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Problem Set 3

due: May 7, 2018

Exercise 1 **6 points**

Let $G = (V, E)$ be an undirected graph with length vector $\ell \in \mathbb{R}^E$, and let $s, t \in V$ be two distinct vertices. For a subset $S \subseteq V$ define

$$\delta(S) := \{\{v, w\} \in E \mid v \in S, w \notin S\},$$

and for $v \in V$ let $\delta(v) := \delta(\{v\})$. Consider the following integer program:

$$(\star) \quad \begin{aligned} \text{Maximize} \quad & \sum_{e \in E} \ell_e x_e & & \\ \text{s.t.} \quad & \sum_{e \in \delta(v)} x_e = 2y_v, & & v \in V \setminus \{s,t\}, \\ & \sum_{e \in \delta(v)} x_e = 1, & & v \in \{s,t\}, \\ & \sum_{e \in \delta(S)} x_e \geq 2 + \sum_{v \in S} (2y_v - 2) & & \emptyset \subsetneq S \subseteq V \setminus \{s,t\}, \\ & x_e \in \{0,1\}, & & e \in E, \\ & y_v \in \{0,1\}, & & v \in V \setminus \{s,t\}. \end{aligned}$$

(a) Let $(x, y) \in \{0,1\}^E \times \{0,1\}^V$ be a feasible solution of $(\star)$. Show that for any non-empty set $S \subseteq V \setminus \{s, t\}$ holds

$$\sum_{e \in \delta(S)} x_e \geq 2 + \sum_{v \in S} (2y_v - 2) \quad \Leftrightarrow \quad \sum_{e \in E[S]} x_e \leq |S| - 1,$$

where $E[S] := \{\{v, w\} \in E \mid v \in S, w \in S\}$.

(b) How can feasible solutions of $(\star)$ be interpreted in the graph G? Prove that your interpretation is correct.

(c) Which graph optimization problem is solved by $(\star)$?

Exercise 2 **8 points**

Consider the following timetable:

Line 10: Berlin → Köln			
Berlin	dep.	06:51	08:51
Hannover	arr.	08:28	10:28
Hannover	dep.	08:31	10:31
Köln	arr.	11:09	13:09

Line 10: Köln → Berlin			
Köln	dep.	06:48	08:48
Hannover	arr.	08:28	10:28
Hannover	dep.	08:31	10:31
Berlin	arr.	11:06	13:06

Line 22: Hamburg → Stuttgart			
Hamburg	dep.	07:06	09:06
Hannover	arr.	08:38	10:38
Hannover	dep.	08:41	10:41
Stuttgart	arr.	12:35	14:35

Line 22: Stuttgart → Hamburg			
Stuttgart	dep.	05:25	07:25
Hannover	arr.	09:17	11:17
Hannover	dep.	09:20	11:20
Hamburg	arr.	10:50	12:50

(a) Draw the line network corresponding to this timetable. Label the vertices with their station name and highlight the line cover.

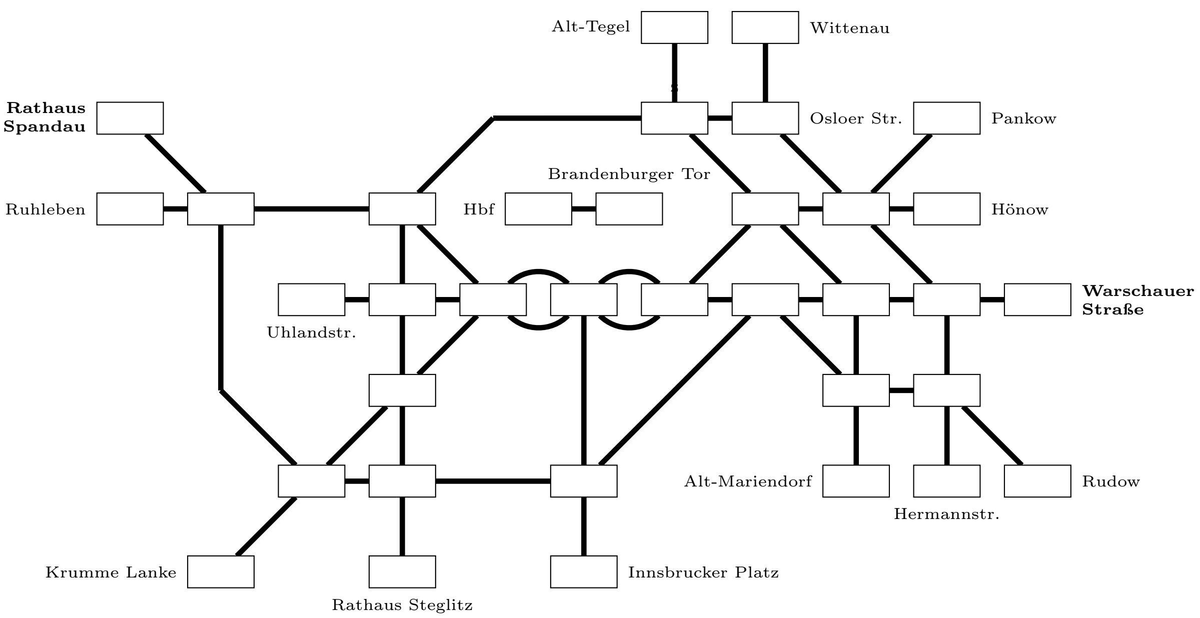
(b) Draw the time-expanded network. Label the events with station name and time, and label each activity with its duration.

(c) Draw the periodic event-activity network with period time $T = 120$ minutes. Use the same labeling as in (b).

(d) Solve the following GATSP: Find the shortest closed walk in the network of (c) visiting Berlin, Hamburg, Hannover, Köln and Stuttgart at least once.

Exercise 3 **6 points**

Consider the following line network (Berlin U-Bahn):



Solve the earliest arrival problem *Rathaus Spandau @ 7 May 2018, 15:32 → Warschauer Straße* as follows:

- Apply the time-dependent Dijkstra algorithm with a minimum transfer time of 2 minutes.
- Stick to the line network. You do not need to draw the route vertices.
- Label each vertex with its current time and mark permanently labeled vertices.
- Do not forget to write down the optimal journey and the earliest arrival time.
- Use `fahrinfo.bvg.de`, `mobil.bvg.de` or a similar trip planner to find out the timetable. Remember to insert the correct departure date and times.