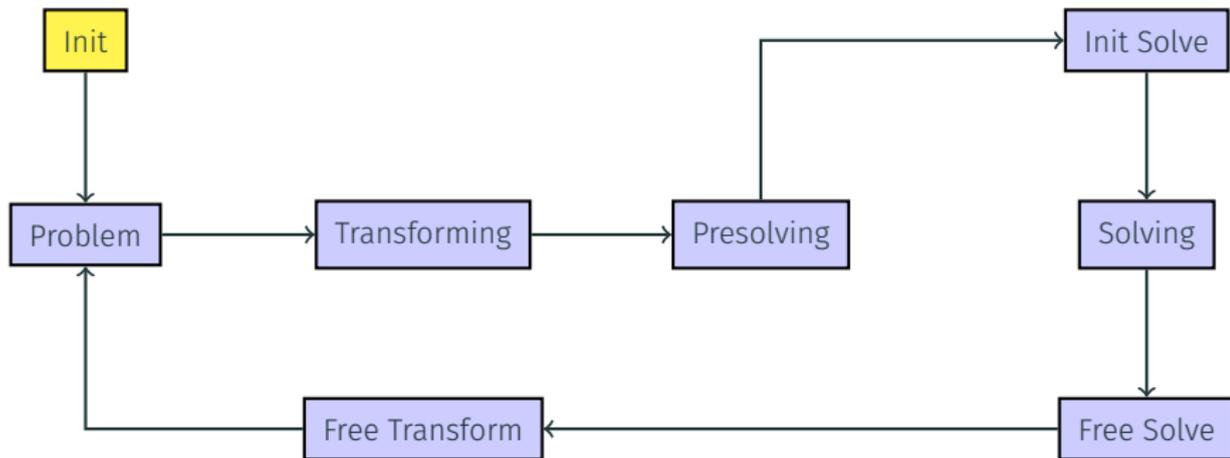
Randomization

- `randomization/randomseedshift`
- `randomization/lpseed`
- `randomization/permutationseed`

Operational Stages

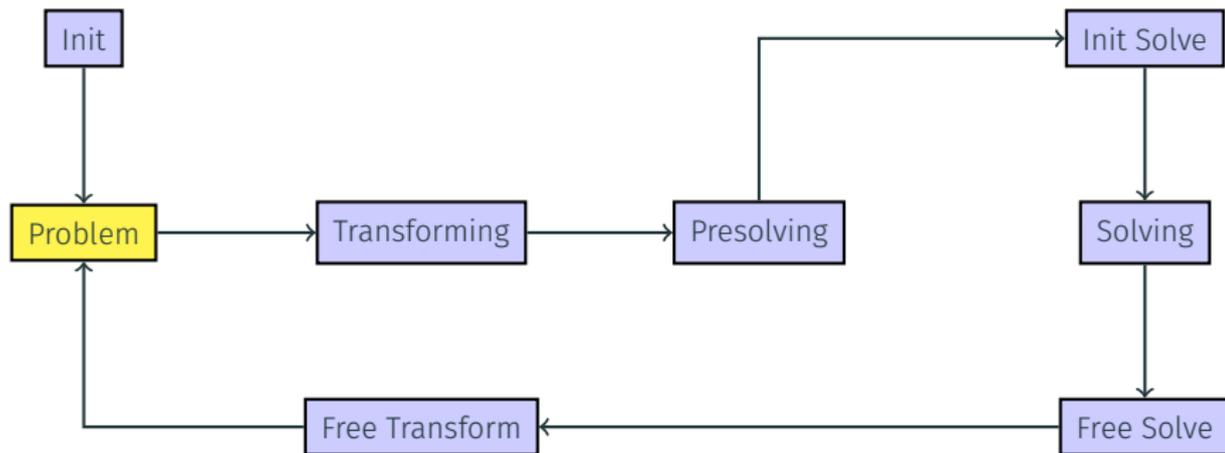


Operational Stages



- Basic data structures are allocated and initialized.
- User includes required plugins (or just takes default plugins).

Problem Specification



- User creates and modifies the original problem instance.
- Problem creation is usually done in file readers.

Define Variables (Binary Example)

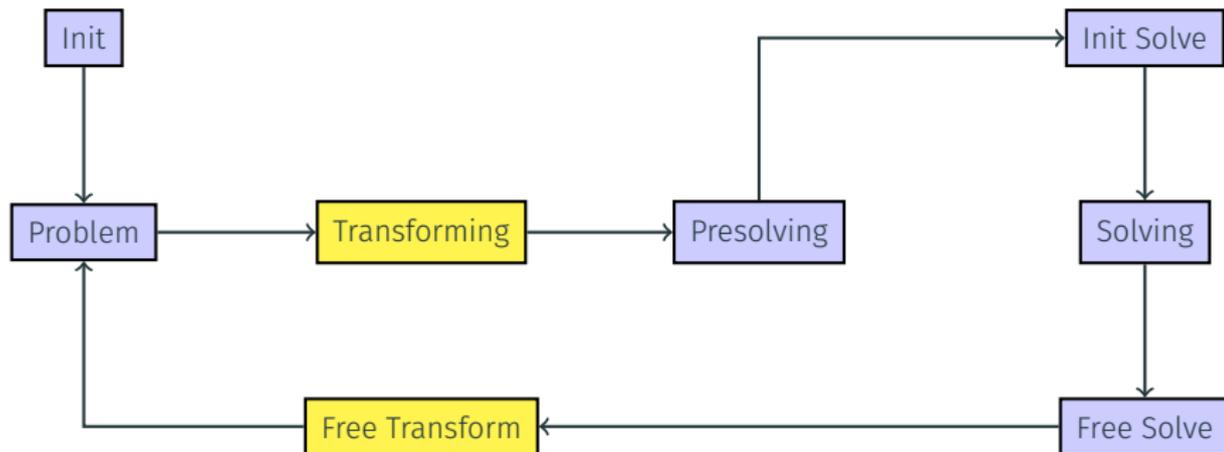
```
SCIP_VAR* var;
SCIP_CALL(                                     // return value macro
    SCIPcreateVar(
        scip,                                 // SCIP pointer
        &var,                                 // save in variable
        "x_1",                                // pass variable name
        0.0,                                  // lower bound
        1.0,                                  // upper bound
        100.0,                                // obj. value
        SCIP_VARTYPE_BINARY,                 // type
        TRUE,                                 // initial
        FALSE,                                // removable
        NULL, NULL, NULL,                    // no callback functions
        NULL                                  // no variable data
    )
);
SCIP_CALL( SCIPaddVar(scip, var) );          // add var.
```

Diet Problem Constraint

```
SCIP_CALL( SCIPcreateConsLinear(
    scip,          // SCIP pointer
    &cons,         // save in cons
    "energy",     // name
    6,            // number of variables
    vars,         // array of variables
    vals,         // array of values (coefficients)
    2000,         // left hand side
    SCIPinfinity(scip), // no right hand side (use for equation)
    TRUE,         // initial?
    FALSE,        // separate?
    TRUE,         // enforce?
    TRUE,         // check?
    TRUE,         // propagate?
    FALSE,        // local?
    FALSE,        // modifiable?
    FALSE,        // dynamic?
    FALSE,        // removable?
    FALSE         // stick at node?
));
SCIP_CALL( SCIPaddCons(scip, cons) ); // add constraint
SCIP_CALL( SCIPreleaseCons(scip, &cons) ); // free cons. space
```

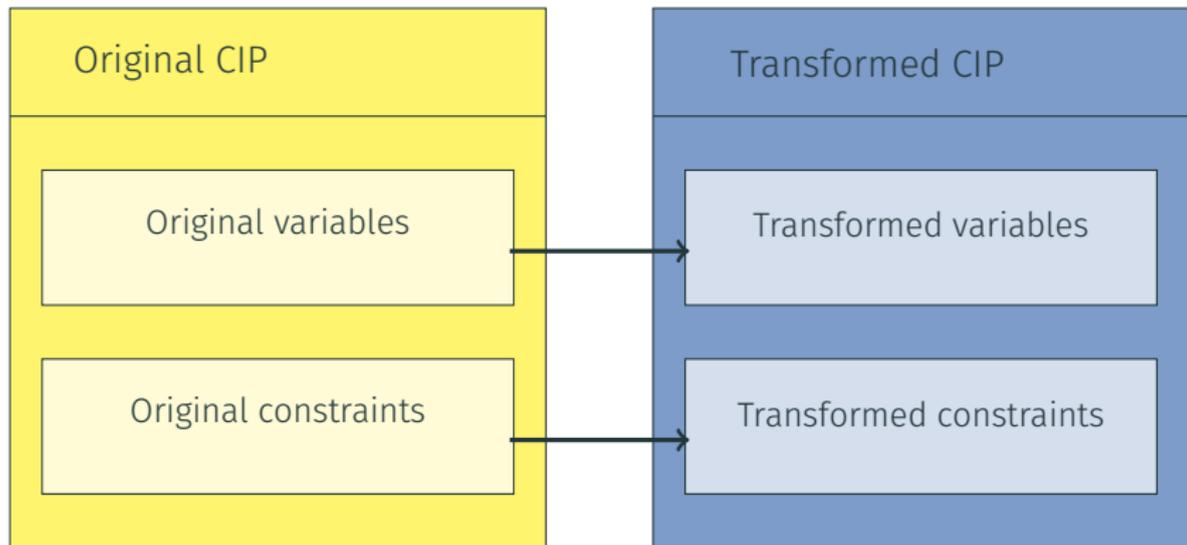
MIPs are specified using linear constraints only (may be “upgraded”).

Transformation



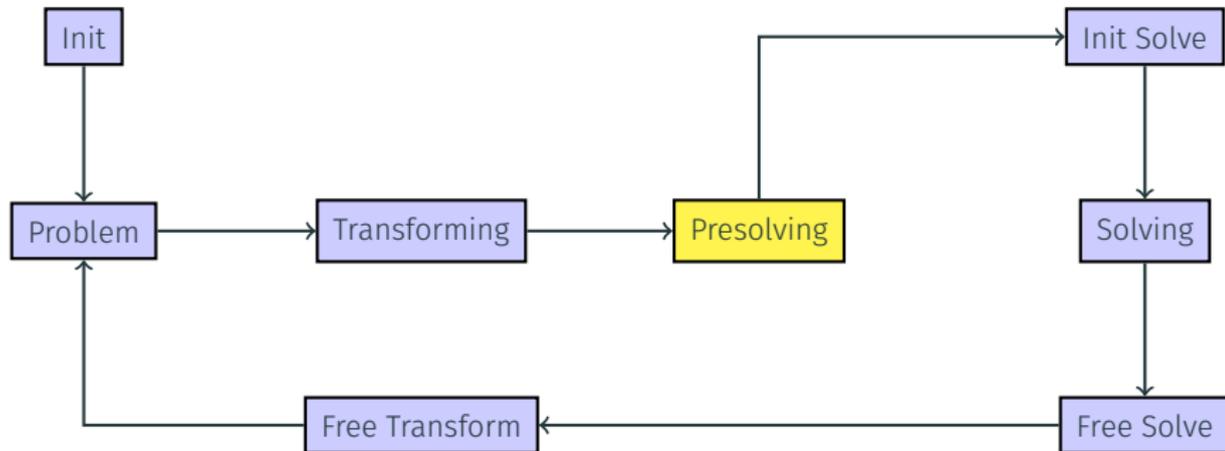
- Creates a working copy of the original problem.

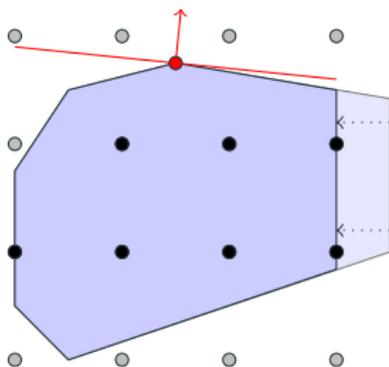
Original and Transformed Problem



- data is copied into separate memory area
- presolving and solving operate on transformed problem
- original data can only be modified in problem modification stage

Presolving





Task

- reduce size of model by removing irrelevant information
- strengthen LP relaxation by exploiting integrality information
- make the LP relaxation numerically more stable
- extract useful information

Primal Reductions:

- based on feasibility reasoning
- no feasible solution is cut off

Dual Reductions:

- consider objective function
- at least one optimal solution remains

Presolving Tips and Parameters

Use `display presolvers` to **list** all presolvers of SCIP.

Disable Presolving

Disable **all** presolving for a model

```
set presolving emphasis off
```

Deactivate **single** techniques

```
set presolving tworowbnd maxrounds 0  
set propagating probing maxprerounds 0  
set constraints components advanced maxprerounds 0
```

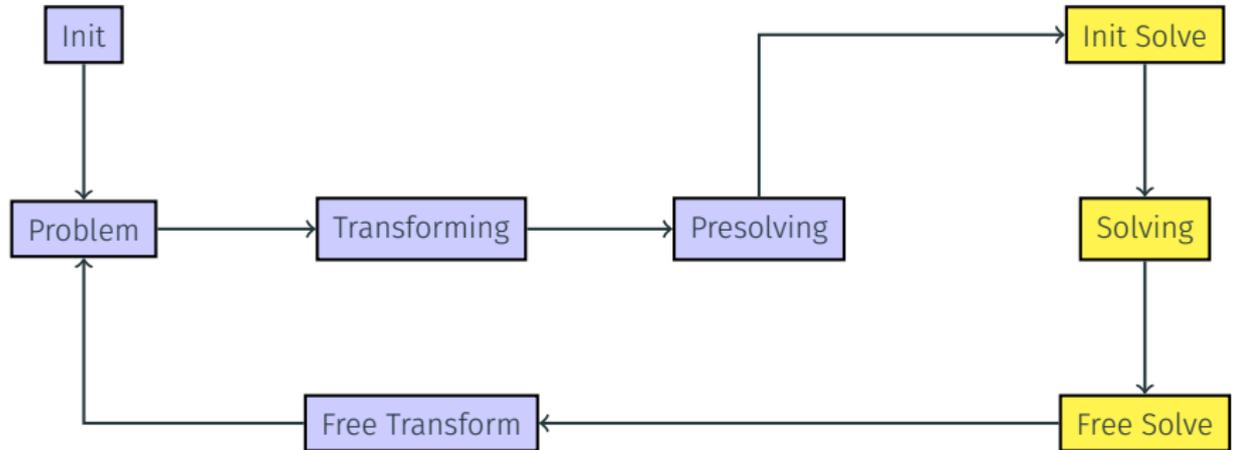
Aggressive Presolving

```
set presolving emphasis aggressive
```

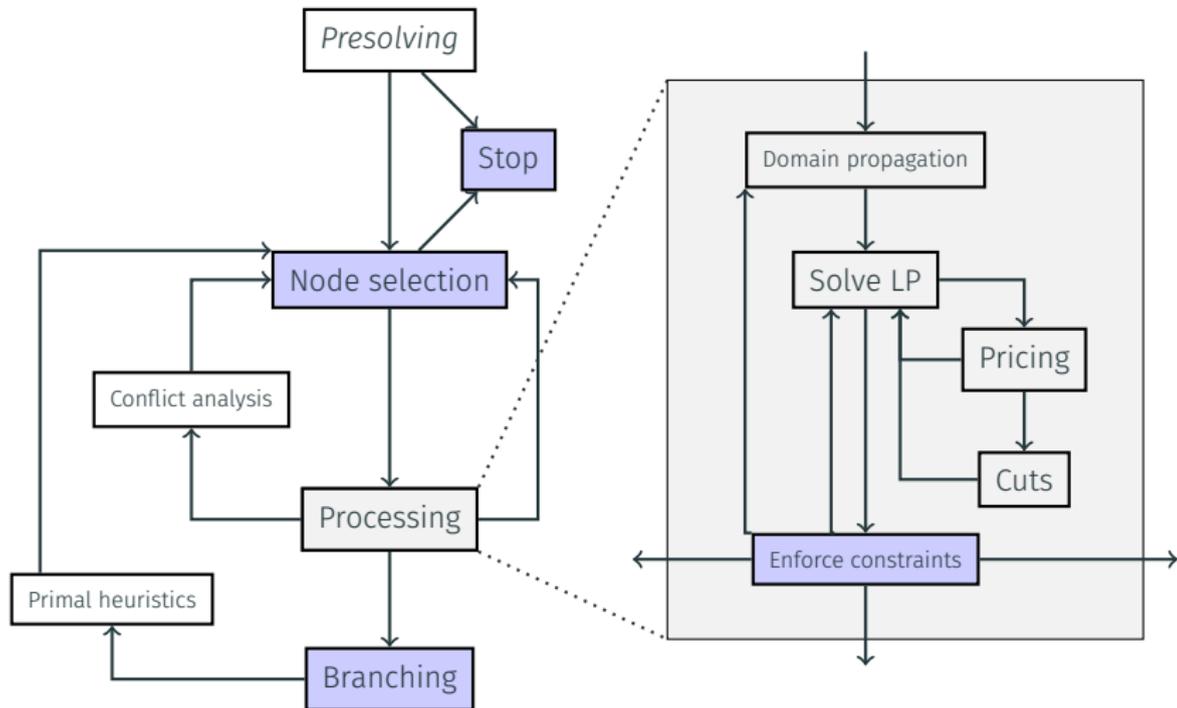
General Rule of Thumb

Only deactivate single presolving techniques if you encounter performance problems.

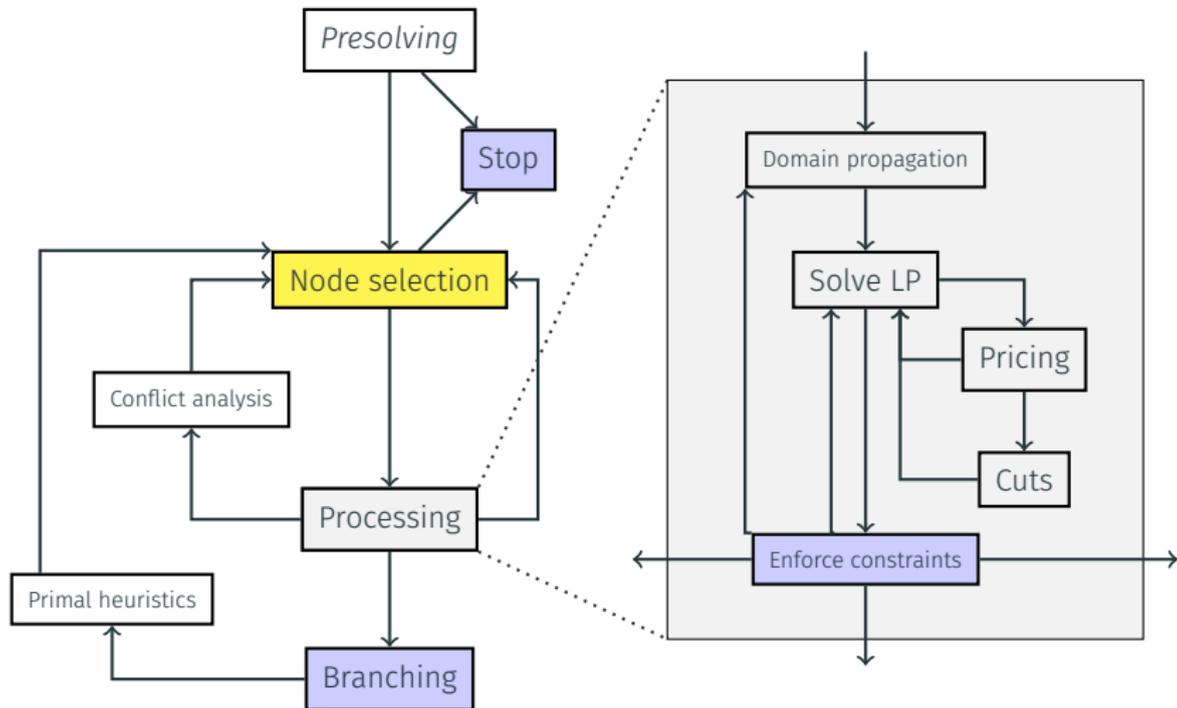
Solving

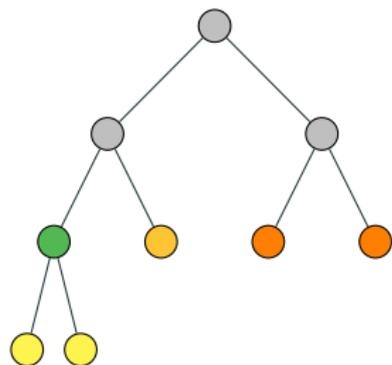


Flow Chart SCIP



Flow Chart SCIP





Task

- improve primal bound
- keep comp. effort small
- improve global dual bound

Techniques

- **basic rules**
 - depth first search (DFS)
 - early feasible solutions
 - best bound search (BBS)
 - improve dual bound
 - best estimate search (BES)
 - improve primal bound
- **combinations**
 - BBS or BES with plunging
 - hybrid BES/BBS
 - interleaved BES/BBS

Available Node Selectors

```
display nodeselectors
```

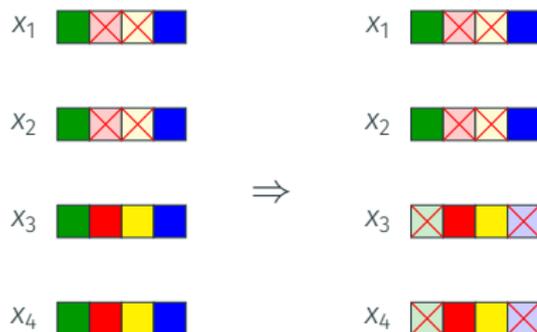
node selector	std priority	memsave	prio	description
estimate	200000		100	best estimate search
bfs	100000		0	best first search
...				
dfs		0	100000	depth first search

Switching Node Selectors

Only the node selector with **highest** standard priority is active. Use

```
set nodeselection dfs stdpriority 1000000
```

to activate depth first search also in non-memsave mode.



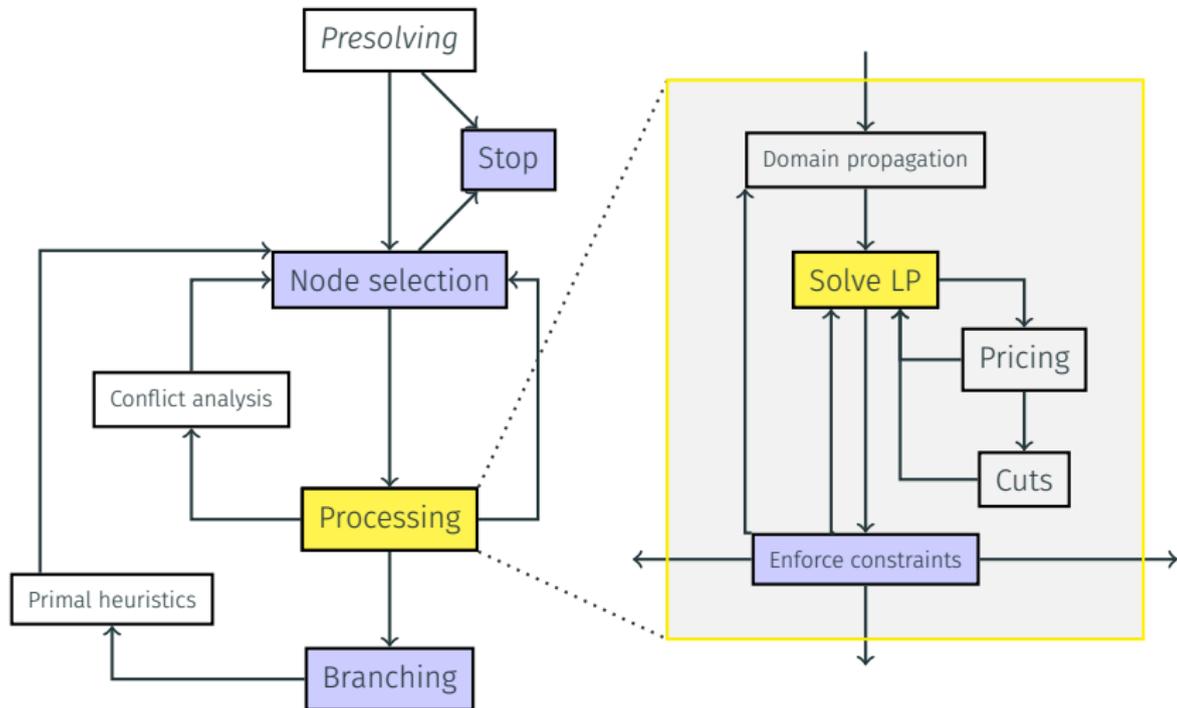
Task

- simplify model locally
- improve local dual bound
- detect infeasibility

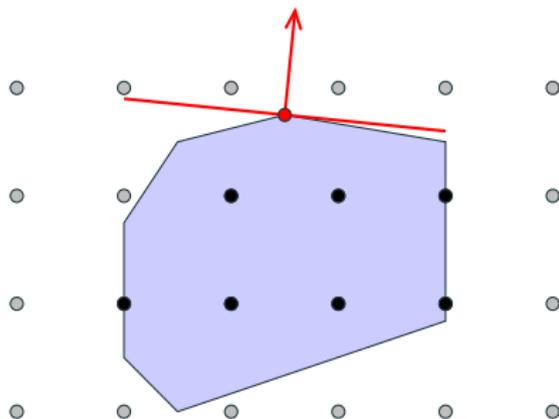
Techniques

- **constraint specific**
 - each cons handler may provide a propagation routine
 - reduced presolving (usually)
- **dual propagation**
 - root reduced cost strengthening
 - objective function
- **special structures**
 - variable bounds

Flow Chart SCIP



- LP solver is a black box
- interface to different LP solvers:
SoPlex, CPLEX, XPress, Gurobi, CLP,
...
- primal/dual simplex
- barrier with/without crossover



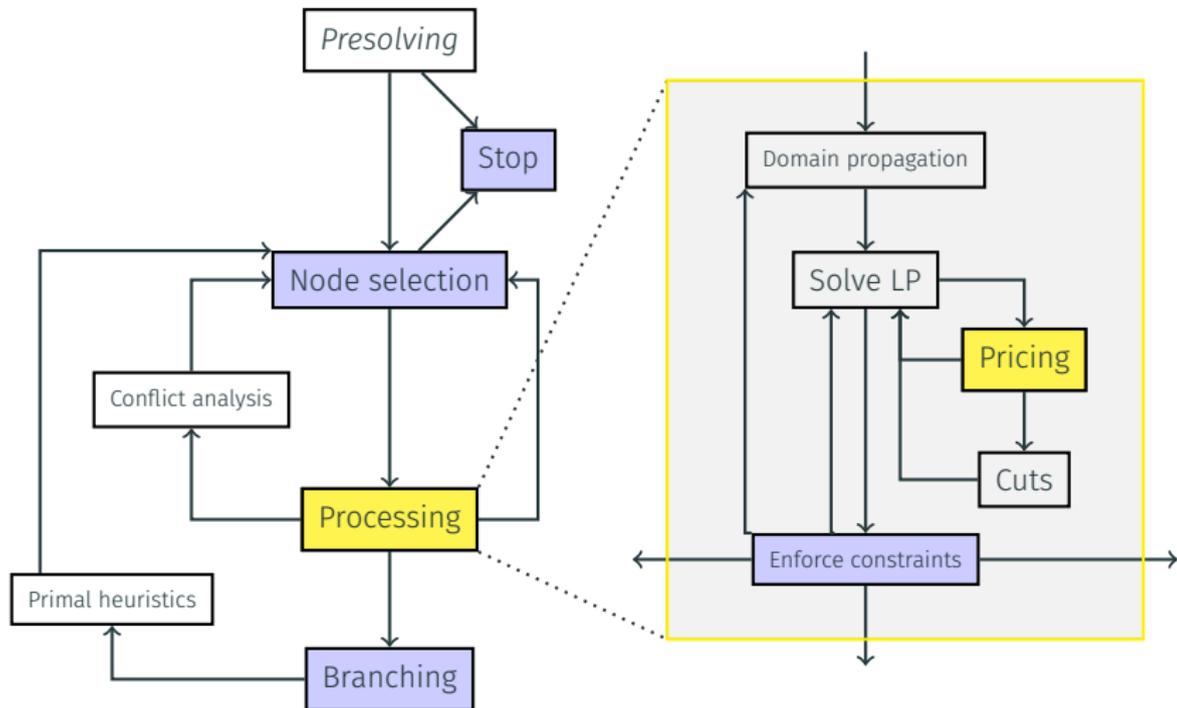
- feasibility double-checked by SCIP
- condition number check
- resolution by changing parameters:
scaling, tolerances, solving from scratch, other simplex

Most Important LP Parameters

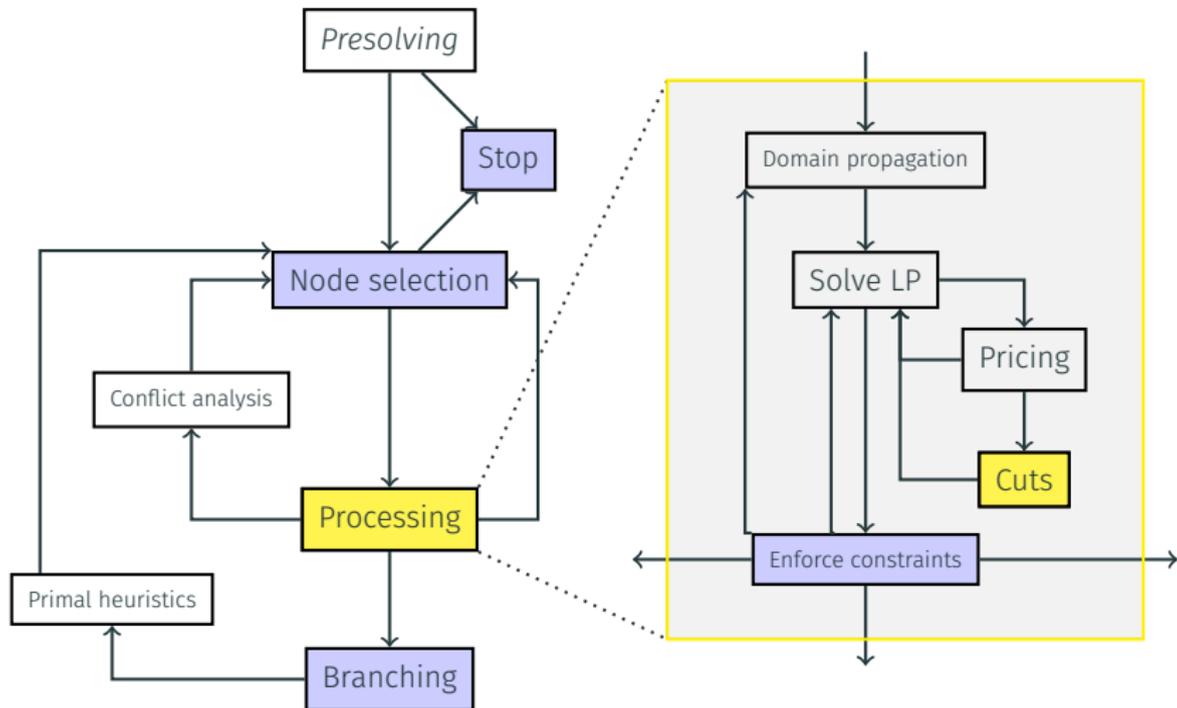
- `lp/initalgorithm`, `lp/resolvealgorithm`
 - Primal/Dual Simplex Algorithm
 - Barrier w and w/o crossover
- `lp/pricing`
 - normally LP solver specific default
 - Devex
 - Steepest edge
 - Quick start steepest edge
- `lp/threads`

Slow LP performance is a blocker for the solving process and can sometimes be manually tuned significantly.

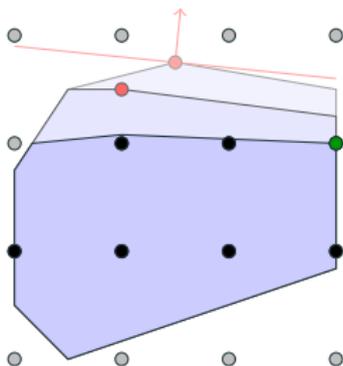
Flow Chart SCIP



Flow Chart SCIP



Cutting Plane Separation



Task

- strengthen relaxation
- add valid constraints
- generate on demand

Techniques

- **general cuts**
 - complemented MIR cuts
 - Gomory mixed integer cuts
 - strong Chvátal-Gomory cuts
 - implied bound cuts
 - reduced cost strengthening
- **problem specific cuts**
 - 0-1 knapsack problem
 - stable set problem
 - 0-1 single node flow problem

Cuts for the 0-1 Knapsack Problem

Feasible region: $(b \in \mathbb{Z}_+, a_j \in \mathbb{Z}_+ \quad \forall j \in N)$

$$X^{BK} := \{ x \in \{0, 1\}^n : \sum_{j \in N} a_j x_j \leq b \}$$

Minimal Cover: $C \subseteq N$

- $\sum_{j \in C} a_j > b$
- $\sum_{j \in C \setminus \{i\}} a_j \leq b \quad \forall i \in C$

Minimal Cover Inequality

$$\sum_{j \in C} x_j \leq |C| - 1$$

$$5x_1 + 6x_2 + 2x_3 + 2x_4 \leq 8$$

Minimal cover:

$$C = \{2, 3, 4\}$$

Minimal cover inequality:

$$x_2 + x_3 + x_4 \leq 2$$

Disable/Speed up/Emphasize All Separation

```
set separating emphasis off/fast/aggressive
```

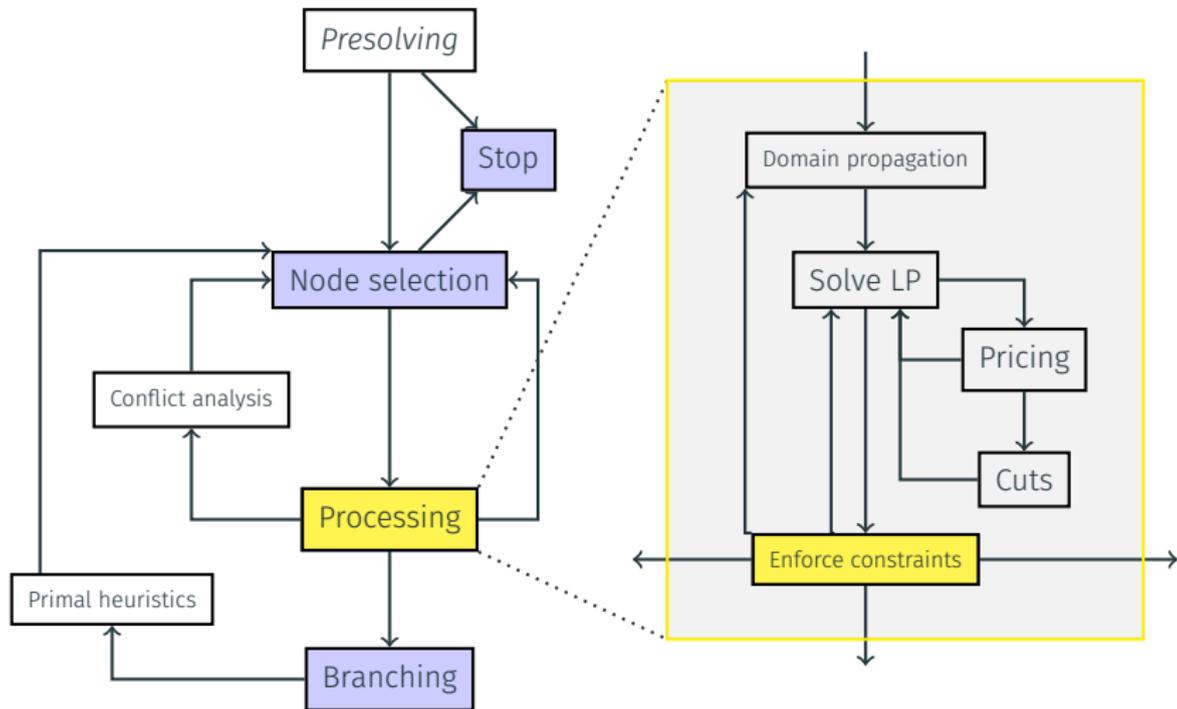
Disable Single Separation Techniques

```
set separating clique freq -1  
set constraints cardinality sepafreq -1
```

Some Important Parameters

- `separating/maxcuts`, `separating/maxcutsroot`
- `separating/maxrounds`, `separating/maxroundsroot`
- `separating/maxstallrounds`,
`separating/maxstallroundsroot`

Flow Chart SCIP



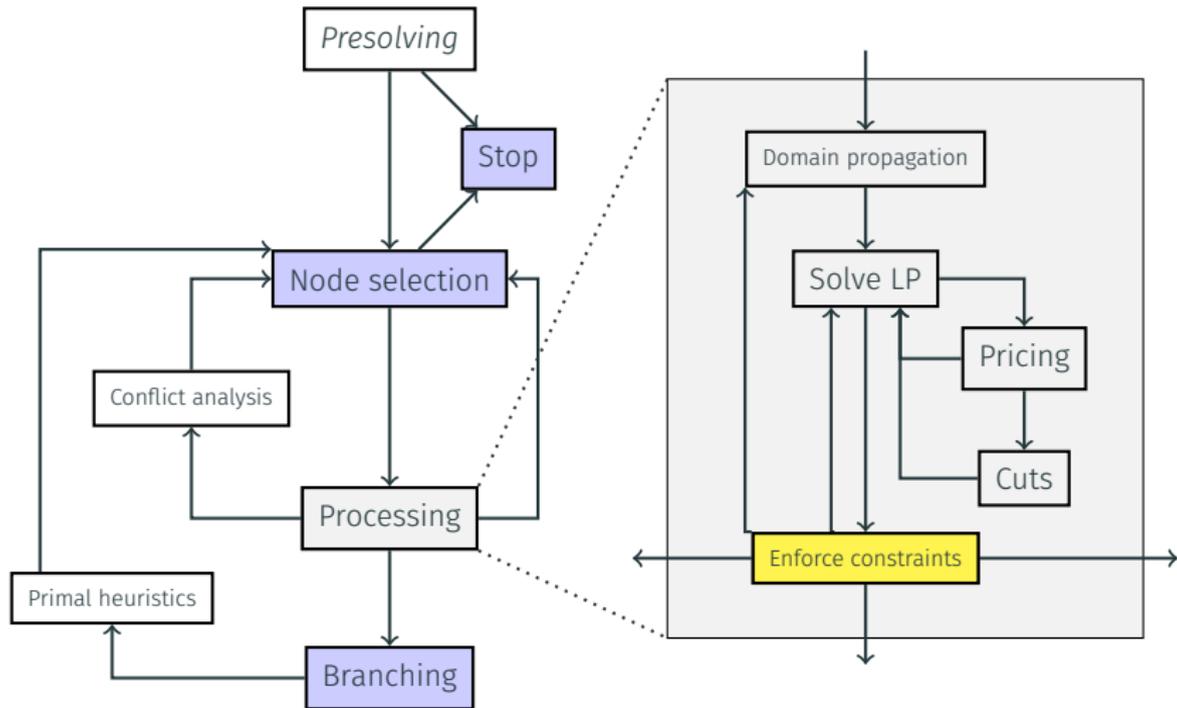
LP solution may violate a constraint not contained in the relaxation.

Enforcing is necessary for a correct implementation!

Constraint handler resolves the infeasibility by ...

- Reducing a variable's domain,
- Separating a cutting plane (may use integrality),
- Adding a (local) constraint,
- Creating a branching,
- Concluding that the subproblem is infeasible and can be cut off, or
- Just saying "solution infeasible".

Constraint Enforcement



- Reduced domain

- Added constraint

- Added cut

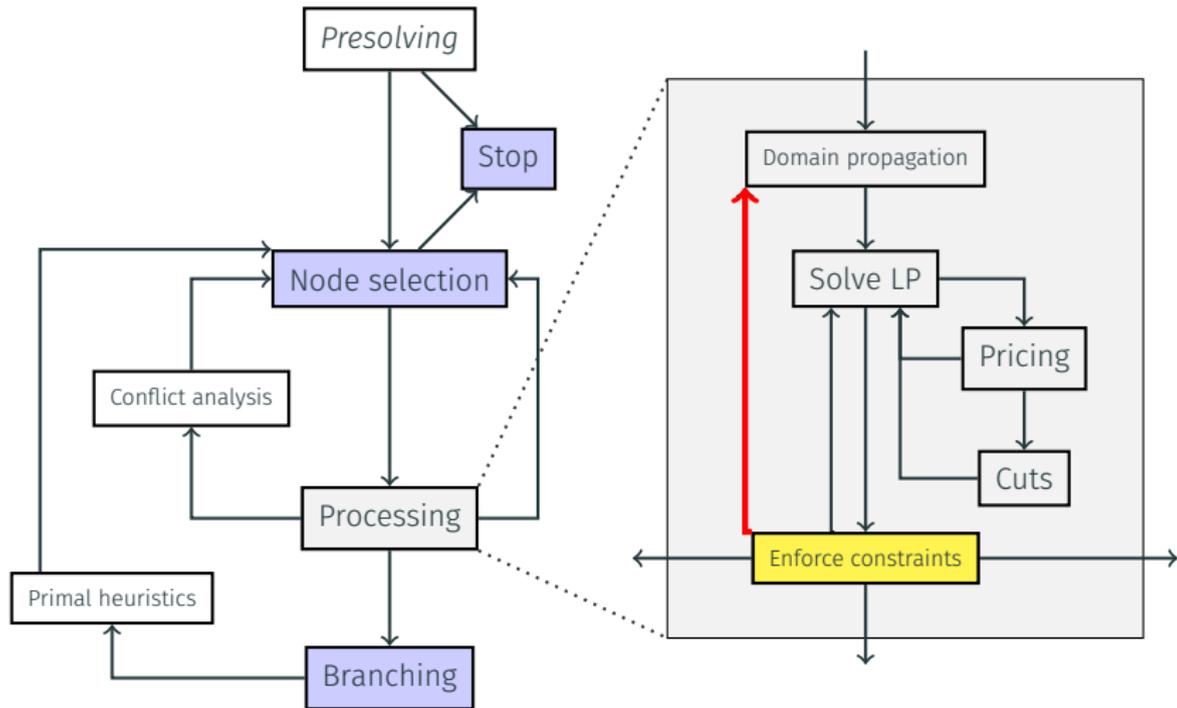
- Branched

- Cutoff

- Infeasible

- Feasible

Constraint Enforcement



• Reduced domain

• Added constraint

• Added cut

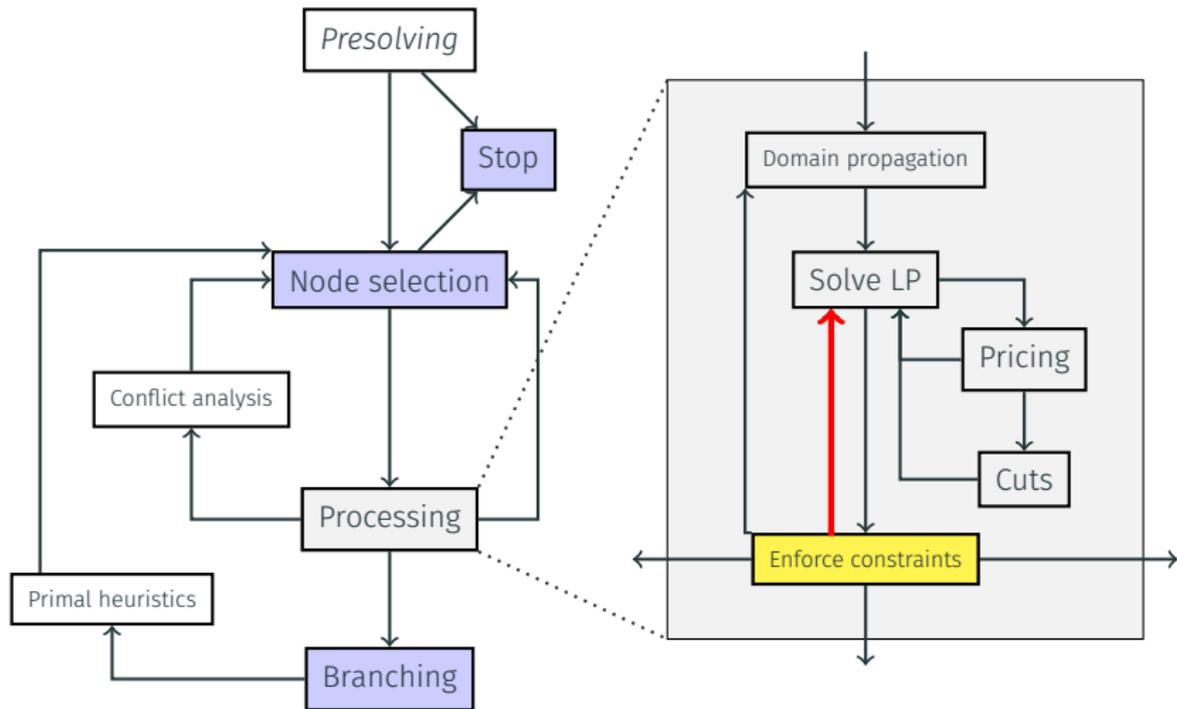
• Branched

• Cutoff

• Infeasible

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Constraint Enforcement



- Reduced domain

- Added constraint

- Cutoff

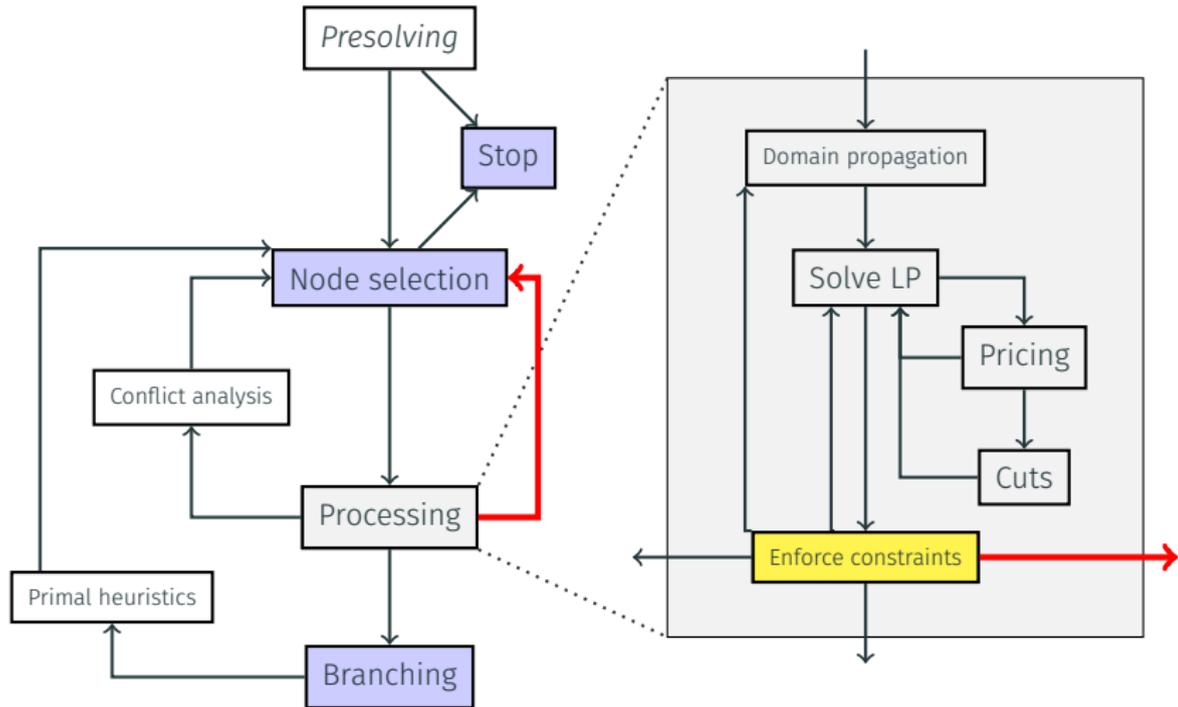
- Infeasible

- Added constraint

- Branched

- Feasible

Constraint Enforcement



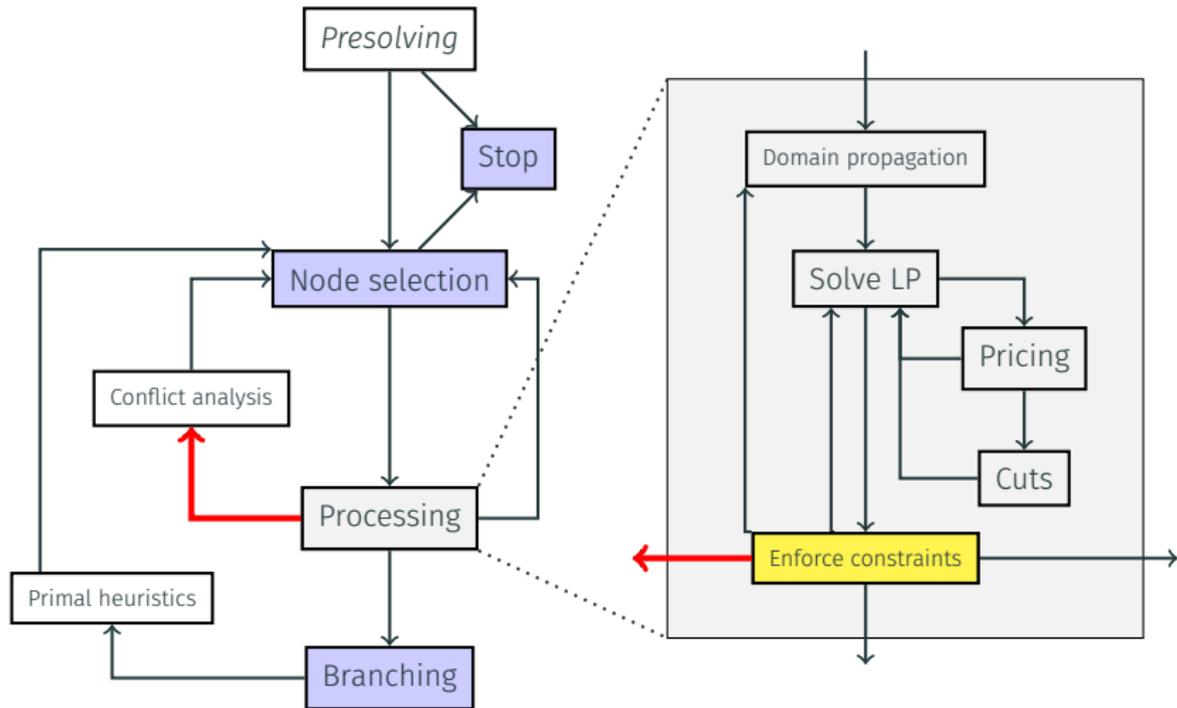
- Reduced domain
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- Added cut
- Branched

- Cutoff

- Infeasible
- Feasible

Constraint Enforcement



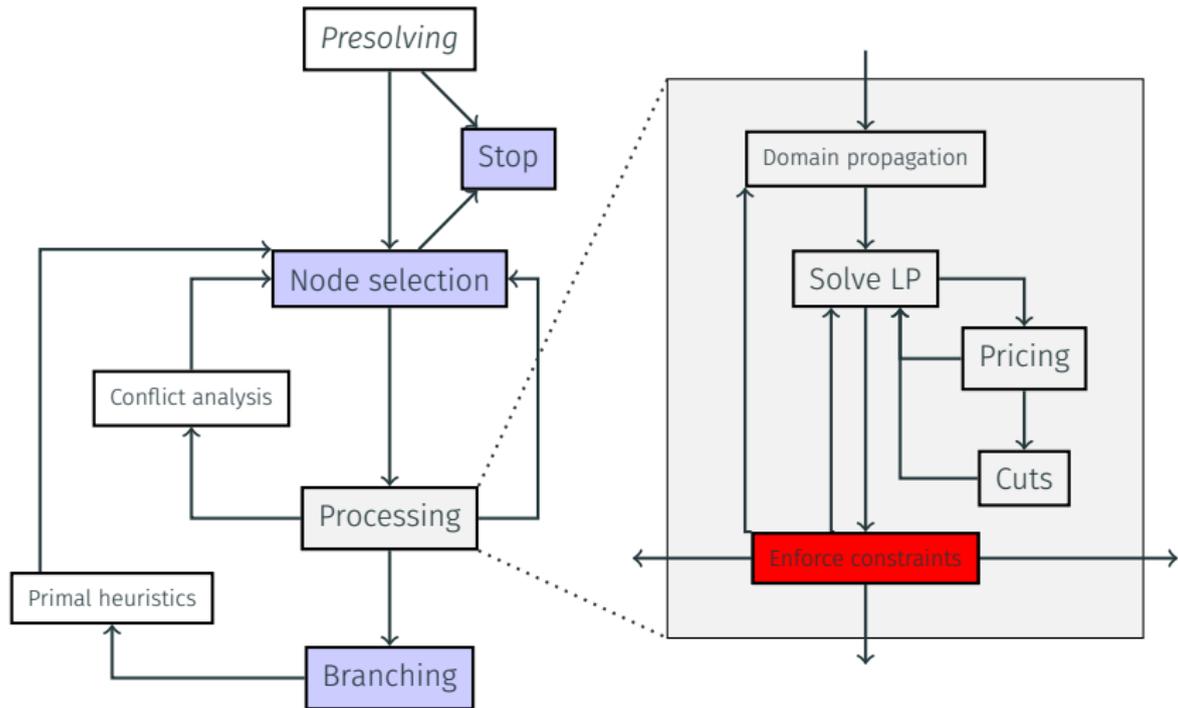
- Reduced domain
- Added constraint

- Added cut
- Branched

• Cutoff

- Infeasible
- Feasible

Constraint Enforcement



- Reduced domain

- Added constraint

- Added cut

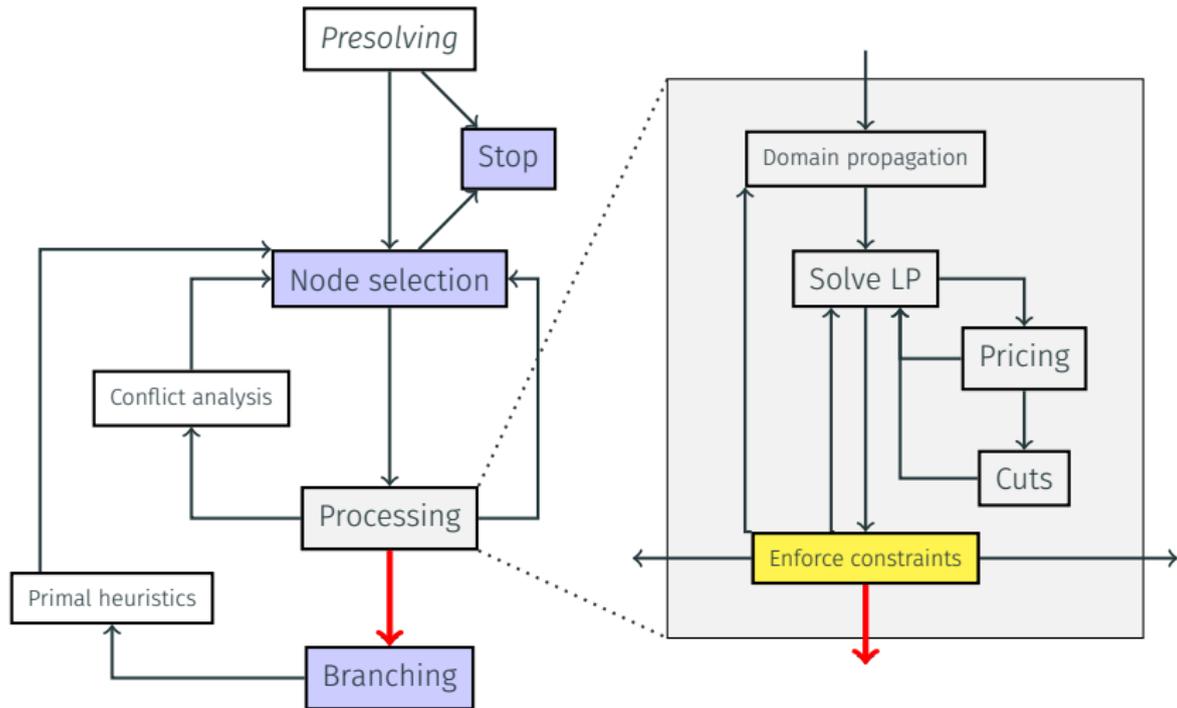
- Branched

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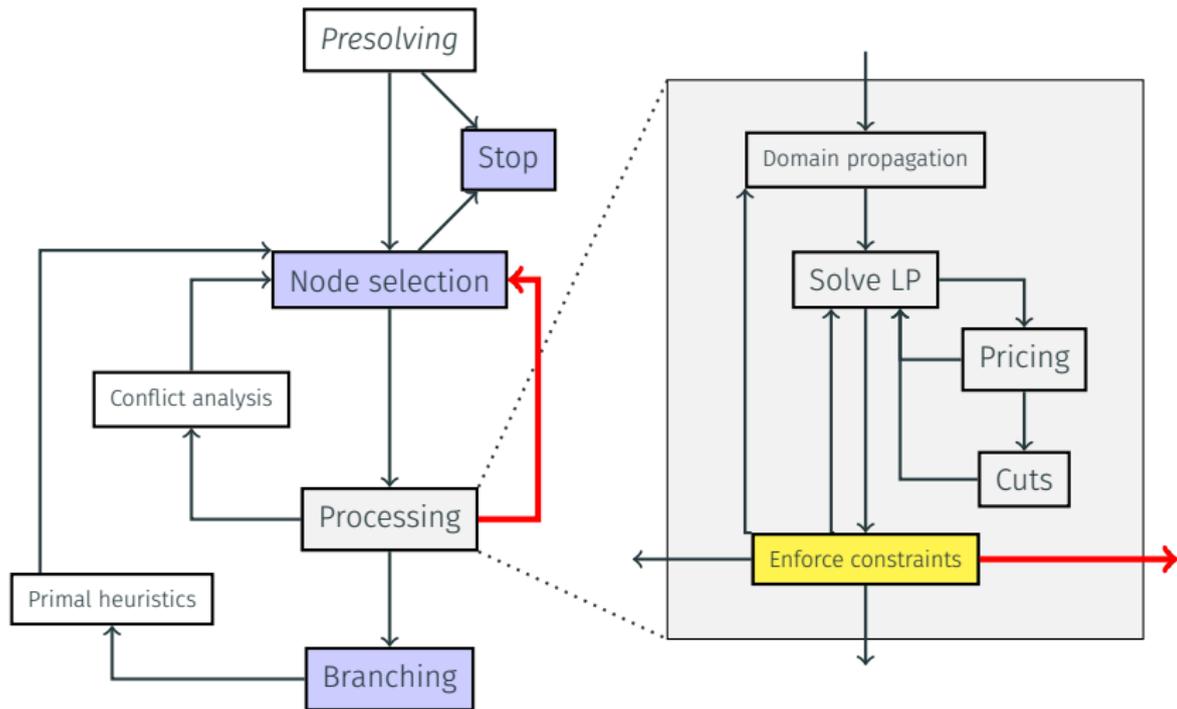
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- Added cut
- Branched

- Cutoff

- Infeasible
- Feasible

Constraint Enforcement



- Reduced domain

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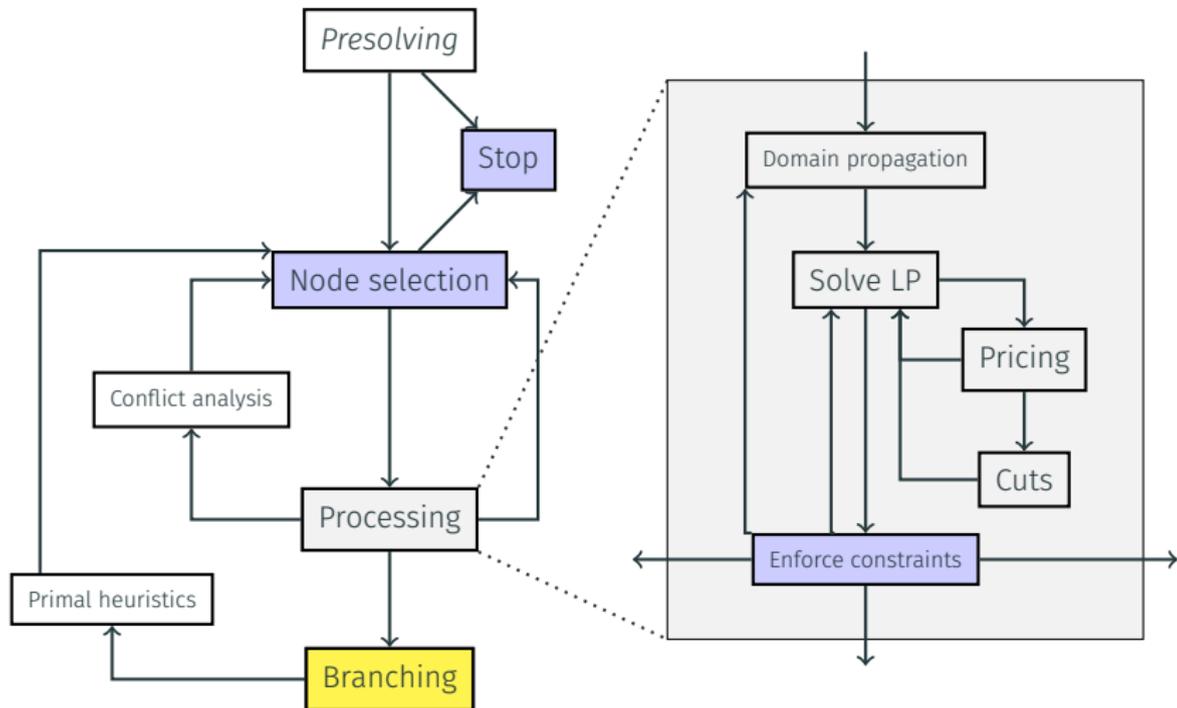
- Branched

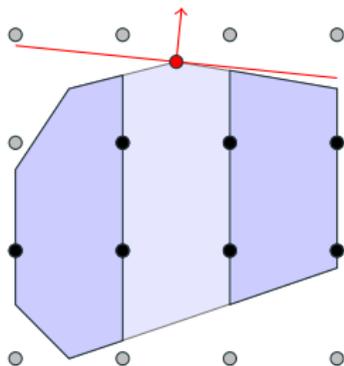
- Cutoff

- Infeasible

- Feasible

Flow Chart SCIP





Task

- divide into (disjoint) subproblems
- improve local bounds

Techniques

- branching on variables
 - most infeasible
 - least infeasible
 - random branching
 - strong branching
 - pseudocost
 - reliability
 - VSIDS
 - hybrid reliability/inference
- branching on constraints
 - SOS1
 - SOS2

Branching Rule Tips and Parameters

Branching Rule Selection

Branching rules are applied in decreasing order of priority.

```
SCIP> display branching
```

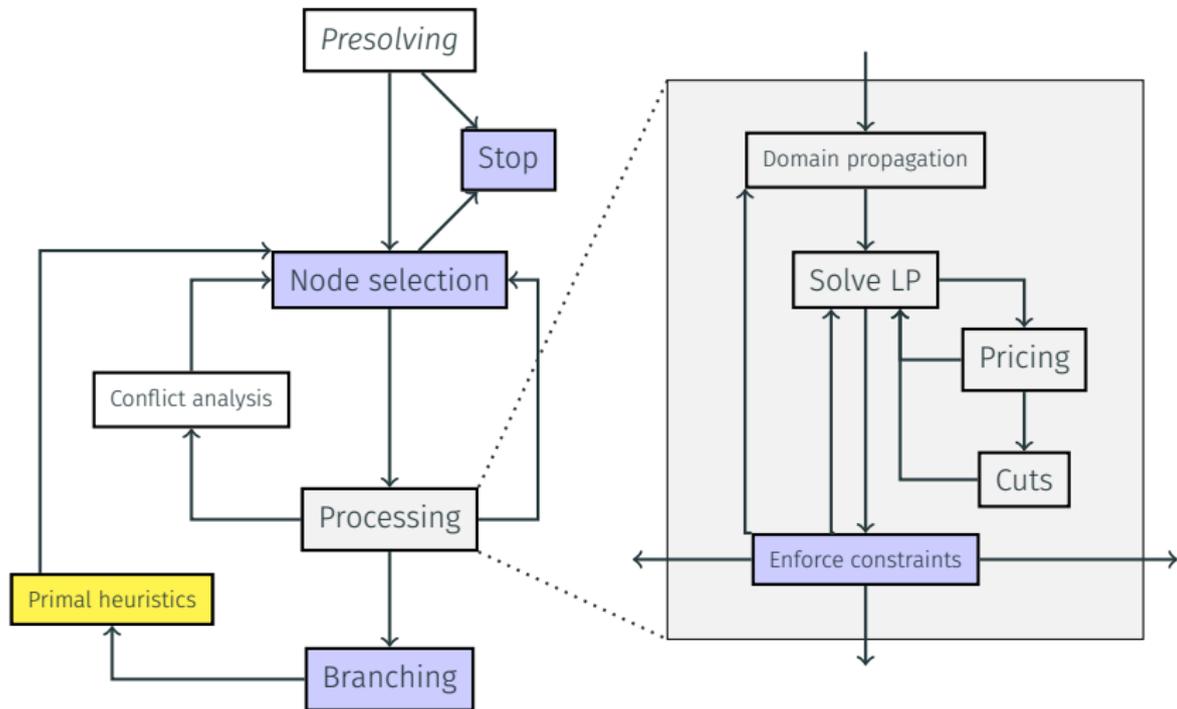
branching rule	priority	maxdepth	maxbddist
relpscost	10000	-1	100.0%
pscost	2000	-1	100.0%
inference	1000	-1	100.0%
mostinf	100	-1	100.0%

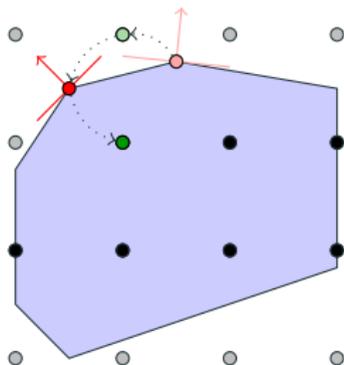
Reliability Branching Parameters

All parameters prefixed with **branching/relpscost/**

- **sbiterquot**, **sbiterofs** to increase the budget for strong branching
- **minreliable** (= 1), **maxreliable** (= 5) to increase threshold to consider pseudo costs as reliable

Flow Chart SCIP





Task

- improve primal bound
- effective on average
- guide remaining search

Techniques

- **structure-based**
 - clique
 - variable bounds
- **rounding**
 - possibly solve final LP
- **diving**
 - least infeasible
 - guided
- **objective diving**
 - objective feasibility pump
- **Large Neighborhood Search**
 - RINS, local branching
 - RENS
 - Adaptive LNS
 - Completion of partial solutions

Disable/Speed Up/Emphasize Heuristics

```
set heuristics emphasis off/fast/aggressive
```

Disable an individual heuristic via

```
set heuristics feaspump freq -1
```

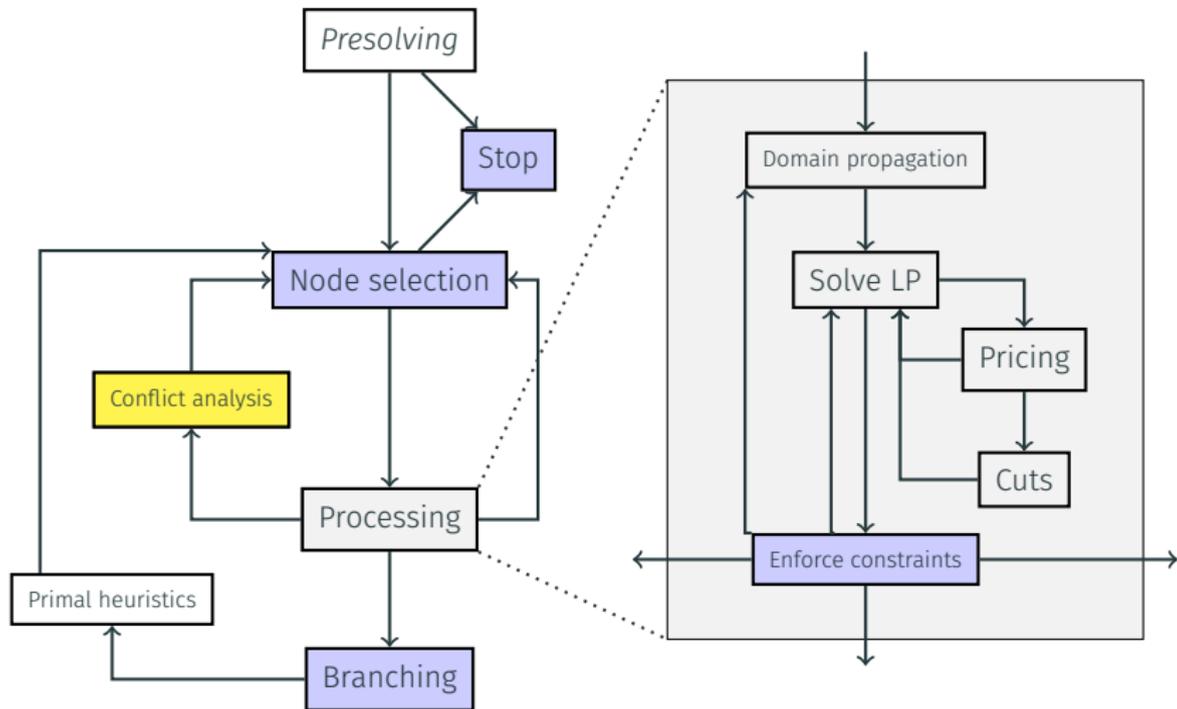
Important Parameters

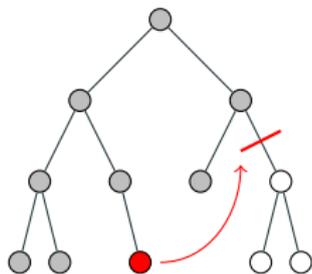
- `heuristics/alns/nodesofs`, `heuristics/alns/nodesquot` to increase the computational budget of this LNS technique
- `heuristics/guideddiving/... lpsolvefreq`, `maxlpiterofs`, `maxlpiterquot` to control the LP solving during this diving technique

Advice

Use emphasis settings. **Do not** attempt to individually tune heuristics by hand.

Flow Chart SCIP





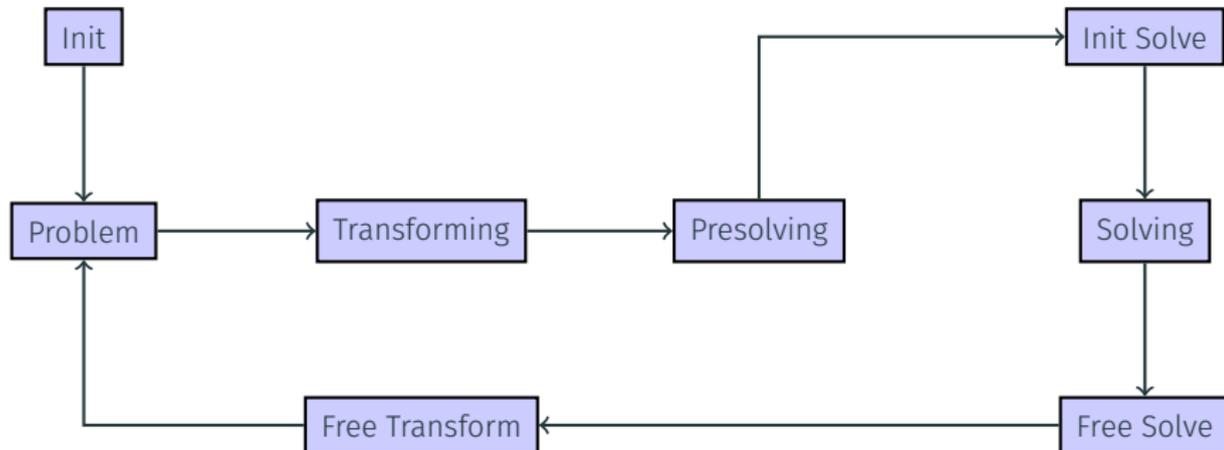
Task

- Analyze infeasibility
- Derive valid constraints
- Help to prune other nodes

Techniques

- **Analyze:**
 - Propagation conflicts
 - Infeasible LPs
 - Bound-exceeding LPs
 - Strong branching conflicts
- **Detection:**
 - Cut in conflict graph
 - LP: Dual ray heuristic
- **Use conflicts:**
 - Only for propagation
 - As cutting planes

Operational Stages



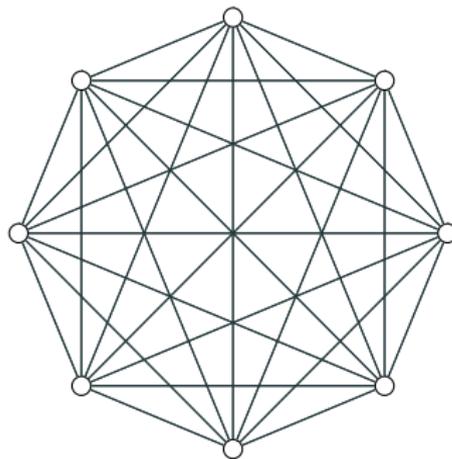
SCIP – Solving Constraint Integer Programs

<http://scip.zib.de>

Definition (TSP)

Given a complete graph $G = (V, E)$
and distances d_e for all $e \in E$:

Find a Hamiltonian cycle (cycle
containing all nodes, tour) of
minimum length.



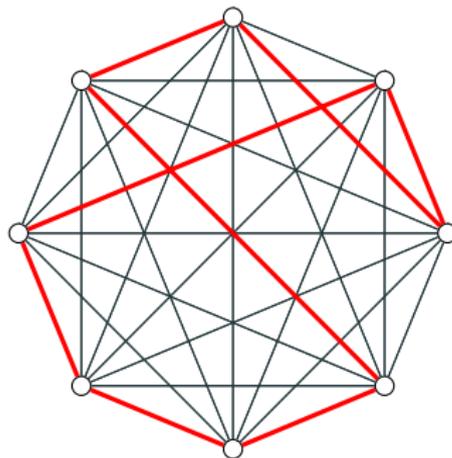
K_8

An Example: the Traveling Salesman Problem

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K_8

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What is a Constraint Integer Program?

Mixed Integer Program

Objective function:

- ▷ linear function

Feasible set:

- ▷ described by linear constraints

Variable domains:

- ▷ real or integer values

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & Ax \leq b \\ & (x_I, x_C) \in \mathbb{Z}^I \times \mathbb{R}^C \end{array}$$

Constraint Program

Objective function:

- ▷ arbitrary function

Feasible set:

- ▷ given by arbitrary constraints

Variable domains:

- ▷ arbitrary (usually finite)

$$\begin{array}{ll} \min & c(x) \\ \text{s.t.} & x \in F \\ & (x_I, x_N) \in \mathbb{Z}^I \times X \end{array}$$

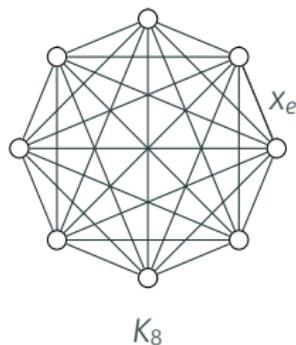
TSP – Integer Programming Formulation

Given

- complete graph $G = (V, E)$
- distances $d_e > 0$ for all $e \in E$

Binary variables

- $x_e = 1$ if edge e is used



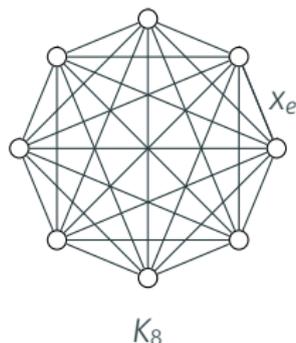
TSP – Integer Programming Formulation

Given

- complete graph $G = (V, E)$
- distances $d_e > 0$ for all $e \in E$

Binary variables

- $x_e = 1$ if edge e is used



$$\begin{array}{ll} \min & \sum_{e \in E} d_e x_e \\ \text{subject to} & \sum_{e \in \delta(v)} x_e = 2 \quad \forall v \in V \\ & \sum_{e \in \delta(S)} x_e \geq 2 \quad \forall S \subset V, S \neq \emptyset \\ & x_e \in \{0, 1\} \quad \forall e \in E \end{array}$$

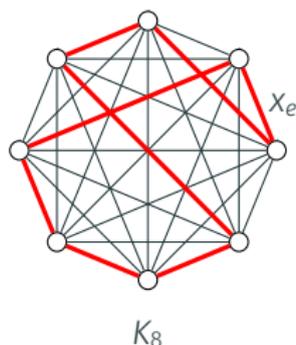
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- distances $d_e > 0$ for all $e \in E$

Binary variables

- $x_e = 1$ if edge e is used



$$\min \sum_{e \in E} d_e x_e$$

$$\text{subject to } \sum_{e \in \delta(v)} x_e = 2 \quad \forall v \in V \quad \text{node degree}$$

$$\sum_{e \in \delta(S)} x_e \geq 2 \quad \forall S \subset V, S \neq \emptyset$$

$$x_e \in \{0, 1\} \quad \forall e \in E$$

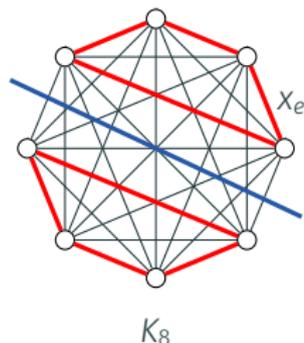
TSP – Integer Programming Formulation

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Binary variables

- $x_e = 1$ if edge e is used



$$\begin{aligned} \min \quad & \sum_{e \in E} d_e x_e \\ \text{subject to} \quad & \sum_{e \in \delta(v)} x_e = 2 & \forall v \in V \\ & \sum_{e \in \delta(S)} x_e \geq 2 & \forall S \subset V, S \neq \emptyset \\ & & \text{subtour elimination} \\ & x_e \in \{0, 1\} & \forall e \in E \end{aligned}$$

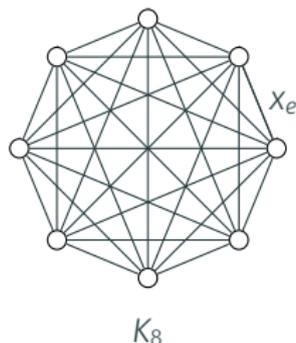
TSP – Integer Programming Formulation

Given

- complete graph $G = (V, E)$
- distances $d_e > 0$ for all $e \in E$

Binary variables

- $x_e = 1$ if edge e is used



$$\min \sum_{e \in E} d_e x_e \quad \text{distance}$$

$$\text{subject to } \sum_{e \in \delta(v)} x_e = 2 \quad \forall v \in V$$

$$\sum_{e \in \delta(S)} x_e \geq 2 \quad \forall S \subset V, S \neq \emptyset$$

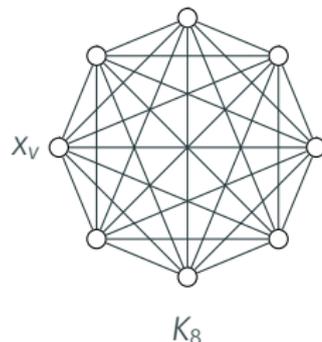
$$x_e \in \{0, 1\} \quad \forall e \in E$$

Given

- complete graph $G = (V, E)$
- for each $e \in E$ a distance $d_e > 0$

Integer variables

- x_v position of $v \in V$ in tour



TSP – Constraint Programming Formulation

Given

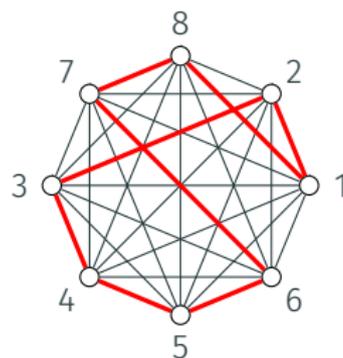
- complete graph $G = (V, E)$
- for each $e \in E$ a distance $d_e > 0$

Integer variables

- x_v position of $v \in V$ in tour

$$\begin{array}{ll} \min & \text{length}(x_1, \dots, x_n) \\ \text{subject to} & \text{alldifferent}(x_1, \dots, x_n) \\ & x_v \in \{1, \dots, n\} \end{array}$$

$$\forall v \in V$$



What is a Constraint Integer Program?

Constraint Integer Program

Objective function:

- ▷ linear function

Feasible set:

- ▷ described by arbitrary constraints

Variable domains:

- ▷ real or integer values

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & x \in F \\ & (x_I, x_C) \in \mathbb{Z}^I \times \mathbb{R}^C \end{aligned}$$

Remark:

- arbitrary objective or variables modeled by constraints

What is a Constraint Integer Program?

Constraint Integer Program

Objective function:

- ▷ linear function

Feasible set:

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Variable domains:

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$$\begin{aligned} \min \quad & \sum_{e \in E} d_e x_e \\ \text{s.t.} \quad & \sum_{e \in \delta(v)} x_e = 2 \quad \forall v \in V \\ & \text{nosubtour}(x) \\ & x_e \in \{0, 1\} \quad \forall e \in E \end{aligned}$$

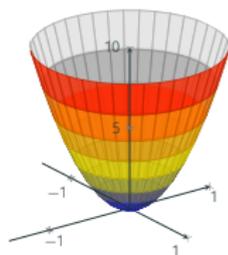
(CIP formulation of TSP)

Single nosubtour constraint rules out subtours (e.g. by domain propagation). It may also separate subtour elimination inequalities.

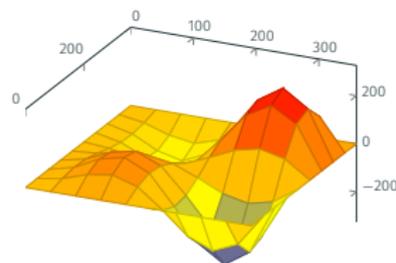
Mixed-Integer Nonlinear Programs (MINLPs)

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & g_k(x) \leq 0 \quad \forall k \in [m] \\ & x_i \in \mathbb{Z} \quad \forall i \in \mathcal{I} \subseteq [n] \\ & x_i \in [\ell_i, u_i] \quad \forall i \in [n] \end{aligned}$$

The functions $g_k \in C^1([\ell, u], \mathbb{R})$ can be



convex or



nonconvex

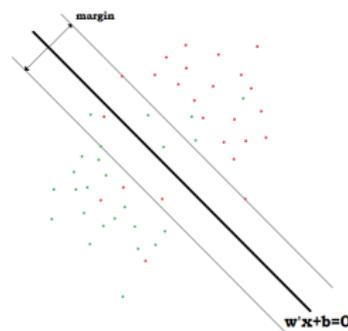
Support Vector Machine, e.g., with **ramp loss**.

$$\min \frac{w^T w}{2} + \frac{C}{n} \sum_{i=1}^n (\xi_i + 2(1 - z_i))$$

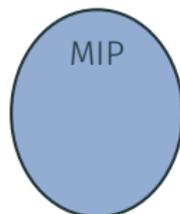
$$\text{s.t. } z_i (y_i (w^T x_i + b) - 1 + \xi_i) \geq 0 \quad \forall i$$

$$\xi_i \in [0, 2], \quad z_i \in \{0, 1\} \quad \forall i$$

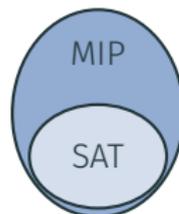
$$w \in \mathbb{R}^d, \quad b \in \mathbb{R}$$



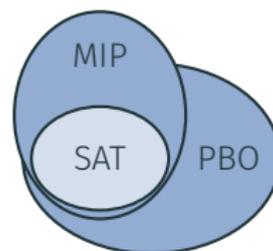
- Mixed Integer Programs



- Mixed Integer Programs
- SATisifiability problems

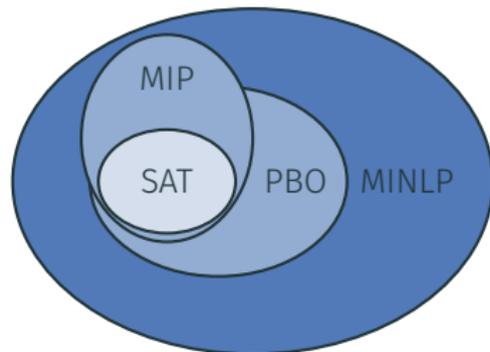


- Mixed Integer Programs
- SATisifiability problems
- Pseudo-Boolean Optimization



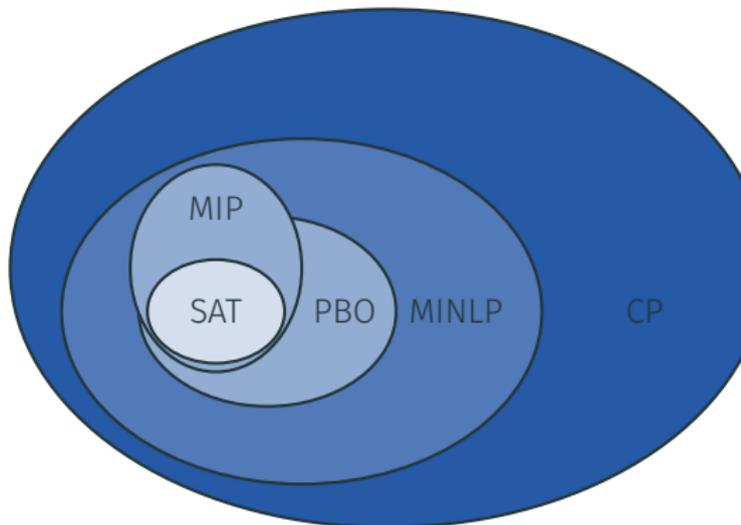
Constraint Integer Programming

- Mixed Integer Programs
- SATisfiability problems
- Pseudo-Boolean Optimization
- Mixed Integer NonLinear Programs



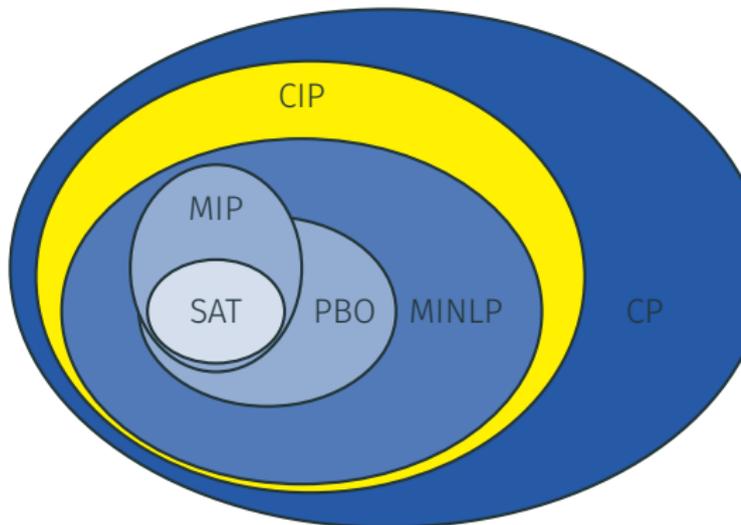
Constraint Integer Programming

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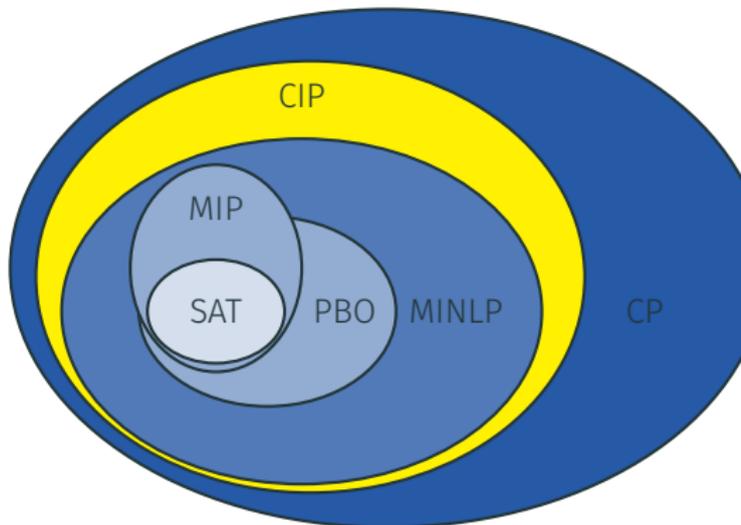
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- Constraint Integer Programming



Constraint Integer Programming

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- Constraint Integer Programming



Relation to CP and MIP

- Every MIP is a CIP. $MIP \subsetneq CIP$
- Every CP over a finite domain space is a CIP. $FD \subsetneq CIP$

SCIP – Solving Constraint Integer Programs

<http://scip.zib.de>

Different plugin classes are responsible of the following tasks.

1. Presolving and node propagation

- Constraint handlers
- Presolvers
- Propagators

2. Separation

- Constraint handlers
- Separators

3. Improving solutions

- Primal heuristics

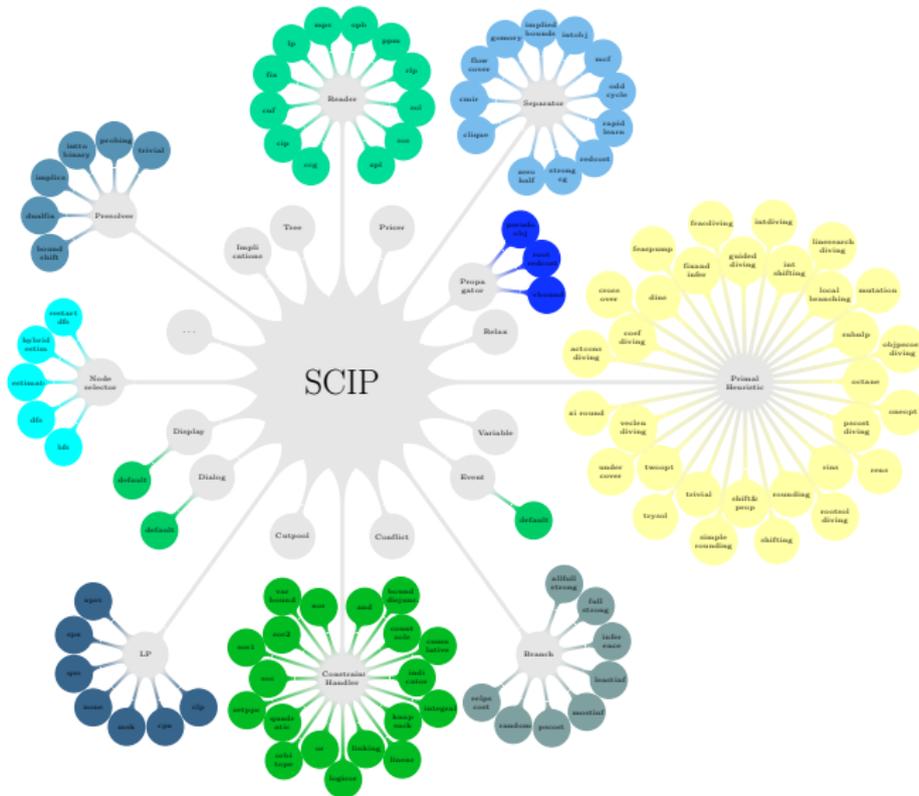
4. Branching

- Constraint handlers
- Branching rules

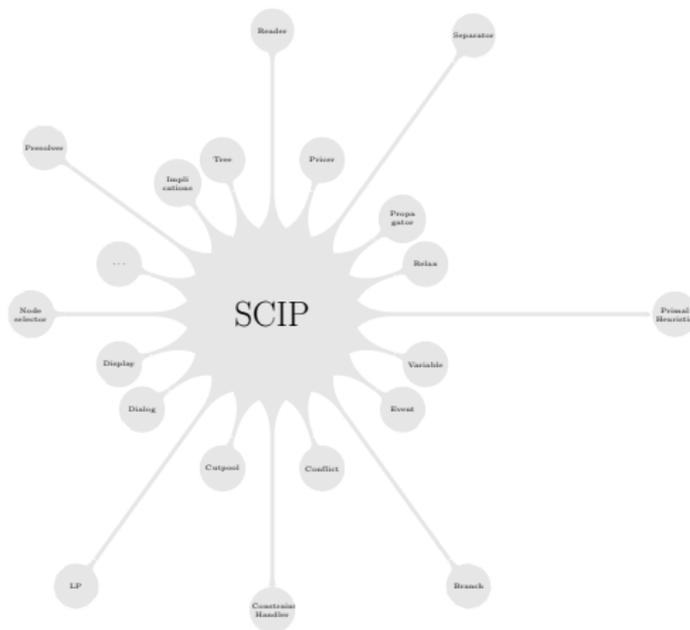
5. Node selection

- Node selectors

Structure of SCIP



Structure of SCIP



SCIP core

- branching tree
- variables
- conflict analysis
- solution pool
- cut pool
- statistics
- clique table
- implication graph
- ...

Plugin based design

SCIP core

- branching tree
- variables
- conflict analysis
- solution pool
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Plugins

- external callback objects
- interact with the framework through a very detailed interface

Plugin based design

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Plugins

- external callback objects
 - interact with the framework through a very detailed interface
 - SCIP knows for each plugin type:
 - the number of available plugins
 - priority defining the calling order (usually)
 - SCIP does not know any structure behind a plugin
- ⇒ plugins are black boxes for the SCIP core

SCIP core

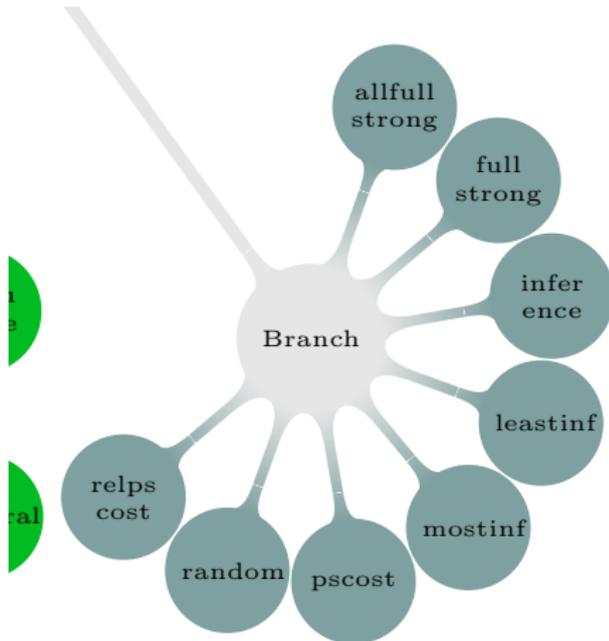
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Plugins

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 - SCIP does not know any structure behind a plugin
- ⇒ plugins are black boxes for the SCIP core
- ⇒ Very flexible branch-and-bound based search algorithm

- Constraint handler: assures feasibility, strengthens formulation
- Separator: adds cuts, improves dual bound
- Pricer: allows dynamic generation of variables
- Heuristic: searches solutions, improves primal bound
- Branching rule: how to divide the problem?
- Node selection: which subproblem should be regarded next?
- Presolver: simplifies the problem in advance, strengthens structure
- Propagator: simplifies problem, improves dual bound locally
- Reader: reads problems from different formats
- Event handler: catches events (e.g., bound changes, new solutions)
- Display: allows modification of output
- ...

A closer look: branching rules



What does SCIP know about branching rules?

- SCIP knows **the number of** available **branching rules**
- each branching rule has a **priority**
- SCIP calls the branching rule in decreasing order of priority
- the interface defines the **possible results** of a call:
 - branched
 - reduced domains
 - added constraints
 - detected cutoff
 - did not run

How does SCIP call a branching rule?

```
/* start timing */
SCIPclockStart(branchrule->branchclock, set);

/* call external method */
SCIP_CALL( branchrule->branchexeclp(set->scip, branchrule,
    allowaddcons, result) );

/* stop timing */
SCIPclockStop(branchrule->branchclock, set);

/* evaluate result */
if( *result != SCIP_CUTOFF
    && *result != SCIP_CONSADDED
    && *result != SCIP_REDUCEDDOM
    && *result != SCIP_SEPARATED
    && *result != SCIP_BRANCHED
    && *result != SCIP_DIDNOTRUN )
{
    SCIPerrorMessage(
        "branching_rule_%s returned invalid result code %d from LP \
        \solution_branching\n",
        branchrule->name, *result);
    return SCIP_INVALIDRESULT;
}
```

What can a plugin access?

Plugins are allowed to access all global (core) information

- branching tree
- variables
- conflict analysis
- solution pool
- cut pool
- statistics
- clique table
- implication graph
- ...

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Ideally, plugins should not access data of other plugins!!!

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Branching Rules

- LP solution
- variables
- statistics

Constraint handlers

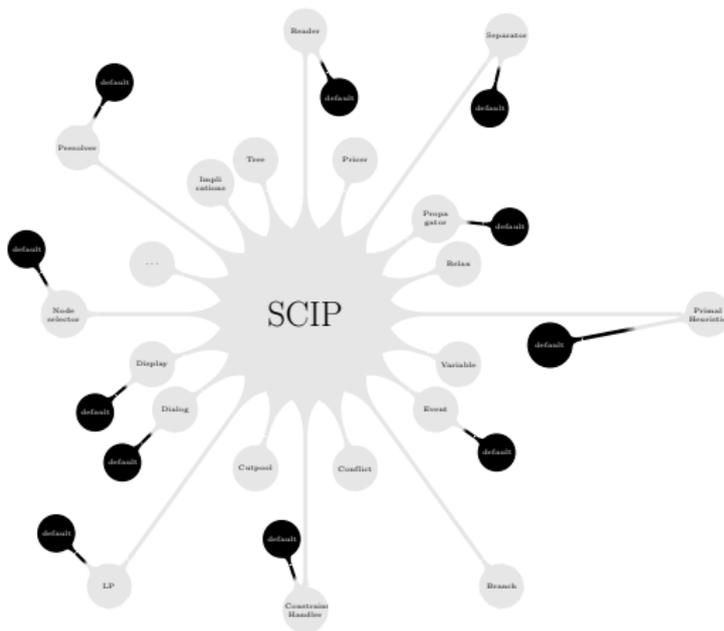
- most powerful plugins in SCIP
- define the feasible region
- a single constraint may represent a whole set of inequalities

Functions

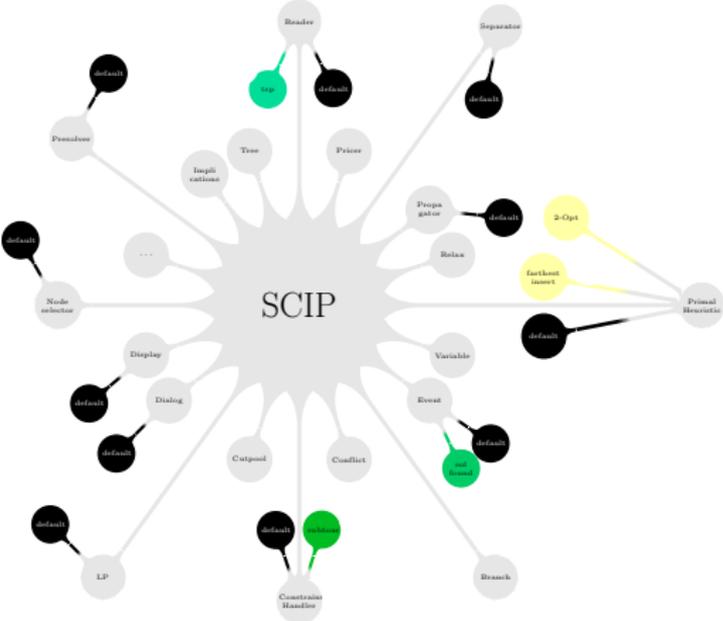
- check and enforce feasibility of solutions
- can add linear representation to LP relaxation
- constraint-specific presolving, domain propagation, separation

Result

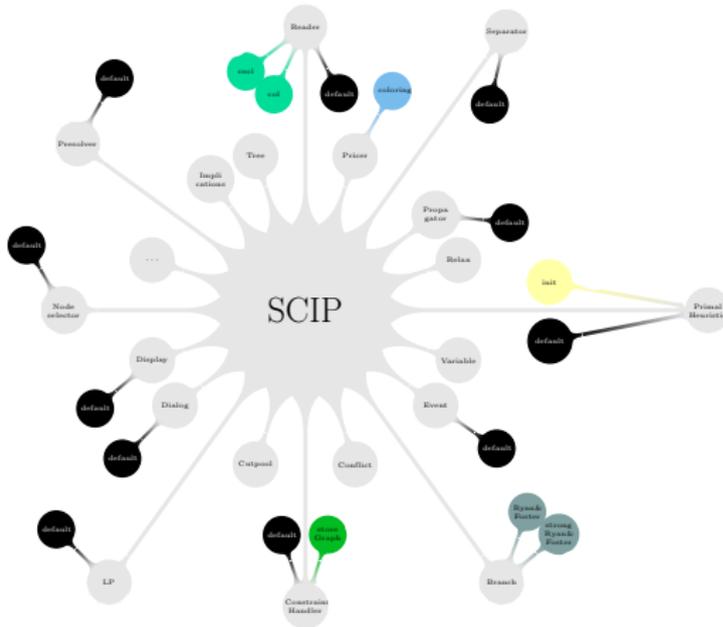
- SCIP is constraint based
 - Advantage: flexibility
 - Disadvantage: limited global view



Extending SCIP: TSP



Extending SCIP: Coloring



How to solve your MINLP optimization problem:

- write down the mathematical description
- modeling language, e.g., ZIMPL, generates input for MINLP solver
- SCIP can even read ZIMPL files directly

MIP and MINLP solving

- ...is a bag of tricks
- new tricks are introduced almost every day
- advantages of SCIP
 - open source, you can see everything that happens
 - hundreds of parameters to play with
 - broad scope
 - easily extendable