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Problem Set 10

due: July 9, 2018

Exercise 1 **6 points**

Let $G = (V, E)$ be an undirected graph. Denote by $\mathcal{L}$ the set of all paths in G. For an edge $e \in E$, define $\mathcal{L}_e := \{\ell \in \mathcal{L} \mid e \in \ell\}$. Further let $f^{\min}, f^{\max} : E \to \mathbb{N}_0$ be frequency bounds with $f^{\min} \leq f^{\max}$, and set $F := \max_{e \in E} f_e^{\max}$.
For $k \in \mathbb{N}$, the *k-line pool generation problem* is to find a vector $x \in \{0,1\}^{\mathcal{L}}$ satisfying the following constraints:

$$\begin{aligned}
\sum_{\ell \in \mathcal{L}} x_\ell &\leq k, && \\
f_e^{\min} \leq \sum_{\ell \in \mathcal{L}_e} f_\ell &\leq f_e^{\max}, && e \in E, \\
f_\ell &\leq F x_\ell, && \ell \in \mathcal{L}, \\
f_\ell &\in \mathbb{N}_0, && \ell \in \mathcal{L}, \\
x_\ell &\in \{0,1\}, && \ell \in \mathcal{L}.
\end{aligned}$$

(a) Give a trivial solution to the k-line pool generation problem for $k \geq |E|$.

(b) Construct a polynomial-time reduction from the directed Hamiltonian path problem to the 1-line pool generation problem.

Exercise 2 **5 points**

Let $G = (V, E)$ be an undirected graph, and let $\mathcal{P}$ be a set of walks in G. For a subset $W \subseteq V$ define

$$\mathcal{P}(W) := \{p \in \mathcal{P} \mid p \text{ contains an edge } \{v, w\} \text{ s.t. } v \notin W \text{ and } w \in W\}.$$

Further let $T \subseteq V$ be a set of terminal nodes, and let $c : \mathcal{P} \to \mathbb{R}_{\geq 0}$ be a cost function. Prove that the following integer program solves the Steiner connectivity problem on $(G, \mathcal{P}, T, c)$:

$$\begin{aligned}
\text{Minimize} \quad & \sum_{p \in \mathcal{P}} c_p x_p && \\
\text{s.t.} \quad & \sum_{p \in \mathcal{P}(W)} x_p \geq 1, && W \subseteq V,\ \emptyset \neq W \cap T \neq T, \\
& x_p \in \{0,1\}, && p \in \mathcal{P}.
\end{aligned}$$

Please turn over!

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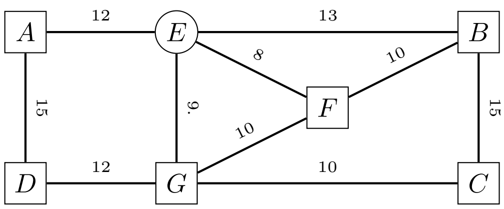
Exercise 3 **9 points**

Consider the following undirected graph with costs on its edges:



(a) Find a Steiner tree of minimum cost connecting $T = \{A, B, C, D, F, G\}$.

(b) Define $\mathcal{L}_0 := \{AD, AEB, AEF, DGC, GEBF, BCGF, DGF\}$ and consider the following OD matrix:

		to			
		A	B	C	F
from	A	0	50	0	50
	D	0	0	80	20
	G	0	40	0	0
	C	30	0	0	0

Every line $\ell \in \mathcal{L}_0$ can be operated with frequency $f_\ell \in \{0, 1, 2\}$ and vehicles of capacity 50. Find a line plan satisfying all demands with the minimum number of lines.