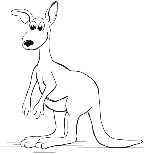


Using SCIP to Solve Your Favorite Integer Optimization Problem

Gregor Hendel, hendel@zib.de, www.zib.de/hendel
Monash University, Melbourne
March 12, 2019

Zuse Institute Berlin – Fast Algorithms, Fast Computers

A research institute and computing center of the State of Berlin with research units:

- Numerical Analysis and Modeling
- Visualization and Data Analysis
- Optimization:
  Energy – Transportation – Health – Mathematical Optimization Methods
- Scientific Information Systems
- Computer Science and High Performance Computing

What is SCIP?

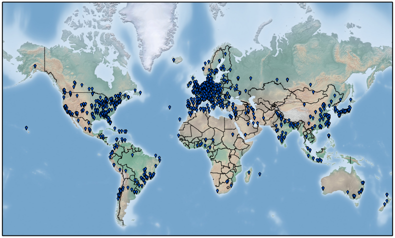
SCIP (Solving Constraint Integer Programs) . . .

- provides a full-scale MIP and MINLP solver,
- is constraint based,
- incorporates
  - MIP features (cutting planes, LP relaxation), and
  - MINLP features (spatial branch-and-bound, OBBT)
  - CP features (domain propagation),
  - SAT-solving features (conflict analysis, restarts),
- is a branch-cut-and-price framework,
- has a modular structure via plugins,
- is free for academic purposes,
- and is available in source-code under `http://scip.zib.de` !

Meet the SCIP Team

31 active developers

- 7 running Bachelor and Master projects
- 16 running PhD projects
- 8 postdocs and professors



4 development centers in Germany

- ZIB: SCIP, SoPlex, UG, ZIMPL
- TU Darmstadt: SCIP and SCIP-SDP
- FAU Erlangen-Nürnberg: SCIP
- RWTH Aachen: GCG

Many international contributors and users

- more than 10 000 downloads per year from 100+ countries

Careers

- 10 awards for Masters and PhD theses: MOS, EURO, GOR, DMV
- 7 former developers are now building commercial optimization software at CPLEX, FICO Xpress, Gurobi, MOSEK, and GAMS

SCIP – Solving Constraint Integer Programs

The SCIP Optimization Suite

(Mixed-) Integer Programs

Constraint Integer Programming

The Solving Process of SCIP

Extending SCIP by Plugins

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SCIP Optimization Suite

- Toolbox for generating and solving constraint integer programs, in particular Mixed Integer (Non-)Linear Programs.
- free for academic use, available in source code

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ZIMPL

- model and generate LPs, MIPs, and MINLPs

SCIP

- MIP, MINLP and CIP solver, branch-cut-and-price framework

SoPlex

- revised primal and dual simplex algorithm

GCG

- generic branch-cut-and-price solver

UG

- framework for parallelization of MIP and MINLP solvers

The SCIP C API

- C code and documentation
  - more than 800 000 lines of C code, 20% documentation
  - over 50 000 assertions and 5 000 debug messages
  - HowTos: plugins types, debugging, automatic testing
  - 10 examples illustrating the use of SCIP
  - 5 problem specific SCIP applications to solve Coloring, Steiner Tree, or Multiobjective problems.
- Interface and usability
  - Cross-platform availability due to CMake
  - user-friendly interactive shell
  - C++ wrapper classes
  - LP solvers: SoPlex, CPLEX, Gurobi, Xpress, CLP, MOSEK, QSopt
  - NLP solvers: IPOPT, FilterSQP, WORHP
  - over 2 300 parameters and 15 emphasis settings

Interfaces to SCIP

- interactive shell supports 11 different input formats
  $\rightarrow$ cip, cnf, flatzinc, rlp, lp, mps, opb, pip, wbo, zimpl, smps
- C API/callable library
- C++ wrapper classes
- Python interface
- Java JNI interface
- AMPL
- GAMS
- Matlab (see also OPTI toolbox, http://www.i2c2.aut.ac.nz/Wiki/OPTI/)

Getting help

If you should ever get stuck, you can . . .

1. type `help` in the interactive shell
2. read the documentation http://scip.zib.de/doc/html
   → FAQ, HowTos for each plugin type, debugging, automatic testing, . . .
3. search or post on Stack Overflow using the tag `scip` (more than 100 questions already answered)
4. active mailing list scip@zib.de (350+ members)
   - search the mailing list archive (append site:listserv/pipermail/scip)
   - register http://listserv.zib.de/mailman/listinfo/scip/ and post

Outline

SCIP – Solving Constraint Integer Programs

The SCIP Optimization Suite

(Mixed-) Integer Programs

Constraint Integer Programming

The Solving Process of SCIP

Extending SCIP by Plugins

http://scip.zib.de

What is a good diet?

Product	Servings	Energy(kj)	Protein(g)	Calcium(g)	Price(ct)
Oatmeal	4	110	4	2	3
Chicken	3	205	32	12	24
Eggs	2	160	13	54	13
Milk	8	160	8	284	9
Pie	2	420	4	22	20
Pork	2	260	14	80	19

Needed per day:

- Energy: 2000 kj
- Protein: 55g
- Calcium: 800 mg

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Task: Find a diet satisfying all nutritional requirements at minimum cost!

The Diet LP

Find a diet satisfying all nutritional requirements at minimum cost!

Our nutrition plan should be expressed as $x \in \mathbb{R}^6_{\geq 0}$.

- Energy: 2000 kj
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The Diet LP

Find a diet satisfying all nutritional requirements at minimum cost!

Our nutrition plan should be expressed as $x \in \mathbb{R}^6_{\geq 0}$.

$$\min 3x_{\text{oat}} + 24x_{\text{Chi}} + 13x_{\text{Egg}} + 9x_{\text{Mil}} + 20x_{\text{Pie}} + 19x_{\text{Por}}$$

- Energy: 2000 kj

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- Energy: 2000 kj

$$110x_{\text{oat}} + 205x_{\text{Chi}} + 160x_{\text{Egg}} + 160x_{\text{Mil}} + 420x_{\text{Pie}} + 260x_{\text{Por}} \geq 2000$$

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$$110x_{\text{oat}} + 205x_{\text{Chi}} + 160x_{\text{Egg}} + 160x_{\text{Mil}} + 420x_{\text{Pie}} + 260x_{\text{Por}} \geq 2000$$

- Protein: 55g

$$4x_{\text{oat}} + 32x_{\text{Chi}} + 13x_{\text{Egg}} + 8x_{\text{Mil}} + 4x_{\text{Pie}} + 14x_{\text{Por}} \geq 55$$

- Calcium: 800 mg

$$2x_{\text{oat}} + 12x_{\text{Chi}} + 54x_{\text{Egg}} + 284x_{\text{Mil}} + 22x_{\text{Pie}} + 80x_{\text{Por}} \geq 800$$

Diet → MIP

Diet Problem

Only an integer vector $x \in \mathbb{Z}^6_{\geq 0}$ is truly feasible, because food such as eggs are unsplittable.

$$
\begin{array}{llr}
\min & \begin{pmatrix} 3 & 24 & 13 & 9 & 20 & 19 \end{pmatrix} & x \\
\text{s.t.} & \begin{pmatrix} 110 & 205 & 160 & 160 & 420 & 260 \\ 4 & 32 & 13 & 8 & 4 & 14 \\ 2 & 12 & 54 & 284 & 22 & 80 \end{pmatrix} & x \geq \begin{pmatrix} 2000 \\ 55 \\ 800 \end{pmatrix} \\
 & & x \geq 0 \\
 & & \mathbf{x_i} \in \mathbb{Z} \qquad \forall \mathbf{i} \in \{\mathbf{1}, \ldots, \mathbf{6}\}
\end{array}
$$

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\end{array}
$$

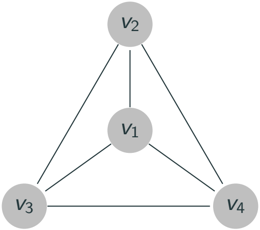
General MIP

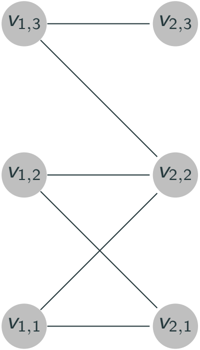
$$
\begin{array}{llll}
\min & c^T x & & \\
\text{s.t.} & Ax \geq & b & \\
& x \geq & 0 & \\
& \mathbf{x_i} \in & \mathbb{Z} & \forall \mathbf{i} \in \mathcal{I} \subseteq \{\mathbf{1}, \ldots, \mathbf{n}\}
\end{array}
$$

(Mixed Integer Program)

Simple graphs

- A simple graph G is a tuple (V, E) of
  - $|V| = n$ nodes and
  - $m = |E|$ edges connecting pairs of nodes, i.e. $E \subseteq \binom{V}{2}$.
  - nodes may have weights/demands $w : V \to \mathbb{R}$
  - edges may have associated costs $c : E \to \mathbb{R}$





Classical Optimization Problems On Graphs

Most of the classical optimization problems on graphs can be formulated as Mixed Integer Programs.

- Max. stable set
- Min. vertex cover
- Max. matching
- Graph coloring
- Traveling salesman problem
- Steiner tree problem
- ...

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Maximum stable set: Find a set of vertices of maximum cardinality without a connecting edge.

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Maximum stable set: Find a set of vertices of maximum cardinality without a connecting edge.

Integer Program:

$$\begin{array}{lll} \max & 1^T x & \\ \text{s.t.} & x_v + x_w \leq 1 & \forall \{v, w\} \in E \\ \text{s.t.} & x_v \in \{0, 1\} & \forall v \in V \end{array}$$

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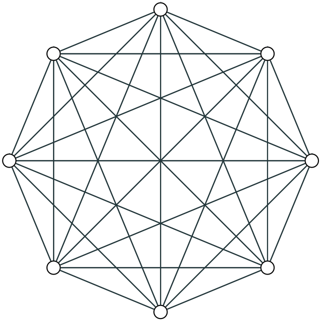
Definition (TSP)
Given a complete graph $G = (V, E)$ and distances d_e for all $e \in E$:

Find a Hamiltonian cycle (cycle containing all nodes, tour) of minimum length.



K_8

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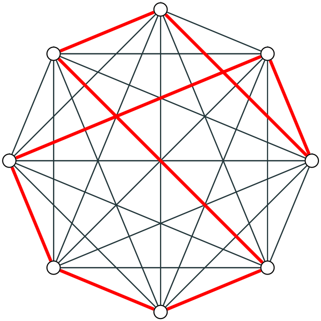

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Definition (TSP)
Given a complete graph $G = (V, E)$ and distances d_e for all $e \in E$:

Find a Hamiltonian cycle (cycle containing all nodes, tour) of minimum length.

What is a Constraint Integer Program?

Mixed Integer Program

Objective function:

- linear function

Feasible set:

- described by linear constraints

Variable domains:

- real or integer values

$$\begin{aligned} \min \quad & c^T x \\ s.t. \quad & Ax \leq b \\ & (x_I, x_C) \in \mathbb{Z}^I \times \mathbb{R}^C \end{aligned}$$

Constraint Program

Objective function:

- arbitrary function

Feasible set:

- given by arbitrary constraints

Variable domains:

- arbitrary (usually finite)

$$\begin{aligned} \min \quad & c(x) \\ s.t. \quad & x \in F \\ & (x_I, x_N) \in \mathbb{Z}^I \times X \end{aligned}$$

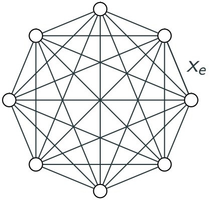
TSP – Integer Programming Formulation

Given

- complete graph $G = (V, E)$
- distances $d_e > 0$ for all $e \in E$

Binary variables

- $x_e = 1$ if edge e is used



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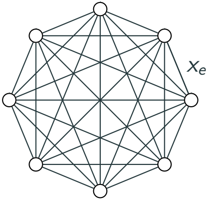

K_8

$$
\begin{aligned}
\min \quad & \sum_{e \in E} d_e\, x_e & & \\
\text{subject to} \quad & \sum_{e \in \delta(v)} x_e = 2 & & \forall v \in V \\
& \sum_{e \in \delta(S)} x_e \geq 2 & & \forall S \subset V, S \neq \varnothing \\
& x_e \in \{0, 1\} & & \forall e \in E
\end{aligned}
$$

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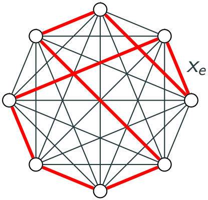

K_8

$$
\begin{array}{llll}
\min & \sum_{e \in E} d_e \, x_e & & \\
\text{subject to} & \sum_{e \in \delta(v)} x_e = 2 & \forall v \in V & \text{node degree} \\
& \sum_{e \in \delta(S)} x_e \geq 2 & \forall S \subset V, S \neq \emptyset & \\
& x_e \in \{0, 1\} & \forall e \in E &
\end{array}
$$

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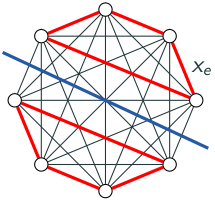

K_8

$$
\begin{aligned}
\min \quad & \sum_{e \in E} d_e \, x_e \\
\text{subject to} \quad & \sum_{e \in \delta(v)} x_e = 2 && \forall v \in V \\
& \sum_{e \in \delta(S)} x_e \geq 2 && \forall S \subset V, S \neq \emptyset \\
& && \text{subtour elimination} \\
& x_e \in \{0, 1\} && \forall e \in E
\end{aligned}
$$

TSP – Integer Programming Formulation

Given

- complete graph $G = (V, E)$
- distances $d_e > 0$ for all $e \in E$

Binary variables

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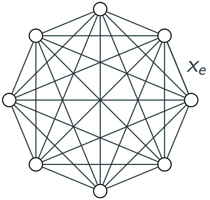


K_8

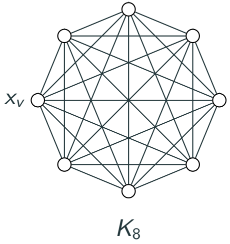
$$
\begin{array}{lll}
\min & \displaystyle\sum_{e \in E} d_e \, x_e & \text{distance} \\
\text{subject to} & \displaystyle\sum_{e \in \delta(v)} x_e = 2 & \forall v \in V \\
& \displaystyle\sum_{e \in \delta(S)} x_e \geq 2 & \forall S \subset V, S \neq \emptyset \\
& x_e \in \{0, 1\} & \forall e \in E
\end{array}
$$

Given

- complete graph $G = (V, E)$
- for each $e \in E$ a distance $d_e > 0$

Integer variables

- x_v position of $v \in V$ in tour



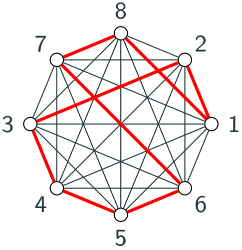
TSP – Constraint Programming Formulation

Given

- complete graph $G = (V, E)$
- for each $e \in E$ a distance $d_e > 0$

Integer variables

- x_v position of $v \in V$ in tour



$$\begin{aligned} \min \quad & \text{length}(x_1, \ldots, x_n) & \\ \text{subject to} \quad & \text{alldifferent}(x_1, \ldots, x_n) & \\ & x_v \in \{1, \ldots, n\} & \forall v \in V \end{aligned}$$

What is a Constraint Integer Program?

Constraint Integer Program

Objective function:

- ▷ linear function

Feasible set:

- ▷ described by arbitrary constraints

Variable domains:

- ▷ real or integer values

$$\begin{array}{ll} \min & c^T x \\ s.t. & x \in F \\ & (x_I, x_C) \in \mathbb{Z}^I \times \mathbb{R}^C \end{array}$$

Remark:

- arbitrary objective or variables modeled by constraints

Constraint Integer Program

Objective function:

- linear function

Feasible set:

- described by arbitrary constraints

Variable domains:

- real or integer values

$$
\begin{array}{llll}
\min & \sum_{e \in E} d_e\, x_e & & \\
s.t. & \sum_{e \in \delta(v)} x_e & = 2 & \forall\, v \in V \\
 & \text{nosubtour}(x) & & \\
 & x_e & \in \{0,1\} & \forall\, e \in E
\end{array}
$$

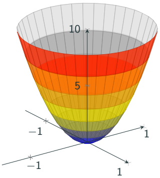
(CIP formulation of TSP)

Single nosubtour constraint rules out subtours (e.g. by domain propagation). It may also separate subtour elimination inequalities.

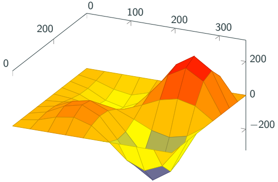
Mixed-Integer Nonlinear Programs (MINLPs)

$$
\begin{aligned}
\min\ & c^{\mathsf{T}} x \\
\text{s.t. } & g_k(x) \le 0 && \forall k \in [m] \\
& x_i \in \mathbb{Z} && \forall i \in \mathcal{I} \subseteq [n] \\
& x_i \in [\ell_i, u_i] && \forall i \in [n]
\end{aligned}
$$

The functions $g_k \in C^1([\ell, u], \mathbb{R})$ can be



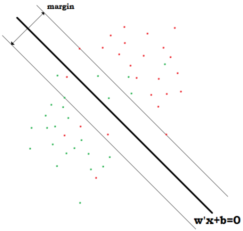
convex or



nonconvex

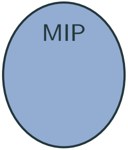
Support Vector Machine, e.g., with ramp loss.

$$
\begin{aligned}
\min\ & \frac{w^\mathsf{T} w}{2} + \frac{C}{n} \sum_{i=1}^{n} (\xi_i + 2(1 - z_i)) \\
\text{s.t. } & z_i(y_i(w^\mathsf{T} x_i + b) - 1 + \xi_i) \geq 0 && \forall i \\
& \xi_i \in [0, 2], \quad z_i \in \{0, 1\} && \forall i \\
& w \in \mathbb{R}^d, \quad b \in \mathbb{R}
\end{aligned}
$$

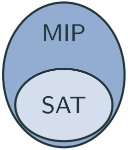


Constraint Integer Programming

- Mixed Integer Programs



- Mixed Integer Programs
- SATisfiability problems



Constraint Integer Programming

- Mixed Integer Programs
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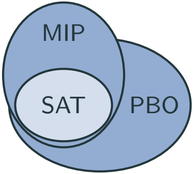
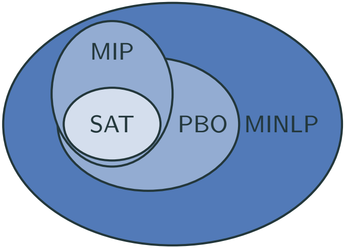
- Pseudo-Boolean Optimization



- Mixed Integer Programs
- SATisfiability problems
- Pseudo-Boolean Optimization
- Mixed Integer NonIinear Programs



Constraint Integer Programming

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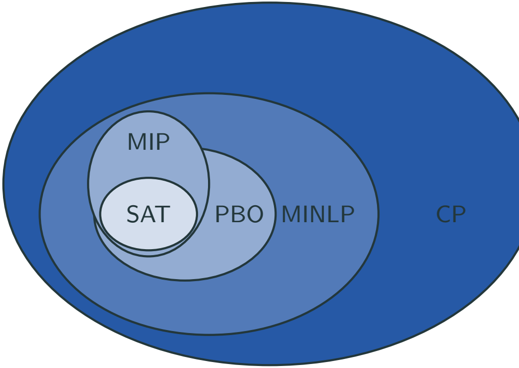
- Constraint Programming



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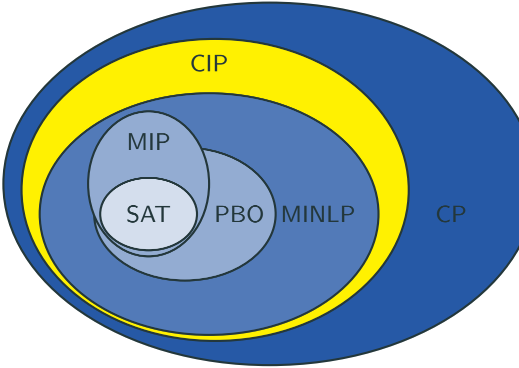

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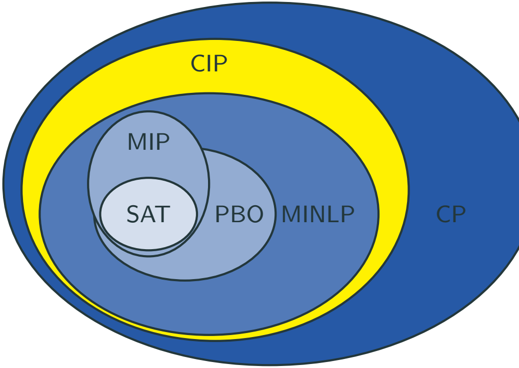


Relation to CP and MIP

- Every MIP is a CIP. *"MIP $\subsetneq$ CIP"*
- Every CP over a finite domain space is a CIP. *"FD $\subsetneq$ CIP"*

SCIP – Solving Constraint Integer Programs

The SCIP Optimization Suite

(Mixed-) Integer Programs

Constraint Integer Programming

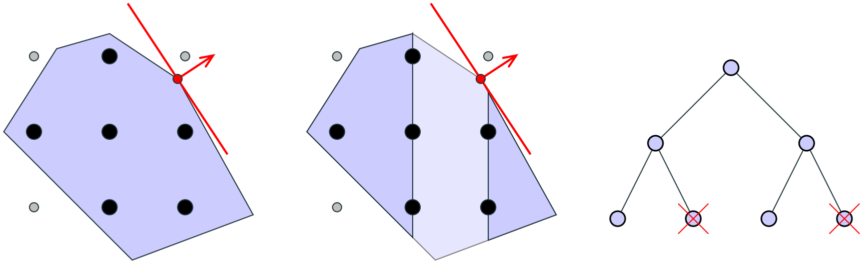
The Solving Process of SCIP

Extending SCIP by Plugins

http://scip.zib.de

$$
\begin{array}{lrcl}
\min & c^T x & & \\
\text{s.t.} & Ax \geq & b & \\
& x \geq & 0 & \\
& x_i \in & \mathbb{Z} & \forall i \in \mathcal{I} \subseteq \{1, \ldots, n\}
\end{array}
$$

(Mixed Integer Program)



SCIP Interactive Shell Basics

Basic Workflow

```
read ../check/instances/MIP/bell5.mps
optimize
write solution mysolution.sol
quit
```

Displaying information

Use the `display ...` command to enter the menu and

- obtain `solution` information
- print the current `transproblem` to the console
- display plugin information, e.g., list all available branching rules

Changing Settings

Use the `set ...` command to list the settings menu.

Important Parameters

Numerical parameters
These must be set before reading a problem.

- `numerics/feastol`, default 10^{-6}
- `numerics/epsilon`, default 10^{-9}
- `numerics/infinity`, default 10^{20}

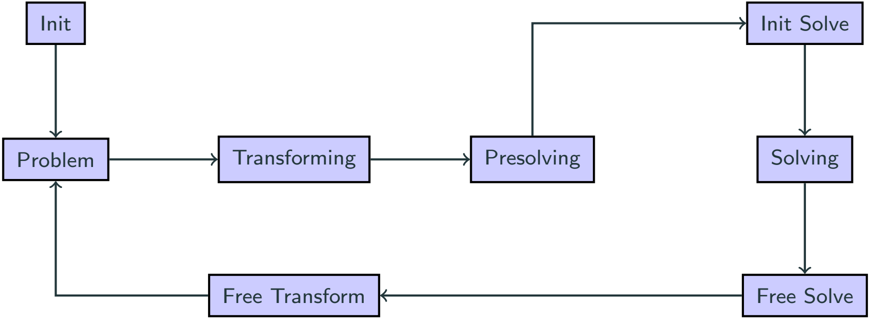
Limits

- `limits/time`
- `limits/nodes`
- `limits/gap`
- ...

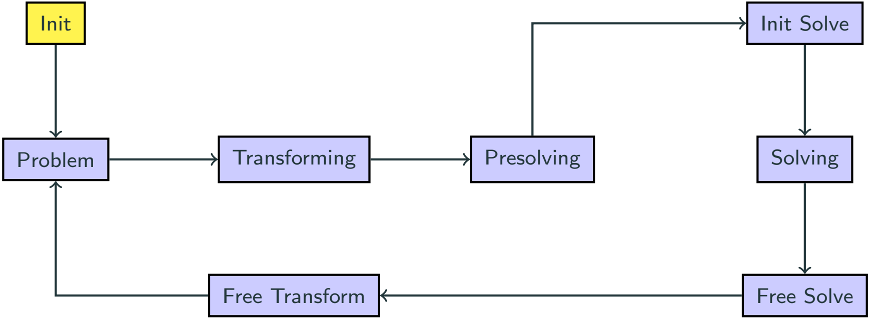
Randomization

- `randomization/randomseedshift`
- `randomization/lpseed`
- `randomization/permutationseed`

Operational Stages

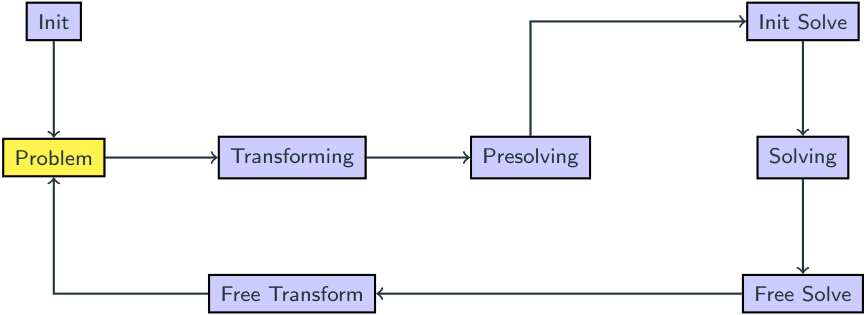


Operational Stages



- Basic data structures are allocated and initialized.
- User includes required plugins (or just takes default plugins).

Problem Specification



- User creates and modifies the original problem instance.
- Problem creation is usually done in file readers.

Define Variables (Binary Example)

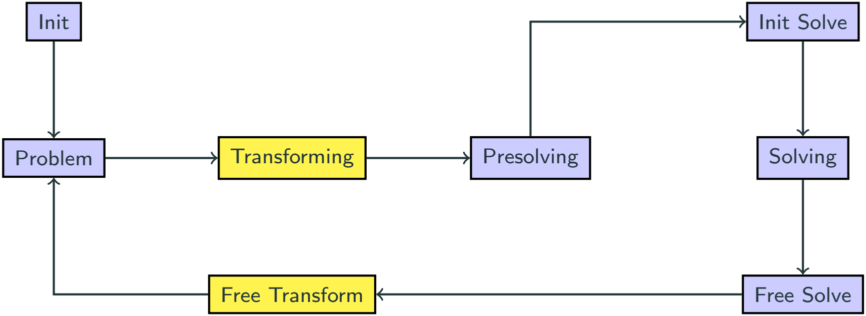
```
SCIP_VAR* var;
SCIP_CALL(                    // return value macro
  SCIPcreateVar(
    scip,                     // SCIP pointer
    &var,                     // save in variable
    "x_1",                    // pass variable name
    0.0,                      // lower bound
    1.0,                      // upper bound
    100.0,                    // obj. value
    SCIP_VARTYPE_BINARY,      // type
    TRUE,                     // initial
    FALSE,                    // removable
    NULL, NULL, NULL,         // no callback functions
    NULL                      // no variable data
  )
);
SCIP_CALL( SCIPaddVar(scip, var) );      // add var.
```

Diet Problem Constraint

```
SCIP_CALL( SCIPcreateConsLinear(
    scip,           // SCIP pointer
    &cons,          // save in cons
    "energy",       // name
    6,              // number of variables
    vars,           // array of variables
    vals,           // array of values (coefficients)
    2000,           // left hand side
    SCIPinfinity(scip), // no right hand side (use for equation)
    TRUE,           // initial?
    FALSE,          // separate?
    TRUE,           // enforce?
    TRUE,           // check?
    TRUE,           // propagate?
    FALSE,          // local?
    FALSE,          // modifable?
    FALSE,          // dynamic?
    FALSE,          // removable?
    FALSE           // stick at node?
  ));
SCIP_CALL( SCIPaddCons(scip, cons) );       // add constraint
SCIP_CALL( SCIPreleaseCons(scip, &cons) ); // free cons. space
```

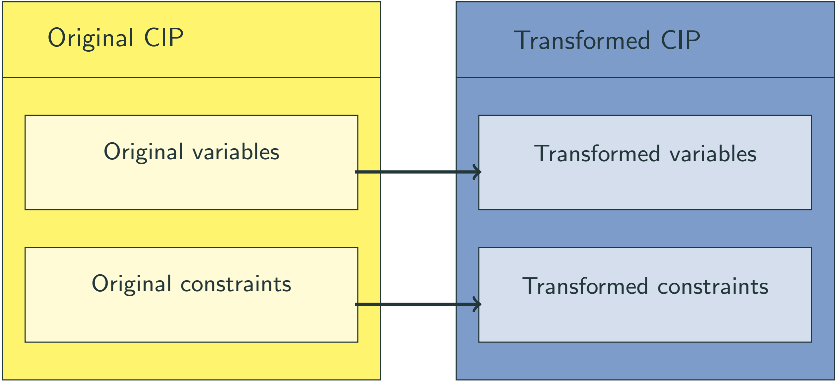
MIPs are specified using linear constraints only (may be “upgraded”).

Transformation



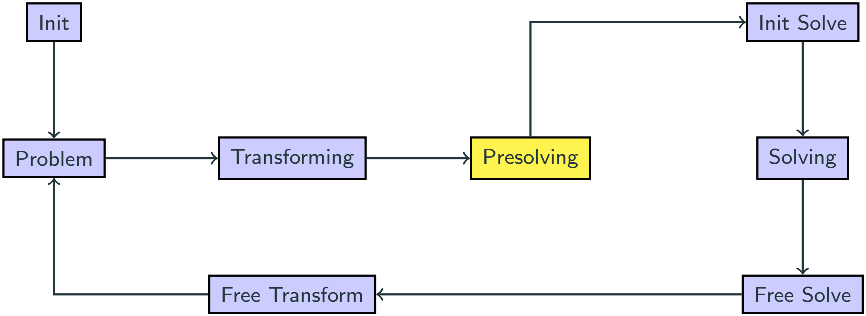
- Creates a working copy of the original problem.

Original and Transformed Problem

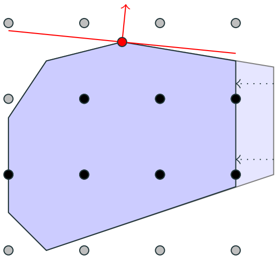


- data is copied into separate memory area
- presolving and solving operate on transformed problem
- original data can only be modified in problem modification stage

Presolving


Init
Problem
Transforming
Presolving
Init Solve
Solving
Free Solve
Free Transform

Presolving



Task

- reduce size of model by removing irrelevant information
- strengthen LP relaxation by exploiting integrality information
- make the LP relaxation numerically more stable
- extract useful information

Primal Reductions:

- based on feasibility reasoning
- no feasible solution is cut off

Dual Reductions:

- consider objective function
- at least one optimal solution remains

Presolving Tips and Parameters

Use `display presolvers` to list all presolvers of SCIP.

Disable Presolving
Disable all presolving for a model

```
set presolving emphasis off
```

Deactivate single techniques

```
set presolving tworowbnd maxrounds 0
set propagating probing maxprerounds 0
set constraints components advanced maxprerounds 0
```

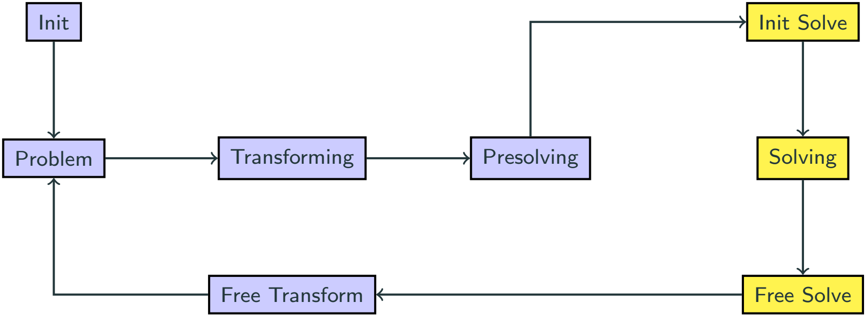
Aggressive Presolving

```
set presolving emphasis aggressive
```

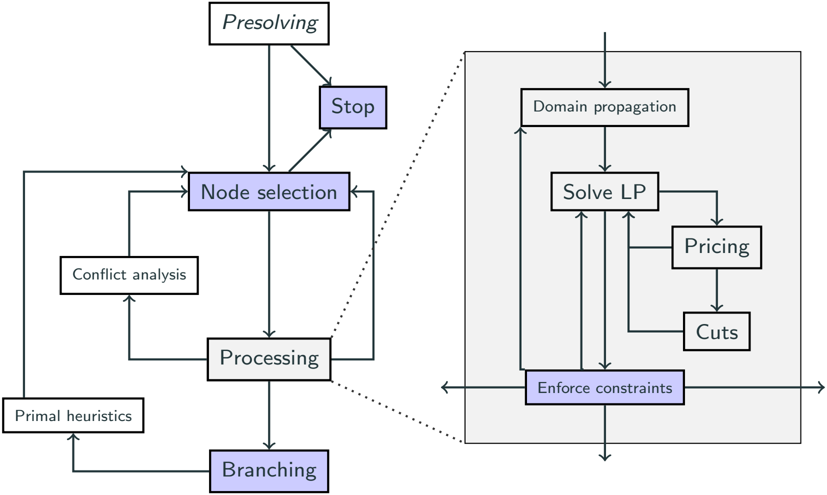
General Rule of Thumb

Only deactivate single presolving techniques if you encounter performance problems.

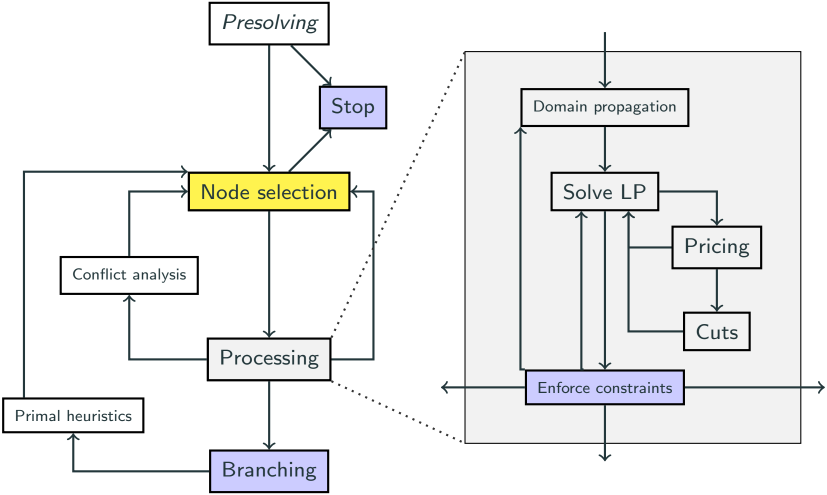
Solving


Init
Problem
Transforming
Presolving
Init Solve
Solving
Free Solve
Free Transform

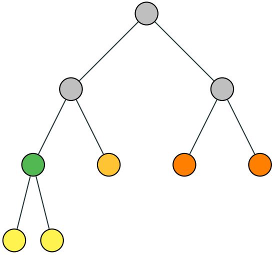
Flow Chart SCIP



Flow Chart SCIP



Node Selection



Task

- improve primal bound
- keep comp. effort small
- improve global dual bound

Techniques

- basic rules
  - depth first search (DFS)
    → early feasible solutions
  - best bound search (BBS)
    → improve dual bound
  - best estimate search (BES)
    → improve primal bound
- combinations
  - BBS or BES with plunging
  - hybrid BES/BBS
  - interleaved BES/BBS

Available Node Selectors

```
display nodeselectors

node selector  std priority  memsave prio   description
-------------  ------------  ------------   -----------
estimate             200000           100   best estimate search
bfs                  100000             0   best first search
...
dfs                       0        100000   depth first search
```

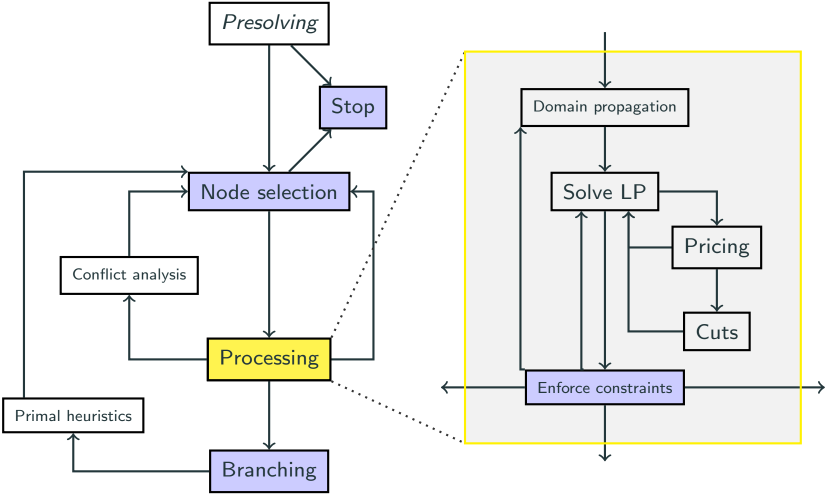
Switching Node Selectors

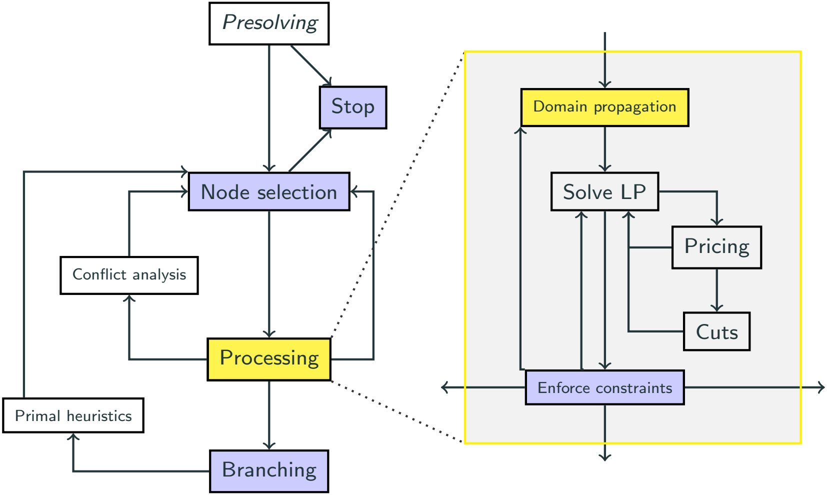
Only the node selector with highest standard priority is active. Use

```
set nodeselection dfs stdpriority 1000000
```

to activate depth first search also in non-memsave mode.

Flow Chart SCIP




Presolving
Stop
Node selection
Conflict analysis
Processing
Primal heuristics
Branching
Domain propagation
Solve LP
Pricing
Cuts
Enforce constraints

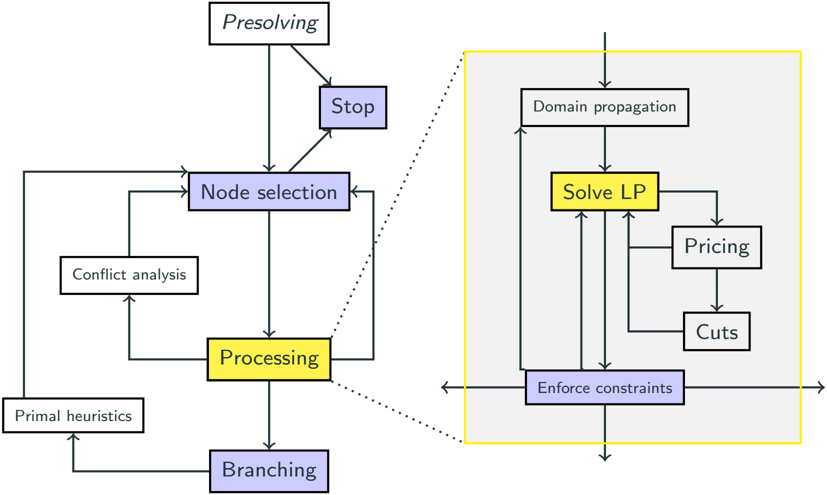
Task

- simplify model locally
- improve local dual bound
- detect infeasibility

Techniques

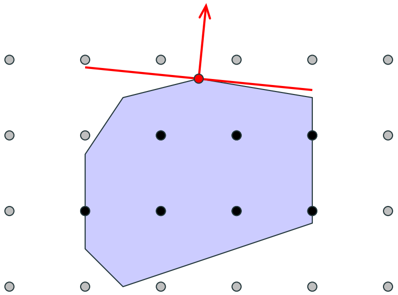
- constraint specific
  - each cons handler may provide a propagation routine
  - reduced presolving (usually)
- dual propagation
  - root reduced cost strengthening
  - objective function
- special structures
  - variable bounds

Flow Chart SCIP



LP Solving

- LP solver is a black box
- interface to different LP solvers: SoPlex, CPLEX, XPress, Gurobi, CLP, . . .
- primal/dual simplex
- barrier with/without crossover



- feasibility double-checked by SCIP
- condition number check
- resolution by changing parameters: scaling, tolerances, solving from scratch, other simplex

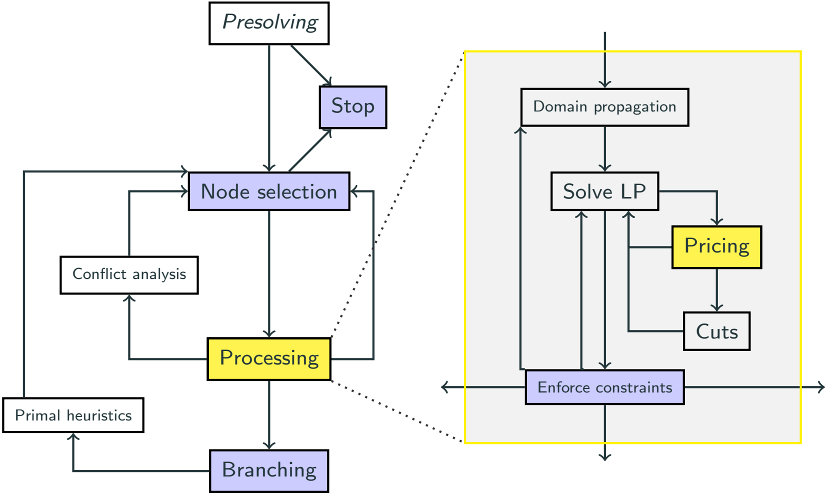
LP Solving Tips and Parameters

Most Important LP Parameters

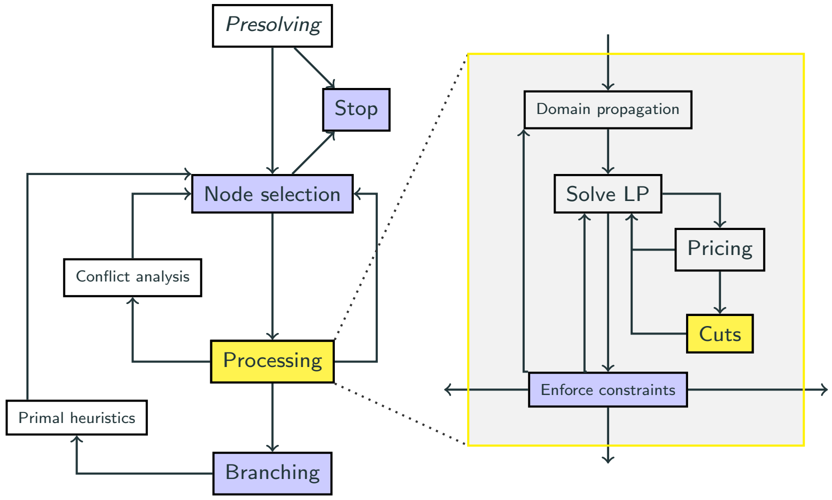
- `lp/initalgorithm`, `lp/resolvealgorithm`
    - Primal/Dual Simplex Algorithm
    - Barrier w and w/o crossover
- `lp/pricing`
    - normally LP solver specific default
    - Devex
    - Steepest edge
    - Quick start steepest edge
- `lp/threads`

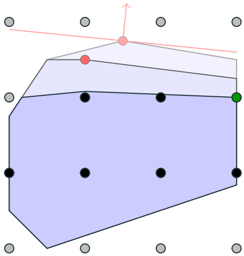
Slow LP performance is a blocker for the solving process and can sometimes be manually tuned significantly.

Flow Chart SCIP



Flow Chart SCIP





Task

- strengthen relaxation
- add valid constraints
- generate on demand

Techniques

- general cuts
  - complemented MIR cuts
  - Gomory mixed integer cuts
  - strong Chvátal-Gomory cuts
  - implied bound cuts
  - reduced cost strengthening
- problem specific cuts
  - 0-1 knapsack problem
  - stable set problem
  - 0-1 single node flow problem

Feasible region: $(b \in \mathbb{Z}_+,\ a_j \in \mathbb{Z}_+ \ \ \forall j \in N)$

$$X^{BK} := \{\ x \in \{0,1\}^n \ :\ \sum_{j \in N} a_j x_j \leq b\ \}$$

Minimal Cover: $C \subseteq N$

- $\sum_{j \in C} a_j > b$
- $\sum_{j \in C \setminus \{i\}} a_j \leq b \quad \forall i \in C$

Minimal Cover Inequality

$$\sum_{j \in C} x_j \leq |C| - 1$$

$$5x_1 + 6x_2 + 2x_3 + 2x_4 \leq 8$$

Minimal cover:

$C = \{2, 3, 4\}$

Minimal cover inequality:

$x_2 + x_3 + x_4 \leq 2$

Disable/Speed up/Emphasize All Separation

```
set separating emphasis off/fast/aggressive
```

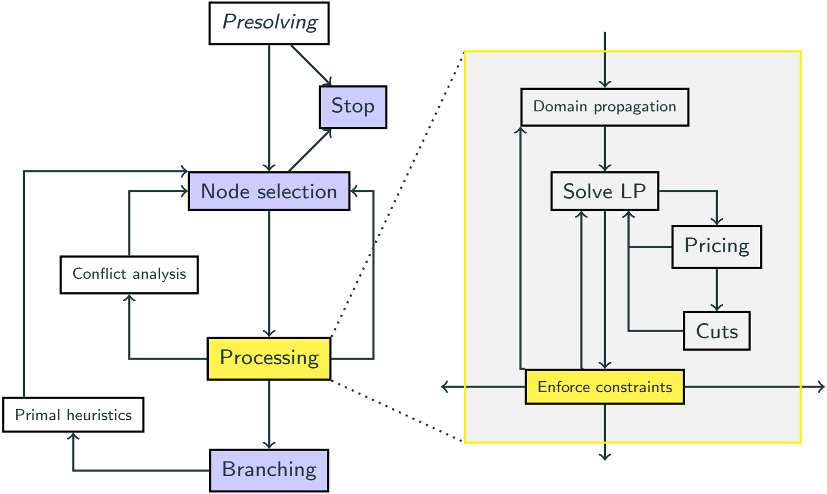
Disable Single Separation Techniques

```
set separating clique freq -1
set constraints cardinality sepafreq -1
```

Some Important Parameters

- `separating/maxcuts`, `separating/maxcutsroot`
- `separating/maxrounds`, `separating/maxroundsroot`
- `separating/maxstallrounds`, `separating/maxstallroundsroot`

Flow Chart SCIP



Constraint Enforcement

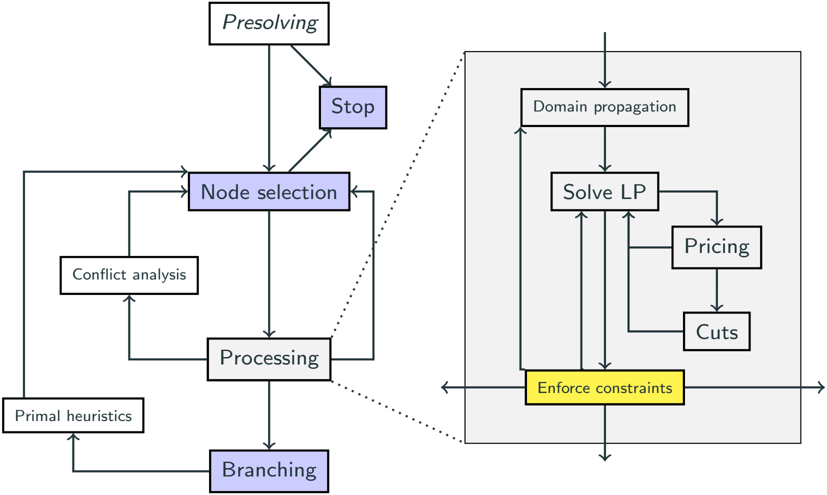
LP solution may violate a constraint not contained in the relaxation.

Enforcing is necessary for a correct implementation!

Constraint handler resolves the infeasibility by ...

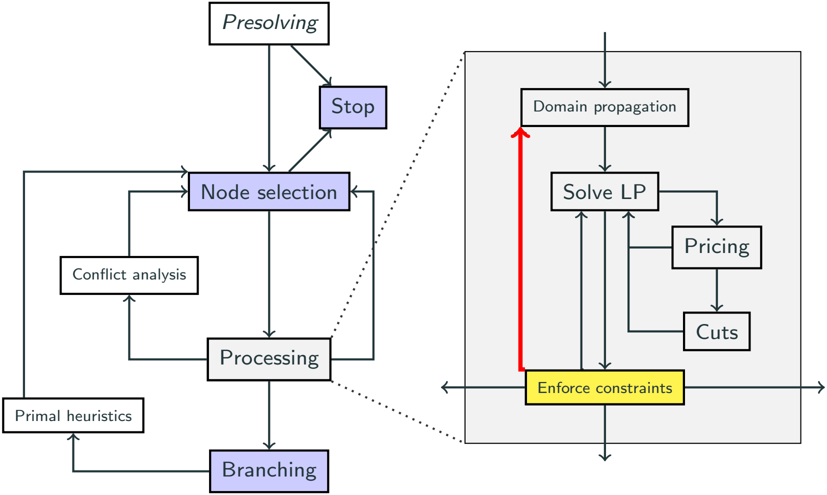
- Reducing a variable's domain,
- Separating a cutting plane (may use integrality),
- Adding a (local) constraint,
- Creating a branching,
- Concluding that the subproblem is infeasible and can be cut off, or
- Just saying "solution infeasible".

Constraint Enforcement



- Reduced domain
- Added constraint
- Added cut
- Branched
- Cutoff
- Infeasible
- Feasible

Constraint Enforcement



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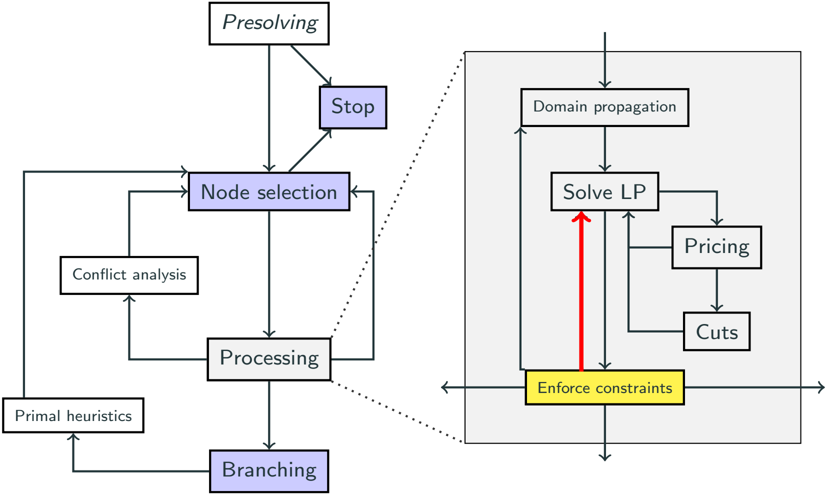
Constraint Enforcement



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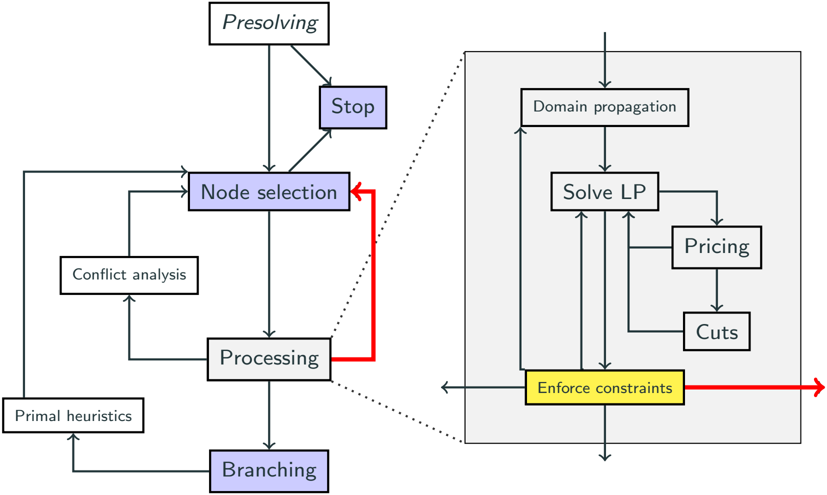
Constraint Enforcement



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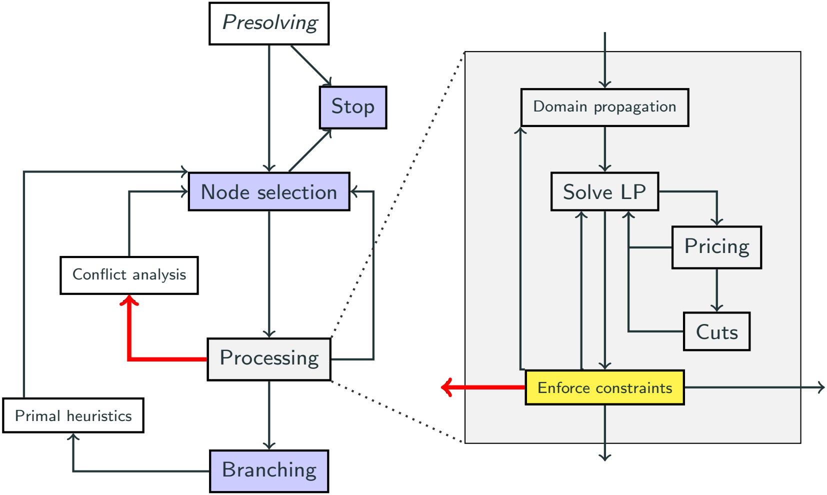
Constraint Enforcement



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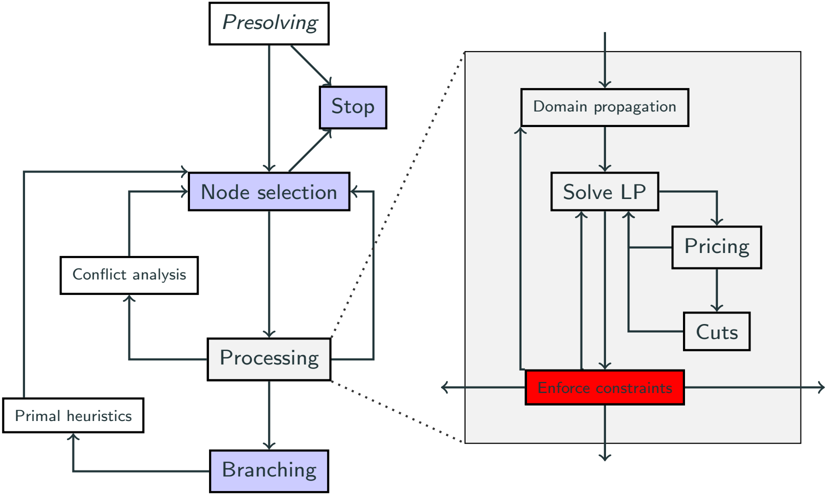
Constraint Enforcement



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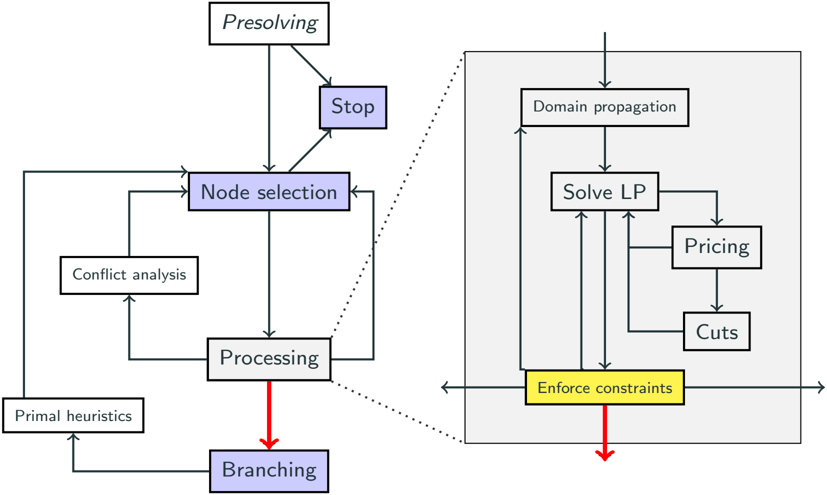

- Reduced domain
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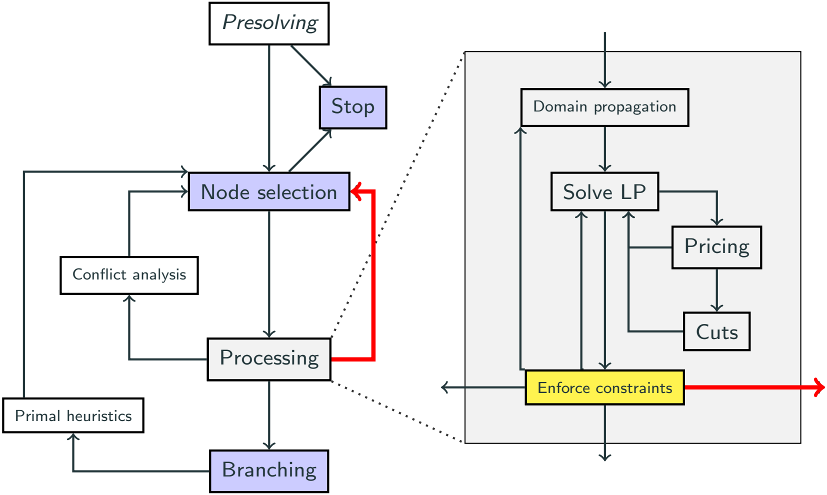
Constraint Enforcement



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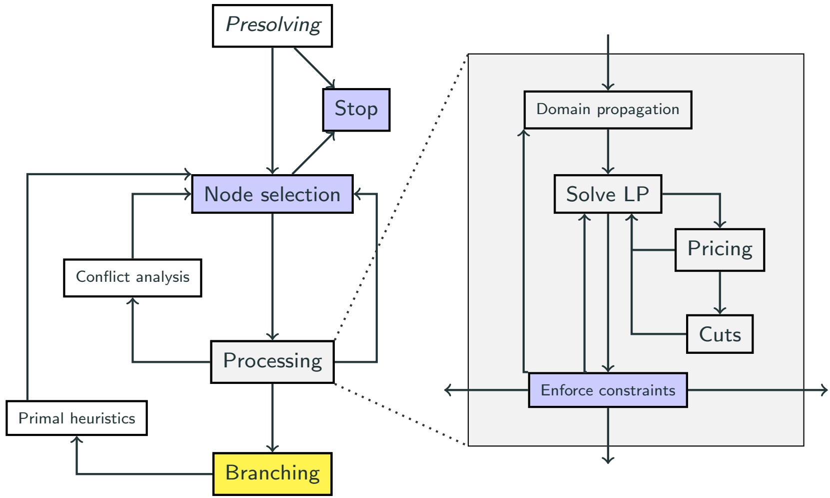
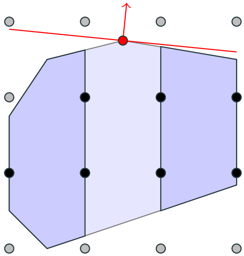
Flow Chart SCIP



Branching Rules



Task

- divide into (disjoint) subproblems
- improve local bounds

Techniques

- branching on variables
  - most infeasible
  - least infeasible
  - random branching
  - strong branching
  - pseudocost
  - reliability
  - VSIDS
  - hybrid reliability/inference
- branching on constraints
  - SOS1
  - SOS2

Branching Rule Tips and Parameters

Branching Rule Selection
Branching rules are applied in decreasing order of priority.

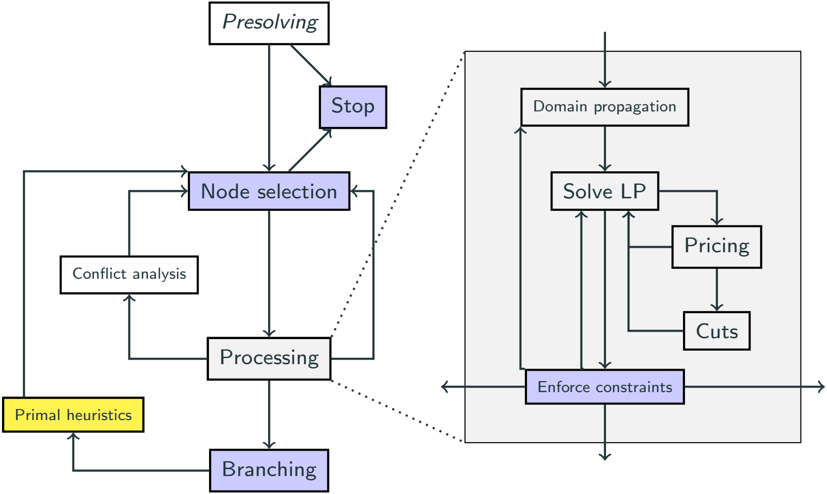
```
SCIP> display branching
```

branching rule	priority	maxdepth	maxbddist
relpscost	10000	−1	100.0%
pscost	2000	−1	100.0%
inference	1000	−1	100.0%
mostinf	100	−1	100.0%

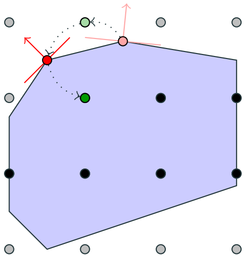
Reliability Branching Parameters
All parameters prefixed with `branching/relpscost/`

- `sbiterquot`, `sbiterofs` to increase the budget for strong branching
- `minreliable` (= 1), `maxreliable` (= 5) to increase threshold to consider pseudo costs as reliable

Flow Chart SCIP



Primal Heuristics



Task

- improve primal bound
- effective on average
- guide remaining search

Techniques

- structure-based
  - clique
  - variable bounds
- rounding
  - possibly solve final LP
- diving
  - least infeasible
  - guided
- objective diving
  - objective feasibility pump
- Large Neighborhood Search
  - RINS, local branching
  - RENS
  - Adaptive LNS
  - Completion of partial solutions

Primal Heuristics Tips and Parameters

Disable/Speed Up/Emphasize Heuristics

```
set heuristics emphasis off/fast/aggressive
```

Disable an individual heuristic via

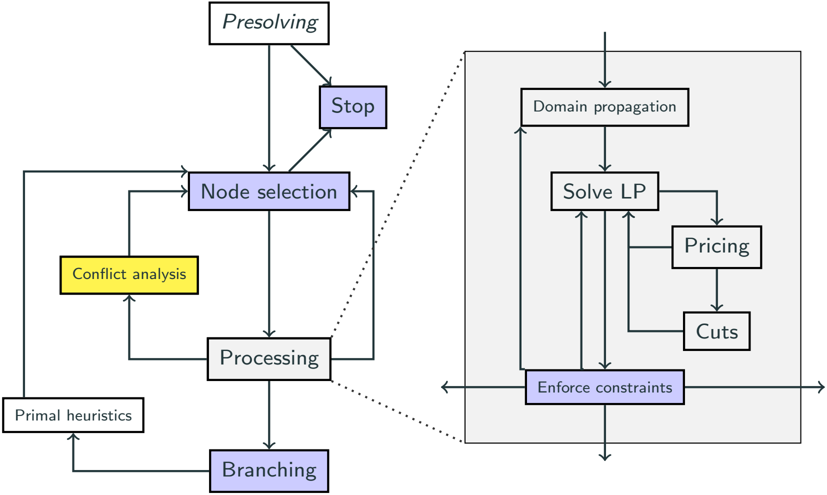
```
set heuristics feaspump freq -1
```

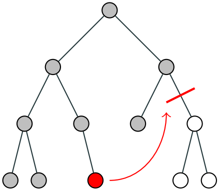
Important Parameters

- `heuristics/alns/nodesofs`, `heuristics/alns/nodesquot` to increase the computational budget of this LNS technique
- `heuristics/guideddiving/... lpsolvefreq`, `maxlpiterofs` `maxlpiterquot` to control the LP solving during this diving technique

Advice
Use emphasis settings. Do not attempt to individually tune heuristics by hand.

Flow Chart SCIP





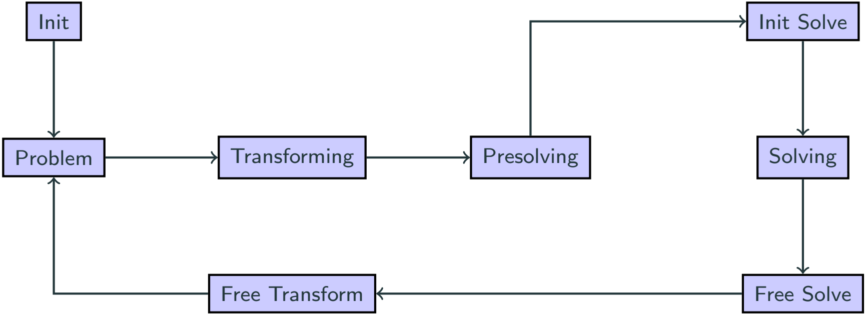
Task

- Analyze infeasibility
- Derive valid constraints
- Help to prune other nodes

Techniques

- Analyze:
  - Propagation conflicts
  - Infeasible LPs
  - Bound-exceeding LPs
  - Strong branching conflicts
- Detection:
  - Cut in conflict graph
  - LP: Dual ray heuristic
- Use conflicts:
  - Only for propagation
  - As cutting planes

Operational Stages



SCIP – Solving Constraint Integer Programs

The SCIP Optimization Suite

(Mixed-) Integer Programs

Constraint Integer Programming

The Solving Process of SCIP

Extending SCIP by Plugins

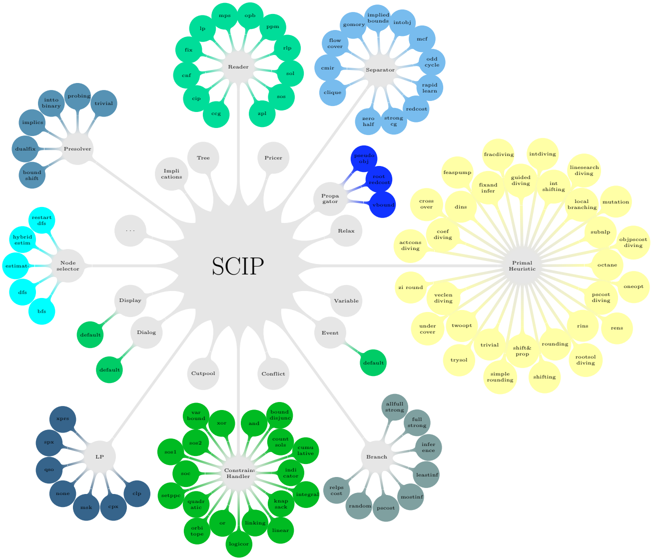
http://scip.zib.de

Different Tasks – Different Plugins

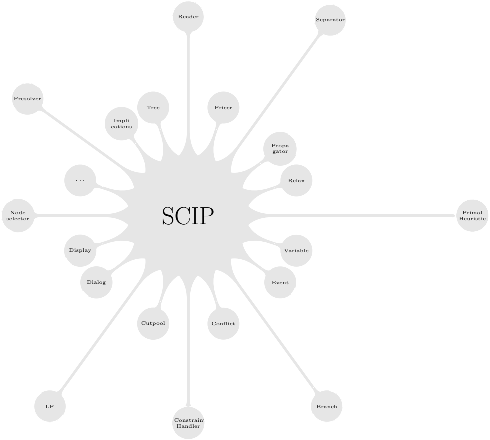
Different plugin classes are responsible of the following tasks.

1. Presolving and node propagation
    - Constraint handlers
    - Presolvers
    - Propagators
2. Separation
    - Constraint handlers
    - Separators
3. Improving solutions
    - Primal heuristics
4. Branching
    - Constraint handlers
    - Branching rules
5. Node selection
    - Node selectors

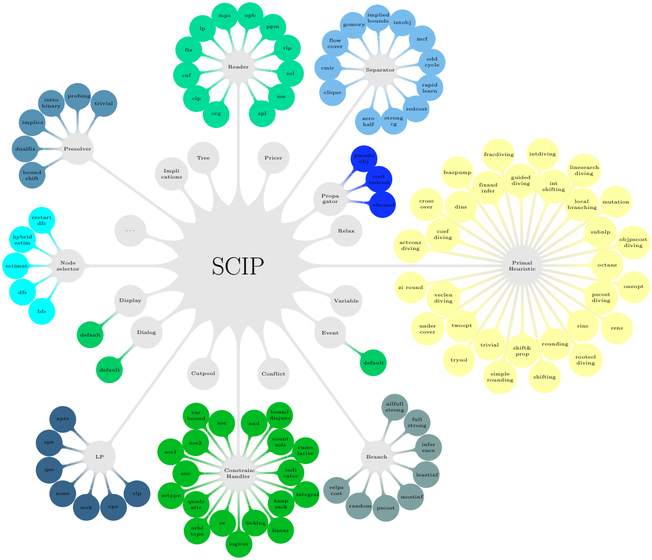
Structure of SCIP



Structure of SCIP



Structure of SCIP



Plugin based design

SCIP core

- branching tree
- variables
- conflict analysis
- solution pool
- cut pool
- statistics
- clique table
- implication graph
- . . .

Plugin based design

SCIP core

- branching tree
- variables
- conflict analysis
- solution pool
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- implication graph
- ...

Plugins

- external callback objects
- interact with the framework through a very detailed interface

Plugin based design

SCIP core

- branching tree
- variables
- conflict analysis
- solution pool
- cut pool
- statistics
- clique table
- implication graph
- ...

Plugins

- external callback objects
- interact with the framework through a very detailed interface
- SCIP knows for each plugin type:
  - the number of available plugins
  - priority defining the calling order (usually)
- SCIP does not know any structure behind a plugin

$\Rightarrow$ plugins are black boxes for the SCIP core

Plugin based design

SCIP core

- branching tree
- variables
- conflict analysis
- solution pool
- cut pool
- statistics
- clique table
- implication graph
- ...

Plugins

- external callback objects
- interact with the framework through a very detailed interface
- SCIP knows for each plugin type:
  - the number of available plugins
  - priority defining the calling order (usually)
- SCIP does not know any structure behind a plugin

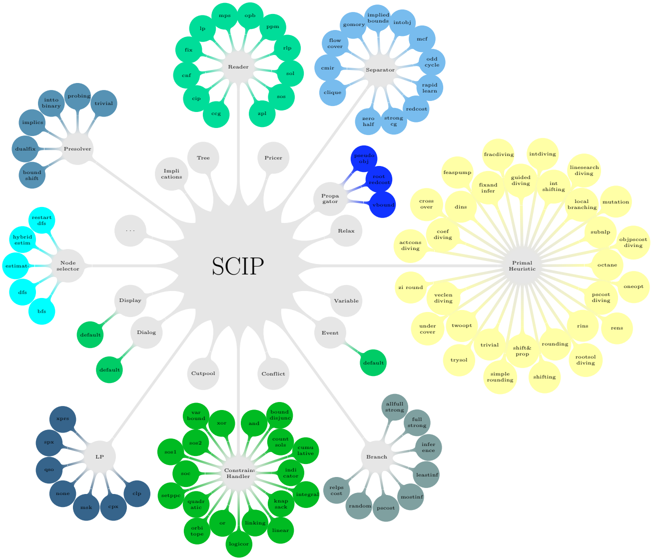
$\Rightarrow$ plugins are black boxes for the SCIP core

$\Rightarrow$ Very flexible branch-and-bound based search algorithm

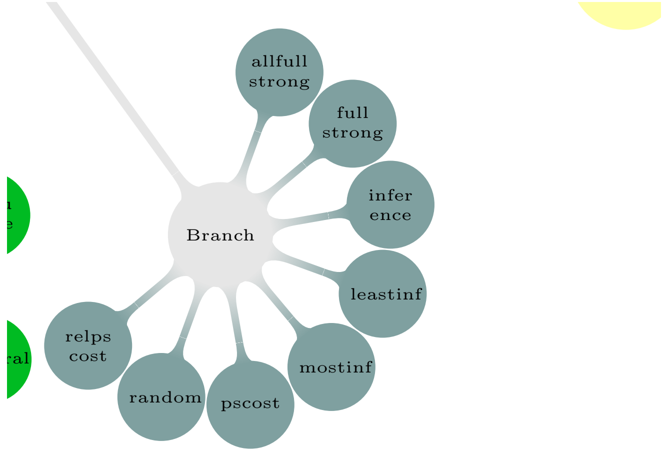
Types of Plugins

- Constraint handler: assures feasibility, strengthens formulation
- Separator: adds cuts, improves dual bound
- Pricer: allows dynamic generation of variables
- Heuristic: searches solutions, improves primal bound
- Branching rule: how to divide the problem?
- Node selection: which subproblem should be regarded next?
- Presolver: simplifies the problem in advance, strengthens structure
- Propagator: simplifies problem, improves dual bound locally
- Reader: reads problems from different formats
- Event handler: catches events (e.g., bound changes, new solutions)
- Display: allows modification of output
- ...

A closer look: branching rules



A closer look: branching rules



What does SCIP know about branching rules?

- SCIP knows the number of available branching rules
- each branching rule has a priority
- SCIP calls the branching rule in decreasing order of priority
- the interface defines the possible results of a call:
  - branched
  - reduced domains
  - added constraints
  - detected cutoff
  - did not run

How does SCIP call a branching rule?

```
/* start timing */
SCIPclockStart(branchrule->branchclock, set);

/* call external method */
SCIP_CALL( branchrule->branchexeclp(set->scip, branchrule,
   allowaddcons, result) );

/* stop timing */
SCIPclockStop(branchrule->branchclock, set);

/* evaluate result */
if( *result != SCIP_CUTOFF
   && *result != SCIP_CONSADDED
   && *result != SCIP_REDUCEDDOM
   && *result != SCIP_SEPARATED
   && *result != SCIP_BRANCHED
   && *result != SCIP_DIDNOTRUN )
{
   SCIPerrorMessage(
      "branching␣rule␣<%s>␣returned␣invalid␣result␣code␣<%d>␣from␣LP␣\
␣␣␣␣␣␣␣solution␣branching\n",
      branchrule->name, *result);
   return SCIP_INVALIDRESULT;
}
```

What can a plugin access?

Plugins are allowed to access all global (core) information

- branching tree
- variables
- conflict analysis
- solution pool
- cut pool
- statistics
- clique table
- implication graph
- ...

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Branching Rules

- LP solution
- variables
- statistics

Constraint Handlers

Constraint handlers

- most powerful plugins in SCIP
- define the feasible region
- a single constraint may represent a whole set of inequalities

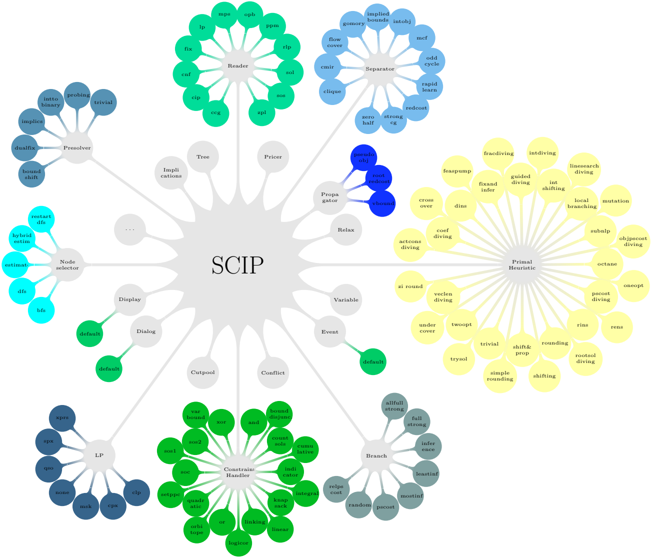
Functions

- check and enforce feasibility of solutions
- can add linear representation to LP relaxation
- constraint-specific presolving, domain propagation, separation

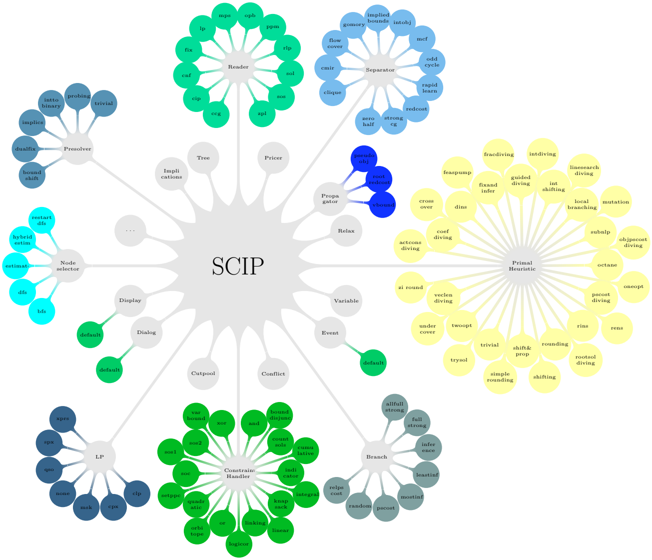
Result

- SCIP is constraint based
  - Advantage: flexibility
  - Disadvantage: limited global view

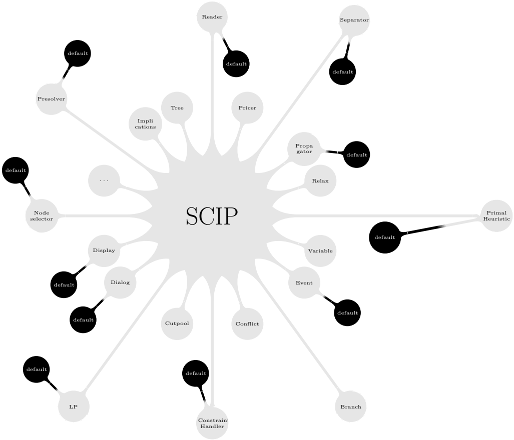
Default Plugins



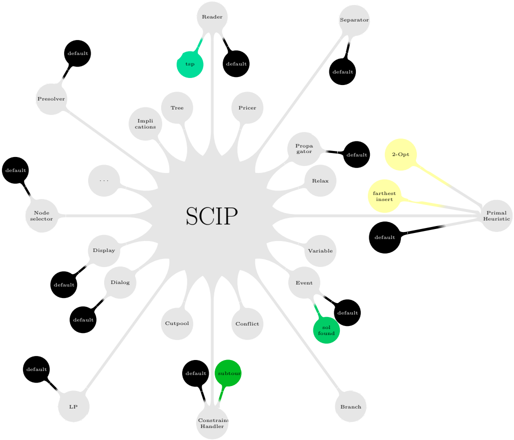
Extending SCIP



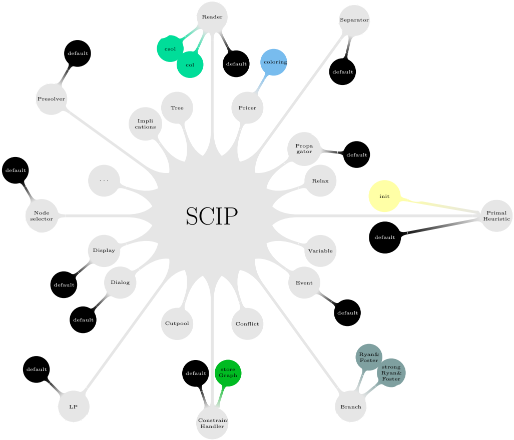
Extending SCIP



Extending SCIP: TSP



Extending SCIP: Coloring



Important SCIP topics not covered in this workshop:

- branch-and-price: SCIP can be extended to a problem-specific branch-cut-and-price solver
  - see The Bin Packing Example in C
  - see also GCG
- allows for Benders decomposition since version 6.0, see Technical Report
- browse older technical reports for details on recently added cutting plane selection, primal heuristics, symmetry breaking, and much more

Wrap-up

How to solve your MINLP optimization problem:

- write down the mathematical description
- modeling language, e.g., ZIMPL, generates input for MINLP solver
- SCIP can even read ZIMPL files directly

MIP and MINLP solving

- . . . is a bag of tricks
- new tricks are introduced almost every day
- advantages of SCIP
  - open source, you can see everything that happens
  - hundreds of parameters to play with
  - broad scope
  - easily extendable